# Indirect Proofs

## Examples and practice questions

Topics include parallel lines and transversals, equidistance theorem, congruent triangles, circles, and more.

Indirect Proofs

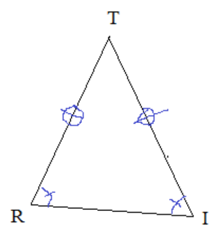
What is it? An indirect proof is a "proof by contradiction"
It's often used when given statement/conclusions are NOT true.

Instead of proving something is true,
you prove it *indirectly* by showing that is *cannot be false.*

The first step is to assume the opposite outcome....
then, proceed with the proof until a contradiction is revealed....

*Example:* Given: $\angle R \ncong \angle I$

Prove: $\overline{TR} \ncong \overline{TI}$




Paragraph Proof

There are 2 possible outcomes: $\overline{TR} = \overline{TI}$ OR $\overline{TR} \neq \overline{TI}$...

Let's assume that TR = TI...

If TR = TI, then $\angle R = \angle I$
(because if congruent sides, then opposite sides are congruent)

However, $\angle R \neq \angle I$
so, there is a contradiction! Therefore, TR $\neq$ TI

2 Column Proof

| Statements | Reasons |
|---|---|
| 1) $\angle R \neq \angle I$ | 1) Given |
| 2) $\overline{TR} \cong \overline{TI}$ | 2) Assume for contradiction |
| 3) $\angle R \cong \angle I$ | 3) If congruent sides, then congruent angles |

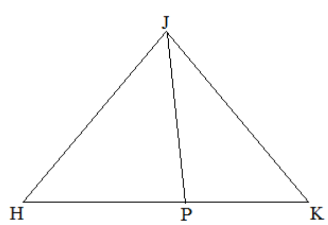
Since statements 1) and 3) contradict each other,
the assumption is incorrect...

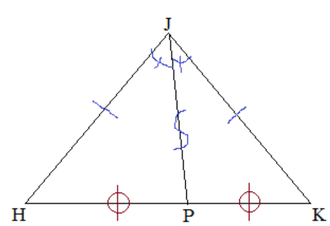
therefore, TR $\neq$ TI

"If there is a contradiction, then the null hypothesis is false..."
and,
"if the null hypothesis is false, then the converse must be true."

*Example:* Given: P is not the midpoint of HK
$\overline{HJ} \cong \overline{JK}$

Prove: $\overline{JP}$ does not bisect $\angle HJK$




| Statements | Reasons |
|---|---|
| 1) $\overline{JP}$ biesects $\angle HJK$ | 1) Assume for contradiction |
| 2) $\angle HJP \cong \angle KJP$ | 2) Definition of angle bisector (bisector divides angle into 2 $\cong$ parts |
| 3) $\overline{HJ} \cong \overline{JK}$ | 3) Given |
| 4) $\overline{JP} \cong \overline{JP}$ | 4) Reflexive property |
| 5) $\triangle HJP = \triangle KJP$ | 5) SAS (Side-Angle-Side) 3, 2, 4 |
| 6) $\overline{HP} \cong \overline{PK}$ | 6) CPCTC (Corresponding Parts of Congruent Triangles are Congruent) |
| 7) P is the midpoint of $\overline{HK}$ | 7) Definition of Midpoint (A midpoint divides a segment into equal parts) |
| 8) P is not the midpoint of $\overline{HK}$ | 8) Given |

Since statements 7) and 8) contradict each other,
the assumption 1) is incorrect....

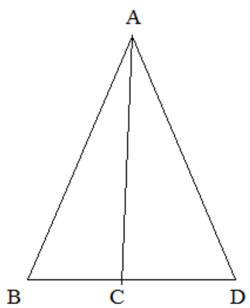
If assumption 1), then contradiction exists...
If contradiction does not exist, then assumption 1) is false!!
("contrapositive")

