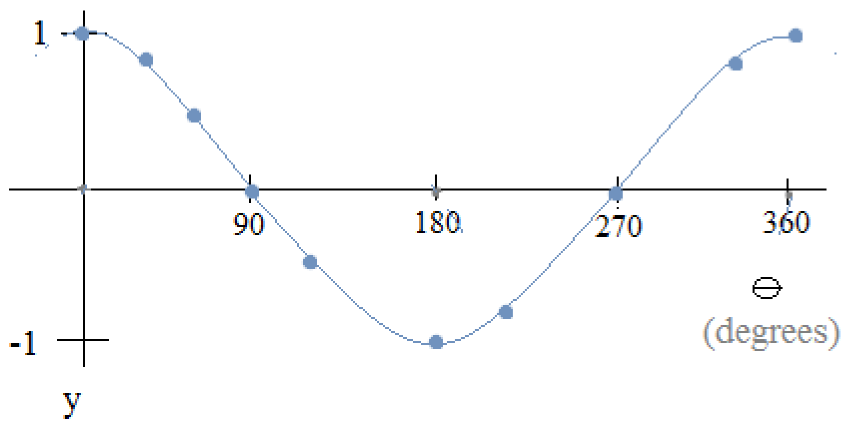
Periodic Trig Functions II: Cosine

Practice Exercises (with Solutions)

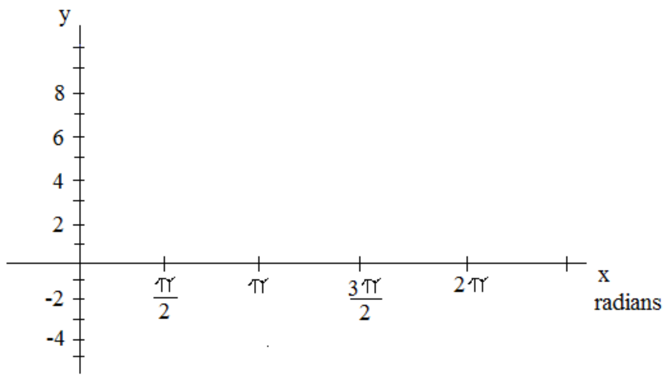


Topics include period, amplitude, phase shift, graphing, maximum and minimum, vertical shift, and more.

Mathplane.com

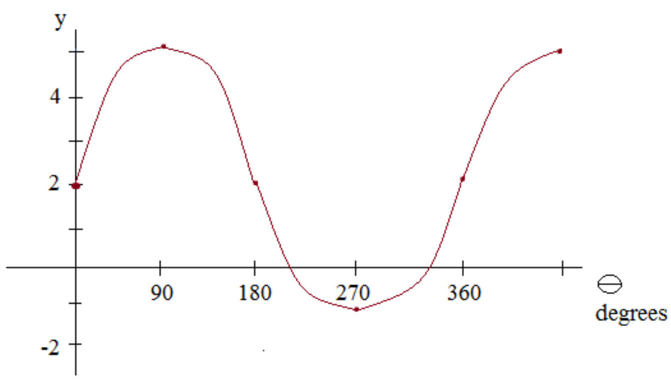
Cosine Function Practice

1) Graph the following function: $4\cos(x - \frac{\pi}{2}) + 3$

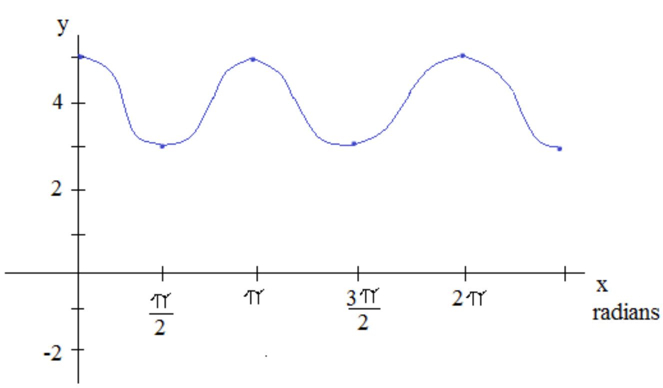


2) Identify the following cosine functions:

A)



B)



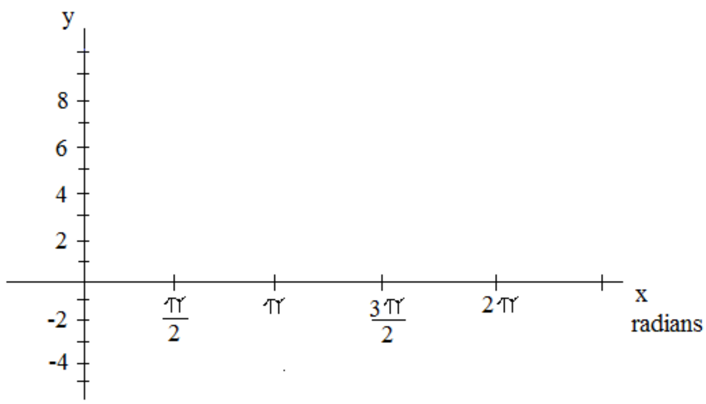
Cosine Function Practice

3) Graph the following Cosine Functions. Then, use the given points to check your answers algebraically and graphically.

A) $y = -5\cos x + 3$

Check: $x = \pi$

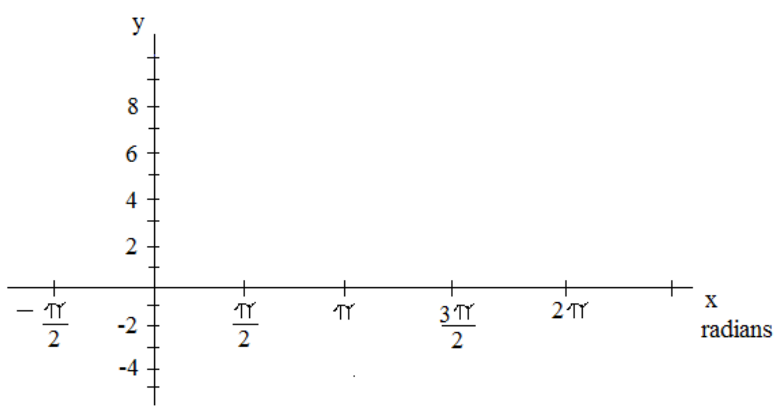
$x = \frac{3\pi}{2}$



B) $y = \cos\left(2x + \frac{\pi}{2}\right)$

Check: $x = \frac{\pi}{4}$

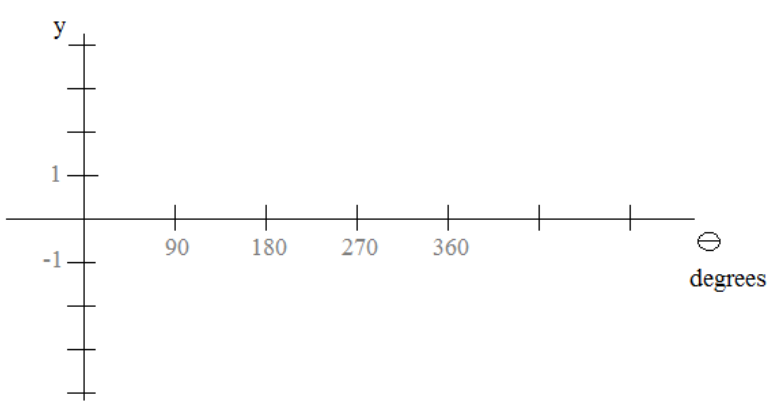
$x = \pi$



C) $y = 2|\cos\theta|$

Check: $\theta = 90^{\circ}$

$\theta = 180^{\circ}$



Cosine Function Practice

4) For the graph $y = \cos x + 3$,

A) Domain:

B) Range:

C) x-intercepts:

D) y-intercept:

***Challenge:

5) What is the cosine equation?

Period: 3

Vertical shift: none

Amplitude: $\frac{\pi}{2}$

Horizontal shift: none

Sketch the graph...

Use the following points to write and graph a sinusoidal model.

