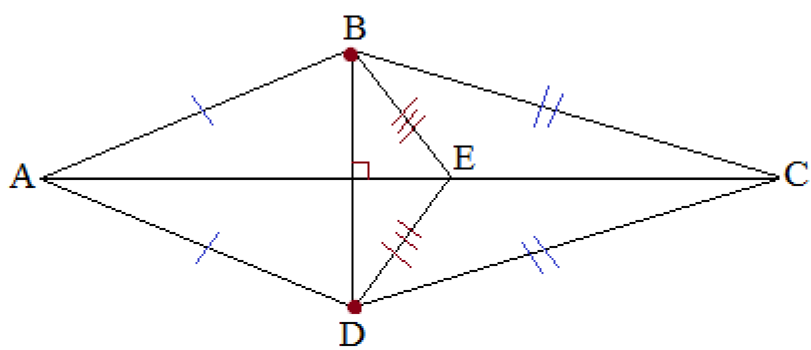
Geometry:
Equidistance Theorem

Notes, Examples, and Practice Test (with Solutions)



Topics include perpendicular bisector, 2-column proofs, kite, isosceles triangle, circles, congruent triangles, and more…

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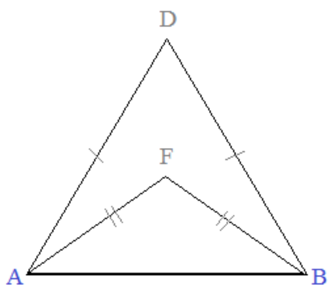
Equidistance Theorem

Definition: If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of the segment.

Illustration 1:



D is equidistant to A and B
F is equidistant to A and B

Therefore, a line through D and F would create the $\perp$ bisector of $\overline{AB}$

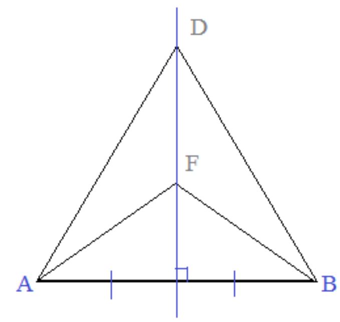
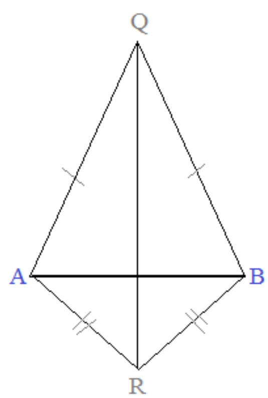


Illustration 2:

(Kite and its diagonals)



Q is equidistant from A and B
R is equidistant from A and B

Therefore, QR is the $\perp$ bisector of $\overline{AB}$

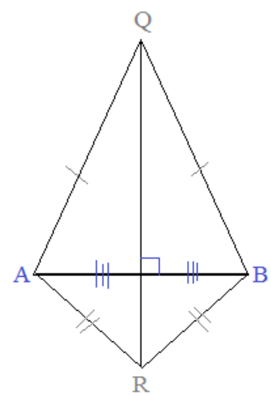
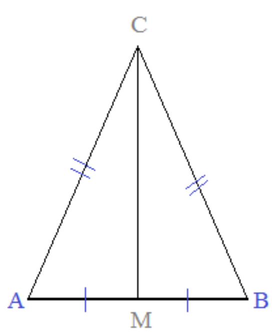


Illustration 3:

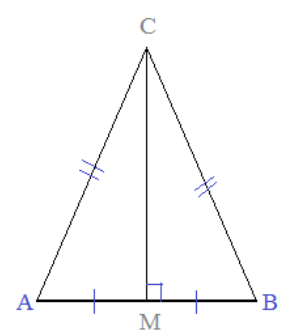
(Isosceles Triangle and its median from vertex to base)



C is equidistant to A and B
M is equidistant to A and B

Therefore, CM is the $\perp$ bisector of AB

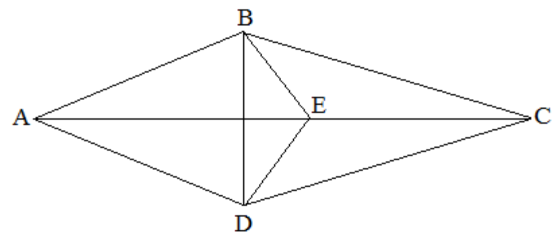
CM is an altitude of $\triangle$ ABC



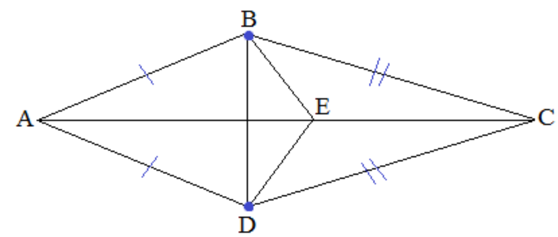
Converse of Perpendicular Bisector Theorem: If a point lies on the perpendicular bisector, then it is equidistant from the endpoints of the bisected segment.

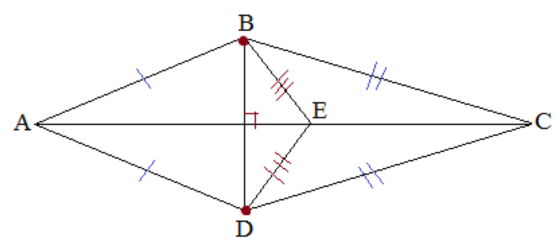
Example: Given: $\overline{AB} = \overline{AD}$
$\overline{CB} = \overline{CD}$

Prove: $\overline{BE} = \overline{DE}$

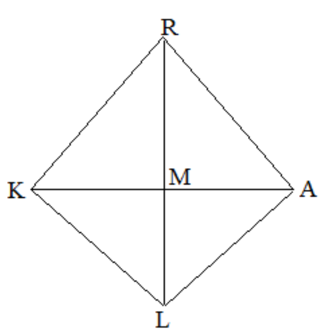


Statements	Reasons
$\overline{AB} = \overline{AD}$ $\overline{CB} = \overline{CD}$	Given
$\overline{AC} \perp$ bisector of $\overline{BD}$	Perpendicular Bisector theorem
$\overline{BE} = \overline{DE}$	Converse of perpendicular bisector theorem (if point lies on $\perp$ bisector, then it is equidistant from endpoints of bisected segment)

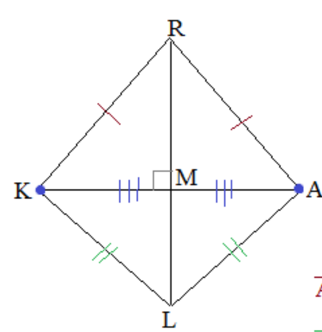




Question: If $\overline{RL}$ is the perpendicular bisector of $\overline{KA}$, which segments are congruent?



