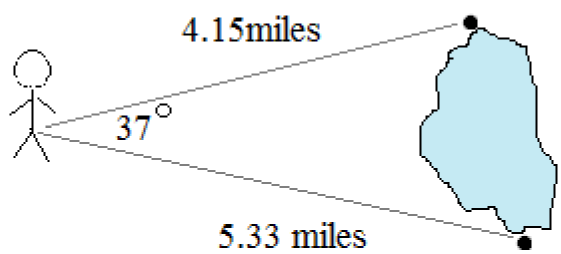
Trigonometry: Law of Sines, Law of Cosines, and Area of Triangles

Formulas, notes, examples, and practice test (with solutions)



Topics include finding angles and sides, the "ambiguous case" of law of Sines, vectors, navigation, and more.

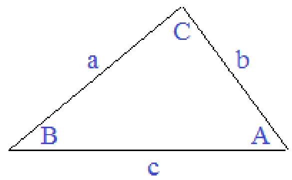
Law of Sines

What is it? Equations that relate the interior angles of a triangle to their corresponding (opposite) sides.

In a triangle, "the ratio of a side to the sine of its opposite angle is the same for all 3 angle/sides"

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

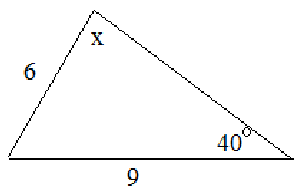


Note: the ratios can be expressed as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Examples:

1) Given the following triangle, find the measure of angle x.



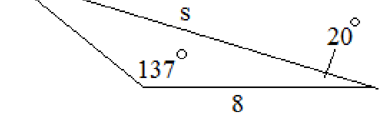
(Acute triangle)

$$\frac{\sin 40}{6} = \frac{\sin x}{9}$$

$$\sin x = \frac{9(\sin 40)}{6} = .964$$

$$x = \arcsin(.964) = 73.74^\circ$$

2) Given the following triangle, find the length of s.



(Obtuse triangle)

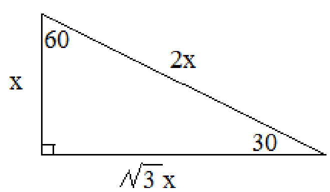
First, we must identify the measure of the angle opposite 8...
180 = 137 + 20 + ?
The angle opposite is 23 degrees...

then, we can use law of sines:

$$\frac{\sin 23^\circ}{8} = \frac{\sin 137^\circ}{s}$$

$$s = \frac{8(\sin 137)}{\sin 23} = 13.96$$

3) Verify the law of sines for a 30-60-90 triangle.



(Right triangle)

$$\frac{\sin 90^\circ}{2x} = \frac{\sin 30^\circ}{x} = \frac{\sin 60^\circ}{\sqrt{3}\,x}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{1}{2x} = \frac{1/2}{x} = \frac{\sqrt{3}/2}{\sqrt{3}\,x}$$

$$\frac{1}{2x}$$

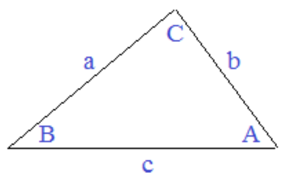
Law of Cosines

What is it? Equations that relate the interior angles of a triangle to their corresponding (opposite) sides.

In a triangle, "two known sides and the <u>included</u> angle can determine the length of the 3rd side"

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$



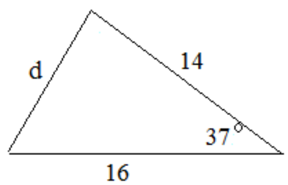
Note the pattern of the formulas:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

side squared; other sides squared; minus 2 times other sides; cosine angle

Examples:

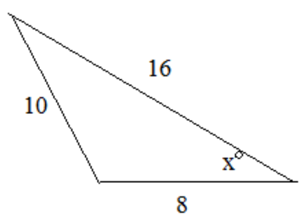
1) Given the following triangle, find the length of d.



Since we know 2 sides and the *included angle*, we can use law of cosines:

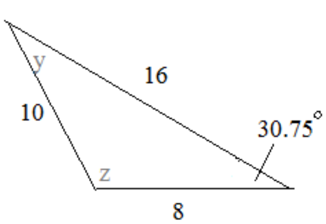
$$d^2 = 14^2 + 16^2 - 2(14)(16)(\cos 37)$$
$$d^2 = 196 + 256 + 448(.7986)$$
$$d^2 = 94.2 \qquad d = 9.7$$

2) Given the following triangle, find the measure of angle x.



$$10^2 = 8^2 + 16^2 - 2(8)(16)(\cos x)$$
$$100 = 64 + 256 - 256(\cos x)$$
$$-220 = -256(\cos x)$$
$$\cos x = .859$$
$$x = 30.75^\circ$$

Let's check the answer by finding the other 2 angles:



Law of Cosines: $8^2 = 10^2 + 16^2 - 2(10)(16)(\cos y)$

$$64 = 100 + 256 - 320(\cos y)$$
$$-292 = -320(\cos y)$$
$$y = 24.15^\circ$$

$$16^2 = 8^2 + 10^2 - 2(8)(10)(\cos z)$$
$$256 = 64 + 100$$
$$92 = -160(\cos z)$$
$$z = 125.1^\circ$$

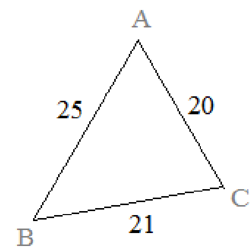
$x + y + z =$

$30.75 + 24.15 + 125.1 = 180$ ✓

(and, smallest angle is opposite the smallest side; largest side is opposite the largest angle)

Three approaches to finding angles of a triangle: An Illustration

Example: Find the 3 angle measurements in triangle ABC



Step 1: Find 1st angle using law of cosines

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$21^2 = 20^2 + 25^2 - 2(20)(25)(\cos A)$$

$$441 = 400 + 625 - 1000(\cos A)$$

$$-584 = -1000(\cos A)$$

$$A = 54.27^\circ$$

Step 2: Find 2nd angle using law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin(54.27)}{21} = \frac{\sin B}{20}$$

$$\sin B = \frac{20(\sin 54.27)}{21} = .773$$

$$B = 50.64^\circ$$

Step 3: Find 3rd angle using geometry theorem:
(sum of interior angles of $\triangle$ is 180 degrees)

$$\angle A + \angle B + \angle C = 180^\circ$$

$$54.27^\circ + 50.64^\circ + \angle C = 180^\circ$$

$$\angle C = 75.09^\circ$$

To check the answer:

a) observe the side lengths/angles

$c > a > b$ and $C > A > B$

$25 > 21 > 20$ and $75.09 > 54.27 > 50.64$

note: angles A and B have close measures, and sides a and b have close measures..

b) use law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(54.27)}{21} = \frac{\sin(50.64)}{20} = \frac{\sin(75.09)}{25} = .03865$$

All have the same ratio

c) use law of cosines to check b or c

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

$$25^2 = 21^2 + 20^2 - 2(21)(20)(\cos 75.09)$$

$$625 = 441 + 400 - 216.13$$

$$625 \qquad 624.87$$

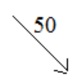
Vectors & Law of Sines/Cosines: Applications

Example: An airplane flies due East at an air speed of 500 miles per hour.
A crosswind flows (from the Northwest) toward the Southeast at a rate of 50 miles per hour.