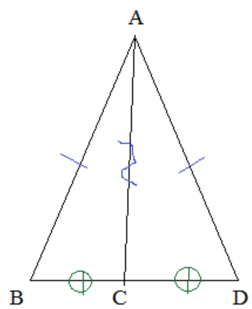
Indirect Proofs

Given: $\overline{AB} \cong \overline{AD}$

$\angle BAC \ncong \angle DAC$

Prove: $\overline{BC} \ncong \overline{DC}$



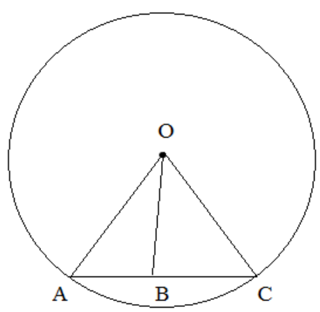

| Statements | Reasons |
|---|---|
| 1) $\overline{AB} \cong \overline{AD}$ | 1) Given |
| 2) $\angle BAC \ncong \angle DAC$ | 2) Given |
| 3) $\overline{BC} \cong \overline{DC}$ | 3) Assume to reach a contradiction |
| 4) $\overline{AC} \cong \overline{AC}$ | 4) Reflexive Property |
| 5) $\triangle ABC \cong \triangle ADC$ | 5) SSS (Side-Side-Side) 1, 3, 4 |
| 6) $\angle BAC = \angle DAC$ | 6) CPCTC (Corresponding Parts of Congruent Triangles are Congruent) |

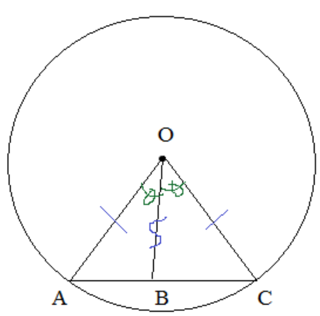
However, statements 2) and 6) contradict each other..

Given: Circle O

$\overline{OB}$ is NOT an altitude

Prove: $\overline{OB}$ does NOT bisect $\angle AOC$




| Statements | Reasons |
|---|---|
| 1) Circle O | 1) Given |
| 2) $\overline{OB}$ is NOT an altitude | 2) Given |
| 3) $\overline{AO} \cong \overline{CO}$ | 3) All radii are congruent |
| 4) $\overline{OB}$ bisects $\angle AOC$ | 4) Assume to reach contradiction |
| 5) $\angle AOB \cong \angle COB$ | 5) Definition of angle bisector |
| 6) $\overline{OB} \cong \overline{OB}$ | 6) Reflexive Property |
| 7) $\triangle AOB \cong \triangle COB$ | 7) SAS (Side-Angle-Side) 3, 5, 6 |
| 8) $\angle OBA \cong \angle OBC$ | 8) CPCTC |
| 9) $\angle$ OBA and OBC are right angles | 9) Right Angle Theorem (If angles are congruent and supplementary, then they are right angles) |
| 10) $\overline{OB}$ is an altitude | 10) Definition of altitude (if a segment from a vertex to opposite side of triangle forms 2 right angles. it is an altitude) |

However, statements 2) and 10) contradict each other!

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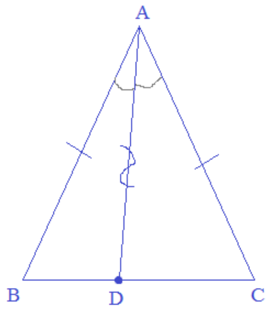
*Example:* Prove that if $\triangle$ ABC is isosceles with base $\overline{BC}$, and if D is a point on $\overline{BC}$ that is not a midpoint, then $\overline{AD}$ does not bisect $\angle$BAC.

First, draw a diagram...




Then, establish "Givens" and "Prove" for 2 column proof...

Given: ABC is isosceles
D is NOT midpoint of $\overline{BC}$

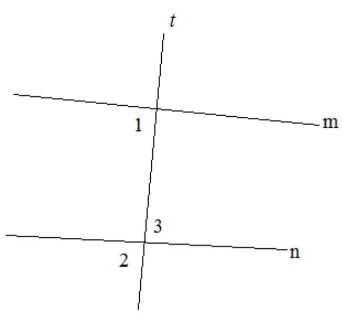
Prove: $\overline{AD}$ does NOT bisect $\angle$BAC

| Statements | Reasons |
|---|---|
| 1) $\triangle$ ABC is isosceles | 1) Given |
| 2) AD bisects $\angle$BAC | 2) Assume for contradiction (AD may or may not bisect angle BAC.. We're assuming it does bisect..) |
| 3) $\overline{AB} \cong \overline{AC}$ | 3) Definition of Isosceles triangle |
| 4) $\angle$BAD $\cong$ $\angle$CAD | 4) Definition of angle bisectorr |
| 5) $\overline{AD} \cong \overline{AD}$ | 5) Reflexive property |
| 6) $\triangle$BAD $\cong$ $\triangle$CAD | 6) SAS (Side-Angle-Side) 3, 4, 5 |
| 7) $\overline{BD} \cong \overline{CD}$ | 7) CPCTC (Corresponding Parts of Congruent Triangles are Congruent) |
| 8) D is NOT a midpoint of $\overline{BC}$ | 8) Given |
| 9) D is midpoint of $\overline{BC}$ | 9) If a point divides a segment $\overline{BC}$ into equal segments, then it is a mipoint |

Since statements 8) and 9) contradict each other, the assumption is incorrect!

