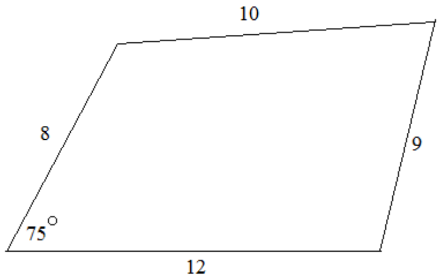
Trigonometry: Law of Sines and Cosines II

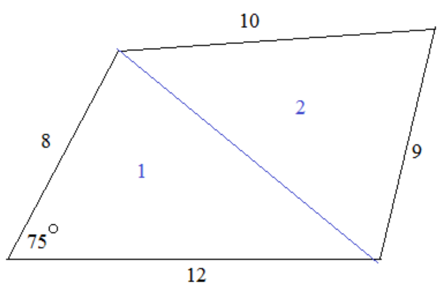
Test Questions and Detailed Solutions

Topics include bearings, law of sines ambiguous case, triangle properties, geometry concepts, quadrilateral area, and more.

Example: Find the area of the quadrilateral



Step 1: Draw a diagonal, dividing quadrilateral into 2 triangles



Step 2: Use Area formula to find area of triangle 1

$$\text{Area} = \frac{1}{2}ab\text{Sin}C$$

$$\text{Area} = \frac{1}{2}(8)(12)\text{Sin}(75^\circ)$$

$$= 46.364$$

Step 3: Find missing parts of triangle 2

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\text{Cos}C$

(triangle 1)

$$c^2 = 8^2 + 12^2 - 2(8)(12)\text{Cos}(75^\circ)$$

$$c^2 = 208 - 49.693$$

$$c = 12.58$$

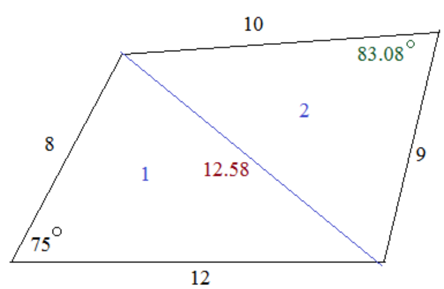
(triangle 2)

$$12.58^2 = 9^2 + 10^2 - 2(9)(10)\text{Cos}(C)$$

$$158.3 = 181 - 180\text{Cos}(C)$$

$$.1205 = \text{Cos}(C)$$

$$C = 83.08 \text{ degrees}$$



Step 4: Find area of triangle 2

$$\text{Area} = \frac{1}{2}ab\text{Sin}C$$

$$\text{Area} = \frac{1}{2}(9)(10)\text{Sin}(83.08)$$

$$= 44.672$$

OR

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$ (Heron's Formula)