Answer: Bisected segment is $\overline{KA}$, so any pair of segments *from endpoints A and K* that *meet on* $\overline{RL}$ would be congruent!!



$\overline{AR} = \overline{KR}$

$\overline{AL} = \overline{KL}$

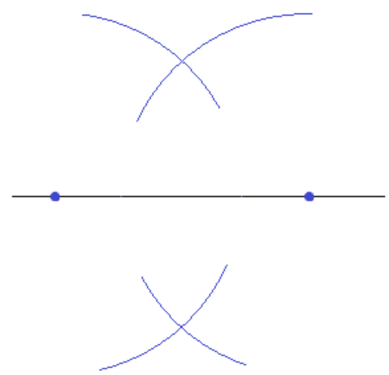
$\overline{AM} = \overline{KM}$

construct perpendicular bisector.....

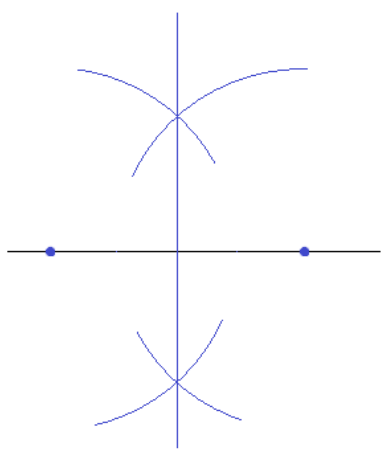
1) pick endpoints on line segment...

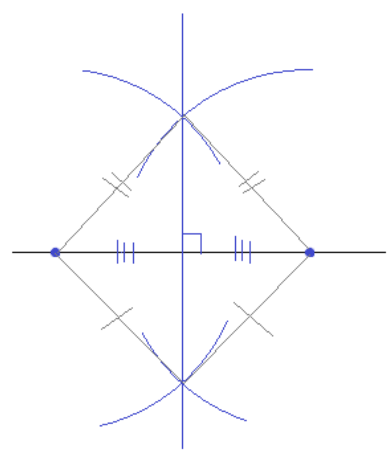
Equidistance Theorem:
If two points are equidistant from the endpoints of a segment,
then the two points determine the perpendicular bisector of the segment.

2) from each endpoint, using a compass, construct arcs above and below...



3) draw line segment through the arc intersections!



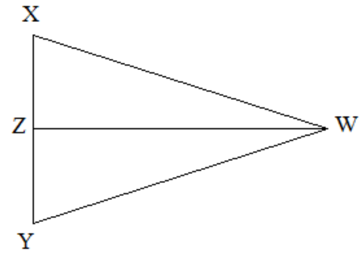


Observation: The arcs create 2 points that are equidistant from the endpoints...

Equidistance Theorem: A Shortcut

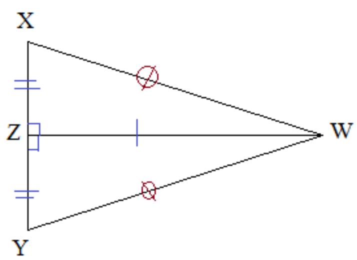
For many proofs, the equidistance theorem is a nice shortcut.

Example: Given: $\overline{WZ}$ is perpendicular bisector of $\overline{XY}$
Prove: $\triangle XWY$ is an isosceles triangle



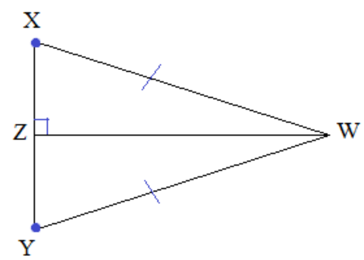
Approach 1:

Statements	Reasons
1. $\overline{WZ} \perp$ bisector of $\overline{XY}$	1. Given
2. $\angle WZX$ and $\angle WZY$ are right angles	2. Definition of perpendicular
3. $\angle WZX \cong \angle WZY$	3. All right angles are congruent
4. $\overline{WZ} = \overline{WZ}$	4. Reflexive Property
5. $\overline{XZ} \cong \overline{YZ}$	5. Definition of Bisector
6. $\triangle XZW \cong \triangle YZW$	6. Side-Angle-Side (4, 3, 5)
7. WX = WY	7. CPCTC
8. $\triangle$ XWY is isosceles	8. Definition of isosceles (two sides of triangle are congruent)



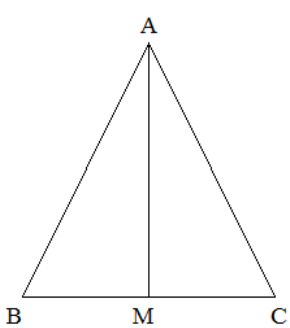
Approach 2: Three steps!

Statements	Reasons
1. WZ $\perp$ bisector of XY	1. Given
2. WY $\cong$ WX	2. If point lies on perpendicular bisector, then it is equidistant to endpoints of bisected segment. (converse) -- perpendicular bisector theorem
3. $\triangle$ XWY is isosceles	3. Definition of isosceles (two sides of triangle are congruent)



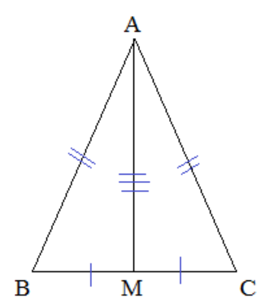
And, for other proofs, the Equidistance Theorem is an alternative.

