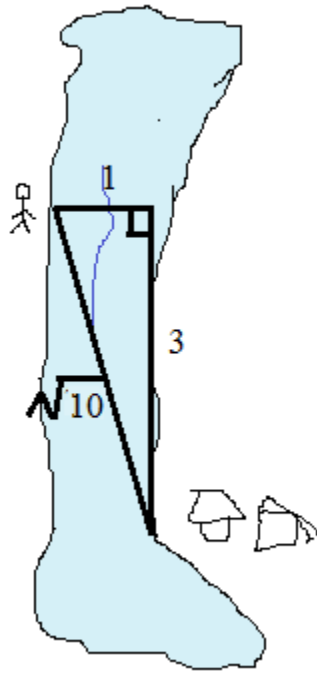


Pythagorean Theorem 2

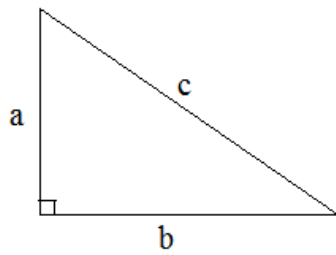
Practice Questions (and Answers)



Topics include Pythagorean Triples, Word Problems, radicals, distance/rate, geometry applications, perimeter, and more.

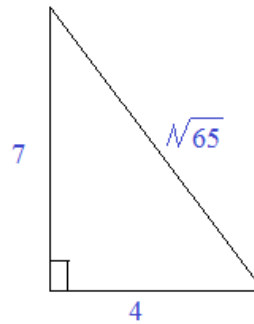
Pythagorean Theorem:

$a^2 + b^2 = c^2$ where a and b are lengths of the legs of a right triangle and c is the length of the hypotenuse



"sum of the squares of the legs is equal to the square of the hypotenuse"

Example:



$$\begin{aligned} (4)^2 + (7)^2 &= c^2 \\ 16 + 49 &= 65 \\ c &= \sqrt{65} \end{aligned}$$

Identifying triangles by their sides:

- $a^2 + b^2 = c^2$ right triangle
- $a^2 + b^2 > c^2$ acute triangle
- $a^2 + b^2 < c^2$ obtuse triangle

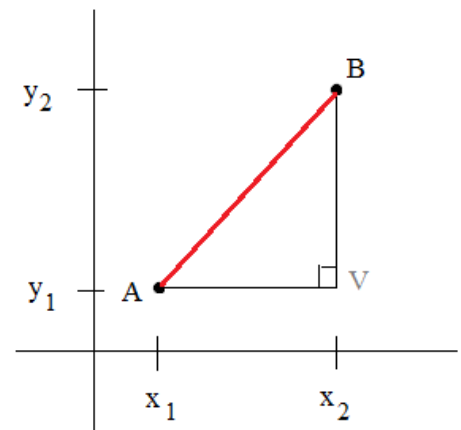
Distance Formula illustrates Pythagorean Theorem!

point A: (x_1, y_1)

point B: (x_2, y_2)

$$\text{distance } AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

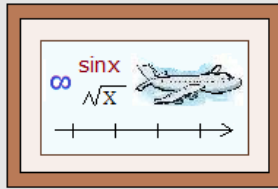
$$AB^2 = AV^2 + BV^2$$



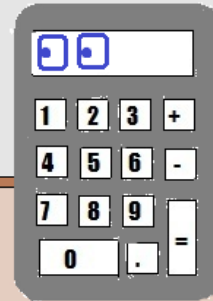
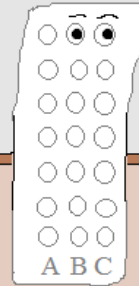
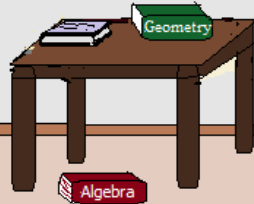
Waiting Room

Patients, thank you
for your patience!

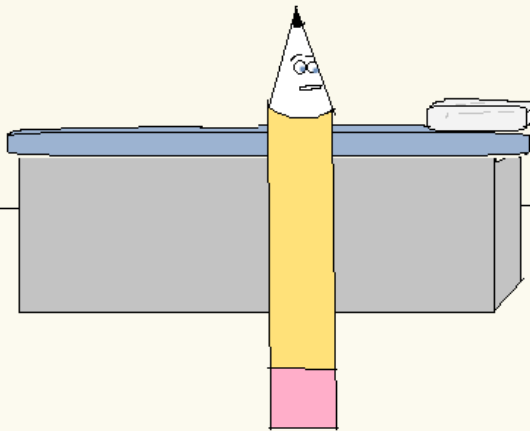
Registration



"I'm feeling drained.
Hopefully, the doc
can give me a boost."



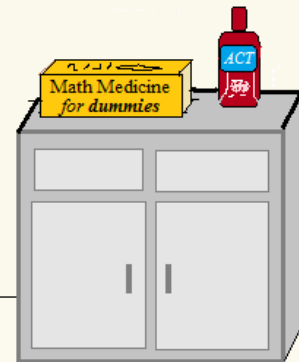
"I have anxiety...
Can't eat.. Can't sleep...
I feel tense and stiff..."



i Chart
" π
" $\ominus R$
" S A T
" E X A M
" H E L P M E

Doctorate
Degree

Masters
in Number
Theory



"I see yellow discoloration, and a bit
of a pink rash -- which is common.
But, let's get to the point, on top,
where it matters, you look sharp!"

Standardized
Test Prep

Practice Questions →

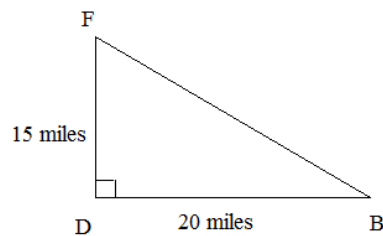
1) The following are sides of a triangle.
Determine whether the triangle is right, obtuse, acute, not possible.

- a) 2, 7, 10
- b) 4, 5, 8
- c) 10, 6, 8
- d) 7, 8, 9
- e) 11, 11, 11

2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

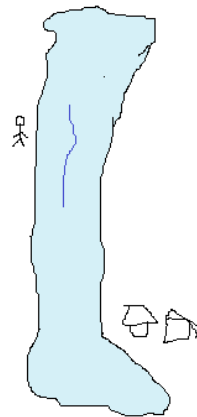
3) Multiple Choice: Jack traveled through D to get from F to B.
How much shorter is the direct route versus the route he took?

