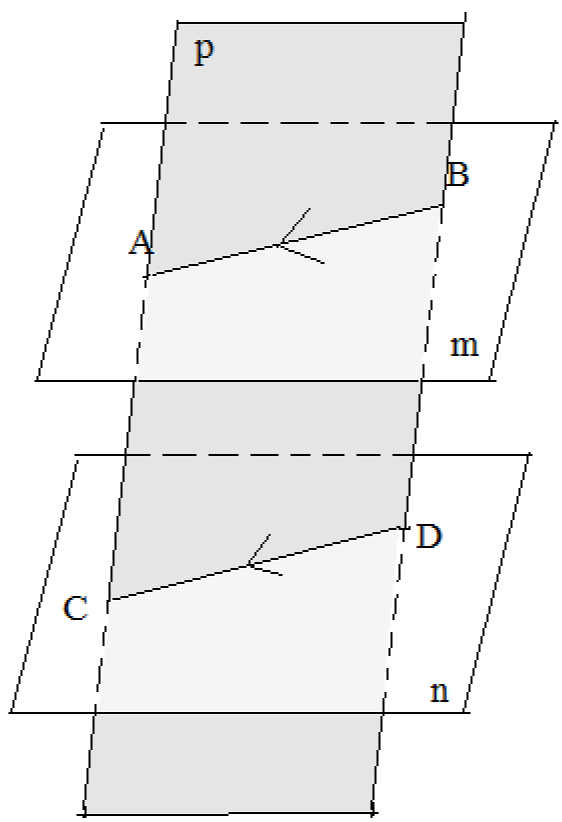
Geometry: Planes, Properties, and Proofs

Notes, Examples, and Exercises (with Solutions)



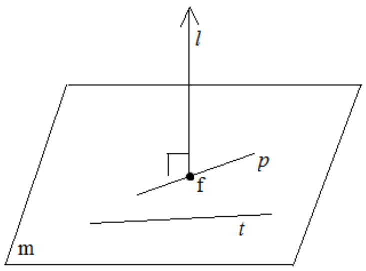
Topics include skew lines, foot, determining a plane, intersections, always/sometimes/never, and more.

Planes, Lines, and Points Theorems

Skew Lines: Lines that are not parallel and do not cross.

Foot (of the perpendicular): Point of intersection where the perpendicular line and plane meet.

1) If a line is perpendicular to a plane, then it is perpendicular to all lines in the plane that meet at the foot.



f is the "foot"

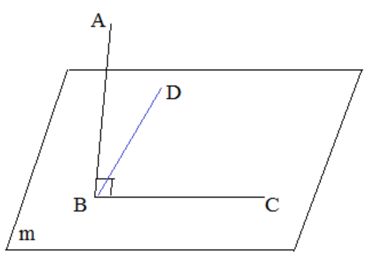
$l \perp m$

$l \perp p$

~~$l \perp t$~~

l and t are skew lines...

2) If a line is perpendicular to one line in a plane, then it is ONLY perpendicular to that line. (It MAY be perpendicular to others, but it's not guaranteed.)



Points D, B, and C are in the plane...

$\angle$ABC is a right angle (that is "tilted" back)

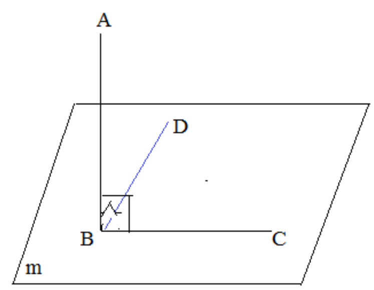
$\angle$ABD is not a right angle

AB $\perp$ BC

AB is not $\perp$ BD

AB is not $\perp$ to plane m

$\angle$ABD is a right angle



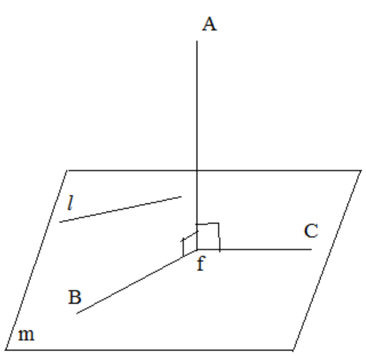
AB $\perp$ BC

AB $\perp$ BD

AB $\perp$ to plane m

3) If a line is perpendicular to 2 (or more lines) in a plane, at a common intersection, then it is perpendicular to all lines in the plane that go through that intersection ("foot")

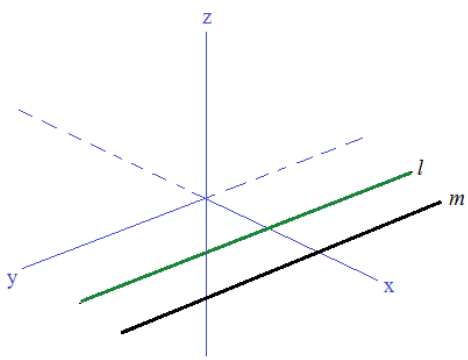
The perpendicular line and any lines in the plane that don't pass throught the foot are skew...



Parallel lines in space..

l and *m* both lie in the xy-plane..