Example: Given: $\overline{AM}$ is a median
$\overline{AB} \cong \overline{AC}$
Prove: $\triangle$ AMB = $\triangle$ AMC



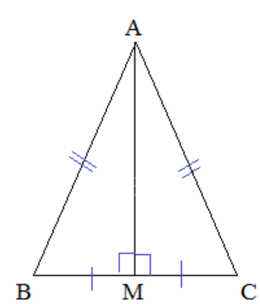
Approach 1:

Statements	Reasons
1. $\overline{AM}$ is a median	1. Given
2. $\overline{BM} \cong \overline{CM}$	2. Definition of Median (and midpoint)
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{AM} \cong \overline{AM}$	4. Reflexive property
5. $\triangle AMB \cong \triangle AMC$	5. SSS (Side-Side-Side) (2, 3, 4)



Approach 2:

Statements	Reasons
1. $\overline{AM}$ is a median	1. Given
2. $\overline{BM} \cong \overline{CM}$	2. Definition of Median (and midpoint)
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{AM}$ is $\perp$ bisector of $\overline{BC}$	4. Equidistance theorem
5. $\angle AMB$ and $\angle AMC$ are right angles	5. Definition of perpendicular
6. $\angle AMB \cong \angle AMC$	6. All right angles are congruent
7. $\triangle AMB \cong \triangle AMC$	7. HL (Hypotenuse-Leg) (6, 3, 2)

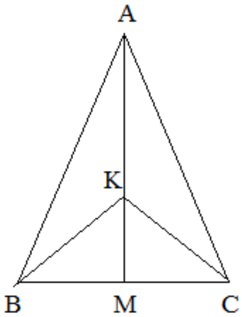


Proofs that utilize the equidistance theorem

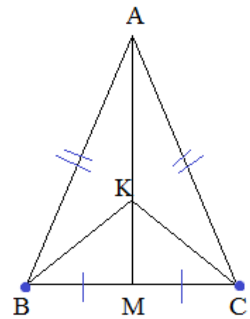
Example: Given: $\overline{AM}$ is a median

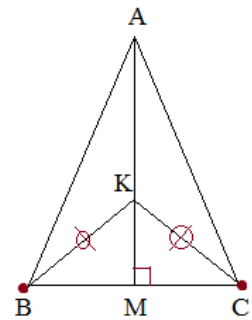
$\overline{AB} \cong \overline{AC}$

Prove: $\overline{BK} \cong \overline{CK}$



Statements	Reasons
1. $\overline{AM}$ is a median	1. Given
2. M is midpoint of $\overline{BC}$	2. Definition of median
3. $\overline{BM} \cong \overline{CM}$	3. Definition of midpoint
4. $\overline{AB} \cong \overline{AC}$	4. Given
5. $\overline{AM}$ is perpendicular bisector of $\overline{BC}$	5. Equidistance theorem (if 2 points are equidistant from the same endpoints of a segment, then the 2 points form a perpendicular bisector of the segment)
6. $\overline{BK} \cong \overline{CK}$	6. (converse) Equidistance theorem (if a point lies on the $\perp$ bisector, then it is equidistant from the endpoints of the bisected segment)

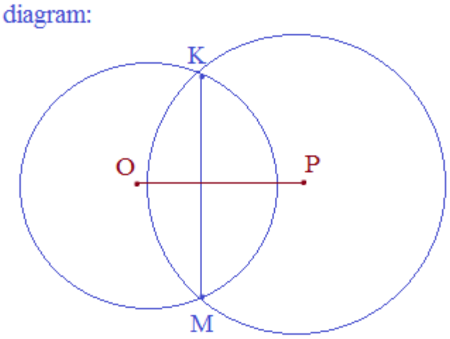


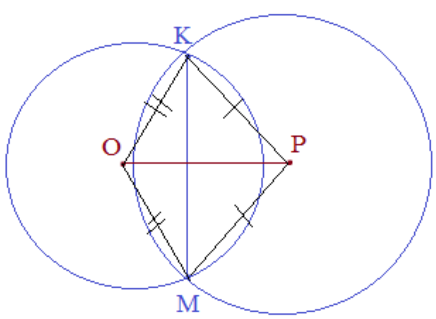


Example: Given: 2 intersecting circles with a segment $\overline{KM}$ connecting the points of intersection.

Prove: The segment joining the centers from each circle bisects $\overline{KM}$.

Draw a diagram:





2 column proof:

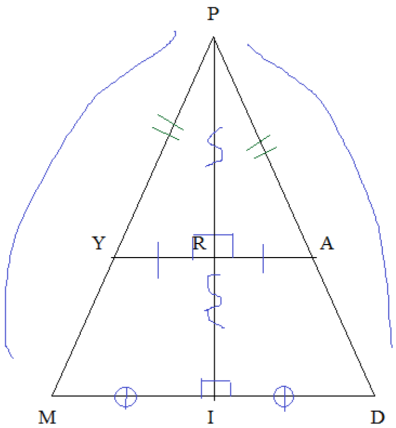
Statements	Reasons
1. Intersecting circles with centers O and P	1. Given (diagram)
2. Draw auxilary lines (radii) $\overline{KP}$, $\overline{PM}$, $\overline{KO}$, $\overline{MO}$	2. A line segment connects two points
3. $\overline{KO} = \overline{MO}$	3. All radii of a circle are congruent
4. $\overline{KP} = \overline{MP}$	4. All radii of a circle are congruent
5. $\overline{OP}$ is perpendicular bisector of $\overline{KM}$	5. Equidistance Theorem (if 2 pts. are equidistant from endpoints of a segment, the 2 pts. form $\perp$ bisector of segment)
6. OP bisects $\overline{KM}$	6. def. of $\perp$ bisector

Example: Equidistance Theorem VS. Detour Proof

Given: $\overline{PI} \perp$ bisector of $\overline{MD}$

$\overline{PI} \perp$ bisector of $\overline{YA}$

Prove: $\overline{MY} = \overline{AD}$



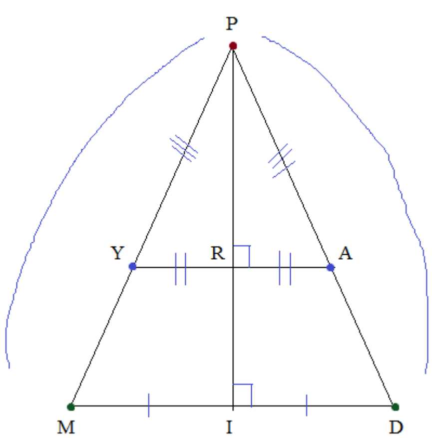
Using Detours....