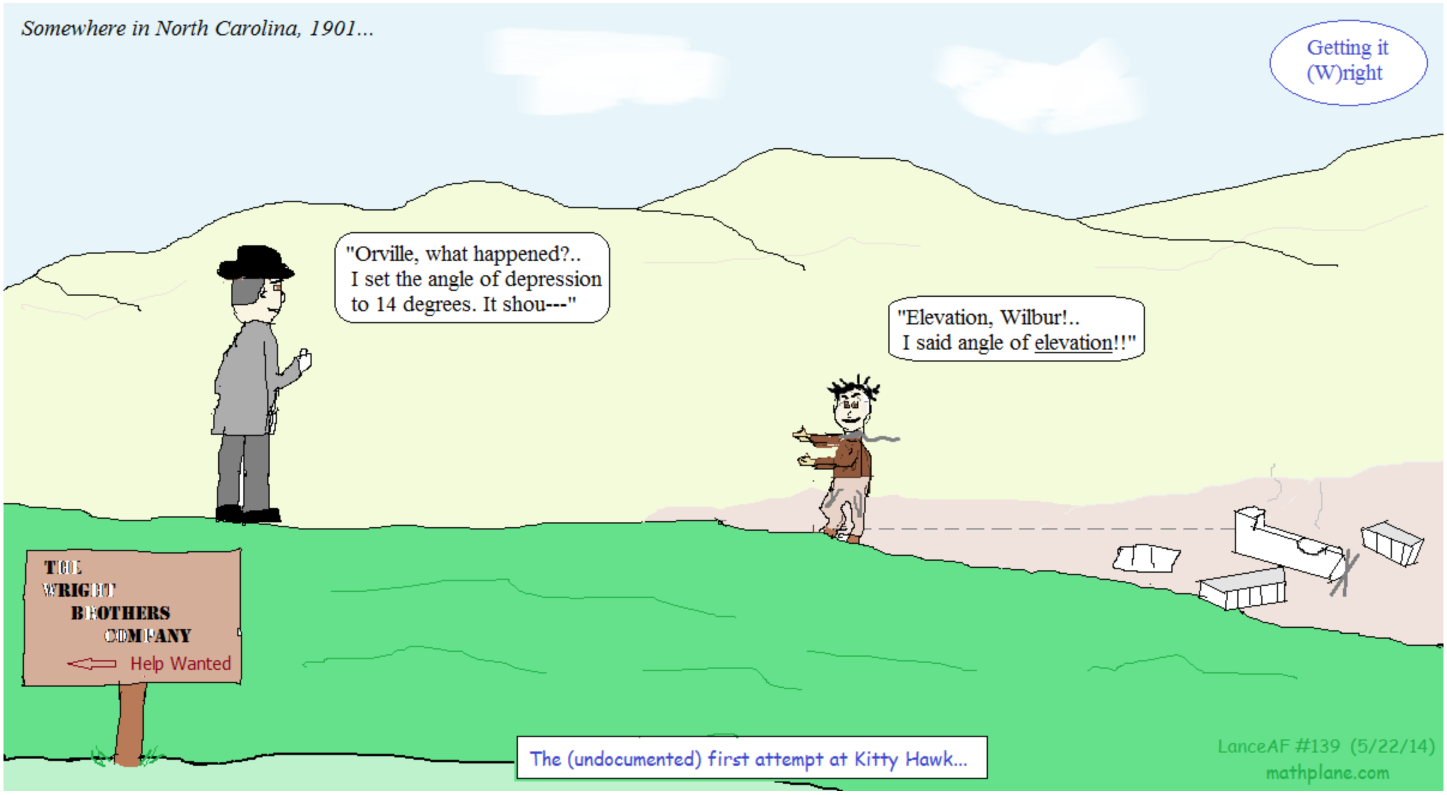
Identifying Cosine and Sine Functions

maximum $(0, 10)$ and minimum $(2\pi, 0)$

maximum $(\pi, 4)$ and minimum $(0, -2)$

maximum $(\frac{\pi}{4}, 8)$ and minimum $(\frac{\pi}{2}, 2)$

maximum $(2, 22)$ and minimum $(8, 14)$

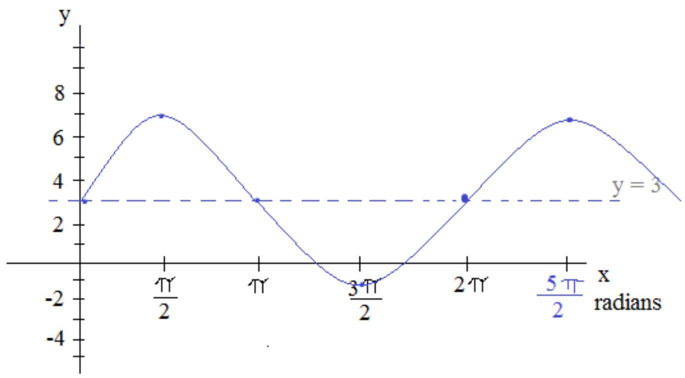

Somewhere in North Carolina, 1901...
Getting it (W)right
"Orville, what happened?.. I set the angle of depression to 14 degrees. It shou---"
"Elevation, Wilbur!.. I said angle of elevation!!"
THE WRIGHT BROTHERS COMPANY
Help Wanted
The (undocumented) first attempt at Kitty Hawk...
LanceAF #139 (5/22/14)
mathplane.com

Solutions →

Cosine Function Practice

SOLUTIONS

1) Graph the following function: $4\cos(x - \frac{\pi}{2}) + 3$



$y = A\cos B(x - C) + D$

$4\cos(x - \frac{\pi}{2}) + 3$

Amplitude (A) = 4