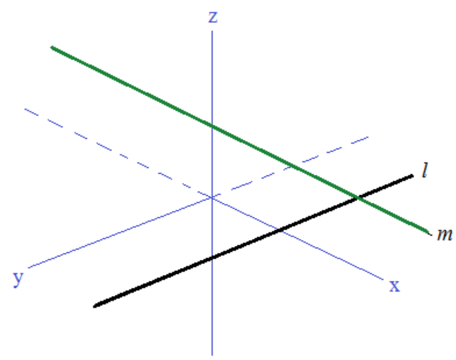
l and *m* never intersect..



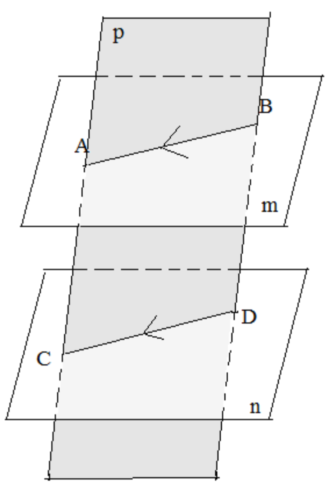
Skew lines in space

l is in the xy-plane
m is in the xz-plane

l and *m* never intersect (line *m* is above line *l*)



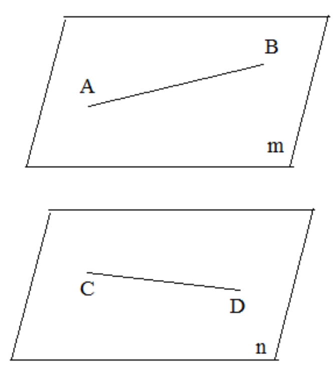
If a plane passes through <u>parallel</u> planes, the lines of intersection must be parallel.



planes m and n are parallel..

plane p intersects plane m at line AB
plane p intersects plane n at line CD

Therefore, AB || CD



planes m and n are parallel.

AB lies in plane m

CD lies in plane n

But, AB may or may not be || to CD

(If the lines are not parallel, then a plane could not pass through both AB and CD --- unless it were warped!)

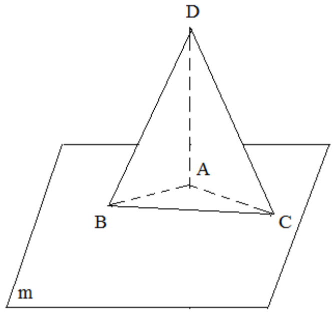
Example Given: $\triangle$ BDC is isosceles

$\overline{BD} \cong \overline{CD}$

$\angle ADB \cong \angle ADC$

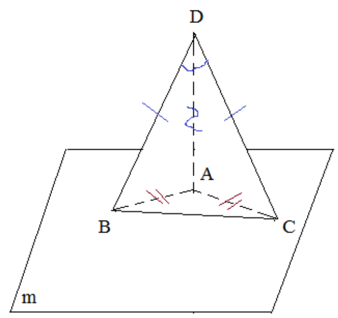
Prove: $\triangle$ ABC is isosceles

Strategy: Use "back" triangles (i.e. prove DAB = DAC) to get congruent sides AB and AC...



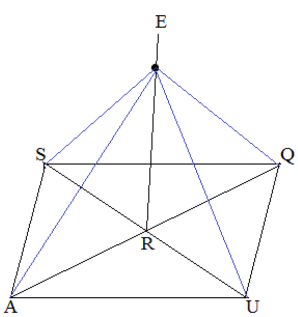
NOTE: You <u>cannot</u> assume DA is perpendicular to plane m. (It may be, but it's not certain) So, you must use SAS (and not HL)

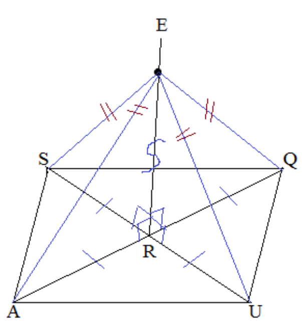
Statements	Reasons
1) $\triangle$ BDC is isosceles $\overline{BD} \cong \overline{CD}$	1) Given
2) $\overline{DA} \cong \overline{DA}$	2) Reflexive Property
3) $\angle ADB \cong \angle ADC$	3) Given
4) $\triangle$ BDA = $\triangle$ CDA	4) SAS (Side-Angle-Side) 1, 3, 2
5) $\overline{AB} \cong \overline{AC}$	5) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
6) $\triangle$ ABC is isosceles	6) Definition of Isosceles (2 or more sides are congruent)