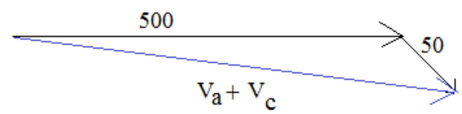
What is the *ground speed* and direction of the airplane?

Airplane can be expressed as a vector: V_a

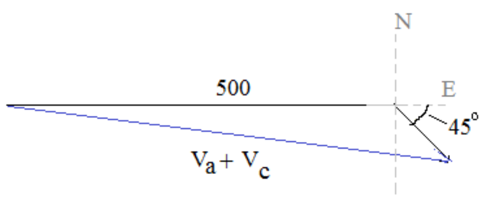
Crosswind can be expressed as a vector: V_c

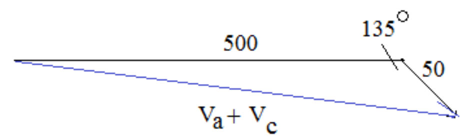


The groundspeed is the sum of the vectors...



We can transform the vectors into a triangle:





Use Law of Cosines to find ground speed of airplane:

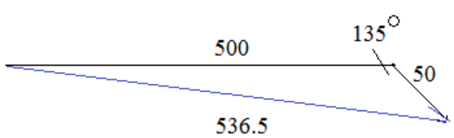
$c^2 = a^2 + b^2 - 2ab(\cos C)$

$= (500)^2 + (50)^2 - 2(500)(50)(\cos 135)$

$= 250000 + 2500 - 50000(.707)$

$= 287{,}855$

$c \cong 536.5$ miles



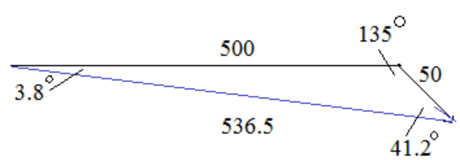
Then, use the Law of Sines to find the direction:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin(135)}{536.5} = \frac{\sin(B)}{50}$$

$$\sin(B) = \frac{50\sin(135)}{536.5}$$

$$B = 3.8^\circ$$



Using Vectors: $V_a = 500i + 0j$

$$V_c = \frac{50}{\sqrt{2}}i - \frac{50}{\sqrt{2}}j = 25\sqrt{2}\,i - 25\sqrt{2}\,j$$

$$V_a + V_c = 535.35i - 35.35j$$

$$\text{groundspeed} = \|V_a + V_c\| = \sqrt{535.35^2 + (-35.35)^2}$$

$$= 536.5$$

$$\text{direction} = \arctan[(-35.35)/535.35] = -3.8^\circ$$

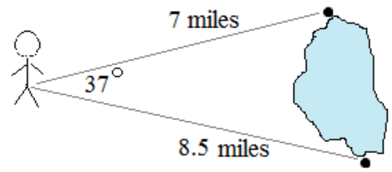
The plane is going N93.8E
or
$\boxed{\text{S86.2E}}$
or
3.8 degrees south of due east

Law of Sines and Cosines Applications: Word Problems

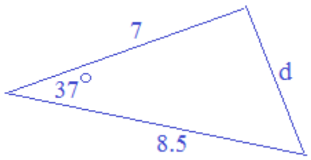
Law of Cosines:

$c^2 = a^2 + b^2 - 2ab(\cos C)$

Example: To find the distance across a lake, a surveyor took the following measurements: What is the distance across the lake?



Looking at the 'triangle', we have Side-Angle-Side...
So, we can use law of cosines to find the other side!



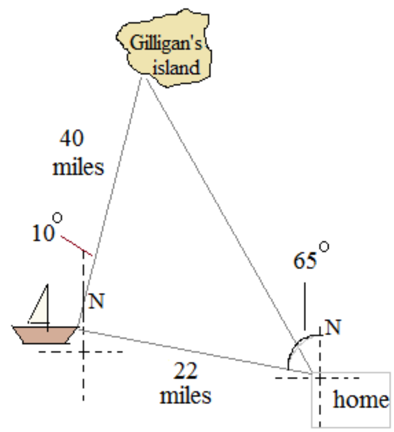
$$d^2 = (7)^2 + (8.5)^2 - 2(7)(8.5)(\cos 37^\circ)$$

$$= 121.25 - 119(.799) = 26.21$$

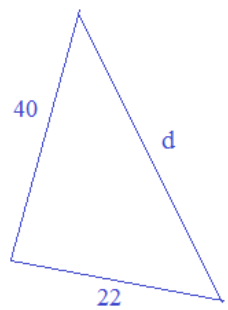
the distance (d) across the lake is approximately $\boxed{5.12 \text{ miles}}$

Note: the answer is 'reasonable', because 37 degrees is likely the smallest angle; therefore, the opposite side should be less than 7 and 8.5... To check, use law of sines to find the other angles...

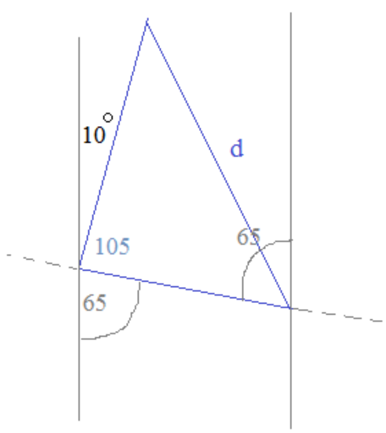
Example: A sailor at sea looks at coordinates on the following map: How far is Gilligan's island from his home?



At first, we have a triangle with 2 sides...



Using geometry properties/theorems, we can find a helpful angle!



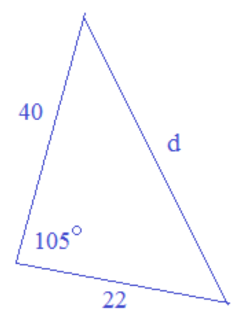
1) "If parallel lines are cut by a transversal, then alternate interior angles are congruent"

65 ----> 65

2) "The sum of adjacent angles on a straight line is 180 degrees"

10 + 65 ----> other angle is 105

3) If you know 2 sides of a triangle and the *included* angle, then you can use law of cosines to determine the 3rd side



$$d^2 = (40)^2 + (22)^2 - 2(40)(22)(\cos 105)$$

$$d^2 = 1600 + 484 - 1760(-.259) = 2539.5$$

therefore, d = $\boxed{50.4 \text{ miles}}$

Example: A plane is approaching an airport runway that is 9000 feet long.
The angle of declination to each end of the runway is 17.7 and 19.1 degrees.

a) What is the air distance to the airport runway?