Period $(2\pi/B) = 2\pi/1 = 2\pi$

Horizontal shift (C) = $\frac{\pi}{2}$ to the right

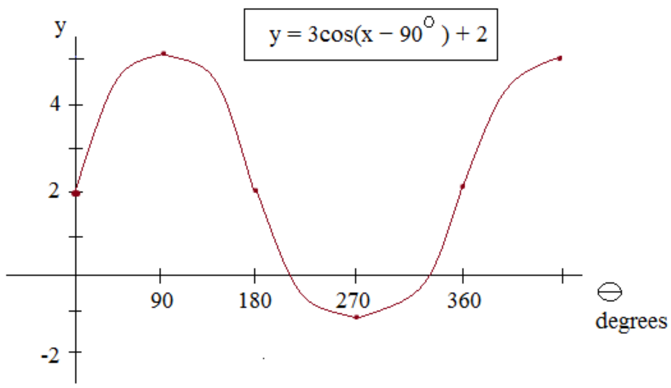
Vertical shift (D) = 3 units UP

The middle of the function will be at y = 3
The range will be from 7 (max) to -1 (min)..

(cosine starts at the max, goes down through the middle to the bottom.. then, goes back up)

2) Identify the following cosine functions:

A)



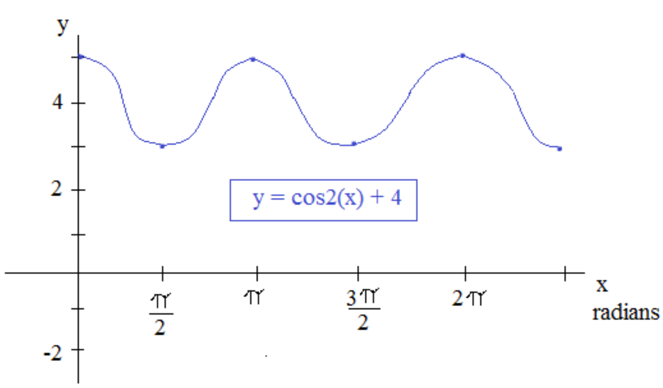
Steps: 1) Identify the center..
max: 5 min: -1
D = 2 midpoint is y = 2 vertical shift: up 2

2) Find the amplitude..
The vertical span of the wave is from
A = 3 5 to -1.. So, the amplitude is 1/2 the range.. 1/2 of 6 is 3

3) Horizontal shift..
$C = -90^{\circ}$ Since the maximum begins at 90 degrees, there is a horizontal shift of 90 to the right..

4) Period..
B = 1 the length of 1 cycle is 360 degrees..