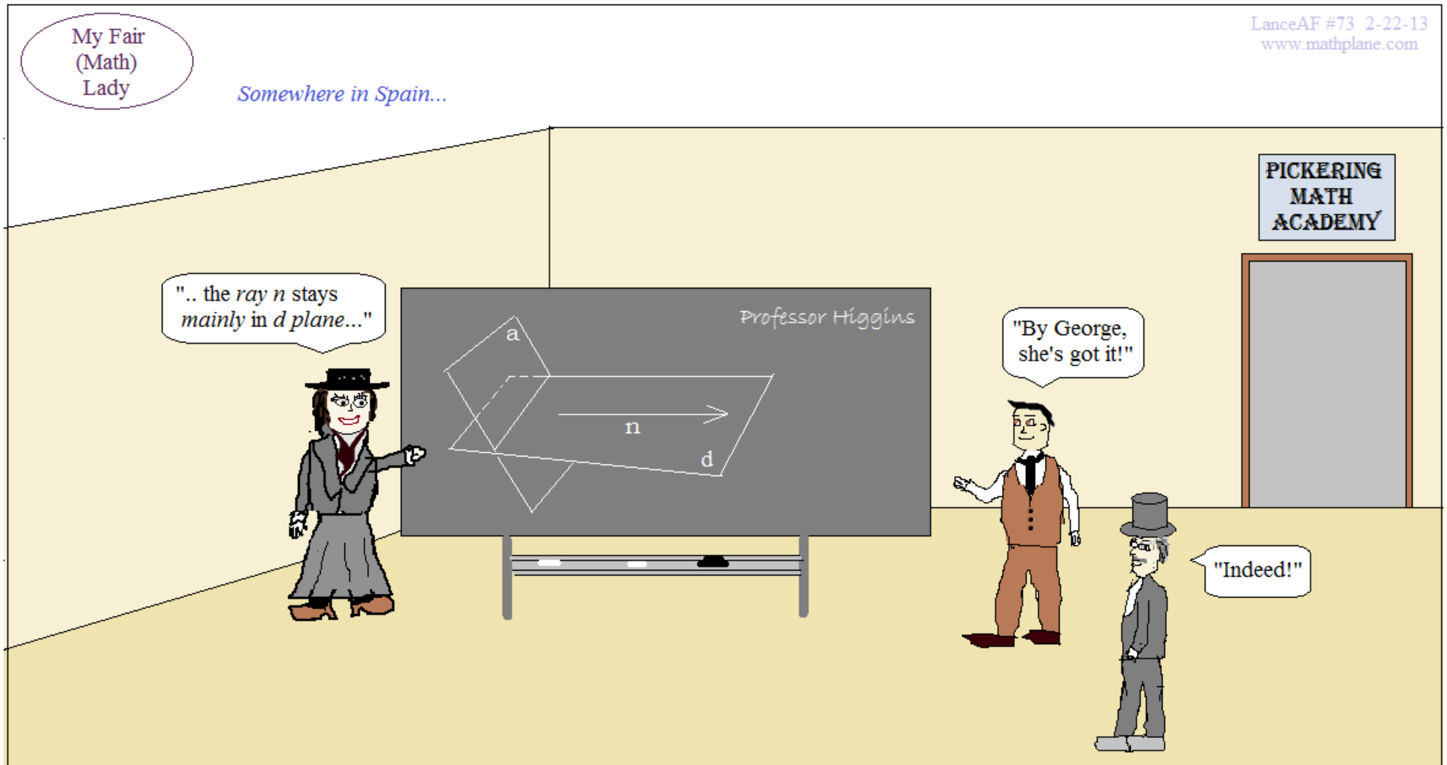
Example: A line is drawn perpendicular to the plane of a square. The point of intersection (the foot), lies at the intersection of the square's diagonals.

Prove that any point on the perpendicular line is equidistant to all 4 vertices of the square.





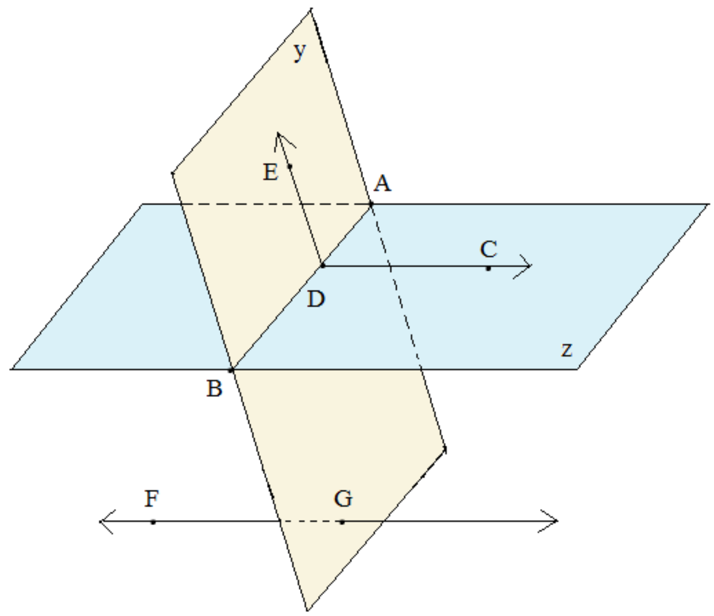
Statements	Reasons
1) SQUA is a square	1) Given
2) R is intersection of diagonals AQ and SU	2) Given
3) SU = AQ	3) Definition of a square (diagonals of square are congruent)
4) AQ and SU are bisectors	4) Definition of square (diagonals bisect each other)
5) QR = AR = UR = SR	5) Definition of Bisector (bisector divides segment into congruent halves)
6) ER = ER	6) Reflexive property
7) ER is perpendicular to plane at foot R	7) Given
8) ERS, ERQ, ERU, ERA are right angles	8) Any segment that intersects the foot of a perpendicular line forms a right angle
9) ERS, ERQ, ERU, and ERA are congruent	9) All right angles congruent
10) Triangles ERS, ERQ, ERU and ERA are congruent	10) SAS (Side-Angle-Side) 6, 9, 5
11) EA, EU, EQ, and ES are congruent	11) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)