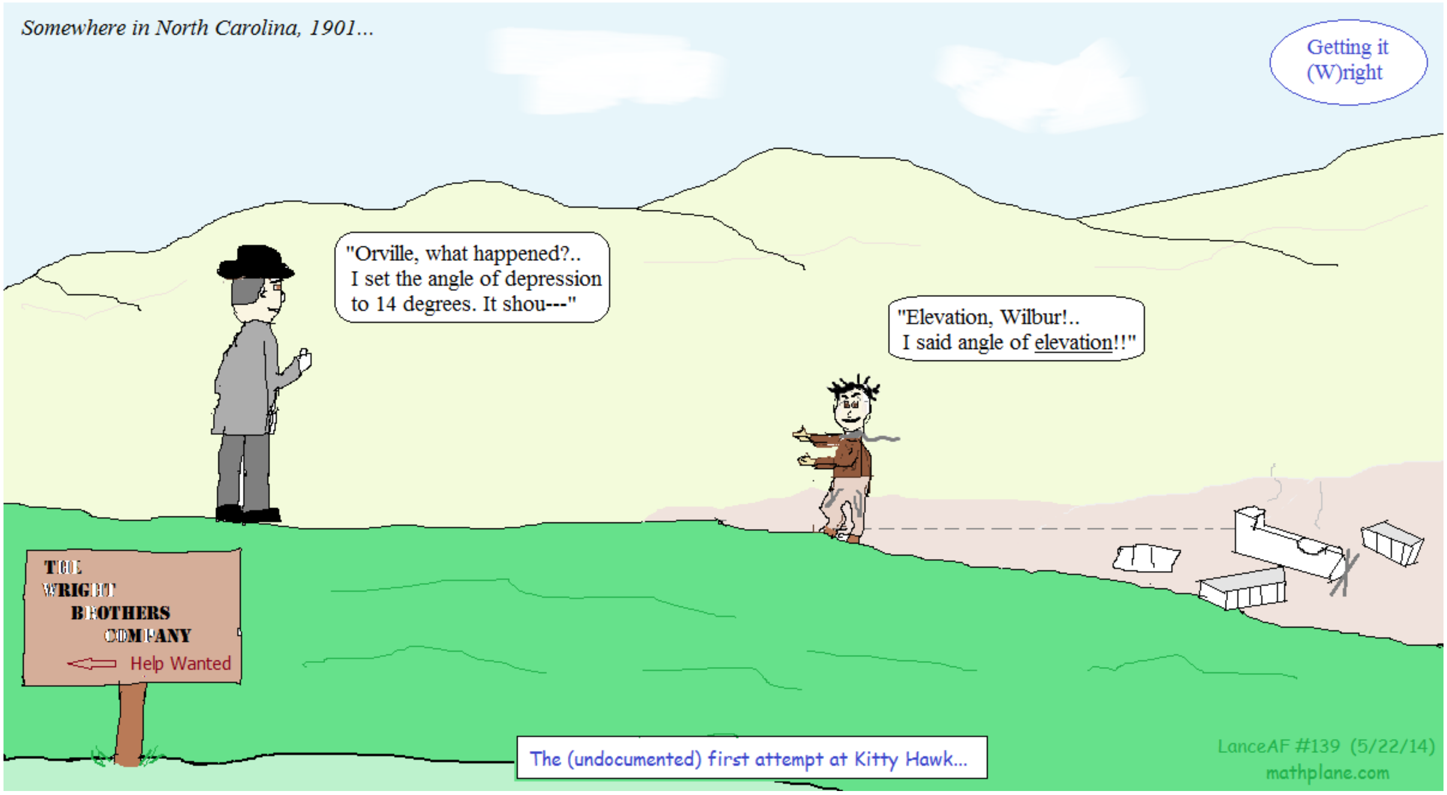
where s is the semiperimeter of the triangle and a, b, and c are the sides...

$$s = \frac{9 + 10 + 12.58}{2} = 15.79$$

$$\text{Area} = \sqrt{15.79(6.79)(5.79)(3.21)} = 44.639$$

Step 5: Add the areas of the triangles

Approximately $46.4 + 44.7 = \boxed{91.1}$


Somewhere in North Carolina, 1901...
Getting it (W)right
"Orville, what happened?.. I set the angle of depression to 14 degrees. It shou---"
"Elevation, Wilbur!.. I said angle of elevation!!"
Help Wanted
The (undocumented) first attempt at Kitty Hawk...
LanceAF #139 (5/22/14)
mathplane.com

Questions-→

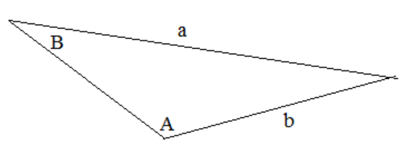
1) A parallelogram has side lengths 12 and 15.
If the longer diagonal has length 20, then
what is the length of the shorter diagonal?

2) A dolphin swims at a bearing of N29E.
Then, it turns and swims at a bearing of N51W.
And, finally, it swims due South 700 meters, returning to its
original starting spot.

How far did the dolphin swim?

3) Why is this triangle not possible?