	Statements	Reasons
	1) $\overline{PI} \perp$ bisector of $\overline{YA}$	1) Given
	2) $\overline{RY} = \overline{RA}$	2) Definition of bisector
	3) $\overline{PR} = \overline{PR}$	3) Reflexive Property
detour	4) $\angle PRY$ and $\angle PRA$ are right angles	4) Definition of perpendicular
	5) $\angle PRY = \angle PRA$	5) All right angles are congruent
	6) $\triangle PRY = \triangle PRA$	6) Side-Angle-Side (3, 5, 2)
	7) PY = PA	7) CPCTC (Corresponding Parts of Congruent Triangles are congruent)
	8) $\overline{PI} \perp$ bisector of $\overline{MD}$	8) Given
	9) $\overline{MI} = \overline{DI}$	9) Definition of bisector
	10) $\overline{PI} = \overline{PI}$	10) Reflexive Property
detour	11) $\angle PIM = \angle PID$ are right angles	11) Definition of perpendicular
	12) $\angle PIM = \angle PID$	12) All right angles are congruent
	13) $\triangle PIM = \triangle PID$	13) Side-Angle-Side (10,-12, 9)
	14) $\overline{PM} = \overline{PD}$	14) CPCTC (Corresponding Parts of Congruent Triangles are congruent)
	15) $\overline{MY} = \overline{AD}$	15) Subtraction Property (If 2 congruent segments are subtracted from congruent segments, then the differences are the same)

Given: $\overline{PI} \perp$ bisector of $\overline{MD}$

$\overline{PI} \perp$ bisector of $\overline{YA}$

Prove: $\overline{MY} = \overline{AD}$



Using Equidistance Theorem...

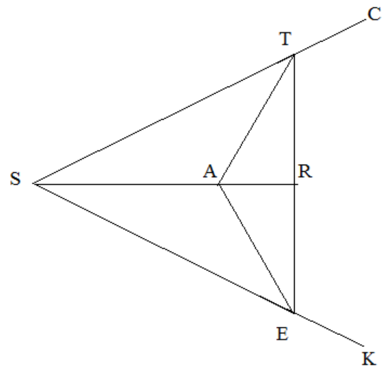
Statements	Reasons
1) $\overline{PI} \perp$ bisector of $\overline{YA}$	1) Given
2) $\overline{PY} = \overline{PA}$	2) Equidistance Theorem (Converse) (If a point lies on a perpendicular bisector, then it is equidistant to the endpoints of the segment)
3) $\overline{PI} \perp$ bisector of $\overline{MD}$	3) Given
4) $\overline{PM} = \overline{PD}$	4) Equidistance Theorem (Converse)
5) $\overline{MY} = \overline{AD}$	5) Subtraction Property (If 2 congruent segments are subtracted from congruent segments, then the differences are the same)

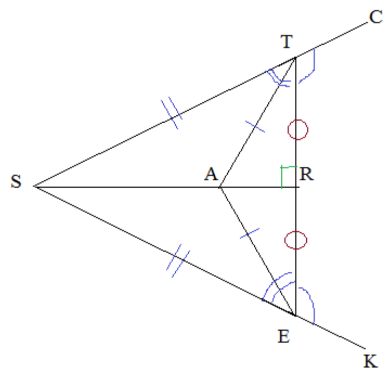
Example: Given: $\angle CTR = \angle KER$

$\overline{TA} = \overline{EA}$

Prove: $\overline{TR} = \overline{ER}$

Uses Supplementary Angles
Equidistance Theorem
(Converse) Equidistance Theorem





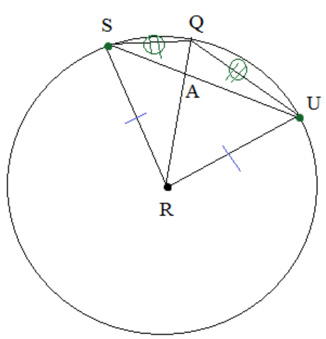
Statements	Reasons
1) $\angle CTR = \angle KER$	1) Given
2) STR and CTR are supplementary angles KER and SER are supplementary angles	2) Definition of Supplementary If (adjacent) angles form a straight angle, then angles are supplementary
3) $\angle SER = \angle STR$	3) If 2 angles are supplementary to congruent angles, then the 2 angles are congruent
4) $\overline{ST} = \overline{SE}$	4) If congruent angles, then congruent sides (in triangle)
5) $\overline{TA} = \overline{EA}$	5) Given
6) $\overline{SR}$ is perpendicular bisector of $\overline{TE}$	6) Equidistance Theorem (If 2 points are equidistant to endpoints of a segment, then the points determine the perpendicular bisector of segment)
7) TR = ER	7) Equidistance Theorem (Converse) If point lies on perpendicular bisector, then point is equidistant from endpoints of segment...