*Example:* Given: Transversal *t* cuts lines m and n
$m\angle 1 \neq m\angle 2$

Prove: $m\angle 1 \neq m\angle 3$




| Statements | Reasons |
|---|---|
| 1) $m\angle 1 = m\angle 3$ | 1) Assume for contradiction (angles 1 and 3 may or may not be equal. We'll assume they are) |
| 2) Transversal *t* cuts m and n | 2) Given |
| 3) m $\parallel$ n | 3) If alternate interior angles are congruent, then the lines are parallel |
| 4) $m\angle 1 = m\angle 2$ | 4) If parallel lines cut by a transversal, then corresponding angles are congruent |
| 5) $m\angle 1 \neq m\angle 2$ | 5) Given |

Proof by contradiction: since statements 4) and 5) contradict each other, our assumption in statement 1) is false...

*Example:* Given: Neighbor's ruined garden
Vet appointment

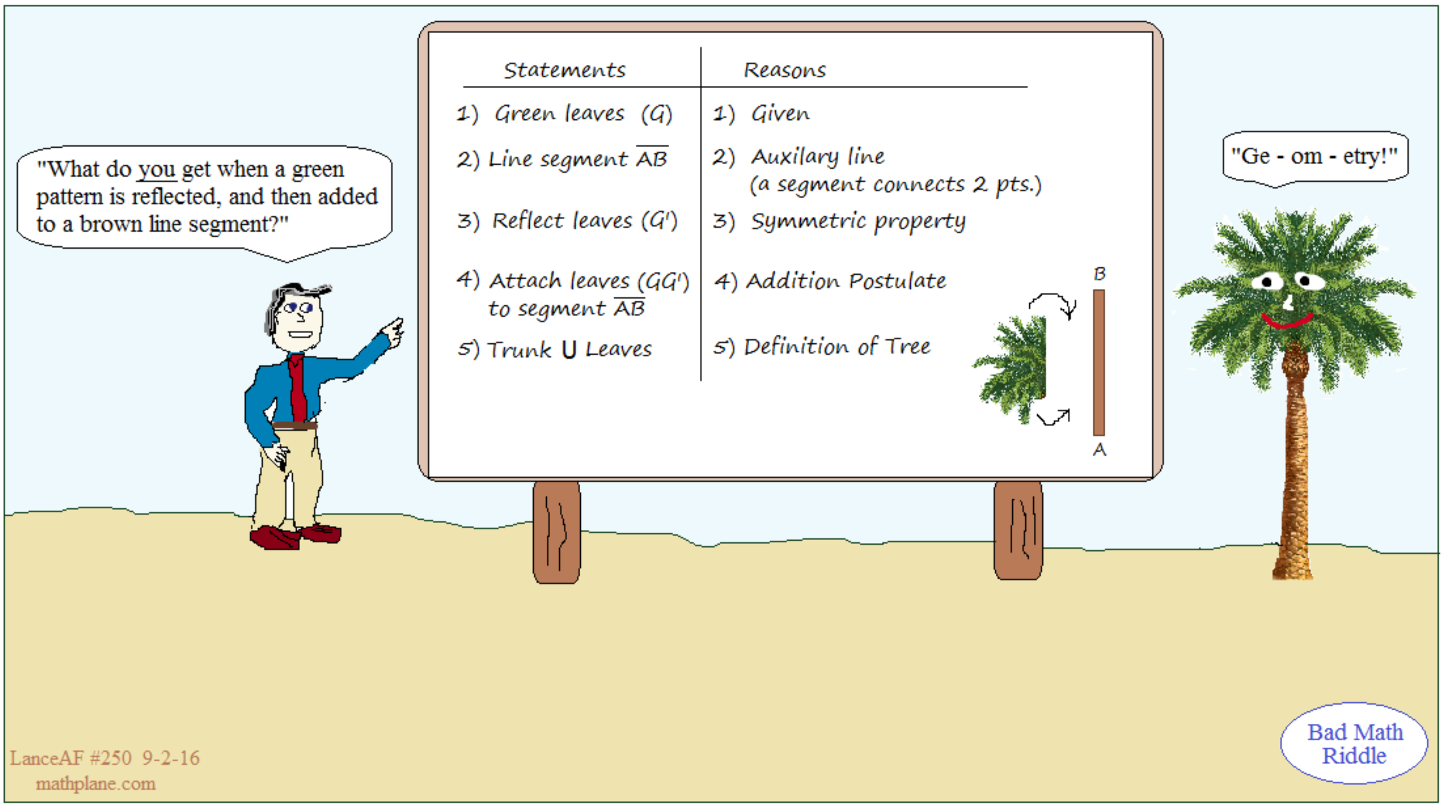
Prove: Oscar the dog did not ruin the garden.

