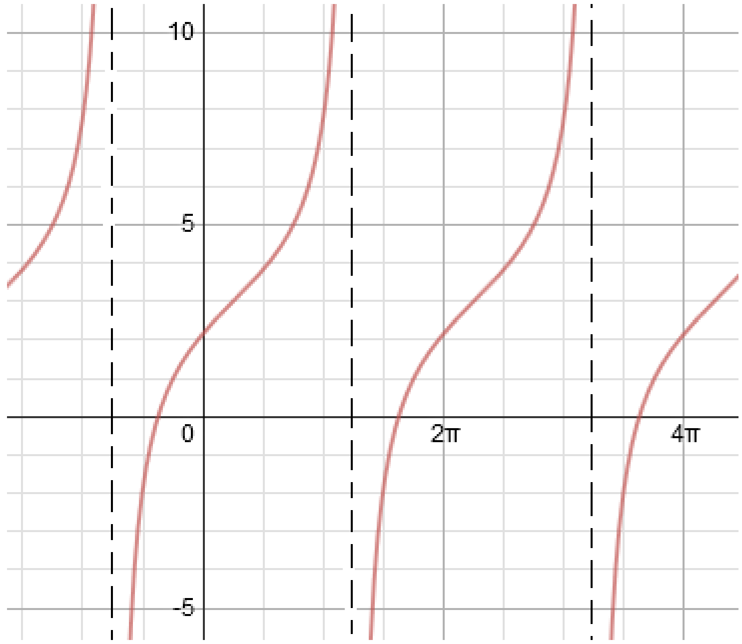
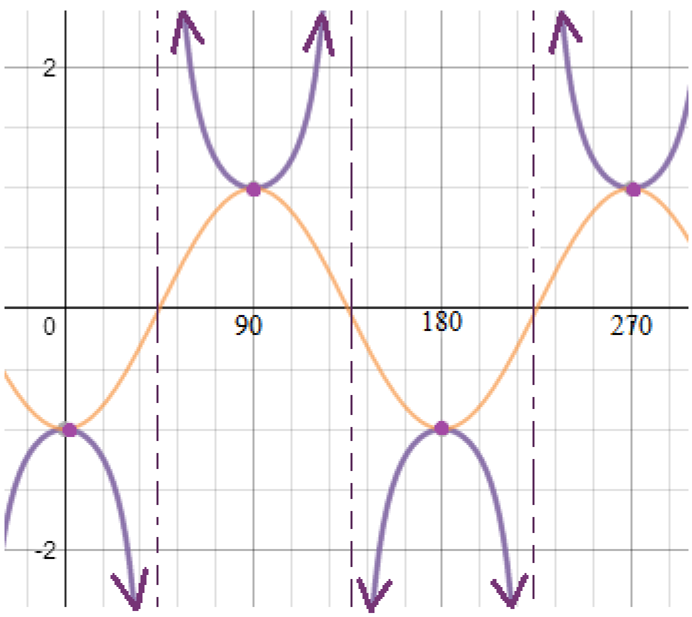
Introduction to Periodic Trig Functions 2:

Tangent Graphs



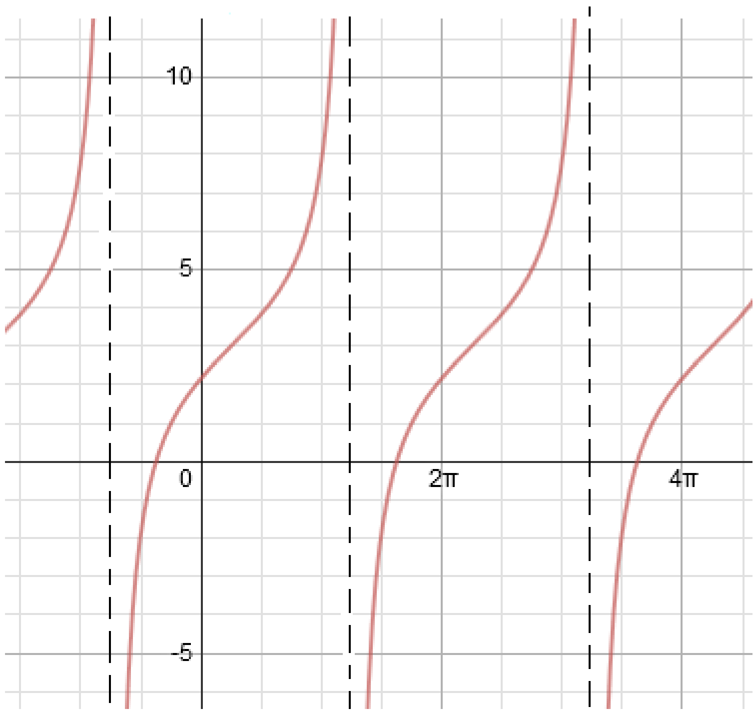


and, Reciprocal Sketches

Notes/examples of trig values and the components of trig graphs

Includes 2 practice tests (and solutions)