Example: Given: Circle R

$\overline{QR}$ is not a perpendicular bisector

Prove: $\overline{SQ} \ncong \overline{UQ}$

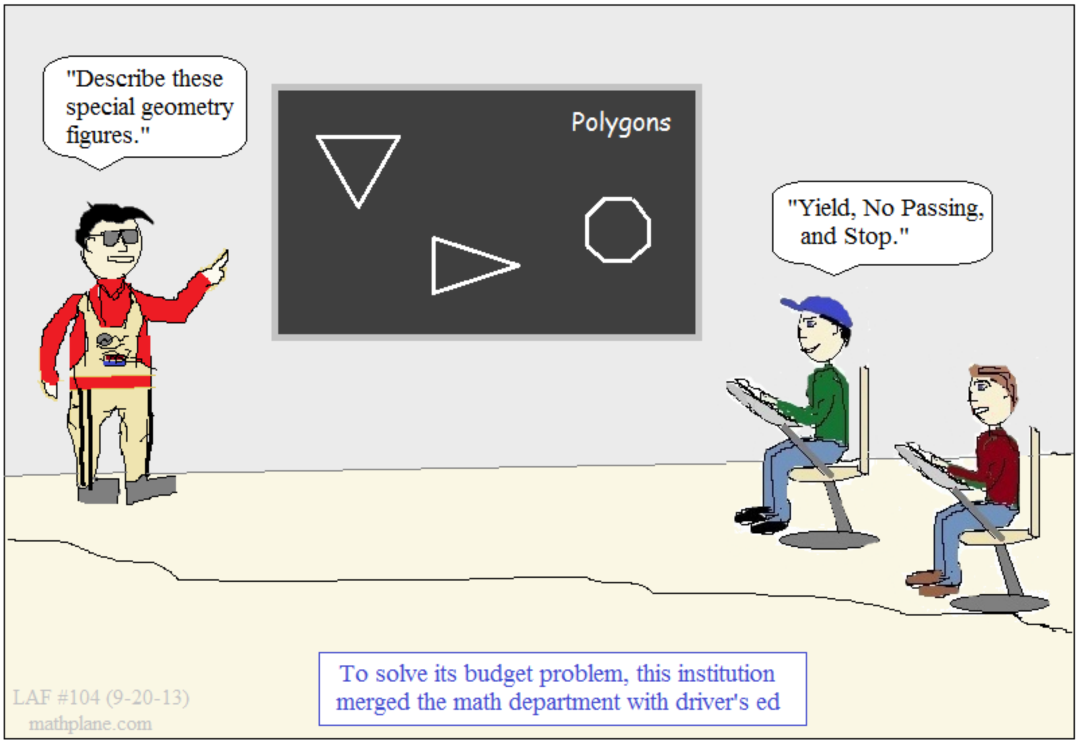
Uses Indirect Proof
Method of Contradiction

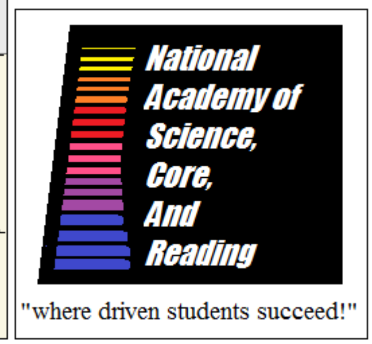


Statements	Reasons
1) Circle R	1) Given
2) QR is not $\perp$ bisector	2) Given
3) $\overline{SQ} \cong \overline{UQ}$	3) Assume to reach a contradiction
4) $\overline{RS} \cong \overline{RU}$	4) All radii are congruent
5) QR is perpendicular bisector of segment SU	5) Equidistance Theorem (If 2 points are equidistant to endpoints of a segment, then the points form a perpendicular bisector of the segment)

However, statements 2) and 5) contradict each other...
Proof by contradiction...

Accelerated Math





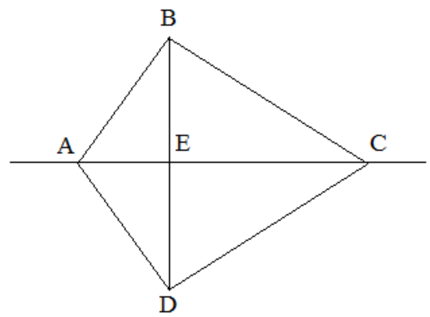
Practice Test: Proofs and Applications

Practice Exercise: Proofs utilizing the Equidistance Theorems

1) Given: $\overline{BD}$ is the base of isosceles triangles ABD and CBD

Prove: $\overline{BE} \cong \overline{ED}$

Statements	Reasons



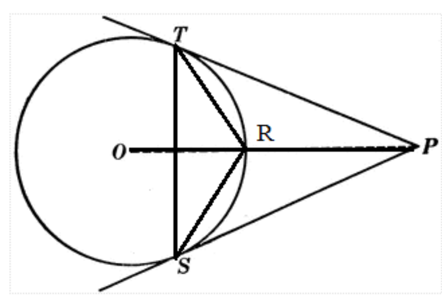
2) Prove the median of an equilateral triangle is also the altitude.

Statements	Reasons

3) Given: $\overline{PT} \cong \overline{PS}$; Circle O

Prove: $\overline{TR} \cong \overline{SR}$

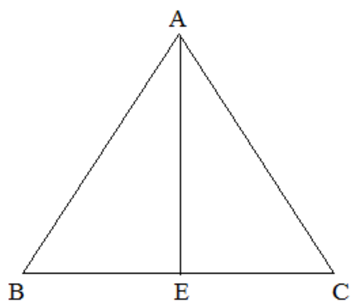
Statements	Reasons



Equidistance Theorem Questions

4) Given: $\triangle ABC$ is isosceles with $\overline{AC} \cong \overline{AB}$; E is midpoint of $\overline{BC}$

Prove: $\overline{AE} \perp \overline{BC}$



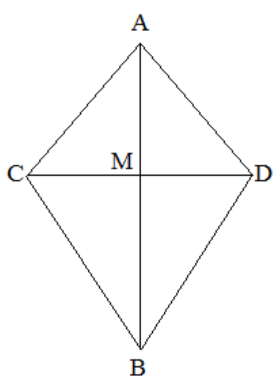
Write 2 proofs: 1 utilizing the Equidistance Theorem, and
1 *without* the Equistance Theorem.

Statements	Reasons

Statements	Reasons

5) $\overline{DC}$ is the perpendicular bisector of $\overline{AB}$.

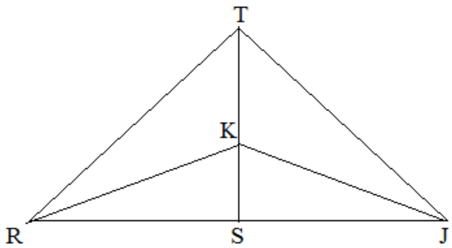
Which segments are congruent?