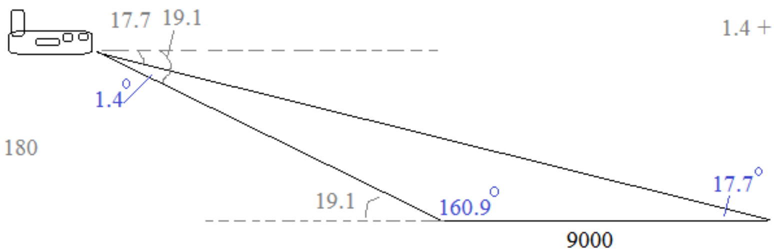
b) What is the altitude of the plane?

c) What is the ground distance to the runway?

$1.4 + 160.9 + x = 180$
$x = 17.7$

Alternate interior angles: 19.1

Supplementary angles: $19.1 + 160.9 = 180$

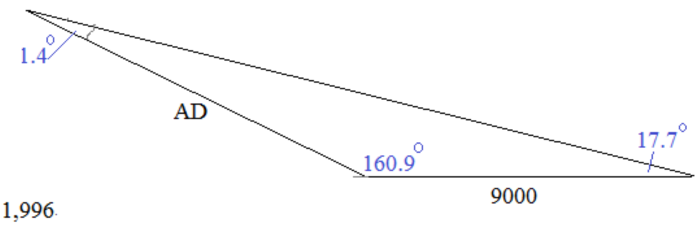


Air distance:

Use Law of Sines

$$\frac{AD}{\text{Sin}(17.7)} = \frac{9000}{\text{Sin}(1.4)}$$

$$AD = \frac{9000\text{Sin}(17.7)}{\text{Sin}(1.4)} \cong 111{,}996$$

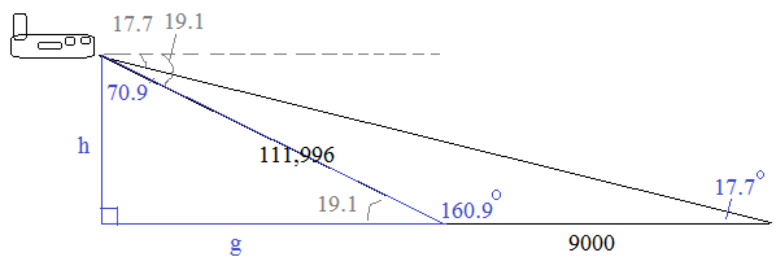


Altitude:

Use trig function Sine:

$$\text{Sin}(19.1) = \frac{h}{111{,}996}$$

$$h \cong 36{,}647$$



Ground distance:

Use trig function Cosine:

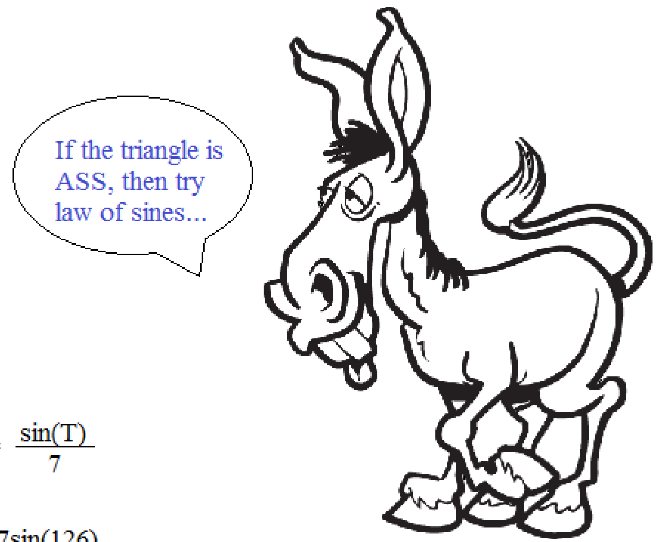
$$\text{Cos}(19.1) = \frac{g}{111{,}996}$$

$$g \cong 105{,}830$$

(also, pythagorean theorem:
$h^2 + g^2 = 111{,}996$)

The "ambiguous case": Law of Sines (SSA)

When given two sides and a *non-included angle*, there may be 3 possible outcomes.



Outcome 1: Zero Solutions

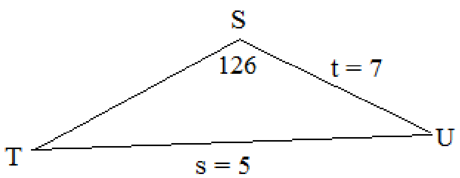
Example: Given triangle STU; $s = 5$
$t = 7$
$\angle S = 126^\circ$

Since 5 is opposite the obtuse angle (and is NOT the largest side), this triangle cannot exist...

$$\frac{\sin(126)}{5} = \frac{\sin(T)}{7}$$

$$\sin(T) = \frac{7\sin(126)}{5} = 1.13$$

$$\sin \not> 1$$



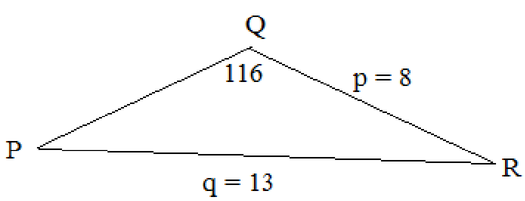
Outcome 2: One Solution

Example: Given triangle PQR; $p = 8$
$q = 13$
$Q = 116^\circ$

$$\frac{\sin(116)}{13} = \frac{\sin(P)}{8}$$

$$\sin(P) = \frac{8\sin(116)}{13} = .553$$

$P = 33.58$



Since $P = 33.58$ and $Q = 116$,

$R = 30.42$

(sum of angles is 180 degrees)

and, side $r = 7.3$

Since given angle is obtuse, the other angles must be acute... (one solution)

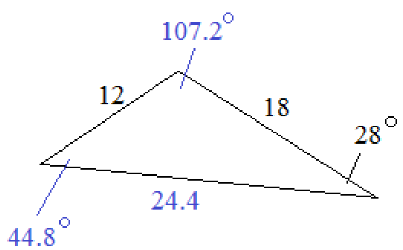
Outcome 3: Two Solutions

Example: Given triangle ABC; $a = 12$
$b = 18$
$\angle A = 28°$

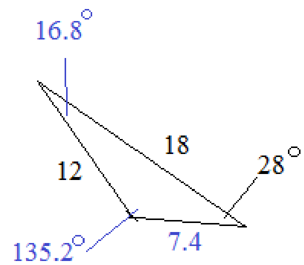
$$\frac{\sin(28)}{12} = \frac{\sin(B)}{18} \qquad \sin(B) = \frac{18\sin(28)}{12} = .704$$

$B = 44.8$ (approximately)

Since $A = 28$ and $B = 44.8$, angle C is 107.2 degrees



Case 1: The missing side is the largest..