A = 120 degrees
a = 19
b = 22



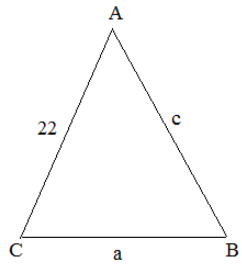
Law of Sines and Cosines

4) In $\triangle ABC$, $\tan A = 1$

$\tan B = \frac{3}{4}$

$b = 22$

Determine the measures of all sides and angles.



a =

b = 22

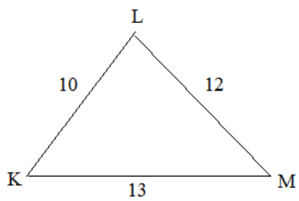
c =

angle A =

angle B =

angle C =

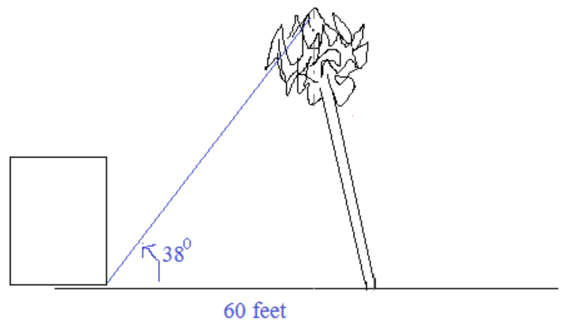
5)



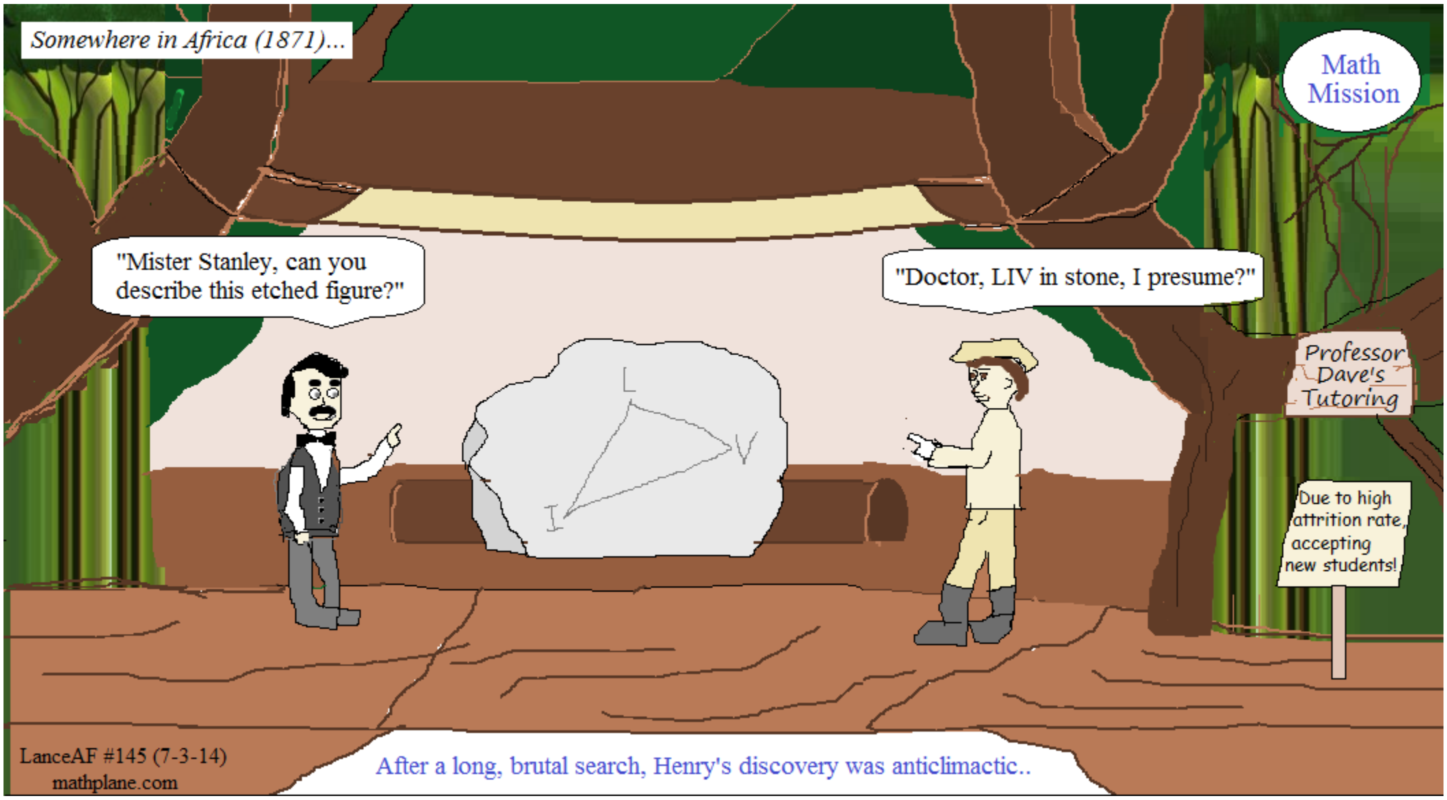
a) What is the length of the median from L to $\overline{KM}$?