- a) 5
- b) 10
- c) 15
- d) 20
- e) 25

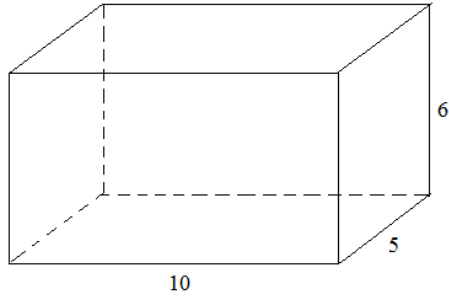


4) Pythagorean Theorem rate question:

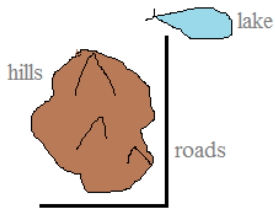
A boy stands on the shore of a one-mile wide lake.
He wants to reach camp down shore 3 miles on the opposite side.
He can swim 2mph and walk 4mph.
Is it quicker to swim across and then walk OR swim directly to the camp?



- 5) Find the length of the diagonal of the rectangular prism.

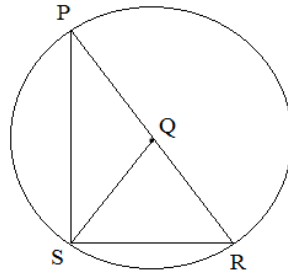


- 6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake. (15 miles due East. Then, 25 miles due North)...
 Meanwhile, a bird flying at 7 miles per hour can go directly over the hills.
 Who would reach the water first?



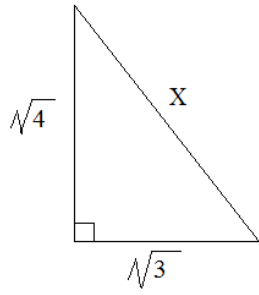
- 7) Given: Circle Q
 $\overline{PS} \perp \overline{SR}$
 $\overline{PS} = 36$
 $\overline{SR} = 15$

Find: The area of circle Q



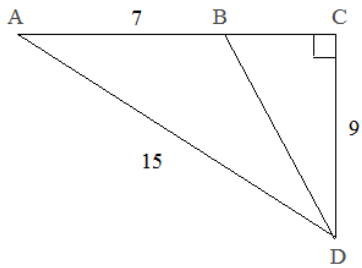
- 8) A 9 x 12 rectangle is inscribed in a circle.
 What is the circumference of the circle?

9) What is X?



10) Find the perimeter of a rectangle whose base is 10 and diagonal measure is 16.

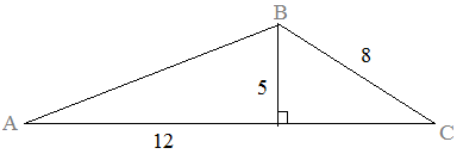
11) What is the perimeter of $\triangle BCD$?



12) \overline{TM} is an altitude of equilateral triangle TRI .

If $\overline{RI} = 7$, what is the measure of \overline{TM} ?

13) Find the area of the triangle:

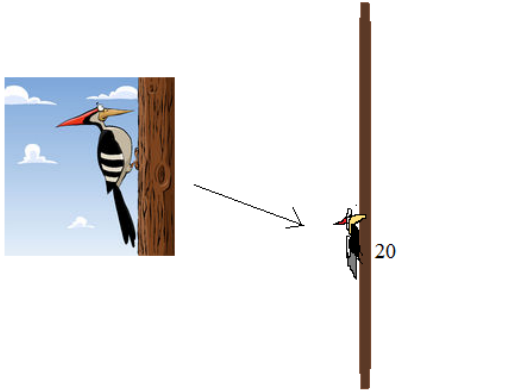


14) A boat is tied to a dock by 25 feet of rope.
 The dock is 15 feet above the water.
 If 8 feet of rope is pulled in, how far will the boat move toward the dock?

15) Sammy the snail and Ted the turtle have lunch together at the jungle cafe.
 At noon, Sammy leaves, heading due north at 15 feet per hour.
 Then, at 1:00pm, Ted leaves, heading due east at 8 yards per hour.
 How far apart are they at 6:00pm?



16) A woodpecker is perched up against a 20-foot pole, pecking away!
 Eventually, he chips away enough of the wood that the pole cracks, buckles, and folds over:
 the top of the pole landing on the ground 12 feet from the bottom of the pole.
 Undeterred, it stands on the top and continues pecking away!
 How high off the ground is the woodpecker?



Identify a related Pythagorean Triple. Then, find x.

a) $15 - 20 - x$ $3 - 4 - 5$ $x = 25$

b) $9 - x - 15$

c) $x - 30 - 34$

d) $24 - 32 - x$

e) $10 - x - 26$

f) $x - 60 - 65$

g) $40 - x - 85$

h) $18 - 80 - x$

i) $14 - x - 50$

j) $100 - 105 - x$

k) $x - 70 - 74$

l) $35 - x - 125$

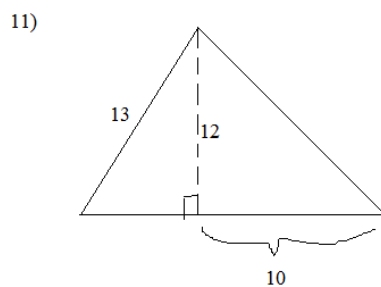
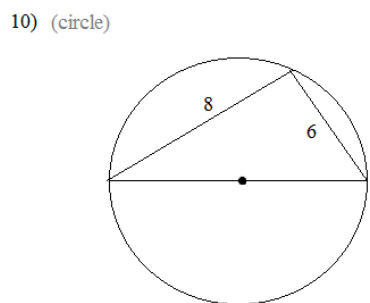
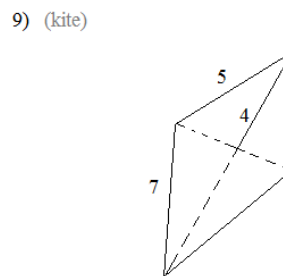
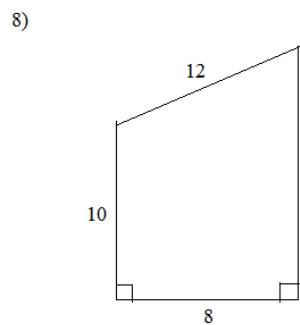
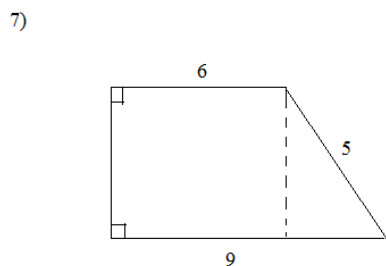
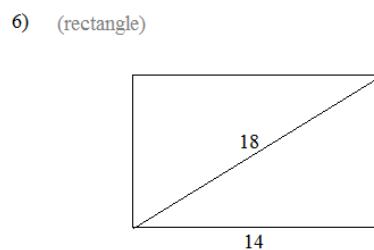
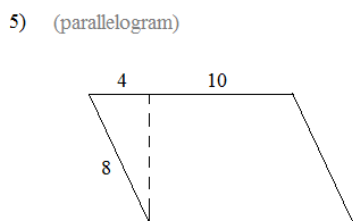
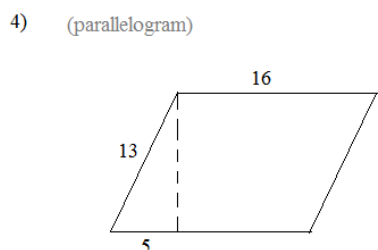
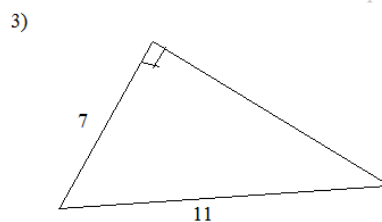
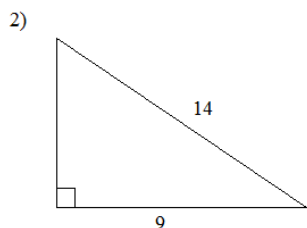
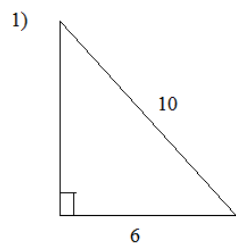
m) $2.5 - x - 6.5$