You say that Oscar the dog ruined your garden on May 30th.
I say Oscar did not ruin the garden...

Proof by Contradiction:

Assume that Oscar did ruin the garden.
Then, he would have been in your garden on May 30th.
I have a bill from the veterinarian showing he was there from May 29 - 31.
Since Oscar was at the vet, he was not in your garden.
(Proof by contradiction: Oscar cannot be in 2 places at once!)
Therefore, Oscar did not ruin the garden....

(Since there is a contradiction, the assumption is false...)

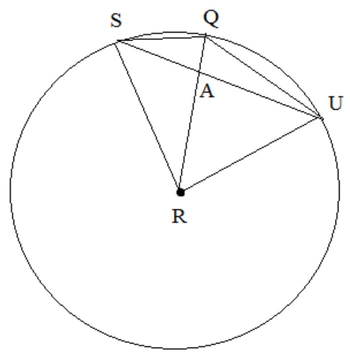



Practice Quiz-→

1) Given: Circle R

$\overline{QR}$ is not a perpendicular bisector

Prove: $\overline{SQ} \ncong \overline{UQ}$




| Statements | Reasons |
| --- | --- |

2) "In an isosceles triangle, if a point on the base is NOT the midpoint, then the segment from the vertex to that point does NOT bisect the vertex angle."

| Statements | Reasons |
| --- | --- |

3 ) "If a radius is NOT perpendicular to a chord, then the radius does NOT bisect chord."

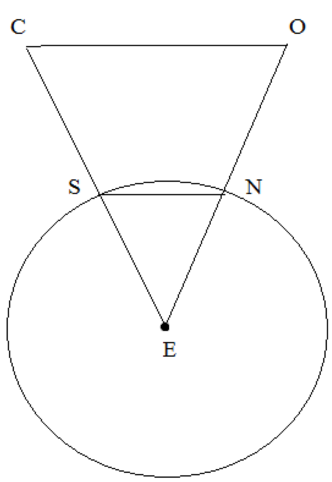
Indirect Proofs

| Statements | Reasons |
|---|---|
| | |

4) Given: Circle E

$\triangle$ COE is scalene

Prove: $\angle C \not\cong \angle ESN$




| Statements | Reasons |
|---|---|
| | |

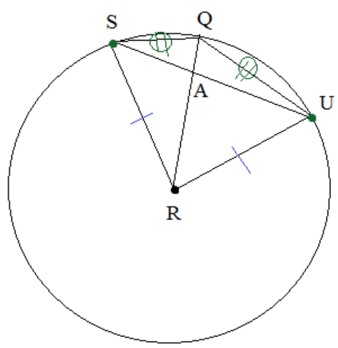
1) Given: Circle R

$\overline{QR}$ is not a perpendicular bisector

Prove: $\overline{SQ} \ncong \overline{UQ}$

SOLUTIONS

Indirect Proofs



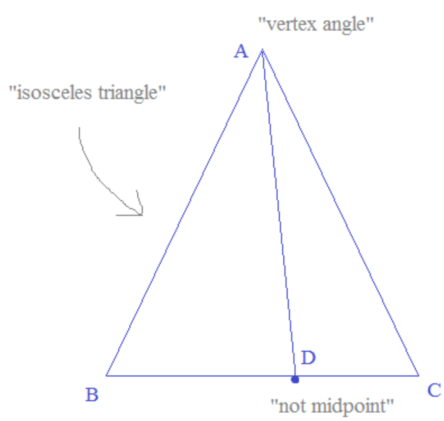

| Statements | Reasons |
|---|---|
| 1) Circle R | 1) Given |
| 2) QR is not $\perp$ bisector | 2) Given |
| 3) $\overline{SQ} \cong \overline{UQ}$ | 3) Assume to reach a contradiction |
| 4) $\overline{RS} \ncong \overline{RU}$ | 4) All radii are congruent |
| 5) QR is perpendicular bisector of segment SU | 5) Equidistance Theorem (If 2 points are equidistant to endpoints of a segment, then the points form a perpendicular bisector of the segment) |

However, statements 2) and 5) contradict each other...
Proof by contradiction...