b) Find the length of the angle bisector from L to $\overline{KM}$

6) After a storm, a tree leans 4 degrees toward a house.
The base of the tree is 60 feet from the steps of the house.
If the angle of elevation from the base of the steps to the top of the tree is 38 degrees, what is the height of the tree?



7) **Challenge Question:
An airplane leaves airport A and flies 210 miles.
Currently, the plane's direction is a bearing of 120 degrees *from airport B*.
Airport B is 270 miles due west of Airport A.
How far is the plane from airport B?

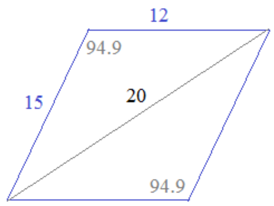

Somewhere in Africa (1871)...
Math Mission
"Mister Stanley, can you describe this etched figure?"
"Doctor, LIV in stone, I presume?"
L
V
I
Professor Dave's Tutoring
Due to high attrition rate, accepting new students!
LanceAF #145 (7-3-14)
mathplane.com
After a long, brutal search, Henry's discovery was anticlimactic..

Solutions -→

SOLUTIONS

1) A parallelogram has side lengths 12 and 15.
If the longer diagonal has length 20, then what is the length of the shorter diagonal?

We need to find the angles of the parallelogram...



Using law of cosines:

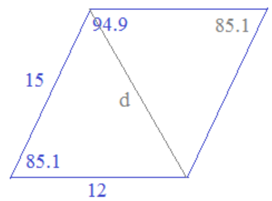
$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$20^2 = 12^2 + 15^2 - 2(12)(15)\cos C$$

$$400 = 144 + 225 - 360\cos C$$

$$\frac{31}{-360} = \cos C \qquad C = 94.9^\circ$$

If C = 94.9 degrees, then the other angles are 180 - 94.9 = 85.1 degrees
(consecutive angles in parallelogram are supplementary)



Then, use law of cosines again to find the other diagonal...

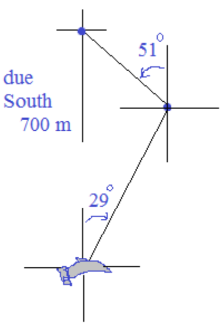
$$d^2 = 12^2 + 15^2 - 2(12)(15)\cos(85.1)$$

$$= 144 + 225 - 360\cos(85.1)$$

$$\boxed{d = 18.39}$$

2) A dolphin swims at a bearing of N29E.
Then, it turns and swims at a bearing of N51W.
And, finally, it swims due South 700 meters, returning to its original starting spot.

How far did the dolphin swim?



Step 1: Sketch the diagram

Step 2: Use geometry properties to identify angles;
Alternate interior angles ---> angle 51 degrees
sum of interior angles of triangle equals 180 ---> 100 degrees

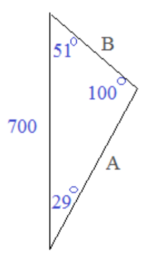
Extract the triangle

Step 3: Use law of sines to find other sides

$$\frac{\sin(100^\circ)}{700} = \frac{\sin(51^\circ)}{A} = \frac{\sin(29^\circ)}{B}$$

$$A = \frac{700(\sin 51)}{(\sin 100)} = 552.4$$

$$B = \frac{700(\sin 29)}{(\sin 100)} = 344.6$$

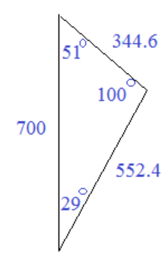


Step 4: Answer the question and check for reasonableness

Sum of the sides: 700 + 344.6 + 552.4 = $\boxed{\text{1597 meters}}$

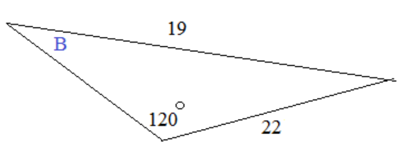
Opposite the smallest angle (29) is the smallest side 344.6
Opposite the medium angle (51) is the medium side 552.4

The measures appear reasonable...