Eliza Doolittle learns Geometry

Practice Exercises-→

Plane Geometry



1) Identify 3 collinear points

2) Select 3 non-collinear points and identify their plane

3) Answer the following:

a) $y \cap z$

b) $\overrightarrow{DE} \cup \overrightarrow{DC}$

c) $\overrightarrow{DE} \cap \overrightarrow{DC}$

4) Assume line FG is parallel to plane z ($\overleftrightarrow{FG} \parallel z$)

a) $\overleftrightarrow{FG} \cap y$

b) $\overleftrightarrow{FG} \cap z$

5) Three non-collinear points determine a plane. Can you name 3 other ways?

A) True/False

1) Two intersecting planes form a point.

2) If two lines don't intersect, then they are parallel.

3) If two lines in a plane don't intersect, then the lines are parallel.

4) In a plane, 2 lines that are perpendicular to a common line are parallel.

5) Two lines that are perpendicular to a common line must be parallel.

6) A triangle is always a planar figure.

7) A square is always a planar figure.

8) Two lines must intersect or be parallel.

9) If a line is perpendicular to a plane, it is perpendicular to every line in that plane.

10) If a line is perpendicular to a line in a plane, then it is perpendicular to the plane.

B) Always/Sometimes/Never

1) Two planes form a line.

2) A given line that is perpendicular to a plane is perpendiculuar to the plane at one point.

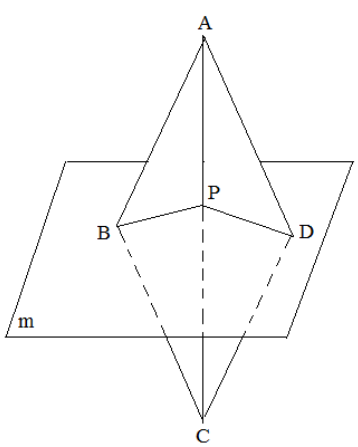
3) Three parallel lines lie in the same plane.

4) Two skew lines have one intersection.

5) Two parallel lines determine a plane.

6) Two skew lines are coplanar.

C)



Given: $\angle$ APB = $x + 72$

$\angle$ APD = $104 - \frac{7}{9}x$

AC is a straight line that contains the point P

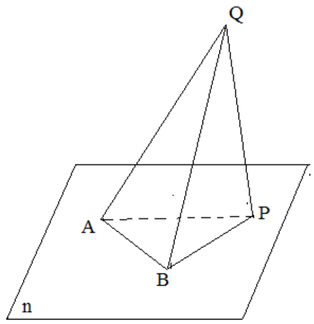
$\angle$ CPD = $3x + 35$

Are the angles congruent? Explain..