Case 2: One of the given sides is the largest..

Remember, $\sin^{-1}(.704)$ has another answer in quadrant II (where sine is also positive!)

$$\sin^{-1}(.704) = 135.2° \qquad \sin(135.2) = .704$$

Assuming the missing angle B is 135.2,

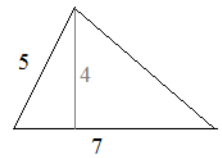
and angle A is 28,

angle C is 16.8 degrees!

Finding area of a triangle (without the height)

The area of a triangle is $\frac{1}{2}$(base)(height)

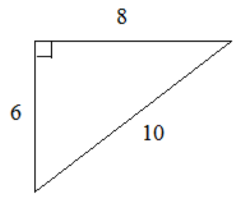
When the height (altitude) is given, simply substitute the values:

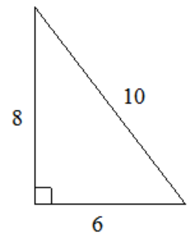


base = 7
height = 4

Area = $\boxed{11/2}$

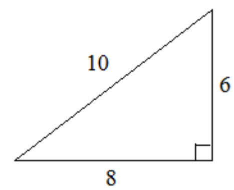
If it's a right triangle, use one of the legs as the base:





base = 6
height = 8

Area = 48/2 = $\boxed{24}$



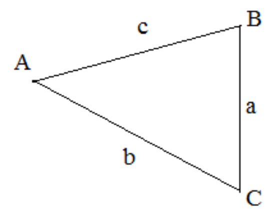
base = 8
height = 6

Area = $\boxed{24}$

What if it's not a right triangle and the height is not given?

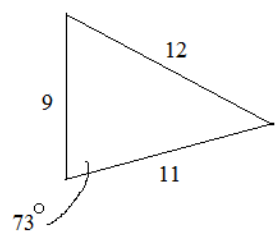
Area of triangle = $\frac{1}{2}$ ab(SinC)

where a and b are sides and C is the <u>included</u> angle



lower case letter is the side <u>opposite</u> the upper case angle

Example: Find the area:

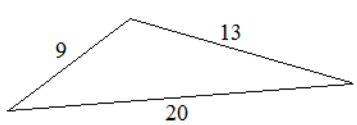


Since just 1 angle is given, the easiest choice is the 2 adjacent sides with that angle:

$$\text{Area} = \frac{1}{2}(9)(11)\sin(73^{\circ}) = \frac{99}{2}(.956) = \boxed{47.34}$$

Example: What is the area of a triangle with sides 9, 13, and 20?

Step 1: Sketch the triangle



**suggestion: find an acute angle rather than an obtuse angle

Step 2: Find an angle

Use law of cosines: $c^2 = a^2 + b^2 - 2ab(\cos C)$

a = 13
b = 20
c = 9

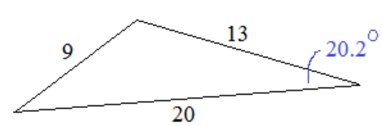
$$9^2 = 13^2 + 20^2 - 2(13)(20)(\cos C)$$

$$81 = 169 + 400 - 520(\cos C)$$

$$-488 = -520\cos C$$

$$.938 = \cos C$$

$$C = 20.2^{\circ}$$



Step 3: Find the area

$$\text{Area} = \frac{1}{2} ab(\sin C)$$

$$\text{Area} = \frac{1}{2}(13)(20)(\sin 20.2)$$

$$\text{Area} = 130(.345) = \boxed{44.9}$$

Finding area of a triangle (without the height)

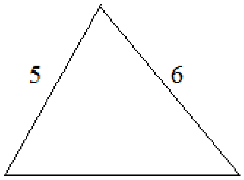
Heron's (or Hero's) Formula

area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where a, b, c are sides of the triangle (s is the *semiperimeter*)

and $s = \dfrac{a+b+c}{2}$

Example: What is the area of the triangle?



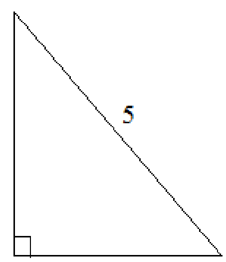
We are given 3 sides (but, no angles), so we'll use Heron's Formula

$s = \dfrac{5+6+7}{2} = 9$

Then, the area = $\sqrt{9(9-5)(9-6)(9-7)}$

$= \sqrt{9 \cdot 4 \cdot 3 \cdot 2} = 14.7$

Example: Use 3 methods to find area of this right triangle



1) Area = 1/2(base)(height)

= 1/2(3)(4) = $\boxed{6}$

2) Heron's Formula: Area = $\sqrt{s(s-a)(s-b)(s-c)}$

semiperimeter $s = \dfrac{12}{2} = 6$

Area = $\sqrt{6(6-5)(6-4)(6-3)} = \sqrt{36} = \boxed{6}$

3) Using Sine: Area = $\dfrac{1}{2}$ ab(sinC)

Since we are given a right angle, we'll use that angle and the adjacent sides:

Area = $\dfrac{1}{2}(3)(4)\sin(90^\circ) = \boxed{6}$

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

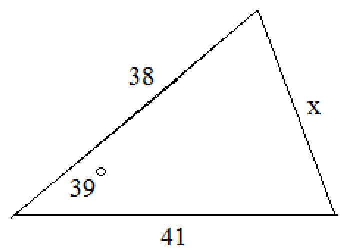
Try with (t), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

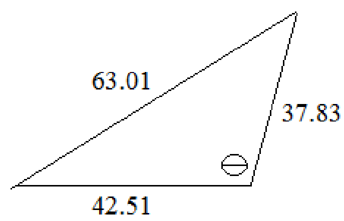
PRACTICE QUIZ -→

Law of Sines and Cosines Quiz

1) Find x:



2) Find θ:

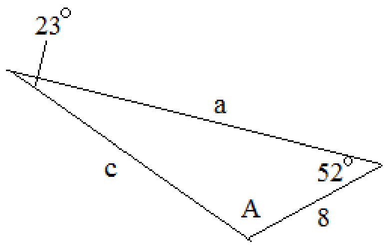


3) $a = 6$
$b = 8$
$\angle c = 53^{\circ}$

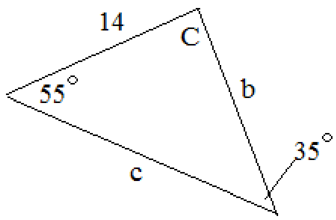
Find the other angle measures and side lengths of the triangle:

Law of Sines and Cosines Quiz

4) Find the other parts of the triangle:



5) Find the missing sides and angles:



6) Given: $\angle A = 28^{\circ}$
$a = 7$
$b = 17$

Find: $\angle B =$
$\angle C =$
$c =$

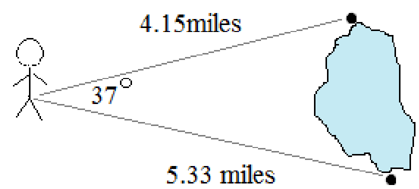
7) Given: $\angle A = 40$
$a = 8$
$b = 11$

Find: $\angle B =$

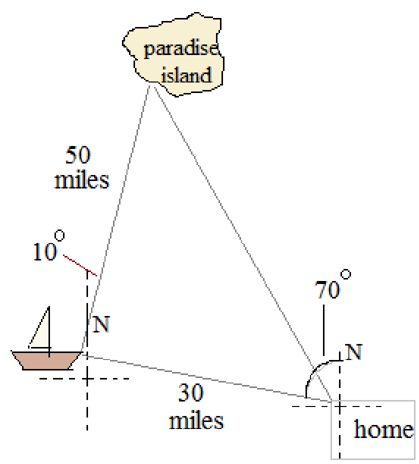
$\angle C =$

$c =$

8) To find the distance across a lake, a surveyor took the following measurements:
What is the distance across the lake?



9) A sailor at sea looks at coordinates on the following map:
How far is the paradise island from his home?



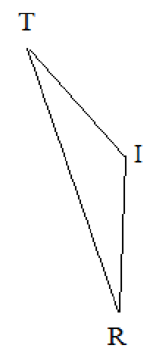
Law of Sines and Cosines Quiz
... and, Area

10) Triangle GUM has sides measuring 7, 8, and 13..
What are the angle measures?
What is the area of the GUM?

11) Find the area of the interior of the triangle.

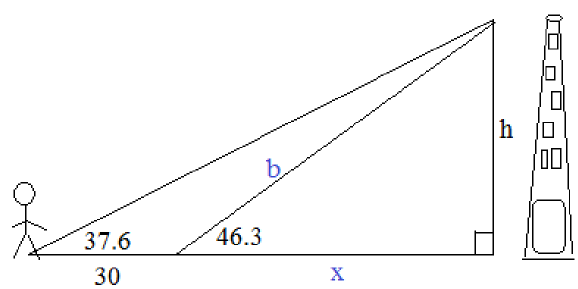
a) Use the trig area formula $\frac{1}{2}$ ab(sin(C))

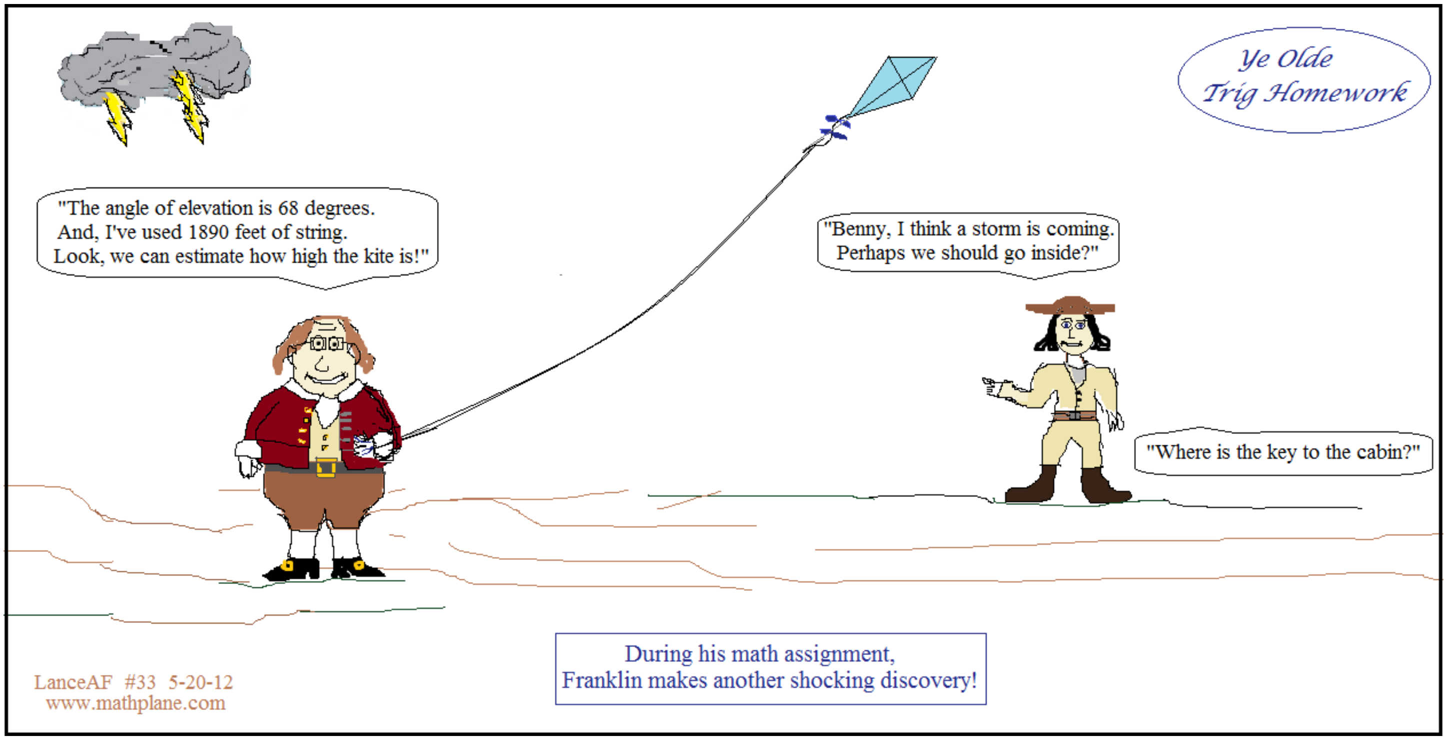
b) Then, use Heron's formula to check your answer.



r = 13
i = 25.75
T = 18 degrees

12) How tall is the tower?




Ye Olde
Trig Homework
"The angle of elevation is 68 degrees.
And, I've used 1890 feet of string.
Look, we can estimate how high the kite is!"
"Benny, I think a storm is coming.
Perhaps we should go inside?"
"Where is the key to the cabin?"
During his math assignment,
Franklin makes another shocking discovery!
LanceAF #33 5-20-12
www.mathplane.com

SOLUTIONS -→

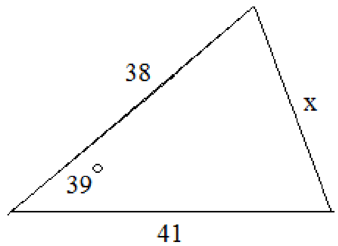
Law of Sines and Cosines Quiz

SOLUTIONS

1) Find x:

Since we're given 2 sides and the included angle, we can use law of cosines..