3) Why is this triangle not possible?

A = 120 degrees
a = 19
b = 22



Since angle A is 120 degrees, the remaining 2 angles must be less than 60 degrees...

However, angle B must be greater than angle A (because side b is greater than side a)...

Angle B cannot be greater than 120 and less than 60!!

$$\frac{\sin B}{22} = \frac{\sin(120)}{19}$$

$$\sin B = \frac{22\sin(120)}{19}$$

sinB = 1.003 since it's greater than 1, angle B is undefined

Law of Sines and Cosines

SOLUTIONS

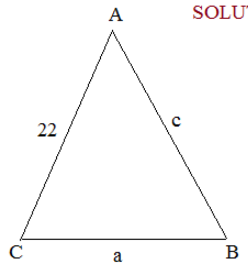
4) In $\triangle ABC$, $\tan A = 1$

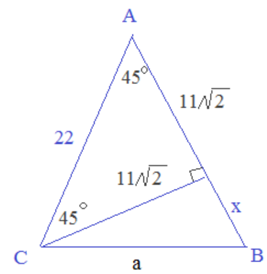
$\tan B = \frac{3}{4}$

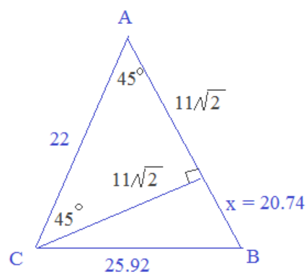
$b = 22$

Determine the measures of all sides and angles.

a =	25.92
b =	22
c =	36.3
angle A =	45 degrees
angle B =	36.9 degrees
angle C =	98.1 degrees







*** Since Tangent is opp/adj, angle A must be 45 degrees and the sides will be 1-1-$\sqrt{2}$ ratio

$$\tan B = \frac{3}{4} = \frac{11\sqrt{2}}{x} \quad \frac{\text{(opposite)}}{\text{(adjacent)}}$$

$$3x = 44\sqrt{2}$$

$$x = \frac{44\sqrt{2}}{3} = 20.74$$

$$c = 11\sqrt{2} + 20.74 = 36.3$$

(Pythagorean Theorem)

$$20.74^2 + (11\sqrt{2})^2 = a^2$$

$$a = 25.92$$

(law of sines)

$$\frac{\sin B}{22} = \frac{\sin A}{25.92}$$

$$\sin B = \frac{22\sin(45)}{25.92}$$

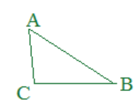
$$B = 36.9^\circ$$

(angles of triangle add up to 180)

$$45 + 36.9 + C = 180$$

$$C = 98.1^\circ$$

NOTE: the diagram of the triangle is not drawn to scale... (eg. side a appears shorter than side b.. But, it is not..)
Here is a better shape...



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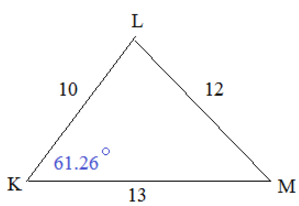


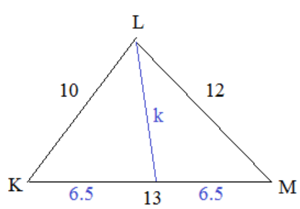
a) What is the length of the median from L to $\overline{KM}$?