1) Given: A and B are equidistant from P

$QP \perp n$

Prove: $\angle QAB \cong \angle QBA$



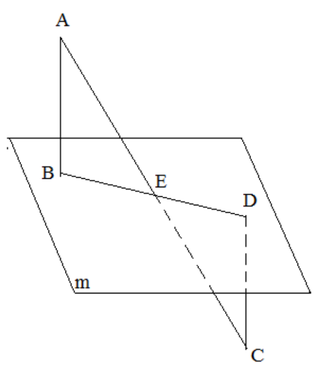
Statements	Reasons

2) Given: $\overline{AB} \perp m$

$\overline{CD} \perp m$

$\overline{AC}$ bisects $\overline{BD}$

Prove: $\overline{BD}$ bisects $\overline{AC}$



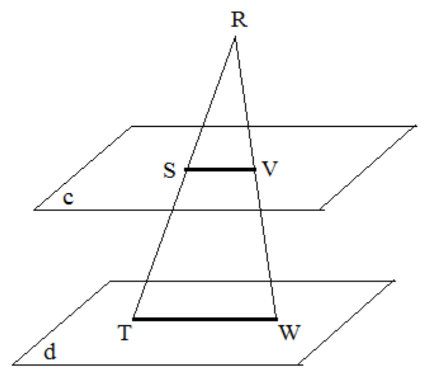
Statements	Reasons

3) Given: $c \parallel d$

$\triangle RTW$ is isosceles with base $\overline{TW}$

Prove: $\triangle RSV$ is isosceles

Statements	Reasons



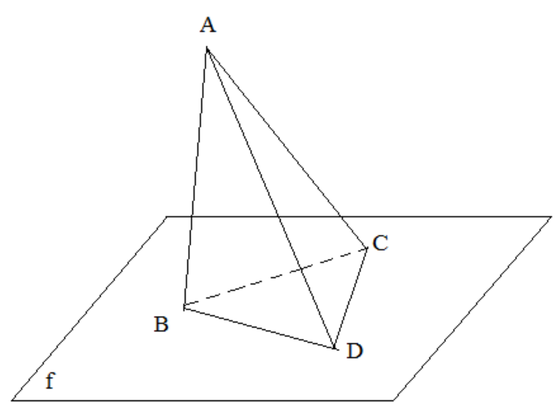
4) Given: $\overline{AB} \perp f$

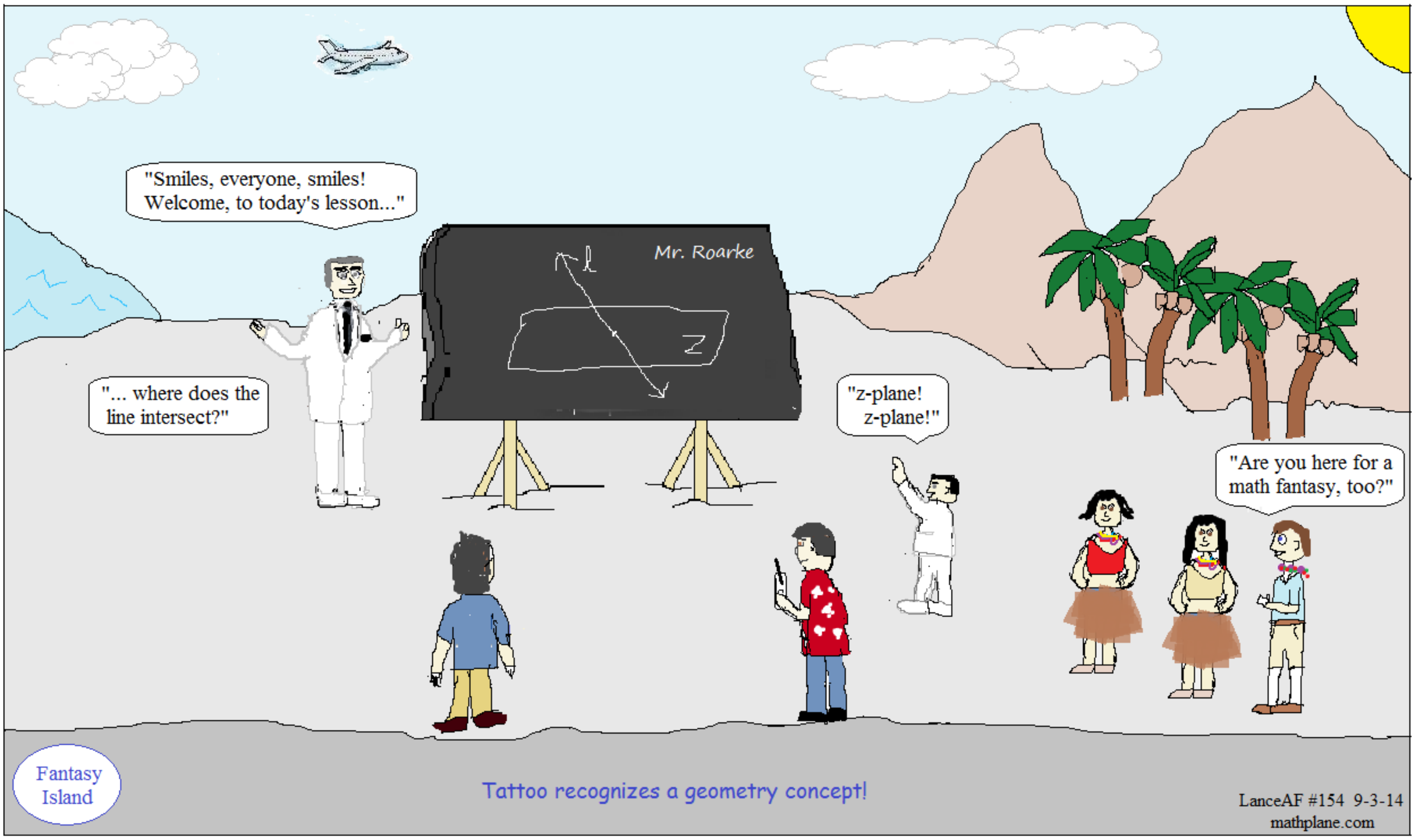
$\triangle BCD$ is an equilateral triangle