B)



The middle of the range is 4...
Vertical shift: UP 4 D = 4

The range goes from 3 to 5,
so the amplitude is 1 A = 1

At x = 0, the function is at its max..
There is no horizontal shift C = 0

One cycle has a length of π.
so, $B = \frac{2\pi}{\pi} = 2$

Cosine Function Practice

SOLUTIONS

3) Graph the following Cosine Functions. Then, use the given points to check your answers algebraically and graphically.

A) $y = -5\cos x + 3$

$y = A\cos B(x - C) + D$

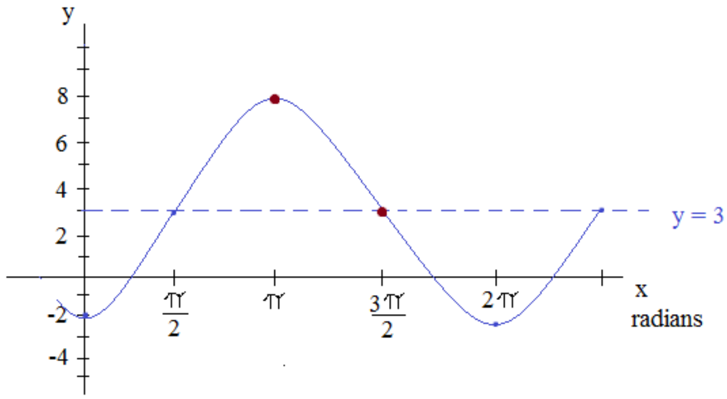
amplitude: $A = -5$ (negative, so "faces down")

period: 2π

Vertical shift: $D = 3$
Up 3

Horizontal shift: C
none..

max 8 ; min -2



Check: $x = \pi$

$x = \frac{3\pi}{2}$

At $x = \pi$,

$y = -5\cos(\pi) + 3$

$= -5(-1) + 3 = 8$ ✓

At $x = \frac{3\pi}{2}$

$y = -5\cos(\frac{3\pi}{2}) + 3$

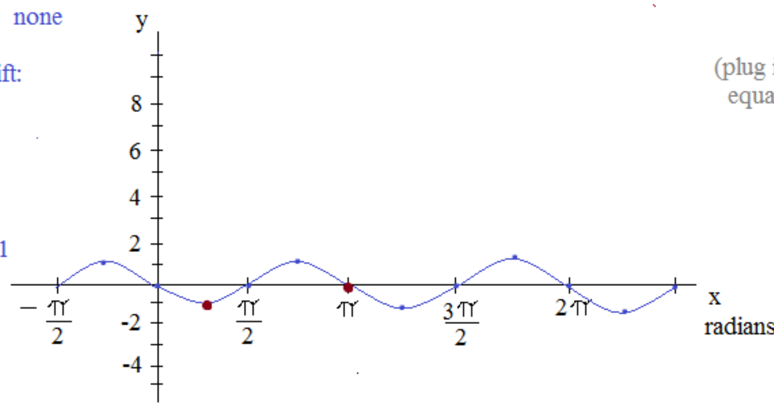
$= -5(0) + 3 = 3$ ✓

B) $y = \cos(2x + \frac{\pi}{2})$ (change to standard form) $y = \cos 2(x + \frac{\pi}{4})$

Vertical shift: none
amplitude: 1
horizontal shift: left $\frac{\pi}{4}$

period: π

max: 1 min: -1



Check: $x = \frac{\pi}{4}$

$x = \pi$

(plug into original equation)