Step 1: Find the measure of angle K (using law of cosines)

$$12^2 = 10^2 + 13^2 - 2(10)(13)\cos K$$

$$-125 = -260\cos K$$

$$K = 61.26^\circ$$



Step 2: Drop a median from L to $\overline{KM}$... (bisecting $\overline{KM}$)

Step 3: Find measure of altitude k (using law of cosines)

$$k^2 = 100 + (6.5)^2 - 2(10)(6.5)(\cos 61.26)$$

$$k^2 = 142.25 - 130(.481)$$

$$\boxed{k = 8.93}$$

b) Find the length of the angle bisector from L to $\overline{KM}$

Step 1: Use law of cosines to find measure of angle L

$$13^2 = 10^2 + 12^2 - 2(10)(12)\cos L$$

$$-75 = -240\cos L$$

$$L = 71.79^\circ$$

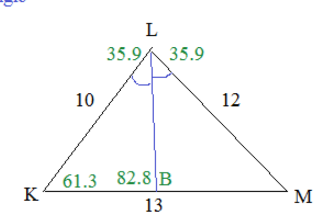
Step 2: Find angle measures of 'left' triangle

angle K = 61.26° (from part a)

angle KLB = (1/2)L = 35.9°

angle LBK = 82.8°

(82.8 + 35.9 + 61.3 = 180)



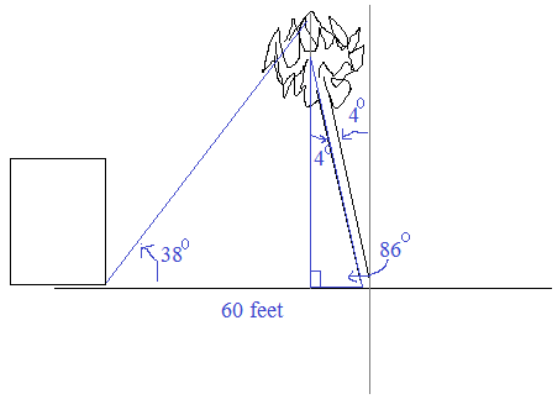
Step 3: Find angle bisector LB (using law of sines)

$$\frac{\sin(61.3)}{LB} = \frac{\sin(82.8)}{10} \qquad LB = \frac{10\sin(61.3)}{\sin(82.8)}$$

$$\boxed{LB = 8.84}$$

SOLUTIONS

6) After a storm, a tree leans 4 degrees toward a house.
The base of the tree is 60 feet from the steps of the house.
If the angle of elevation from the base of the steps to the top of the tree is 38 degrees, what is the height of the tree?



After drawing a sketch, we can determine the missing angles using geometry concepts...

if parallel lines cut by transversal, then alternate interior angles are congruent..

then, sum of the interior angles of a triangle is 180 degrees..
Therefore, the bottom right angle is 86 degrees...

Since the bottom angles are 38 and 86, the top angle is 56 degrees... $38 + 86 + 56 = 180$

To find the length of the tree, use law of sines:

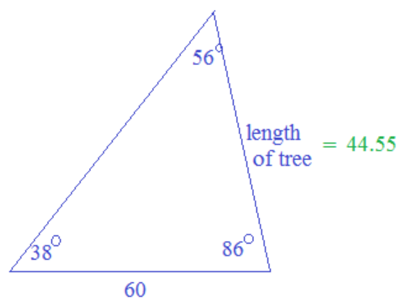
$$\frac{\sin(38)}{\text{tree}} = \frac{\sin(56)}{60} \qquad \text{tree} = \frac{60\sin(38)}{\sin(56)} = \boxed{44.55 \text{ length of tree}}$$

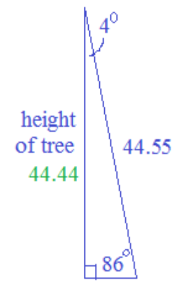
Then, to find the height of the tree, we look at the right triangle:

$$\sin(86) = \frac{\text{height}}{44.55}$$

$$\cos(4) = \frac{\text{height}}{44.55}$$

$$\boxed{\text{height above ground} = 44.44 \text{ feet}}$$





7) **Challenge Question:
An airplane leaves airport A and flies 210 miles.
Currently, the plane's direction is a bearing of 120 degrees *from airport B*.
Airport B is 270 miles due west of Airport A.
How far is the plane from airport B?

Step 1: Sketch a diagram

Note: a bearing of 120 degrees is equivalent to -30 degrees.