B, C, D are coplanar

Prove: ACD is isosceles

Statements	Reasons

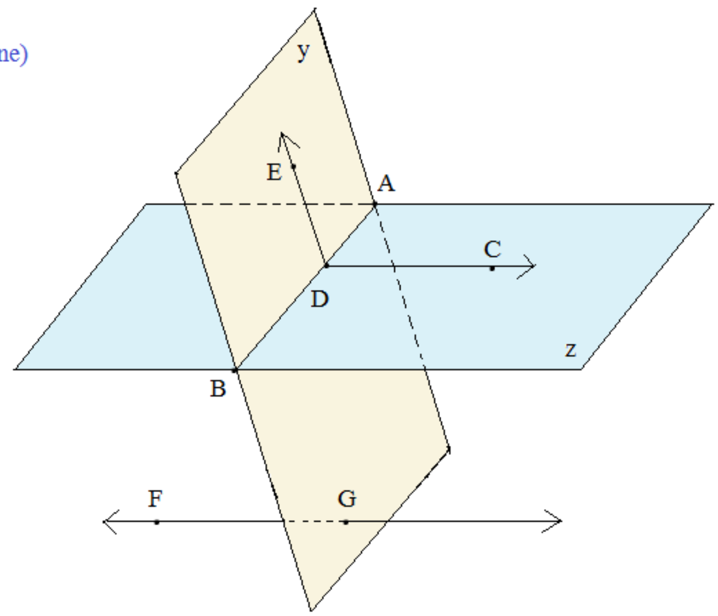



"Smiles, everyone, smiles!
Welcome, to today's lesson..."
"... where does the
line intersect?"
Mr. Roarke
l
Z
"z-plane!
z-plane!"
"Are you here for a
math fantasy, too?"
Fantasy
Island
Tattoo recognizes a geometry concept!
LanceAF #154 9-3-14
mathplane.com

SOLUTIONS-→

Plane Geometry

Solutions



1) Identify 3 collinear points (3 points that lie on the same line)

A-D-B

2) Select 3 non-collinear points and identify their plane

Examples: E-D-G (plane y)
B-D-C (plane z)
F-D-C (not labeled)

3) Answer the following:

a) $y \cap z$ $\overleftrightarrow{AB}$ (or $\overleftrightarrow{BD}$ or $\overleftrightarrow{AD}$)
(intersecting planes form a line)

b) $\overrightarrow{DE} \cup \overrightarrow{DC}$ $\angle$ EDC
(2 rays that share the same endpoint form an angle)

c) $\overrightarrow{DE} \cap \overrightarrow{DC}$ The point D
(The only common point of the rays)

4) Assume line FG is parallel to plane z ($\overleftrightarrow{FG} \parallel z$)

a) $\overleftrightarrow{FG} \cap y$ Point G (The line goes through point G)

b) $\overleftrightarrow{FG} \cap z$ $\emptyset$ (Since they are parallel, there is no intersection)

5) Three non-collinear points determine a plane. Can you name 3 other ways?

1) Two intersecting lines

2) Two parallel lines (Skew lines do not work)

3) A line and a point not on that line

A) True/False

SOLUTIONS

1) Two intersecting planes form a point. False; 2 intersecting planes form a line

2) If two lines don't intersect, then they are parallel. False. In 3-d space, skew lines do not intersect. But, they are not parallel

3) If two lines in a plane don't intersect, then the lines are parallel. True

4) In a plane, 2 lines that are perpendicular to a common line are parallel. True.

5) Two lines that are perpendicular to a common line must be parallel. False

6) A triangle is always a planar figure. True

7) A square is always a planar figure. False

8) Two lines must intersect or be parallel. False. Exception: skew lines

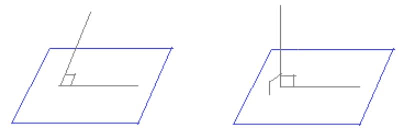
9) If a line is perpendicular to a plane, it is perpendicular to every line in that plane. False

It is perpendicular to lines that go through the foot..
If a line doesn't go through the foot, then they are skew.

10) If a line is perpendicular to a line in a plane, then it is perpendicular to the plane. False

perpendicular to line only ("tilted")

perpendicular to plane



B) Always/Sometimes/Never

1) Two planes form a line. Sometimes. If they are parallel, there is no intersection.
If they intersect, they form a line.

2) A given line that is perpendicular to a plane is perpendicului to the plane at one point. Always... The point is the "foot" of the perpendicular line

3) Three parallel lines lie in the same plane. Sometimes.

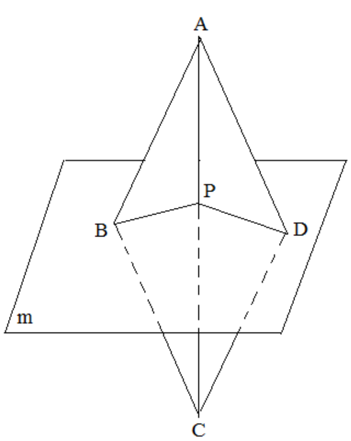
4) Two skew lines have one intersection. Never

(Skew lines have 0 intersections)

5) Two parallel lines determine a plane. Always

6) Two skew lines are coplanar. Never

C)



Given: $\angle$ APB $=$ $x + 72$ 90 degrees

$\angle$ APD $=$ $104 - \frac{7}{9}x$ 90 degrees

AC is a straight line that contains the point P

$\angle$ CPD $=$ $3x + 35$ 89 degrees

Are the angles congruent? Explain... No...

$x + 72 = 104 - \frac{7}{9}x$

$\frac{16}{9}x = 32$

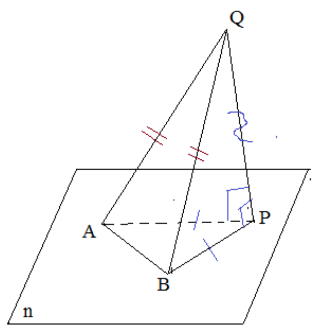
$x = 18$

If APB and APD are congruent, then they must be right angles..
If they are right angles, then AC is perpendicular to plane m...
And, then CPD must be a right angle, too...