At $x = \frac{\pi}{4}$

$y = \cos(2\left(\frac{\pi}{4}\right) + \frac{\pi}{2})$

$= \cos(\frac{\pi}{2} + \frac{\pi}{2}) = -1$ ✓

At $x = \pi$

$y = \cos(2\pi + \frac{\pi}{2}) = 0$ ✓

C) $y = 2|\cos\theta|$

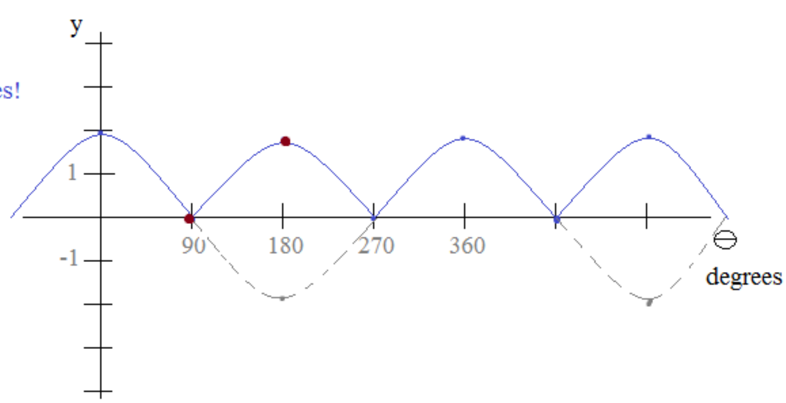
Amplitude $A = 2$

period is 180 degrees!

vertical shift: none

horizontal (phase) shift: none

max: 2 min: 0



Check: $\theta = 90°$

$\theta = 180°$

At 90 : $y = 2|\cos 90°|$

$y = 0$ ✓

At 180: $y = 2|\cos 180°|$

$y = 2|-1| = 2 \times 1 = 2$ ✓

Cosine Function Practice

SOLUTIONS

4) For the graph $y = \cos x + 3$,

A) Domain: all real numbers... (any number can go into x)

B) Range: [2, 4] center is 3 and amplitude is 1

C) x-intercepts: none

$0 = \cos x + 3$

$-3 = \cos x$

no solution

D) y-intercept: (0, 4)

***Challenge:

5) What is the cosine equation?

Period: 3

Vertical shift: none

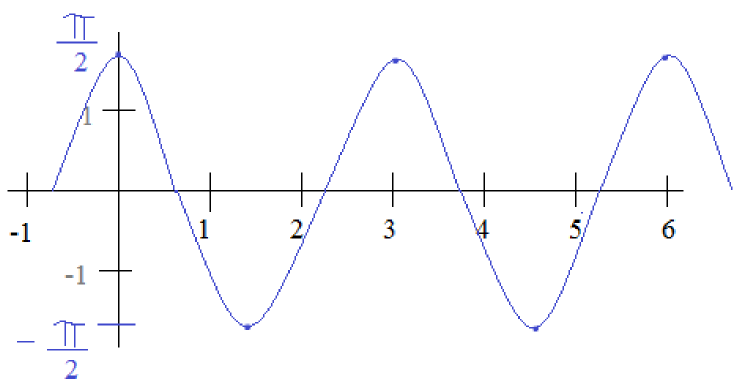
Amplitude: $\frac{\pi}{2}$

Horizontal shift: none

$y = A\cos B(x - C) + D$

$B = \frac{2\pi}{3}$

Sketch the graph...



$$y = \frac{\pi}{2} \cos \frac{2\pi}{3}(x)$$

$\frac{\pi}{2} = 1.57$ (approx)

Use the following points to write and graph a sinusoidal model.

SOLUTIONS

maximum $(0, 10)$ and minimum $(2\pi, 0)$

midline or axis of wave: $y = 5$ (midpoint between max and min)
amplitude: 5
period: 4π (one cycle is max to min and min to max)

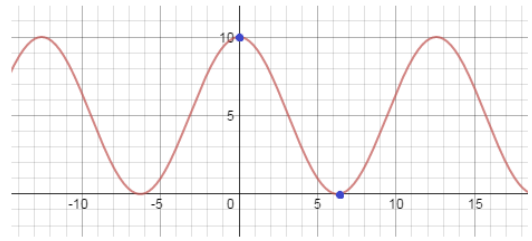
since a relative maximum occurs at $x = 0$, we'll use cosine graph with no horizontal shift...

Note: There are many other solutions. For example, suppose the max and min were not in the same cycle....