$c^2 = a^2 + b^2 - 2ab(\cos c)$



$x^2 = (38)^2 + (41)^2 - 2(38)(41)(\cos 39^\circ)$

$x^2 = 1444 + 1681 - 3116(.777)$

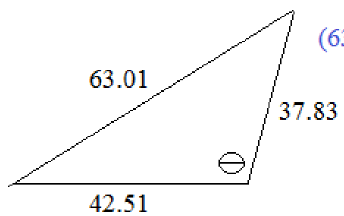
$x^2 = 3125 - 2421.59 = 703.41$

$\boxed{x = 26.52}$ (note: since x is opposite the smallest angle, it must be the smallest side!)

2) Find θ:

Since we know 3 sides, we can use law of cosines to find the (included) angle..

$c^2 = a^2 + b^2 - 2ab(\cos c)$



$(63.01)^2 = (37.83)^2 + (42.51)^2 - 2(37.83)(42.51)(\cos\theta)$

$3970.3 = 3238.2 - 3216.3(\cos\theta)$

$732.1 = -3216.3(\cos\theta)$

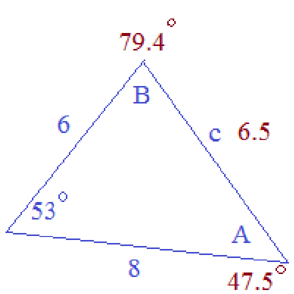
$-0.2276 = \cos\theta$

$\boxed{\theta = 103^\circ}$ (approximately)

3) a = 6
b = 8
$\angle c = 53^\circ$

Find the other angle measures and side lengths of the triangle:

First, draw a sketch of the triangle:



Check: $53 + 79.4 + 47.5 = 179.9 \cong 180^\circ$ ✓

smallest side a is opposite smallest angle A
middle side c is opposite middle angle C ✓
largest side b is opposite largest angle B

To find side "c" (opposite $\angle$c), use law of cosines:

$c^2 = (6)^2 + (8)^2 - 2(6)(8)(\cos 53^\circ)$

$c^2 = 100 - 96(.602)$

$c^2 = 42.2$ $\boxed{c = 6.5}$ (obviously, length is NOT −6.5)

To find another side, use law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{8} = \frac{\sin(53)}{6.5} \qquad \frac{\sin A}{6} = \frac{\sin(53)}{6.5}$$

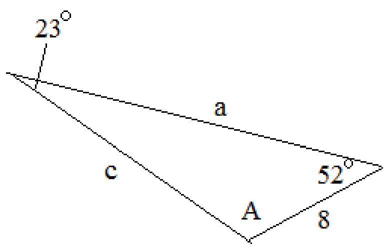
$$\sin B = \frac{8\sin(53)}{6.5} = .983 \qquad \sin A = \frac{6\sin(53)}{6.5} = .737$$

$\boxed{B = 79.4^\circ}$ $\boxed{A = 47.5^\circ}$

Law of Sines and Cosines Quiz

SOLUTIONS

4) Find the other parts of the triangle:



sum of interior angles of triangle = 180°

23 + 52 + A = 180 so, $\boxed{\angle A = 105^\circ}$

Then, use law of sines

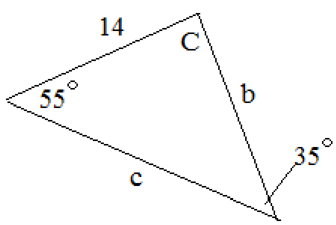
$\frac{\sin A}{a} = \frac{\sin 23}{8}$ $\quad 8\sin(105) = a\sin(23)$ $\quad a = \frac{8(.966)}{(.391)}$

$\boxed{19.78}$

$\frac{\sin 23}{8} = \frac{\sin 52}{c}$ $\quad 8\sin(52) = c\sin(23)$ $\quad c = \frac{8(.788)}{(.391)}$

$\boxed{16.13}$

5) Find the missing sides and angles:



Since the given angles, 55 and 35, add up to 90, angle $\boxed{\text{C is a right angle!}}$ so, we can use basic trig functions:

$\tan 55 = \frac{b}{14}$ $\quad b = 14(\tan 55) = 14(1.428) = \boxed{19.99}$

$\sin 35 = \frac{14}{c}$ $\quad c = \frac{14}{\sin 35} = \frac{14}{.574} = \boxed{24.41}$

note: to check, use pythagorean theorem:

$(14)^2 + (19.99)^2 = (24.41)^2$ ✓

Suppose we want to use law of sines:

$\frac{\sin 35}{14} = \frac{\sin 55}{b}$ $\quad b = \frac{14\sin 55}{\sin 35} = 19.99$

6) Given: $\angle A = 28^\circ$

$a = 7$

$b = 17$

Find: $\angle B =$

$\angle C =$

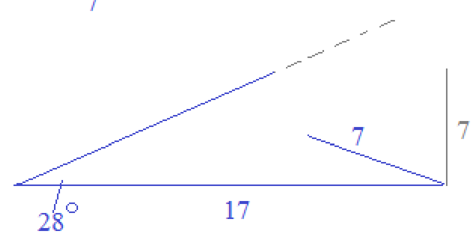
$c =$

Since this is a SSA case, we may have 0, 1, or 2 solutions...

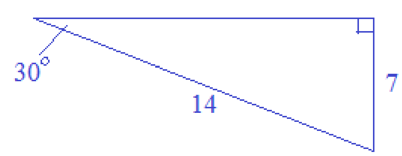
$\frac{\sin 28}{7} = \frac{\sin B}{17}$ $\quad \sin B = \frac{17(.469)}{7} = 1.14$

since sinB > 1, there is no solution!

Here is a sketch:



observe the similarities



Law of Sines and Cosines Quiz

SOLUTIONS

7) Given: $\angle A = 40$
$a = 8$
$b = 11$

Find: $\angle B =$
$\angle C =$
$c =$

Since this is a SSA case, we may have 0, 1, or 2 solutions...

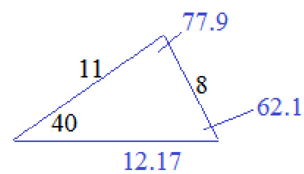
Using law of sines:

$\frac{\sin 40}{8} = \frac{\sin B}{11}$ $\sin B = \frac{11\sin(40)}{8} = .884$

If B = 62.1, then $C = 180 - (62.1 + 40)$

$\frac{\sin 40}{8} = \frac{\sin 77.9}{c}$ $c = \frac{8(\sin 77.9)}{\sin 40}$

$\angle B = 62.1$
$\angle C = 77.9$
$c = 12.17$



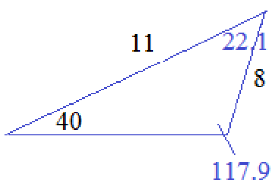
Instead of B = 62.1, B can equal 117.9°

arcsin(.884) = 62.1 OR 117.9

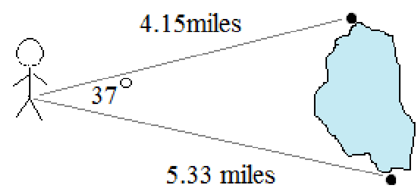
therefore, if B = 117.9, then C = 22.1°

$\frac{\sin 22.1}{c} = \frac{\sin 40}{8}$ $c = \frac{8(\sin 22.1)}{(\sin 40)} = 4.68$

$\angle B = 117.9^\circ$
$\angle C = 22.1^\circ$
$c = 4.68$



8) To find the distance across a lake, a surveyor took the following measurements:
What is the distance across the lake?