Pythagorean Triple (or, Triplet)

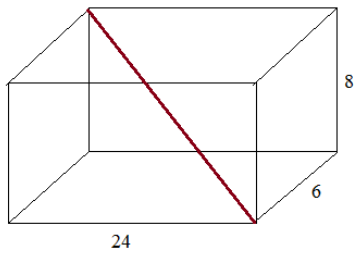
consists of 3 *positive integers* a, b, c that satisfy the Pythagorean Theorem

$$a^2 + b^2 = c^2$$

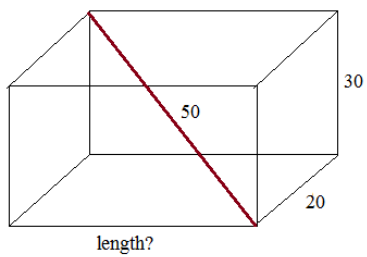
Find the area and perimeter of each figure.



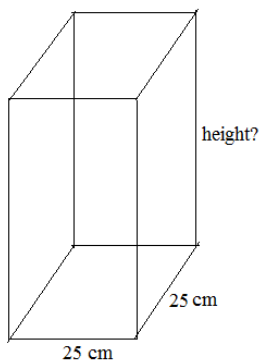
- 1) Find the diagonal of the rectangular prism
(figure not necessarily drawn to scale)



- 2) Find the length of the base of the rectangular prism.

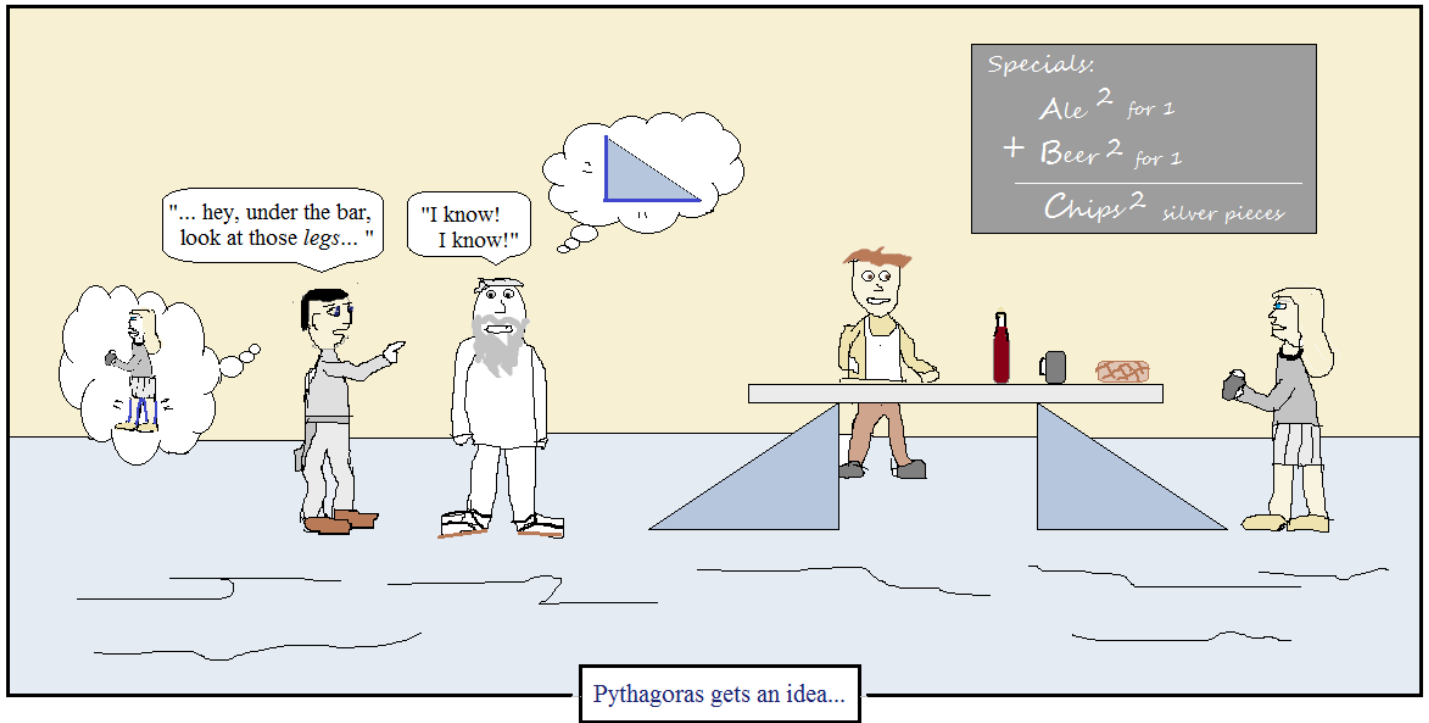
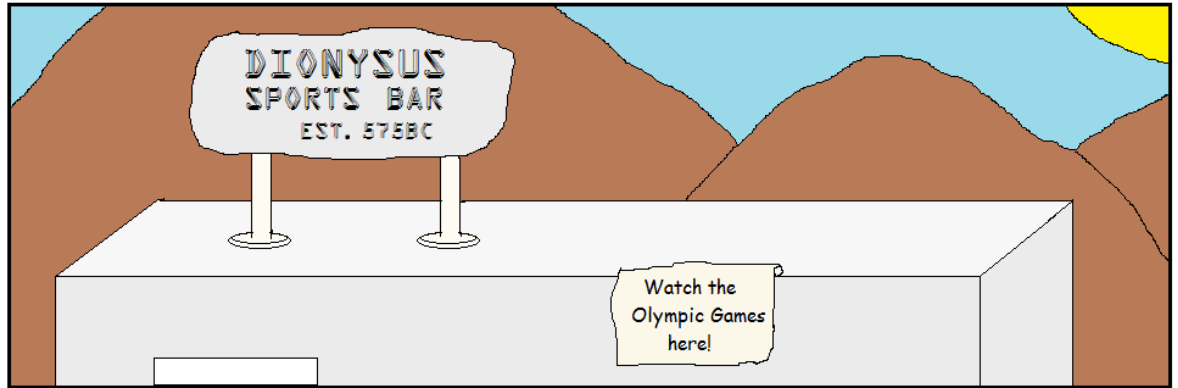