Practice Exercise: Proofs utilizing the Equidistance Theorems

6) Given: $\overline{TS}$ is a perpendicular bisector of $\overline{RJ}$

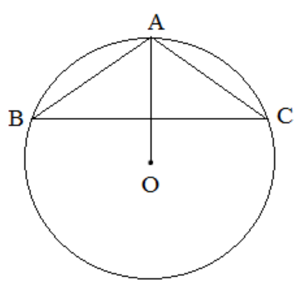
Prove: $\triangle TRK = \triangle TJK$



Statements	Reasons

7) Given: Circle O; $\angle B \cong \angle C$

Prove: $\overline{AO}$ bisects $\overline{BC}$



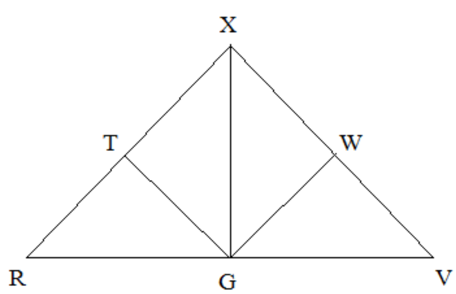
Statements	Reasons

8) Given: G is the midpoint of $\overline{RV}$

$\overline{TG} \perp \overline{RX}$ and $\overline{WG} \perp \overline{VX}$

$\overline{TR} \cong \overline{WV}$

Prove: $XG \perp RV$

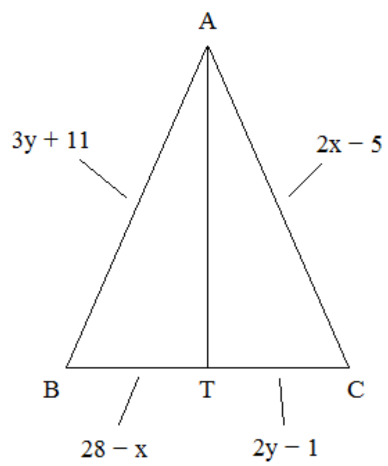


Statements	Reasons

Equidistance Theorem Questions

9) $\overline{AT}$ is the perpendicular bisector of $\overline{BC}$.

What is the perimeter of $\triangle ABC$?



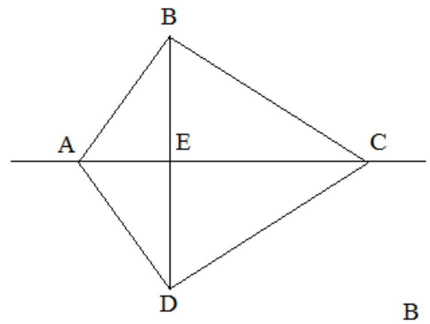
SOLUTIONS

Practice Exercise: Proofs utilizing the Equidistance Theorems

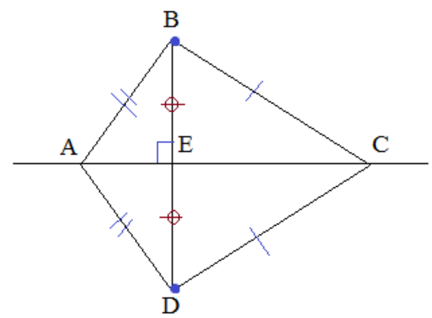
SOLUTIONS

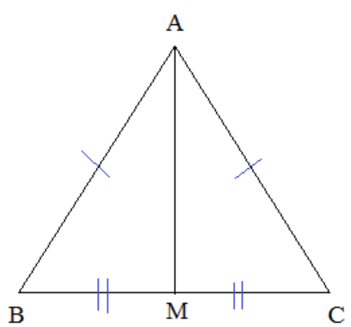
1) Given: $\overline{BD}$ is the base of isosceles triangles ABD and CBD

Prove: $\overline{BE} \cong \overline{ED}$



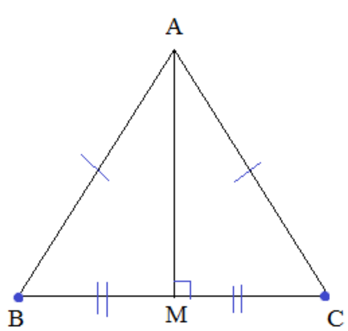
Statements	Reasons
1. ABD and CBD are isosceles $\triangle$s	1. Given
2. $\overline{BA} \cong \overline{DA}$	2. Definition of Isosceles
3. $\overline{BC} \cong \overline{DC}$	3. Definition of Isosceles
4. $\overline{AC}$ is $\perp$ bisector of $\overline{BD}$	4. Equidistance Theorem
5. $\overline{BE} \cong \overline{ED}$	5. Definition of Bisector





2) Prove the median of an equilateral triangle is also the altitude.

Statements	Reasons
1. $\triangle$ ABC is equilateral	1. Given (diagram)
2. $\overline{AB} \cong \overline{AC}$	2. Definition of equilateral (all sides congruent)
3. $\overline{AM}$ is median	3. Given (diagram)
4. M bisects $\overline{BC}$	4. Definition of median (segment from vertex to midpoint of opposite side)
5. $\overline{BM} = \overline{MC}$	5. Definition of midpoint
6. $\overline{AM}$ is perpendicular bisector of $\overline{BC}$	6. Equidistant theorem (if 2 pts. are each equidistant to the endpoints of a segment, then the 2 pts. determine the perpendicular bisector of the segment)
7. $\overline{AM}$ is altitude	7. Definition of altitude (segment from vertex that is perpendicular to opposite side)



B and C are the endpoints of the segment

equidistance pair 1: BA and CA
equidistance pair 2: BM and CM

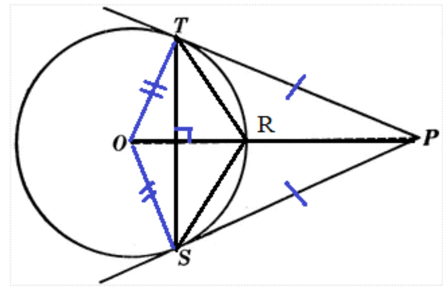
Therefore, AM is the perpendicular bisector

Practice Exercise: Proofs utilizing the Equidistance Theorems

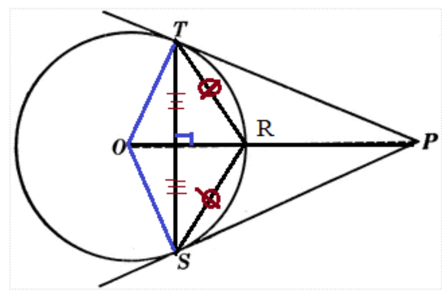
SOLUTIONS

3) Given: $\overline{PT} \cong \overline{PS}$; Circle O

Prove: $\overline{TR} \cong \overline{SR}$

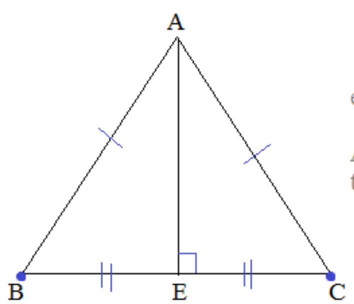


Statements	Reasons
1. $\overline{PT} = \overline{PS}$	1. Given
2. $\overline{TO}$ and $\overline{OS}$ are auxilary line segments	2. Line segment connects two points
3. $\overline{TO} = \overline{OS}$	3. All radii are congruent
4. $\overline{PO}$ is $\perp$ bisector of $\overline{TS}$	4. Equidistance Theorem
5. $\overline{TR} = \overline{RS}$	5. Converse of Equidistance Theorem (If a point lies on a perpendicular bisector, then it is equidistant from the endpoints of the bisected segment.)