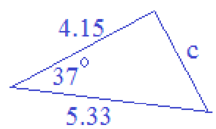


Once we recognize that the distance across the lake is the 3rd side of a triangle, we can use law of cosines to solve.

$c^2 = a^2 + b^2 - 2ab(\cos C)$

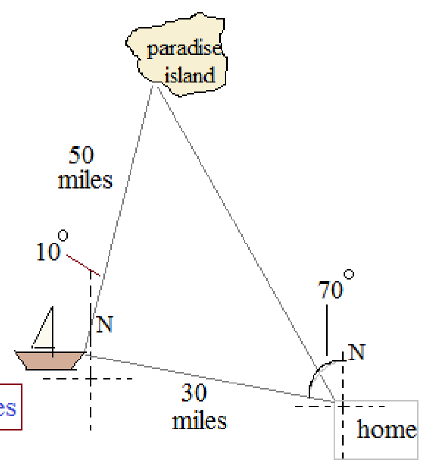
$c^2 = 17.22 + 28.41 - 44.24(.799)$

$c^2 = 10.28$

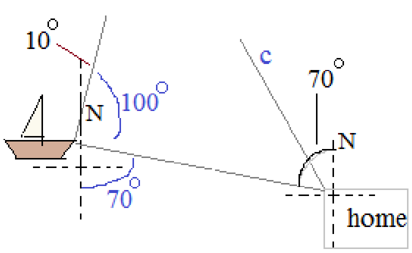


the distance is approximate 3.21 miles

9) A sailor at sea looks at coordinates on the following map:
How far is the paradise island from his home?



We have 2 sides of the triangle: 50 and 30...
Then, using geometry, we can figure out the included angle!
("parallel lines cut by a transversal -- alternate interior angles congruent"; then, angles on straight line add up to 180")



law of cosines:

$c^2 = (50)^2 + (30)^2 - 2(50)(30)(\cos 100)$

$c^2 = 3400 - 3000(-.174) = 3921$

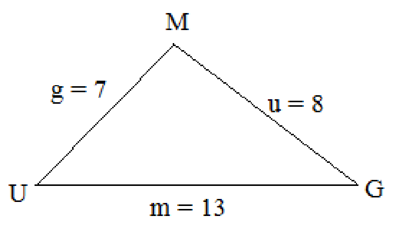
therefore, the distance is approx. 62.6 miles

10) Triangle GUM has sides measuring 7, 8, and 13..
What are the angle measures?
What is the area of the GUM?

SOLUTIONS

Law of Sines and Cosines Quiz
... and, Area

Step 1: Sketch the triangle



Step 2: Find an angle

given 3 sides, we'll use law of cosines

$g^2 = u^2 + m^2 - 2um(\text{Cos}G)$

$49 = 64 + 169 - 2(104)(\text{Cos}G)$

$-184 = -208(\text{Cos}G)$

$\boxed{G = 27.8 \text{ degrees}}$

Step 3: Find a second angle

given 2 sides and an included angle, we can use law of sines

$\frac{\sin G}{g} = \frac{\sin U}{u}$

$\frac{\sin(27.8)}{7} = \frac{\sin U}{8}$

$\sin U = \frac{8\sin(27.8)}{7}$

$\boxed{U = 32.2 \text{ degrees}}$

Step 4: Find last angle

$G + U + M = 180$ degrees

$27.8 + 32.2 + M = 180$

$\boxed{M = 120 \text{ degrees}}$

To check solutions,
can use law of sines for all..
Also, note: u is slightly larger than g
and, angle U is slightly larger than G.
Then, M is much larger than G and U
(as is m is larger than g and u)

AREA of GUM:

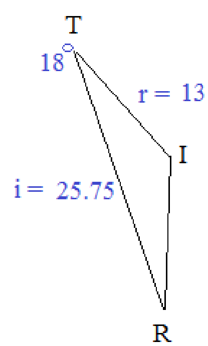
$\text{Area} = \frac{1}{2}um\text{Sin}(G) = \frac{1}{2}(8)(13)\sin(27.8)$

$= \boxed{24.25 \text{ sq. units}}$

11) Find the area of the interior of the triangle.

a) Use the trig area formula $\frac{1}{2}ab(\sin(C))$

b) Then, use Heron's formula to check your answer.



r = 13
i = 25.75
T = 18 degrees

a) since we have 2 sides and *included angle*, we can use the trig area formula

$\text{Area} = \frac{1}{2}(13)(25.75)\text{Sin}(18^\circ) = \boxed{51.7 \text{ sq. units}}$

b) To use Heron's formula, we need to know all 3 sides...

We can use law of cosines to get side t:

$t^2 = (13)^2 + (25.75)^2 - 2(13)(25.75)\cos(18^\circ)$

$= 169 + 663 - 669.5(.951)$

$= 195.27$

$t = 13.976$

so, r = 13
i = 25.75
t = 13.976

$s = \frac{r + i + t}{2} = 26.363$

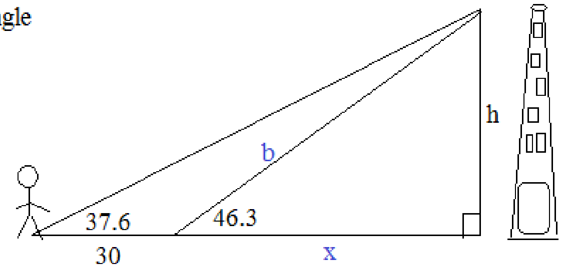
$\text{area} = \sqrt{26.363(26.363 - 13)(26.363 - 25.75)(26.363 - 13.976)}$

$= \sqrt{2675} = 51.7$ ✓

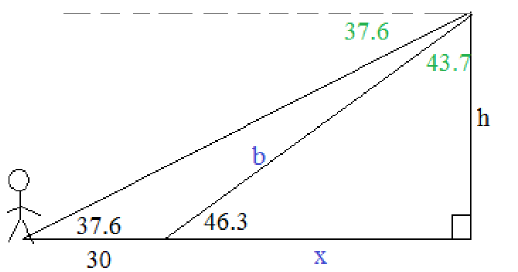
12) How tall is the tower?

method 1: use law of sines in the left triangle to find b . Then, use trig functions (sine) to find h in the right triangle

method 2: use tangent in large right triangle
use tangent in small right triangle
set equations equal to each other to find x
then, use trig functions (tangent) to find h