- 3) Paolo needs to send ski poles to Swen in Sweden.
The poles are 110 cm long, and the shipping box has a square base 25cm x 25cm.
What is the minimum height of the box required to ship the poles?



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Greece

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www.mathplane.com



ANSWERS-→

SOLUTIONS

1) The following are sides of a triangle.
Determine whether the triangle is right, obtuse, acute, not possible.

- a) 2, 7, 10 not possible $2 + 7 < 10$
- b) 4, 5, 8 obtuse $16 + 25 < 64$
- c) 10, 6, 8 right ('3-4-5' triangle) $36 + 64 = 100$
- d) 7, 8, 9 acute $49 + 64 > 81$
- e) 11, 11, 11 acute (equilateral triangle)
 $121 + 121 > 121$



- $a^2 + b^2 = c^2$ right
- $a^2 + b^2 > c^2$ acute
- $a^2 + b^2 < c^2$ obtuse
- $a + b < c$ not possible

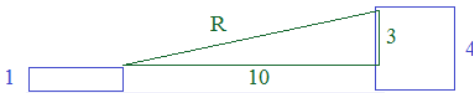
2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

Ramp (R)² = (height)² + (distance)²

$R^2 = 9 + 100$

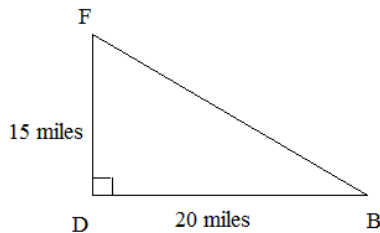
$R = \sqrt{109}$

Diagram:



3) Multiple Choice: Jack traveled through D to get from F to B.
How much shorter is the direct route versus the route he took?

- a) 5
- b) 10**
- c) 15
- d) 20
- e) 25



long route = 15 miles + 20 miles = 35 miles

short route: $15^2 + 20^2 = \overline{FB}^2$
 $\overline{FB} = 25$ miles

FB is 10 miles shorter than FDB

4) Pythagorean Theorem rate question:

A boy stands on the shore of a one-mile wide lake.
He wants to reach camp down shore 3 miles on the opposite side.
He can swim 2mph and walk 4mph.
Is it quicker to swim across and then walk OR swim directly to the camp?

distance = rate(time)

time = $\frac{\text{distance}}{\text{rate}}$

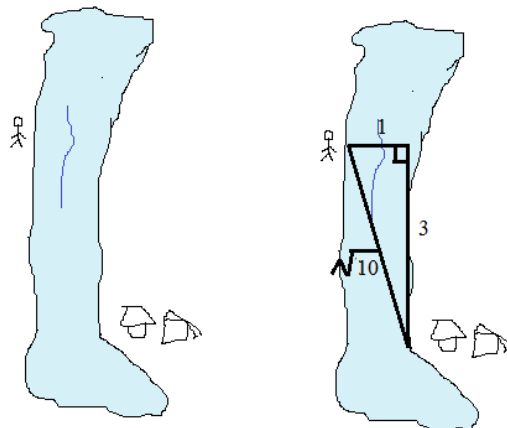
swim directly:

time = $\frac{\sqrt{10} \text{ miles}}{2 \text{ mph}} = 1.58$ hours

swim and walk:

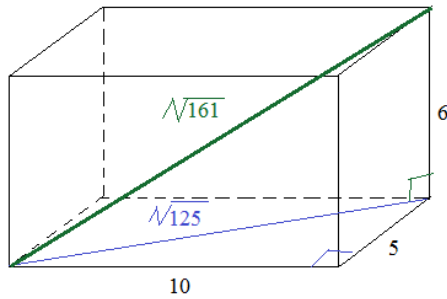
time (swim) = $\frac{1 \text{ mile}}{2 \text{ mph}} = .5$ hours

time (walk) = $\frac{3 \text{ miles}}{4 \text{ mph}} = .75$ hours



It's faster to swim across and then walk....

5) Find the length of the diagonal of the rectangular prism.



SOLUTIONS

one method: first, find diagonal of bottom:

$$d^2 = a^2 + b^2$$

$$d^2 = 10^2 + 5^2 = 125$$

$$d = 5\sqrt{5}$$

then, find the prism's diagonal:

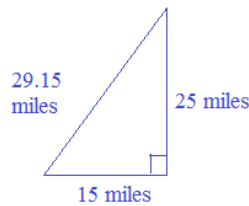
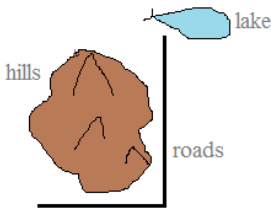
$$D^2 = d^2 + c^2$$

$$D^2 = 125 + 6^2$$

$$D = \sqrt{161}$$

shortcut: $\sqrt{10^2 + 5^2 + 6^2} = \sqrt{161}$

6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake. (15 miles due East. Then, 25 miles due North)...
 Meanwhile, a bird flying at 7 miles per hour can go directly over the hills.
 Who would reach the water first?



Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$15^2 + 25^2 = c^2$$

$$c = 5\sqrt{34} \text{ or approx. } 29.15 \text{ miles}$$

distance = rate x time

biker: $40 \text{ miles} = (10 \text{ m/hr})(\text{time})$
 $\text{time} = 4 \text{ hours}$

bird: $29.15 \text{ miles} = (7 \text{ m/hr})(\text{time})$
 $\text{time} = 4.16 \text{ hours (approx.)}$

The biker will reach the lake first!

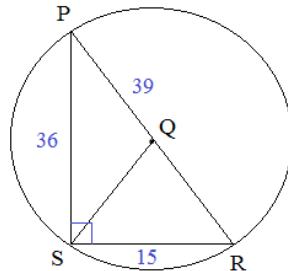
7) Given: Circle Q
 $\overline{PS} \perp \overline{SR}$
 $\overline{PS} = 36$
 $\overline{SR} = 15$

Find: The area of circle Q

$$\text{Area} = \pi (\text{radius})^2$$

$$\text{Area} = \pi (19.5)^2$$

$$= 380.25\pi \text{ square units}$$



$$15 - 36 - X \quad X = 39$$

(5 - 12 - 13 right triangle)

diameter: 39 radius: 19.5

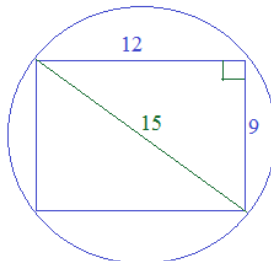
8) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?

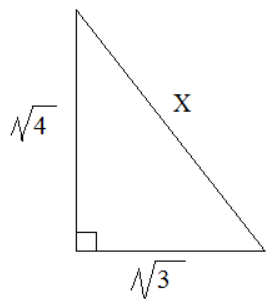
diameter of circle is 15

$$\text{circumference} = \pi (\text{diameter})$$

$$15\pi$$



9) What is X?



This is NOT a 3-4-5 Pythagorean Triple!

$$X \neq \sqrt{5}$$

SOLUTIONS

$$\sqrt{3}^2 + \sqrt{4}^2 = X^2$$

$$3 + 4 = X^2$$

$$X = \sqrt{7}$$

Since X is a side length, it cannot be negative...

10) Find the perimeter of a rectangle whose base is 10 and diagonal measure is 16.

Use Pythagorean Theorem...

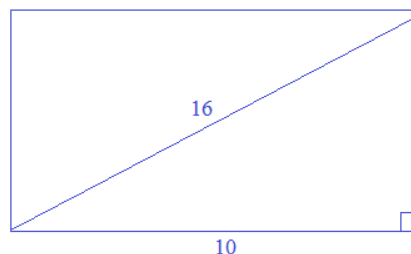
$$a^2 + 10^2 = 16^2$$

$$a^2 = 156$$

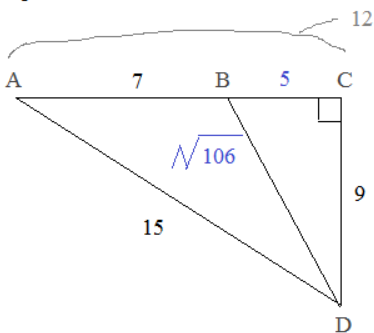
$$a = 2\sqrt{39}$$

So, the perimeter is

$$10 + 10 + 2\sqrt{39} + 2\sqrt{39} = 20 + 4\sqrt{39}$$



11) What is the perimeter of $\triangle BCD$?



9-12-15 right triangle

then, Pythagorean Theorem:

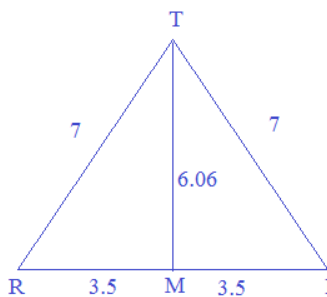
$$5^2 + 9^2 = \overline{BD}^2$$

$$\overline{BD} = \sqrt{106}$$

$$\text{Perimeter} = 14 + \sqrt{106}$$

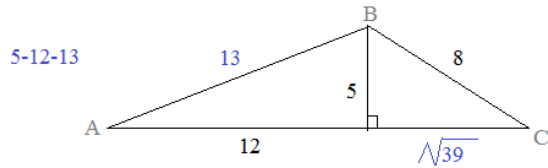
12) \overline{TM} is an altitude of equilateral triangle TRI.

If $\overline{RI} = 7$, what is the measure of \overline{TM} ?



$$\overline{TM} = 6.06$$

13) Find the area of the triangle:



$$a^2 + b^2 = c^2$$

$$25 + b^2 = 64$$

$$b = \sqrt{39}$$

SOLUTIONS

$$\text{Area} = \frac{1}{2} (\text{base})(\text{height})$$

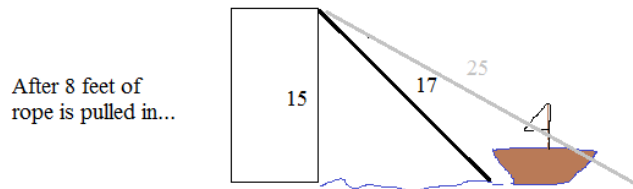
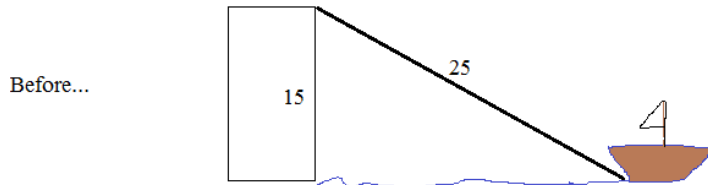
$$= \frac{1}{2} (12 + \sqrt{39})(5)$$

$$= 45.6 \text{ (approx)}$$

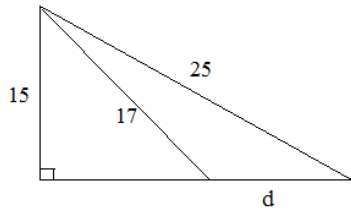
14) A boat is tied to a dock by 25 feet of rope. The dock is 15 feet above the water.

If 8 feet of rope is pulled in, how far will the boat move toward the dock?