Tangent Functions

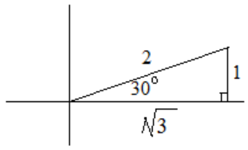


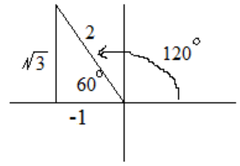
Topics include asymptotes, period, amplitude, horizontal and vertical shifts, reflection, and more.

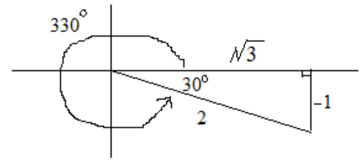
Sketching the Parent Function: $y = \tan(x)$ (radians) $y = \tan\theta$ (degrees)

Tangent = $\dfrac{\text{opposite}}{\text{adjacent}}$ → $\dfrac{y}{x}$

A few examples of common angles:







$\tan 30^\circ = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ $\tan 120^\circ = \dfrac{\sqrt{3}}{-1} = -\sqrt{3}$ $\tan 330^\circ = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$

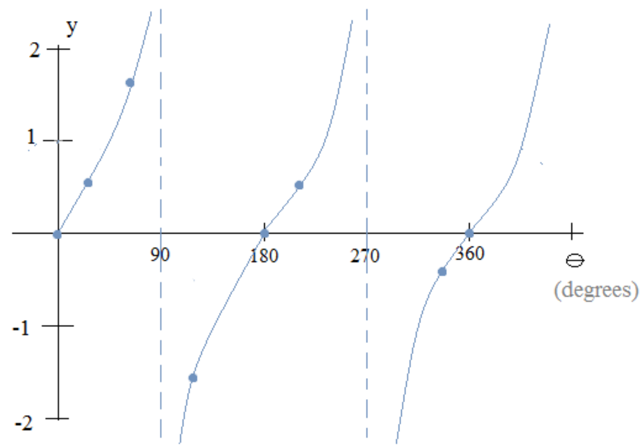
The following is a table of chosen values:

$y = \tan\theta$

θ	0	30	60	90	120	180	210	270	330	360
y	0	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{1}{0}$	$-\sqrt{3}$	0	$\frac{-1}{\sqrt{3}}$	$\frac{-1}{0}$	$\frac{-1}{\sqrt{3}}$	0

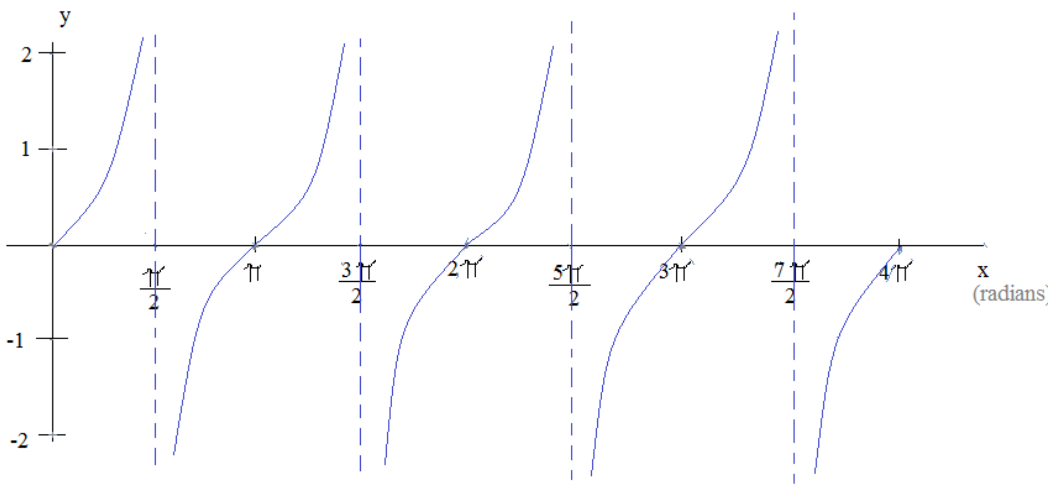
Undefined (90) Undefined (270)

Then, plot the points...