Step 2: Extract the triangle and solve using law of sines

$$\frac{\sin(30)}{210} = \frac{\sin P}{270}$$

$$\sin P = \frac{270\sin(30)}{210}$$

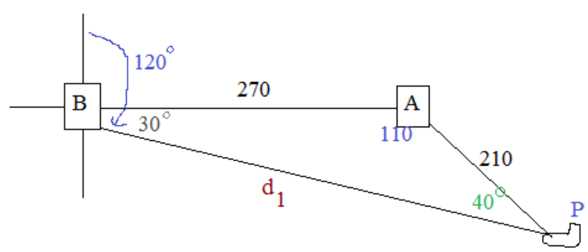
P = 40 degrees

$$A + B + P = 180$$
$$A + 30 + 40 = 180$$
$$A = 110^{\circ}$$

$$\frac{d_1}{\sin(110)} = \frac{210}{\sin(30)}$$

$$d_1 = \frac{210\sin(110)}{\sin(30)}$$

$$\boxed{= 394.7 \text{ miles}}$$



***Step 3: Recognize there are 2 possible answers!
(SSA -- ambiguous case)

$$\frac{\sin(30)}{210} = \frac{\sin P}{270}$$

$$\sin P = \frac{270\sin(30)}{210}$$

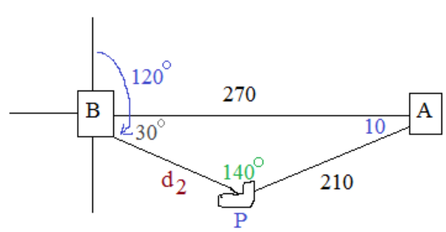
P = 140 degrees

$$A + B + P = 180$$
$$A + 30 + 140 = 180$$
$$A = 10^{\circ}$$

$$\frac{d_2}{\sin(10)} = \frac{210}{\sin(30)}$$

$$d_2 = \frac{210\sin(10)}{\sin(30)}$$

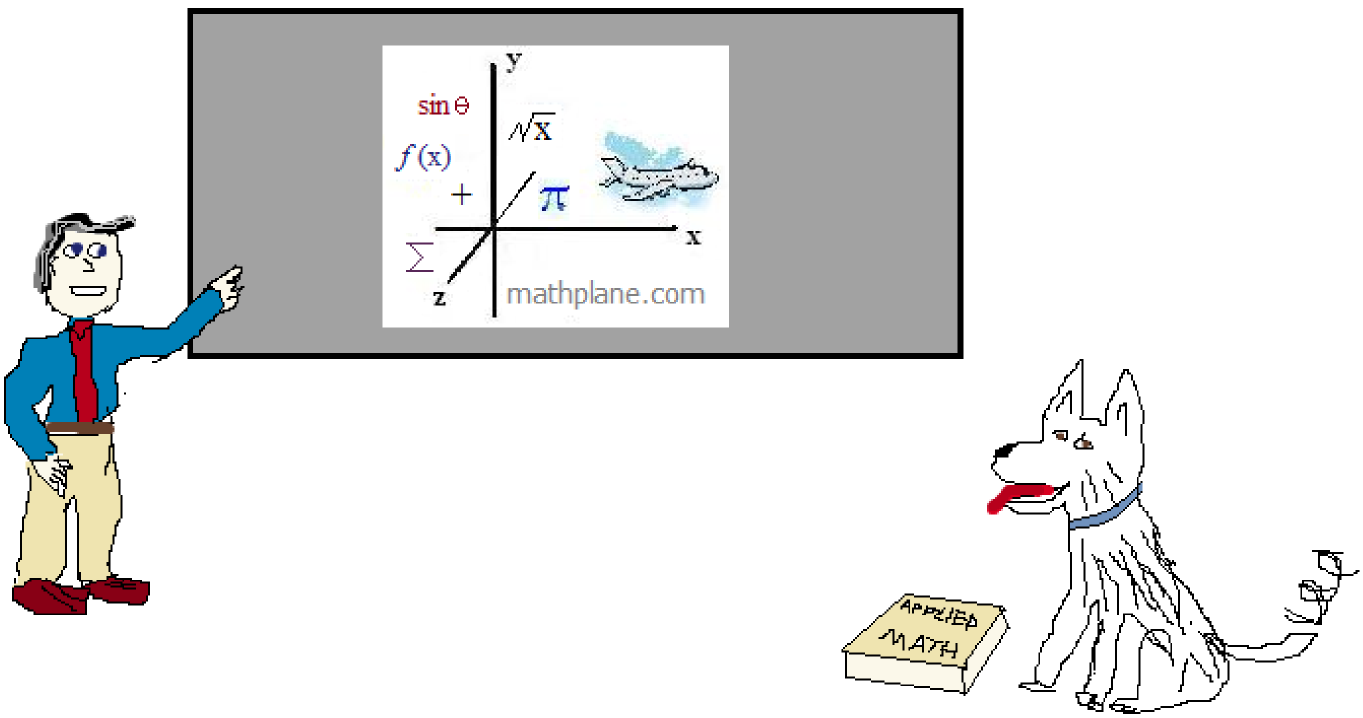
$$\boxed{= 72.9 \text{ miles}}$$



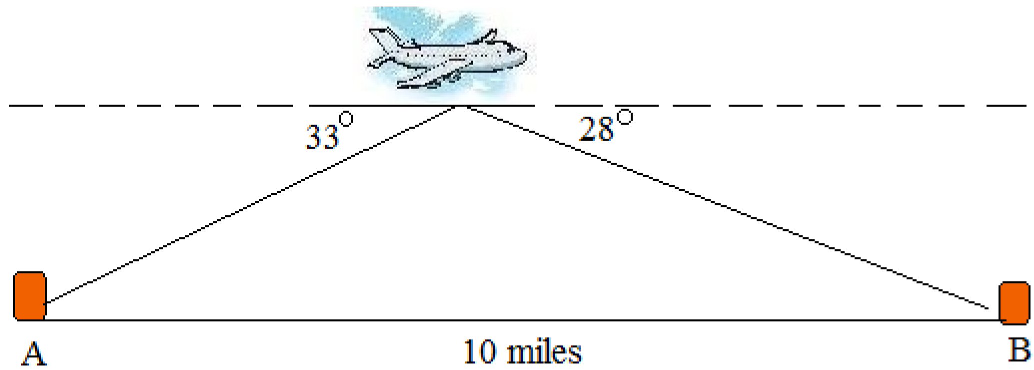
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at TeachersPayTeachers, Facebook, Google+, TES, and Pinterest