Step 1: Sketch a picture

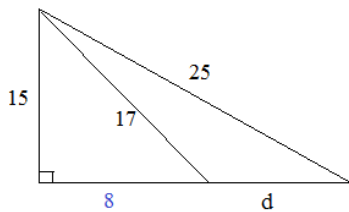


Step 2: Diagram with Right Triangles



We want to find d
(the distance the boat moved)

Step 3: Solve



8-15-17 Pythagorean Triplet

Then,

$$(8 + d)^2 + 15^2 = 25^2$$

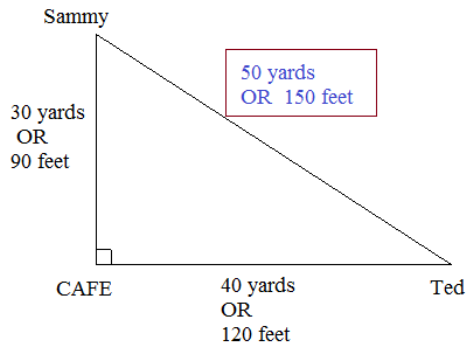
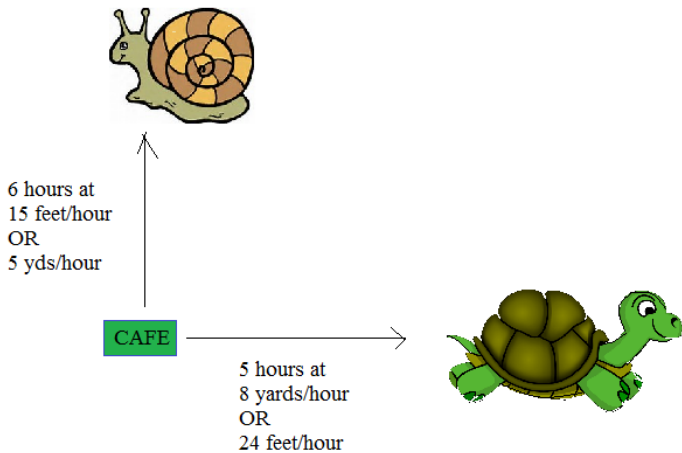
$$(8 + d)^2 = 400$$

$$d = 12$$

The boat moved 12 feet toward the dock

- 15) Sammy the snail and Ted the turtle have lunch together at the jungle cafe.
 At noon, Sammy leaves, heading due north at 15 feet per hour.
 Then, at 1:00pm, Ted leaves, heading due east at 8 yards per hour.
 How far apart are they at 6:00pm?

SOLUTIONS



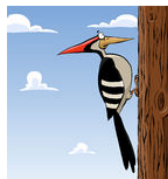
Pythagorean Theorem

$$30^2 + 40^2 = 50^2 \quad \text{Yards}$$

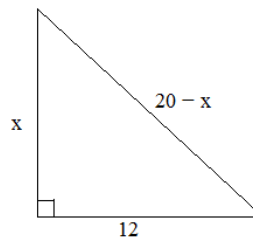
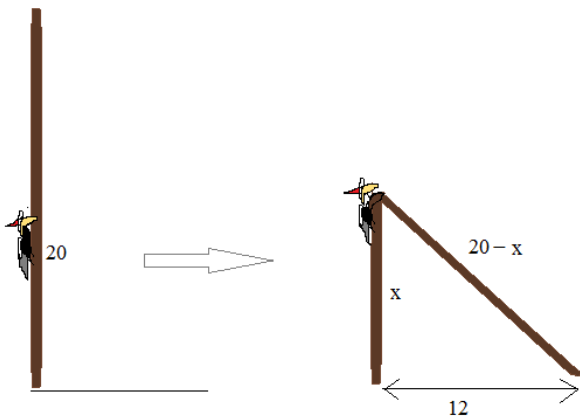
OR

$$90^2 + 120^2 = 150^2 \quad \text{Feet}$$

- 16) A woodpecker is perched up against a 20-foot pole, pecking away!
 Eventually, he chips away enough of the wood that the pole cracks, buckles, and folds over:
 the top of the pole landing on the ground 12 feet from the bottom of the pole.
 Undeterred, it stands on the top and continues pecking away!



How high off the ground is the woodpecker?



$$x^2 + 12^2 = (20 - x)^2$$

$$x^2 + 144 = 400 - 40x + x^2$$

$$40x = 256$$

$$x = 6.4 \text{ feet}$$

Identify a related Pythagorean Triple. Then, find x.

SOLUTIONS

a) $15^2 - 20^2 = x^2$ $3^2 - 4^2 = 5^2$ $x = 25$

b) $9^2 - x^2 = 15^2$ $3^2 - 4^2 = 5^2$ $x = 12$

c) $x^2 - 30^2 = 34^2$ $8^2 - 15^2 = 17^2$ $x = 16$

d) $24^2 - 32^2 = x^2$ $3^2 - 4^2 = 5^2$ $x = 40$

e) $10^2 - x^2 = 26^2$ $5^2 - 12^2 = 13^2$ $x = 24$

f) $x^2 - 60^2 = 65^2$ $5^2 - 12^2 = 13^2$ $x = 25$

g) $40^2 - x^2 = 85^2$ $8^2 - 15^2 = 17^2$ $x = 75$

h) $18^2 - 80^2 = x^2$ $9^2 - 40^2 = 41^2$ $x = 82$

i) $14^2 - x^2 = 50^2$ $7^2 - 24^2 = 25^2$ $x = 48$

j) $100^2 - 105^2 = x^2$ $20^2 - 21^2 = 29^2$ $x = 145$

k) $x^2 - 70^2 = 74^2$ $12^2 - 35^2 = 37^2$ $x = 24$

l) $35^2 - x^2 = 125^2$ $7^2 - 24^2 = 25^2$ $x = 120$

m) $2.5^2 - x^2 = 6.5^2$ $5^2 - 12^2 = 13^2$ $x = 6$

A few Pythagorean Triples:

3, 4, 5

5, 12, 13

8, 15, 17

7, 24, 25

9, 40, 41

12, 35, 37

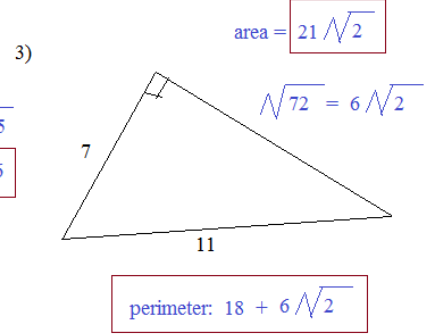
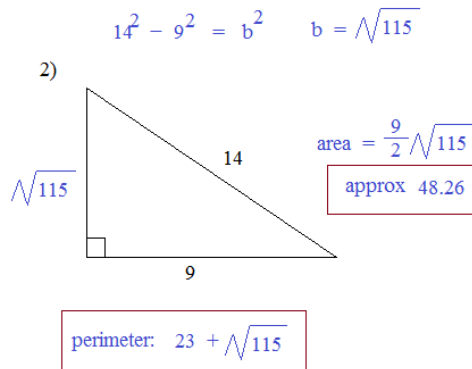
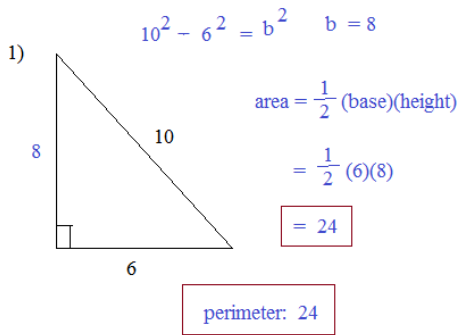
20, 21, 29

Find the area and perimeter of each figure.