4) Given: $\triangle$ ABC is isosceles with $\overline{AC} \cong \overline{AB}$; E is midpoint of $\overline{BC}$

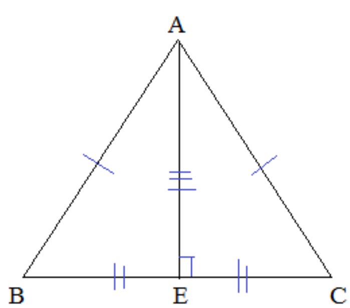
Prove: $\overline{AE} \perp \overline{BC}$



endpoints are B and C

A and E are each equidistant to the endpoints

Statements	Reasons
1. $\overline{AC} \cong \overline{AB}$	1. Given
2. E is midpoint	2. Given
3. $\overline{BE} \cong \overline{EC}$	3. definition of midpoint
4. $\overline{AE}$ is $\perp$ ___ bisector of BC	4. Perpendicular Bisector Theorem
5. $\overline{AE} \perp \overline{BC}$	5. Def. of $\perp$ bisector



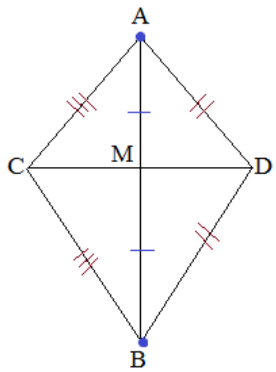
Statements	Reasons
1. $\overline{AC} \cong \overline{AB}$	1. Given
2. E is midpoint	2. Given
3. $\overline{BE} \cong \overline{EC}$	3. definition of midpoint
4. $\overline{AE} \cong \overline{AE}$	4. reflexive property
5. $\triangle$AEB = $\triangle$AEC	5. Side-Side-Side (4, 3, 1)
6. $\angle$AEB = $\angle$AEC	6. CPCTC
7. $\angle$AEB and $\angle$AEC are right angles	7. Right angle theorem (if 2 angles are both supplementary and congruent, then they are right)
8. AE $\perp$ BC	8. Definition of perpendicular

Equidistance Theorem Questions

SOLUTIONS

5) $\overline{DC}$ is the perpendicular bisector of $\overline{AB}$.

Which segments are congruent?



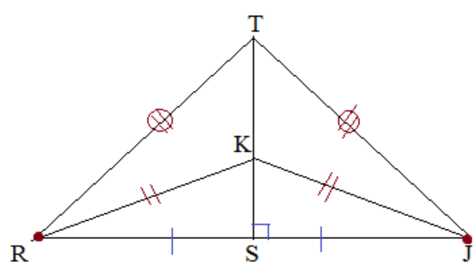
Although $\overline{AB}$ *appears* to bisect $\overline{CD}$,
$\overline{DC}$ bisects $\overline{AB}$!!

therefore, $\boxed{\overline{MB} \cong \overline{MA}}$

also, every point on perpendicular bisector is equidistant to endpoints A and B.

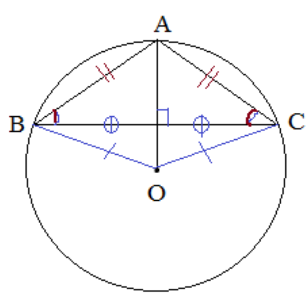
therefore, $\boxed{\overline{CB} \cong \overline{CA} \quad \overline{DB} \cong \overline{DA}}$

6) Given: $\overline{TS}$ is a perpendicular bisector of $\overline{RJ}$
Prove: $\triangle TRK = \triangle TJK$



Statements	Reasons
1. $\overline{TS}$ is $\perp$ bisector of $\overline{RJ}$	1. Given
2. $\overline{RK} \cong \overline{JK}$ $\overline{RT} \cong \overline{JT}$	2. Equidistance Theorem (If point lies on $\perp$ bisector, then it is equidistant from endpoints of bisected segment)
3. TK = TK	3. Reflexive property
4. $\triangle$ TRK = $\triangle$ TJK	4. Side-Side-Side (SSS) (2, 2, 3)

7) Given: Circle O; $\angle B \cong \angle C$
Prove: $\overline{AO}$ bisects $\overline{BC}$



Statements	Reasons
1. $\angle B \cong \angle C$	1. Given
2. Circle with center O	2. Given (diagram)
3. Auxilary line segments $\overline{OB}$ and $\overline{OC}$	3. line segment joins 2 points
4. $\overline{OB} \cong \overline{OC}$	4. All radii are congruent
5. $\overline{AB} \cong \overline{AC}$	5. If congruent angles, then congruent sides
6. $\overline{AO}$ $\perp$ bisector of $\overline{BC}$	6. Equidistance theorem
7. $\overline{AO}$ bisects $\overline{BC}$	7. Definition of perpendicular bisector

Equidistance Theorem Questions

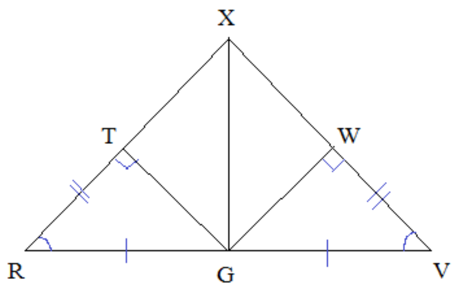
SOLUTIONS

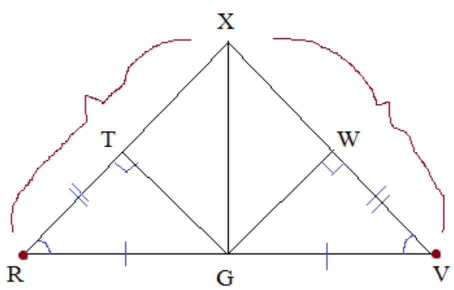
8) Given: G is the midpoint of $\overline{RV}$