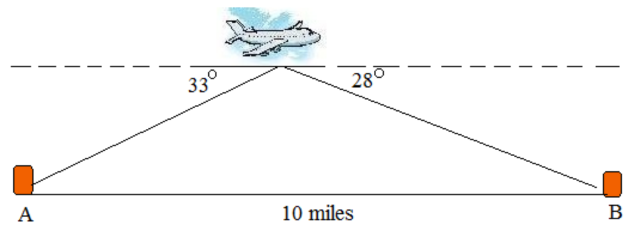
a) What is the distance from the plane to milepost A?

b) What is the elevation of the plane?

ANSWERS→

a) What is the distance from the plane to milepost A?

b) What is the elevation of the plane?



Step 1: Use Geometry to identify angle values

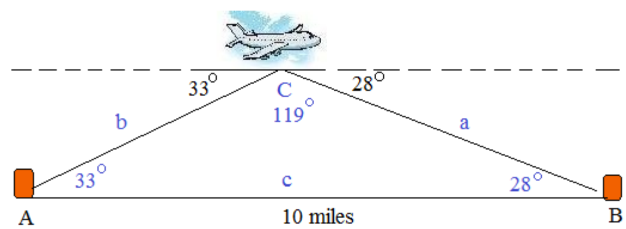
If parallel lines are cut by a transversal,
then alternate interior angles are congruent...

(Horizon is parallel to the ground,) so base angles are 33° and 28° ..

Then, sum of adjacent angles on a line eqauls 180 degrees.

$33 + C + 28 = 180$

$C = 119$ degrees



Step 2: Use Law of Sines to find distance to A

$$\frac{\sin(28)}{b} = \frac{\sin(119)}{10}$$

$$\boxed{b = 5.37}$$

Step 3: Use trig functions to find elevation

The elevation is the altitude extending from plane
to base of the triangle.

And, the altittude forms right angles.

$$\sin A = \frac{e}{5.37}$$

$$5.37(\sin 33) = e$$

$$\boxed{\text{elevation } e = 2.92}$$