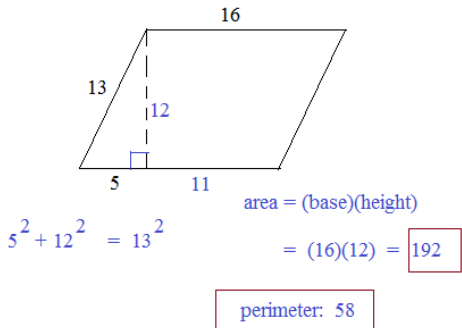
mathplane.com

SOLUTIONS

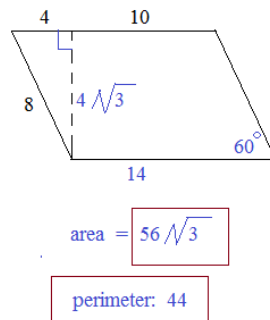
Pythagorean Theorem



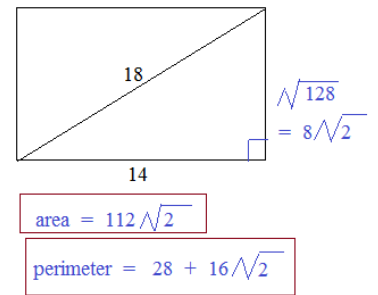
4) (parallelogram)



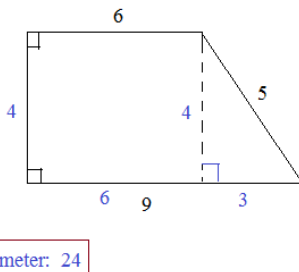
5) (parallelogram)



6) (rectangle)

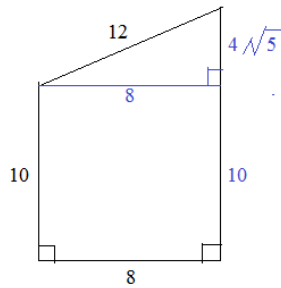


7)

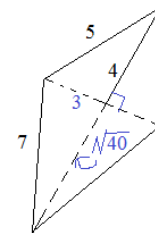


area = $\frac{1}{2}$ (base1 + base2)(height)
 $= \frac{1}{2} (9 + 6)(4) = 30$

8)

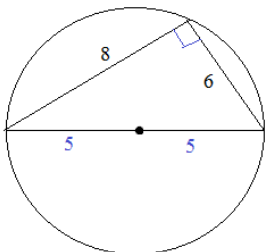


9) (kite)

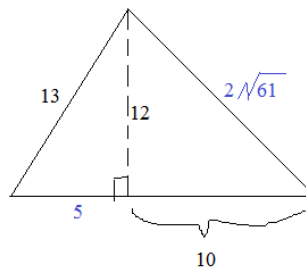


area = $\frac{1}{2}$ (diagonal 1)(diagonal 2)
 $= \frac{1}{2} (6)(4 + 2\sqrt{10}) \approx 40.97$

10) (circle)



11)



Pythagorean Theorem: $a^2 + b^2 = c^2$

rectangle: area = (length)(width)
 perimeter = 2(length) + 2(width)

triangle: area = $(1/2)$ (base)(height)
 perimeter = (side) + (side) + (side)

circle: area = π (radius)²
 circumference = 2π (radius)

kite: area = $(1/2)$ (diagonal 1)(diagonal 2)
 (or, find area of each triangle)

trapezoid: area = $(1/2)$ (base1 + base2)(height)

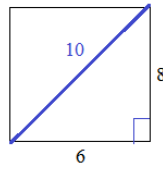
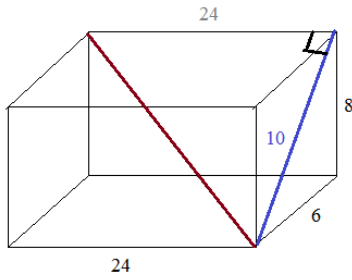
- 1) Find the diagonal of the rectangular prism
(figure not necessarily drawn to scale)

SOLUTIONS

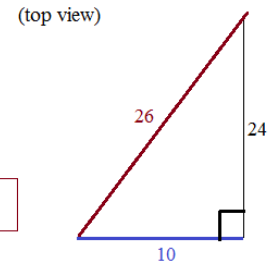
Pythagorean Theorem in 3-D Space

For ease, we'll apply Pythagorean Theorem to the side first (because it's a Triple)

Then, we'll apply Pythagorean Theorem to other part of prism to get the diagonal...

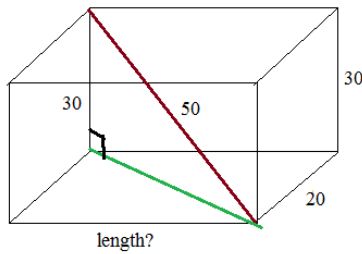


The diagonal is 26.

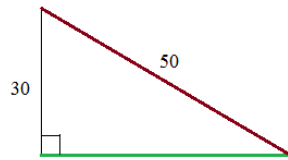


The more difficult route: apply Pythagorean Theorem to bottom first...

- 2) Find the length of the base of the rectangular prism.

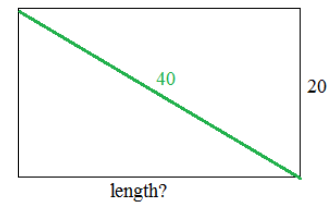


(side view)



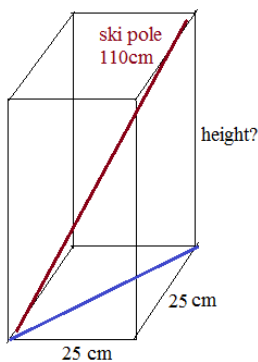
Using Pythagorean Theorem,
 $d^2 + 30^2 = 50^2$
we can see the base diagonal is 40 cm...