It's a <u>periodic function</u> -- it will repeat the pattern of y- values at a regular interval of 180°. (The period is 180°)

$y = \tan x$



A cycle is a complete pattern repetition. This sketch contains 4 cycles.

The period is π (Horizontal length of one cycle)

Tangent Functions: 4 components

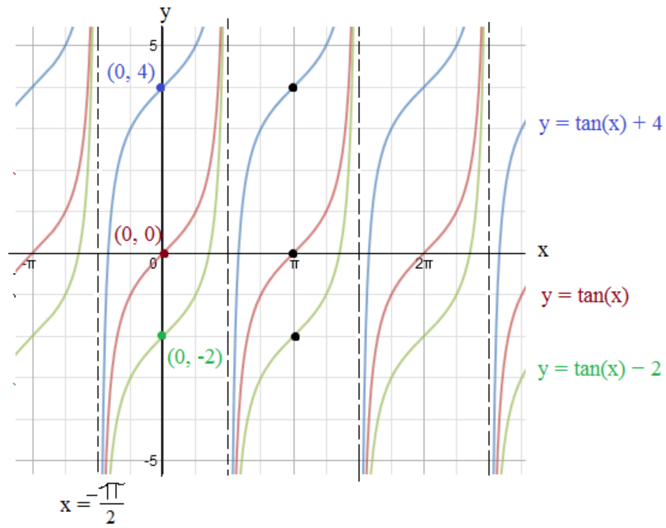
The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

$y = \tan(x)$ is the parent function

Vertical Shift:



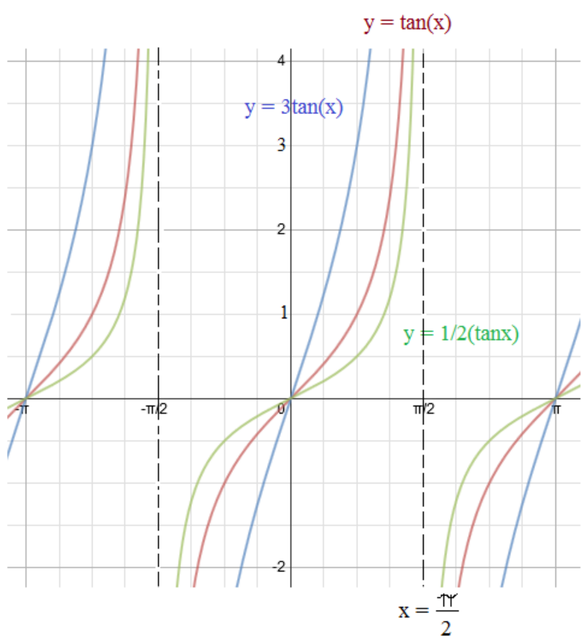
If $x = \pi$

$y = \tan(x)$ -----> $\tan(\pi) = 0$

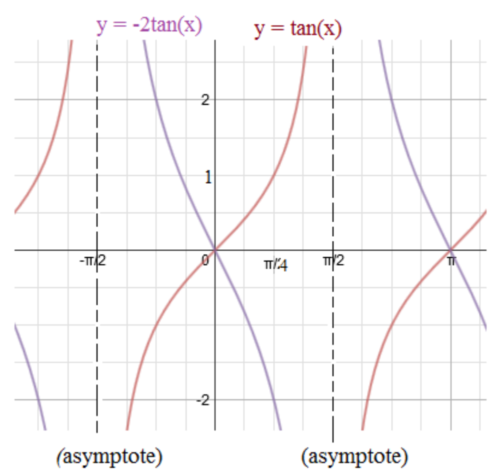
$y = \tan(x) + 4$ ----> $\tan(\pi) + 4 = 4$

$y = \tan(x) - 2$ -----> $\tan(\pi) - 2 = -2$

Amplitude:



x	tanx	3tanx	$\frac{1}{2}$ tanx	−2tanx
0	0	0	0	0
$\frac{\pi}{4}$	1	3	$\frac{1}{2}$	-2
$\frac{\pi}{2}$	undefined	undefined	undefined	
π	0	0	0	0
$\frac{3\pi}{4}$	-1	-3	$-\frac{1}{2}$	2