2) "In an isosceles triangle, if a point on the base is NOT the midpoint, then the segment from the vertex to that point does NOT bisect the vertex angle."

Step 1: Sketch diagram by picking out key phrases...

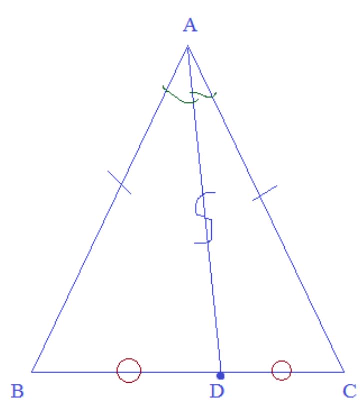
Step 2: Write out "givens" using IF statements...
And, write "prove" using THEN statements...




Given: Isosceles Triangle ABC

D is NOT a midpoint

Prove: $\overline{AD}$ is NOT an angle bisector




| Statements | Reasons |
|---|---|
| 1) $\triangle$ABC is isosceles | 1) Given |
| 2) $\overline{AB} = \overline{AC}$ | 2) Definition of Isosceles (2 or more congruent sides) |
| 3) D is NOT a midpoint of $\overline{BC}$ | 3) Given |
| 4) $\overline{AD}$ is angle bisector | 4) Assume for contradiction |
| 5) $\overline{AD} = \overline{AD}$ | 5) Reflexive Property |
| 6) $\triangle$ ABD = $\triangle$ ACD | 6) Side-Angle-Side (SAS) (2, 3, 4) |
| 7) $\overline{BD} = \overline{CD}$ | 7) CPCTC (Corresponding parts of congruent triangles are congruent) |
| 8) D is midpoint of BC | 8) Definition of Midpoint (If point divides segment into congruent halves, then it is a midpoint) |

However, statements 3) and 8) contradict each other!

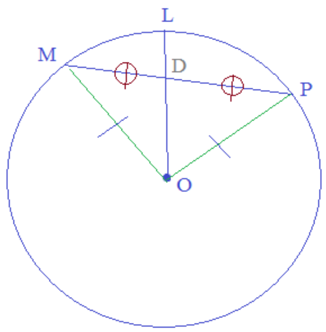
Indirect Proofs

SOLUTIONS

Uses Equidistance Theorem
Auxilary Lines
Indirect Proof

3) "If a radius is NOT perpendicular to a chord, then the radius does NOT bisect chord."

Step 1: Sketch a diagram




Step 2: Design the proof

Given: Circle O

$\overline{LO}$ is NOT perpendicular to $\overline{MP}$ "If"

Prove: $\overline{LO}$ does NOT bisect $\overline{MP}$ "Then"

Step 3: Use indirect proof to solve

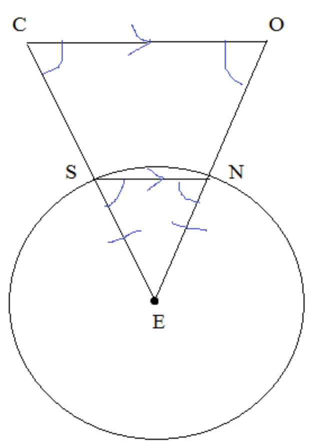
| Statements | Reasons |
|---|---|
| 1) Circle O | 1) Given |
| 2) $\overline{LO}$ NOT perpendicular to $\overline{MP}$ | 2) Given |
| 3) Draw radii $\overline{OP}$ and $\overline{OM}$ | 3) Auxilary lines (line joins 2 points) |
| 4) $\overline{OP} \cong \overline{OM}$ | 4) All radii congruent |
| 5) $\overline{LO}$ bisects $\overline{MP}$ | 5) Assume for contradiction |
| 6) $\overline{MD} = \overline{PD}$ | 6) Definition of bisector (Bisector divides segment into congruent halves) |
| 7) $\overline{LO}$ is perpendicular bisector of $\overline{MP}$ | 7) Equidistance Theorem |

However, statements 2) and 7) contradict each other!