(top view)

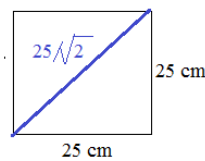


Then, using Pythagorean Theorem,
 $20^2 + (\text{length})^2 = 40^2$
we can see the length is $20\sqrt{3}$

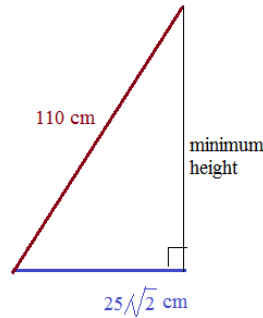
- 3) Paolo needs to send ski poles to Swen in Sweden.
The poles are 110 cm long, and the shipping box has a square base 25cm x 25cm.
What is the minimum height of the box required to ship the poles?



(top view)



(side view)



$$(\text{base diagonal})^2 + (\text{height})^2 > (110 \text{ cm})^2$$

$$1250 + (\text{height})^2 > 12,100$$

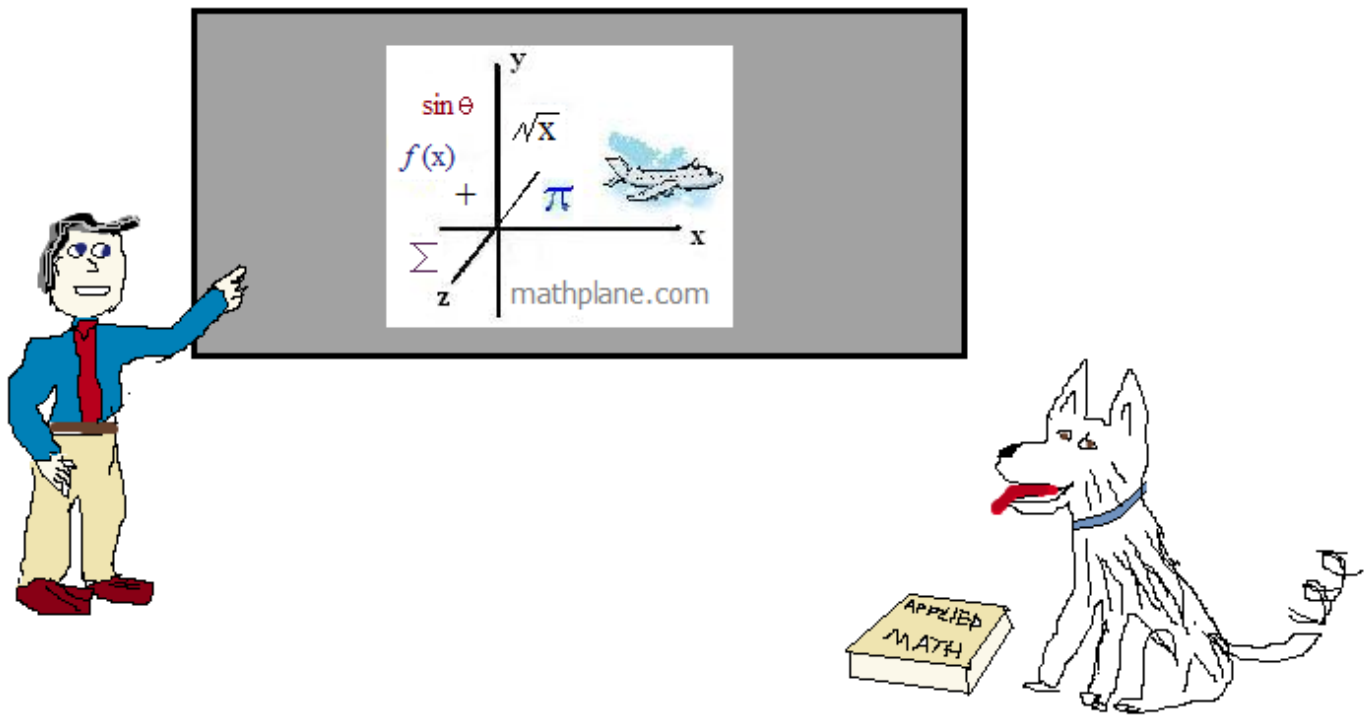
$$\text{height} > 104.16 \text{ cm}$$

The height must be greater than 104.16, in order to fit the 110 cm ski poles..

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, Google+, Pinterest, TES, and TeachersPayTeachers

And, *Mathplane Express* for mobile at mathplane.ORG

One more question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10. What is the measure of each leg and base?

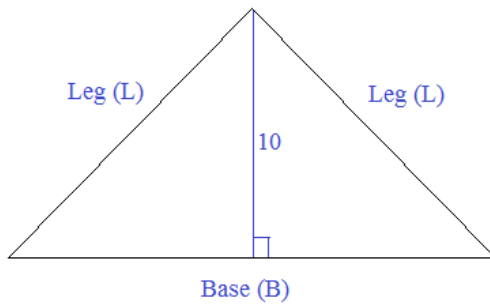
ANSWER-→

One More Question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10.

What is the measure of each leg and base?

Step 1: Draw a picture and label parts

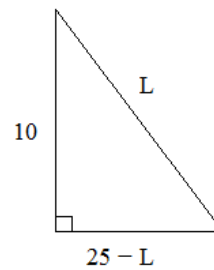


Step 2: Solve (applying Pythagorean Theorem)

$$\text{base} + \text{leg} + \text{leg} = 50$$

$$\text{Therefore, base} = 50 - 2L$$

$$\text{and half the base is } \frac{1}{2}(50 - 2L) = 25 - L$$



$$a^2 + b^2 = c^2$$

$$10^2 + (25 - L)^2 = L^2$$

$$100 + 625 - 50L + L^2 = L^2$$

$$725 = 50L$$

$$L = 14.5$$

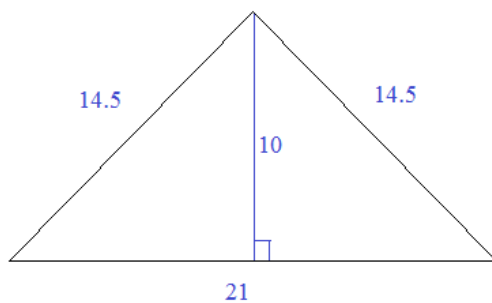
Each leg is 14.5

$$B = 50 - 2L$$

$$= 50 - 29 = 21$$

The base is 21

Step 3: Check answers



$$\text{Perimeter of triangle} = 14.5 + 14.5 + 21 = 50 \quad \checkmark$$

Since legs are congruent, it's an isosceles triangle. \checkmark

$$(10.5)^2 + (10)^2 = (14.5)^2$$

$$110.25 + 100 = 210.25 \quad \checkmark$$

Pythagorean Theorem confirms right and left triangles