$\overline{TG} \perp \overline{RX}$ and $\overline{WG} \perp \overline{VX}$

$\overline{TR} \cong \overline{WV}$

Prove: $XG \perp RV$

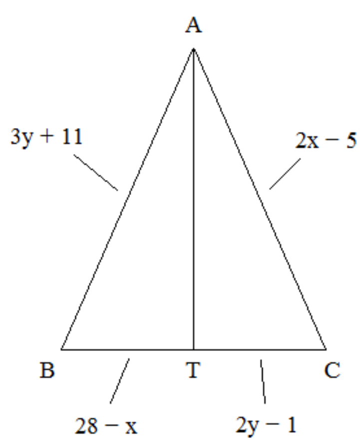




Statements	Reasons
1. G is midpt. of $\overline{RV}$	1. Given
2. $\overline{RG} = \overline{VG}$	2. Definition of midpoint
3. $TG \perp RX$ $WG \perp VX$	3. Given
4. $\angle GTR$ & $\angle GWV$ are right angles	4. Definition of perpendicular
5. $\angle GTR \cong \angle GWV$	5. All right angles are congruent
6. $\overline{TR} \cong \overline{WV}$	6. Given
7. $\triangle GTR = \triangle GWV$	7. Hypotenuse Leg (HL) (4, 2, 6)
8. $\angle R \cong \angle V$	8. CPCTC (corresponding parts of congruent triangles are congruent)
9. $\overline{RX} = \overline{VX}$	9. If congruent angles, then congruent sides
10. $\overline{XG} \perp$ bisector $\overline{RV}$	10. Equidistance Theorem
11. $\overline{XG} \perp \overline{RV}$	11. Definition of perpendicular bisector

9) $\overline{AT}$ is the perpendicular bisector of $\overline{BC}$.

What is the perimeter of $\triangle ABC$?



Since $\overline{AT}$ is $\perp$ bisector of $\overline{BC}$,

$\overline{AC} \cong \overline{AB}$ and $\overline{TC} \cong \overline{TB}$

$3y + 11 = 2x - 5$

and

$28 - x = 2y - 1$

$2x - 3y = 16$

$x + 2y = 29$

$-2x - 4y = -58$

system with 2 equations and 2 unknowns

$-7y = -42$

$y = 6$

$x + 2(6) = 29$

$x = 17$

Since x = 17 and y = 6,

AB = 29 AC = 29 BT = 11 and TC = 11

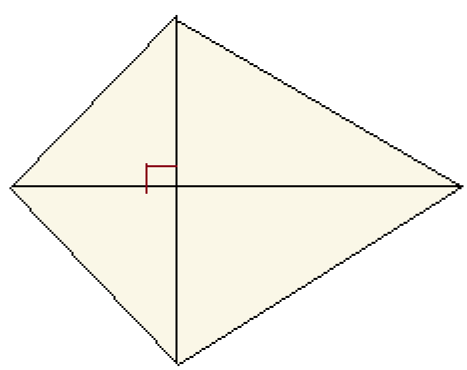
$\boxed{\text{Perimeter of triangle ABC = 80}}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

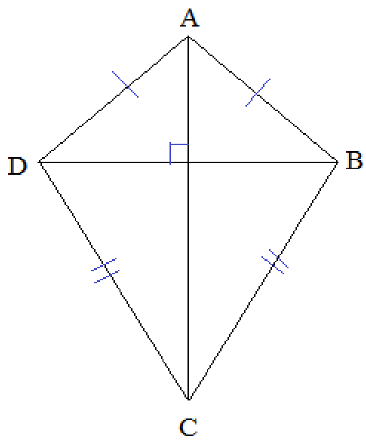
Cheers, LAF

One more question: Prove the diagonals of a kite are perpendicular.



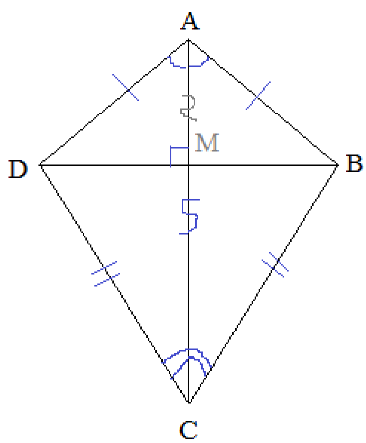
Prove the Diagonals of a Kite are Perpendicular

Utilizing the Equidistance Theorem



Statements	Reasons
1. Kite ABCD	1. Given (diagram)
2. $\overline{AB} \cong \overline{AD}$ $\overline{CB} \cong \overline{CD}$	2. Definition of Kite (2 pairs of adjacent sides are congruent)
3. $\overline{AC}$ is perpendicular bisector of $\overline{DB}$	3. Equidistance Theorem (if 2 points are equidistant from the endpoints of a segment, then the 2 points determine the perpendicular bisector of the segment)
4) $AC \perp DB$	4. Definition of perpendicular bisector

An alternative:



Statements	Reasons
1. Kite ABCD	1. Given (diagram)
2. $\overline{AB} \cong \overline{AD}$ $\overline{CB} \cong \overline{CD}$	2. Definition of Kite (2 pairs of adjacent sides are congruent)
3. AC = AC	3. Reflexive property
4. $\triangle DAC = \triangle BAC$	4. Side-Side-Side (SSS) (2, 2, 3)
5. $\angle DAC = \angle BAC$	5. CPCTC
6. $\overline{AM} \cong \overline{AM}$	6. Reflexive property
7. $\triangle AMD \cong \triangle AMB$	7. Side-Angle-Side (SAS) (2, 5, 6)
8. $\angle AMD \cong \angle AMB$	8. CPCTC
9. $\angle AMD$ & $\angle AMB$ are right angles	9. If angles are supplementary and congruent, then they are right angles
10. $AC \perp DB$	10. If right angles, then segments are perpendicular

Eventually, Noah realizes that this assignment was NOT a geometry construction

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