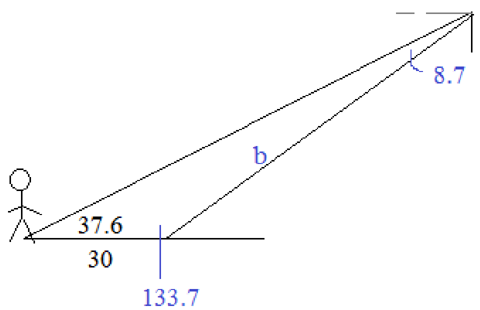


method 1:



Using geometry properties:
parallel lines cut by transversal, alternate interior angles are congruent --- 37.6
sum of interior angles of triangle is 180 --- 43.7

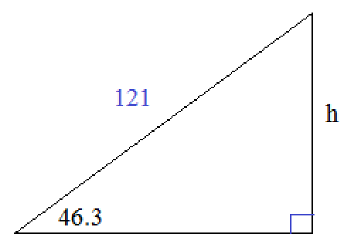
therefore, upper angle of left triangle is 8.7°



Use law of sines to find length of b:

$$\frac{\sin(37.6)}{b} = \frac{\sin(8.7)}{30}$$

$$b = \frac{30(\sin(37.6))}{\sin(8.7)} = 121.0$$



$$\sin(46.3) = \frac{h}{121}$$

$$\boxed{h = 87.48}$$

method 2:

small triangle: $\tan(46.3) = \frac{h}{x}$ $\rightarrow$ $x = \frac{h}{\tan(46.3)}$

large triangle: $\tan(37.6) = \frac{h}{x + 30}$

$$\tan(37.6) = \frac{h}{\frac{h}{\tan(46.3)} + 30}$$

$$.770 = \frac{h}{\frac{h}{1.046} + 30}$$

$$h = \frac{.770h}{1.046} + 23.1$$

$.2638h = 23.1$

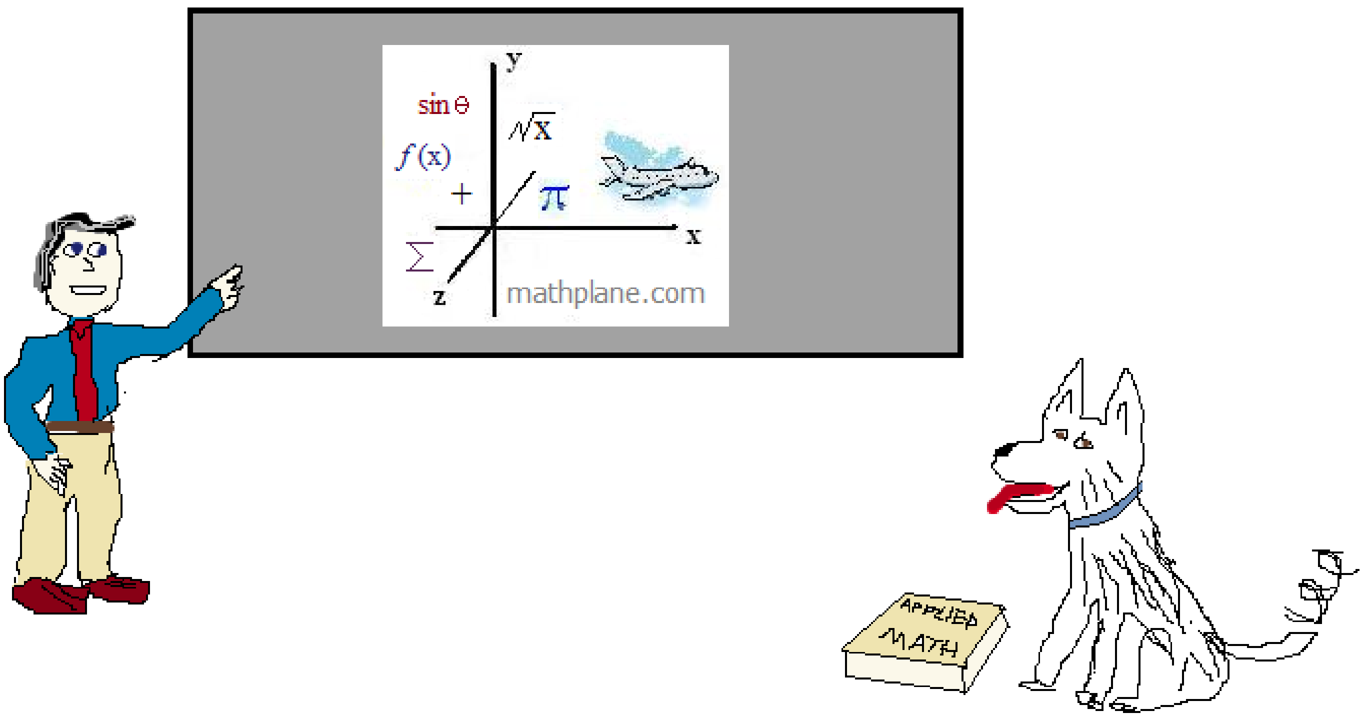
$h = 87.5$ ✓

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

We appreciate your support!

Cheers



Also, find at Facebook, Google+, and TeachersPayTeachers

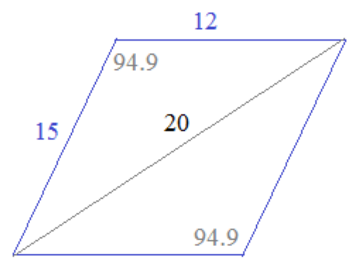
One more question:

A parallelogram has side lengths 12 and 15.

If the longer diagonal has length 20, what is the length of the <u>shorter</u> diagonal?

SOLUTION on next page→

A parallelogram has side lengths 12 and 15.
The longer diagonal has length 20...
What is the length of the shorter diagonal?



We need to find the angles of the parallelogram...

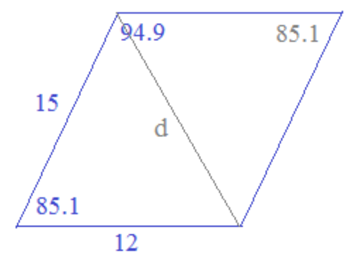
Using law of cosines:

$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$20^2 = 12^2 + 15^2 - 2(12)(15)\cos C$$

$$400 = 144 + 225 - 360\cos C$$

$$\frac{31}{-360} = \cos C \qquad C = 94.9^{\circ}$$



If C = 94.9 degrees, then the other angles are 180 - 94.9 = 85.1 degrees
(consecutive angles in parallelogram are supplementary)

Then, use law of cosines again to find the other diagonal...

$$d^2 = 12^2 + 15^2 - 2(12)(15)\cos(85.1)$$

$$= 144 + 225 - 360\cos(85.1)$$

$$d = 18.39$$