1) Given: A and B are equidistant from P

$QP \perp n$

Prove: $\angle QAB \cong \angle QBA$



SOLUTIONS

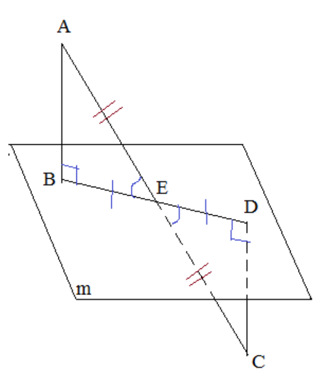
Statements	Reasons
1) A and B are equidistant from P	1) Given
2) $\overline{PB} \cong \overline{PA}$	2) Definition of equidistant
3) $\overline{QP} \perp n$	3) Given
4) $\overline{QP} \perp \overline{PB}$ $\overline{QP} \perp \overline{PA}$	4) If a line is perpendicular to a plane, then it is perpendicular to any line in the plane that passes through the point of intersection (foot)
5) QPA and QPB are right angles	5) Definition of perpendicular (Perpendicular lines form right angles)
6) $\angle QPA \cong \angle QPB$	6) All right angles are congruent
7) $\overline{QP} \cong \overline{QP}$	7) Reflexive property
8) $\triangle QPA = \triangle QPB$	8) SAS (Side-Angle-Side) 7, 6, 2
9) $\overline{QA} \cong \overline{QB}$	9) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
10) $\angle QAB \cong \angle QBA$	10) If congruent sides in a triangle, then opposite angles are congruent

2) Given: $\overline{AB} \perp m$

$\overline{CD} \perp m$

$\overline{AC}$ bisects $\overline{BD}$

Prove: $\overline{BD}$ bisects $\overline{AC}$



Statements	Reasons
1) $AB \perp m$ $CD \perp m$	1) Given
2) ABD and CDB are right angles	2) Definition of perpendicular (perpendicular lines form right angles at intersection)
3) $\angle ABD \cong \angle CDB$	3) All right angles are congruent
4) $\overline{AC}$ bisects $\overline{BD}$	4) Given
5) $\overline{BE} \cong \overline{DE}$	5) Definition of segment bisector (a bisector divides a segment into congruent halves)
6) $\angle AEB \cong \angle CED$	6) Vertical angles congruent
7) $\triangle AEB = \triangle CED$	7) ASA (Angle-Side-Angle) 3, 5, 6
8) $\overline{AE} \cong \overline{CE}$	8) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
9) $\overline{BD}$ bisects $\overline{AC}$	9) Definition of segment bisector (If congruent halves are divided by a segment, then the segment is a bisector)

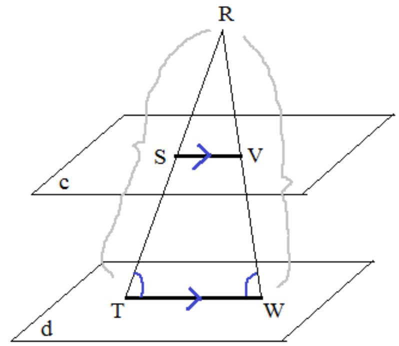
SOLUTIONS

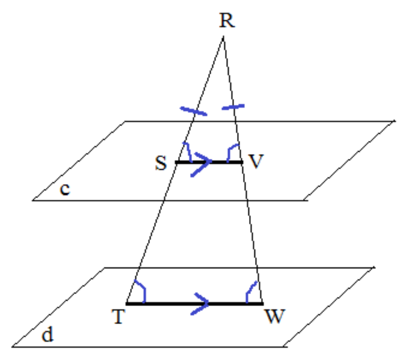
3) Given: c ∥ d

$\triangle$RTW is isosceles with base $\overline{TW}$

Prove: $\triangle$RSV is isosceles

Statements	Reasons
1) c ∥ d	1) Given
2) RTW is isosceles triangle	2) Given
3) $\overline{RT} \cong \overline{RW}$	3) Definition of isosceles
4) $\angle T \cong \angle W$	4) If congruent sides, then congruent angles
5) RTW form a plane	5) 3 points form a plane
6) $\overline{SV} \parallel \overline{TW}$	6) If a plane intersects 2 parallel planes, then the intersected lines are parallel
7) $\angle RSV \cong \angle T$ $\angle RVS \cong \angle W$	7) Corresponding angles
8) $\angle RSV \cong \angle RVS$	8) Substitution
9) $\overline{RS} \cong \overline{RV}$	9) If congruent angles, then congruent sides
10) $\triangle$ RSV is isosceles	10) Definition of Isosceles