EX: $y = 5\cos\frac{7}{2}x + 5$

$y = 5\cos\frac{1}{2}x + 5$

also, $y = 5\sin\frac{1}{2}(x + \pi) + 5$



maximum $(\pi, 4)$ and minimum $(0, -2)$

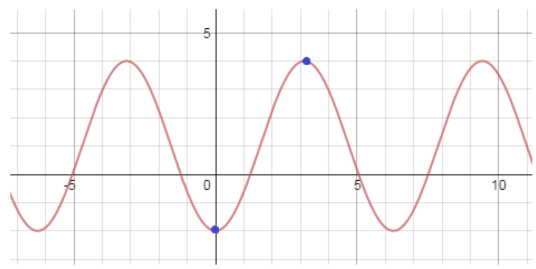
midline or axis of wave: $y = 1$ (midpoint between max and min)
amplitude: 3 (distance from axis of wave to an extreme. OR, 1/2 of distance from max to min)
period: 2π (one cycle is max to min and min to max)

since a relative minimum occurs at $x = 0$, we'll use a cosine graph with no horizontal (phase) shift...

$y = -3\cos x + 1$

also,

$y = 3\sin(x - \frac{\pi}{2}) + 1$



maximum $(\frac{\pi}{4}, 8)$ and minimum $(\frac{\pi}{2}, 2)$

midline or axis of wave: $y = 5$ (midpoint between max and min)
amplitude: 3 (distance from axis of wave to an extreme. OR, 1/2 of distance from max to min)
period: $\frac{\pi}{2}$ (one period is 2 x (max to min))

For convenience, we'll use the maximum and choose a cosine function with shift $\frac{\pi}{4}$

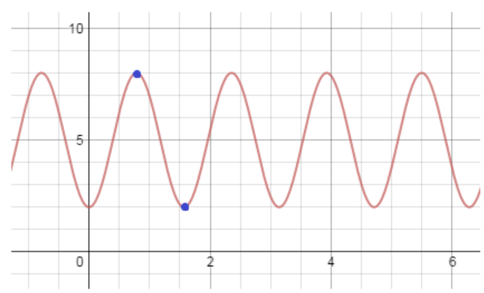
$y = A\cos B(x - C) + D$

A: Amplitude (magnitude)
B: Period
C: Horizontal Shift
D: Vertical Shift

period $= \frac{2\pi}{B}$

$B = \frac{2\pi}{\text{period}}$

$y = 3\cos 4(x - \frac{\pi}{4}) + 5$



maximum $(2, 22)$ and minimum $(8, 14)$

midline or axis of wave: $y = 18$
amplitude: 4

period: 12 'B' value is $\frac{2\pi}{12} = \frac{\pi}{6}$

Suppose we prefer a sine function...

$y = A\sin B(x - C) + D$

$y = 4\sin\frac{\pi}{6}(x - C) + 18$

then, to find C, substitute either point...