4) Given: Circle E

$\triangle$ COE is scalene

Prove: $\angle C \ncong \angle ESN$




$C = ESN$ or $C \neq ESN$

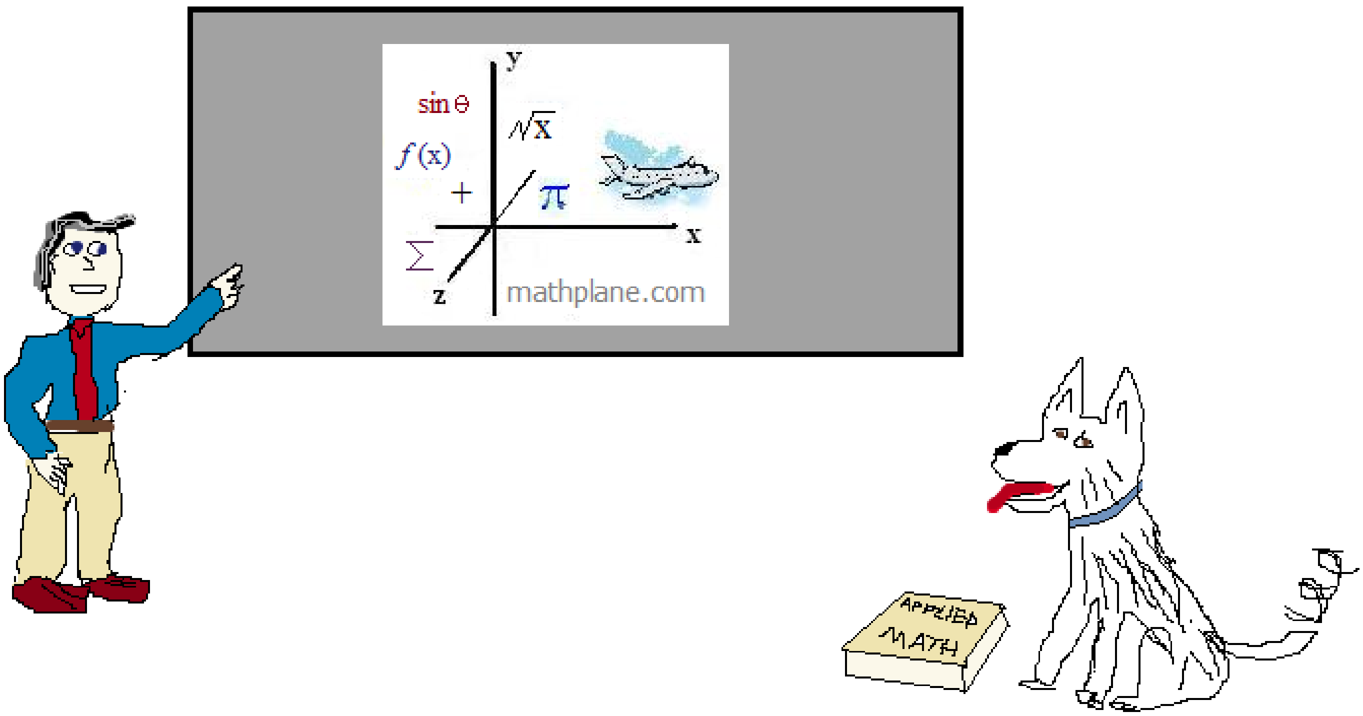
| Statements | Reasons |
|---|---|
| 1) Circle E | 1) Given |
| 2) $\triangle$ COE is scalene | 2) Given |
| 3) $\angle C = \angle ESN$ | 3) Assume for Contradiction |
| 4) $\overline{ES} = \overline{EN}$ | 4) All radii are congruent |
| 5) $\overline{CO} \parallel \overline{SN}$ | 5) If corresponding angles are congruent, then lines are parallel |
| 6) $\angle O = \angle ENS$ | 6) If lines are parallel, then corresponding angles are congruent |
| 7) $\angle ESN = \angle ENS$ | 7) If congruent sides, then congruent angles |
| 8) $\angle O = \angle C$ | 8) Transitive property |
| 9) $\triangle$ COE is isosceles | 9) If base angles are congruent, then triangle is isosceles |

However, 2) and 9) contradict each other

Thanks for visiting! (Hope it helped)

If you have questions, suggestions, or requests, let us know.

Cheers.

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