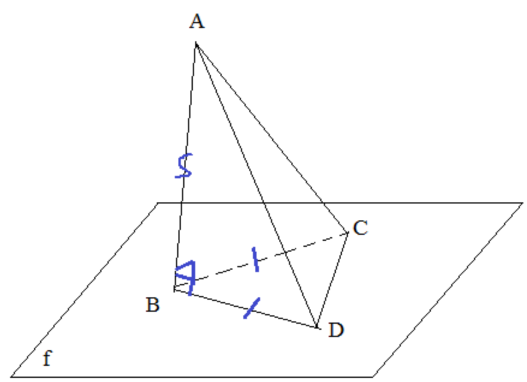
4) Given: $\overline{AB} \perp$ f

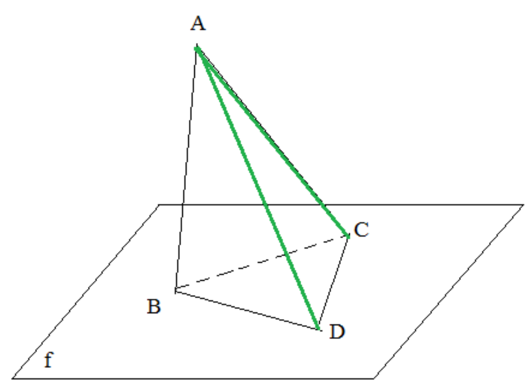
$\triangle$ BCD is an equilateral triangle

B, C, D are coplanar

Prove: ACD is isosceles

Statements	Reasons
1) AB $\perp$ f	1) Given
2) $\angle ABD$ and $\angle ABC$ are right angles	2) If a line is perpendicular to a plane, it is perpendicular to any line passing through its foot; and, it forms right angles
3) $\angle ABD \cong \angle ABC$	3) All right angles are congruent
4) $\triangle$ BCD is equilateral	4) Given
5) $\overline{BC} \cong \overline{BD}$	5) Definition of Equilateral
6) $\overline{AB} \cong \overline{AB}$	6) Reflexive property
7) $\triangle ABC \cong \triangle ABD$	7) Side-Angle-Side (SAS) (6, 3, 5)
8) $\overline{AD} \cong \overline{AC}$	8) Corresponding parts of congruent triangles are congruent (CPCTC)
9) ACD is isosceles triangle	9) Definition of Isosceles

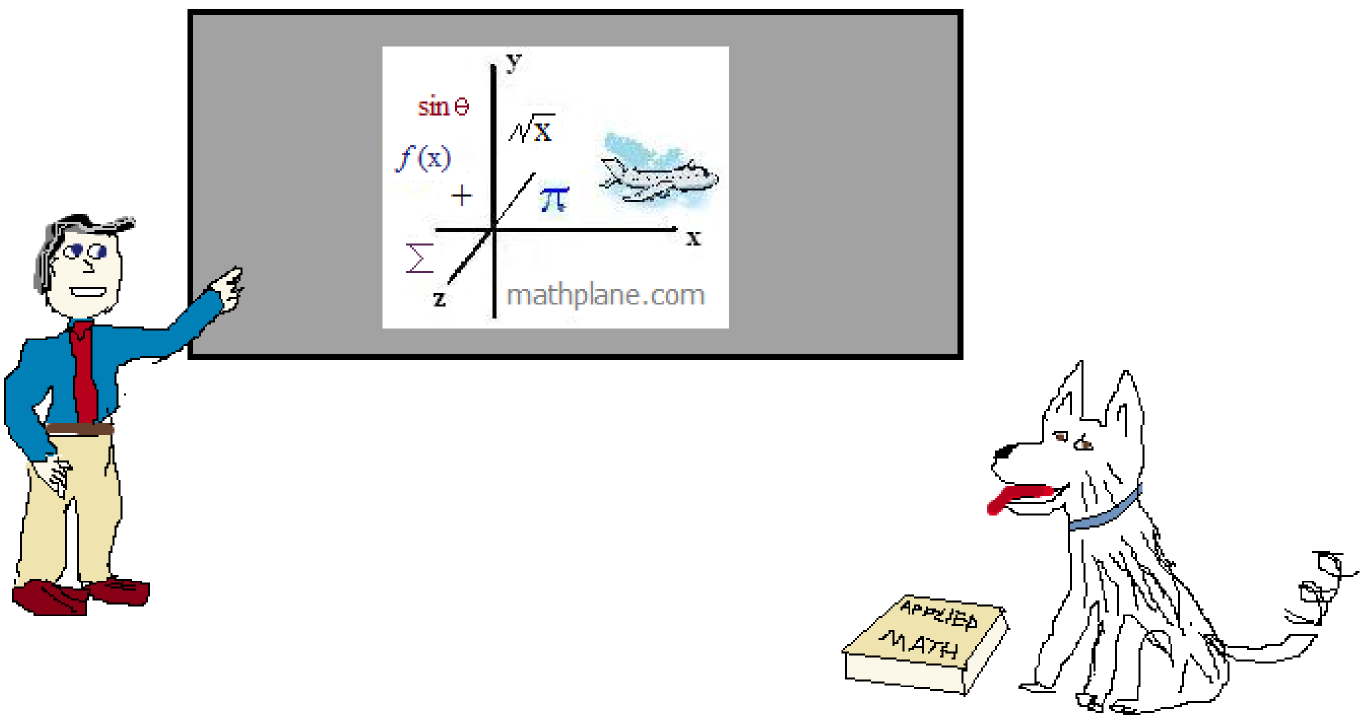




Thanks for visiting. Hope it helped!

If you have questions, suggestions, or requests, let us know.

Cheers



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