$14 = 4\sin\frac{\pi}{6}(8 - C) + 18$

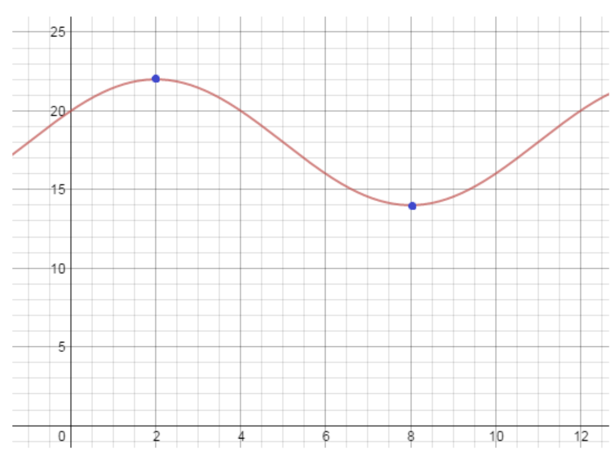
$-1 = \sin\frac{\pi}{6}(8 - C)$

$\sin^{-1}(-1) = \frac{\pi}{6}(8 - C)$

$-\frac{\pi}{2} = \frac{\pi}{6}(8 - C)$ $C = 11$

$y = 4\sin\frac{\pi}{6}(x - 11) + 18$

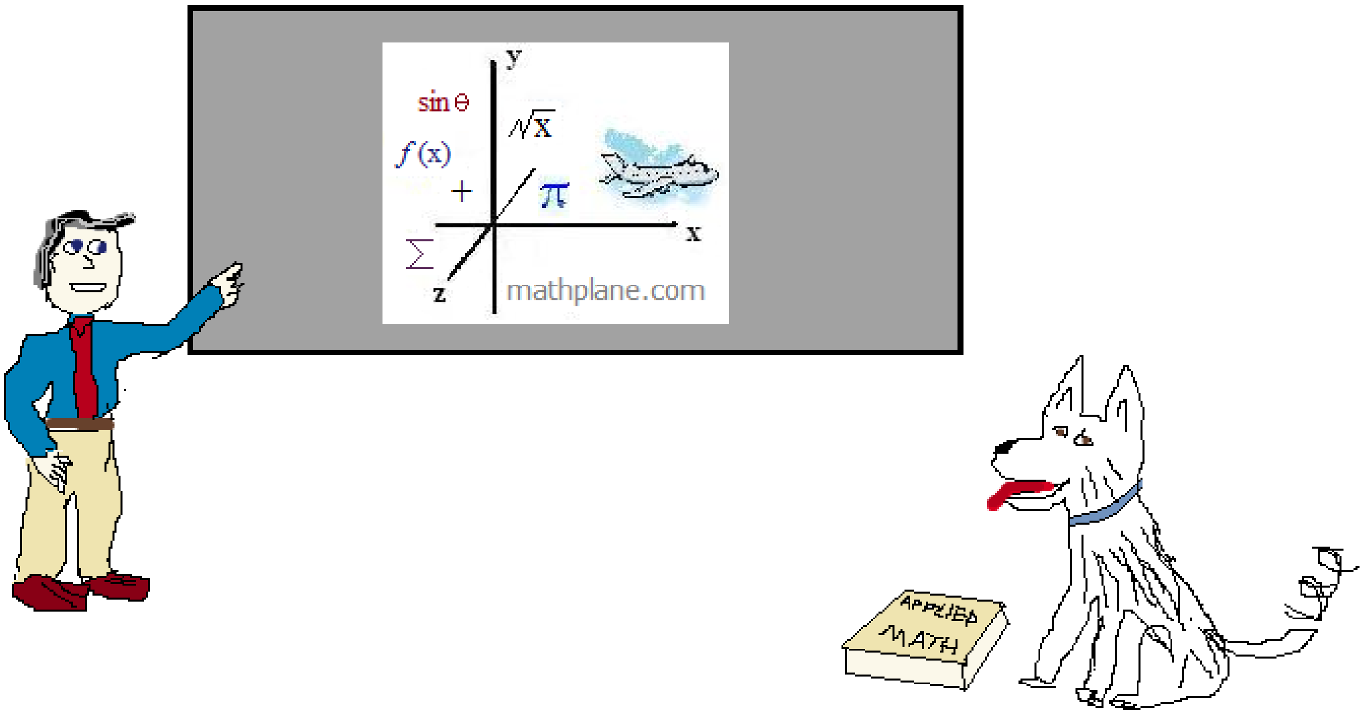
or $y = 4\sin\frac{\pi}{6}(x + 1) + 18$



Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers.



Find more trigonometry materials and comics at the mathplane.com

Also, find us at teacherspayteachers, facebook, google+, and pinterest…

One more function: Identify the transformations. Then, graph:

$$y = -2\cos(\pi x + \frac{\pi}{2})$$

Identify the transformations of the following cosine function. Then, graph.

$$y = -2\cos(\pi x + \frac{\pi}{2})$$

The "A" value is -2, so the amplitude is 2

To find the period and phase (horizontal) shift, we must simplify ---> put in standard form!

$$y = -2\cos \pi (x + \frac{1}{2}) + 0$$

A B C D

$$y = A\cos B(x - C) + D$$

A: Amplitude (magnitude)
B: Period
C: Horizontal Shift
D: Vertical Shift

Amplitude: 2

Period: $\frac{2\pi}{B} = 2$

Horizontal shift: $\frac{1}{2}$ to the left

Vertical shift: None

Reflection: Since the "A" value is negative there is reflection over the x-axis