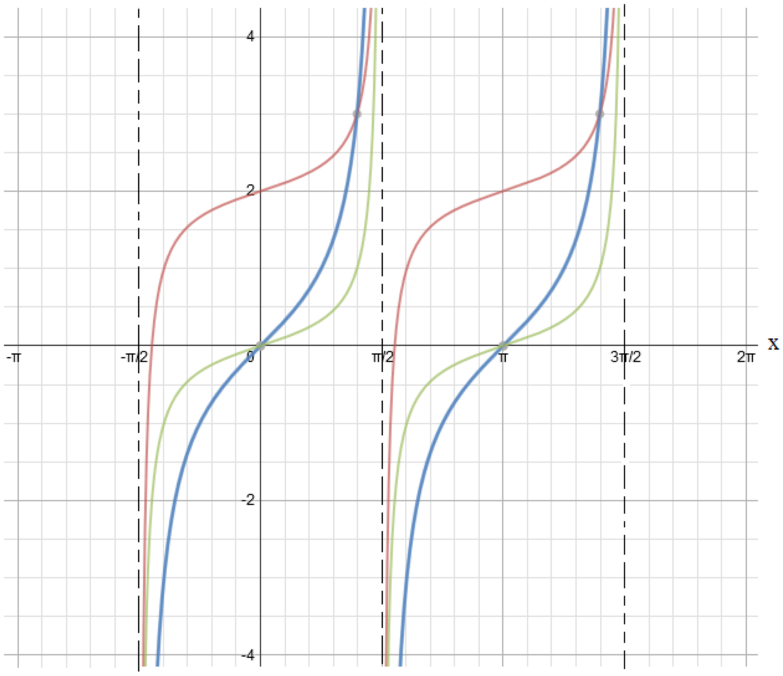
When the coefficient gets larger, the tangent curves *stretch...*

When the coefficient is negative, the output is reflected over the x-axis.

NOTE: The amplitude (A) and vertical shift (D) are numbers *outside the function.* So, they affect changes that are *up and down.*

Tangent Functions: Amplitude and Vertical Shift Illustrations

I. Sketch two cycles of the function $f(x) = \frac{1}{3}\tan x + 2$



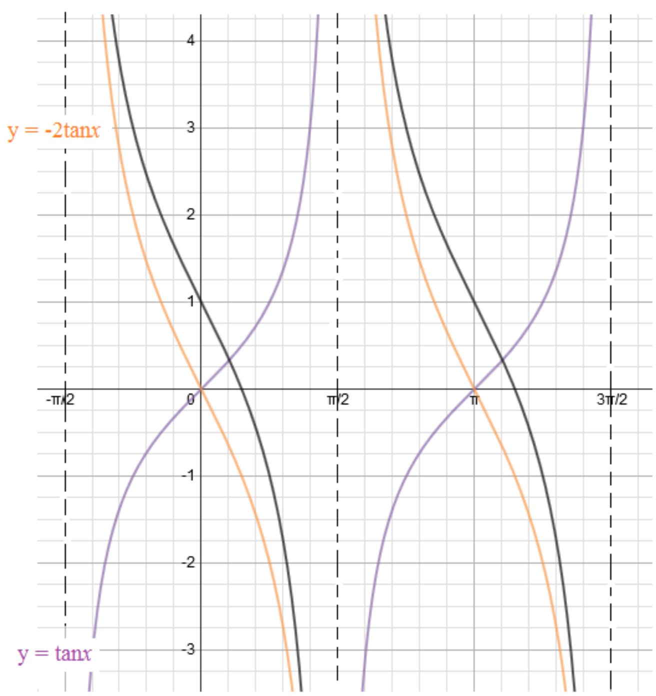
1) Sketch the parent function $y = \tan x$

2) *Decrease ("shrink") the amplitude* by a factor of 3 $y = \frac{1}{3}\tan x$

3) *Shift the graph* up 2 units $y = \frac{1}{3}\tan x + 2$

Note: The asymptotes provide a good outline for your sketch...

II. Sketch $y = -2\tan x + 1$



1) Parent Function $y = \tan x$

2) "Stretch" and "Reflect" $y = -2\tan x$

3) Vertical Shift $y = -2\tan x + 1$

Notice, the "new middle" is at $y = 1$
(instead of the x-axis)

And, because the (A) value is 'negative',
the curve gets 'flipped'/'reflected'
(i.e. goes from upper left to lower right)

Tangent Functions: 4 components

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

> Since the A and D terms are <u>outside</u> the function, the changes affect the vertical components..
> (vertical "stretch", reflection, and shift)
>
> Since the B and C terms are <u>inside</u> the function, they will affect the horizontal shape of the graph....

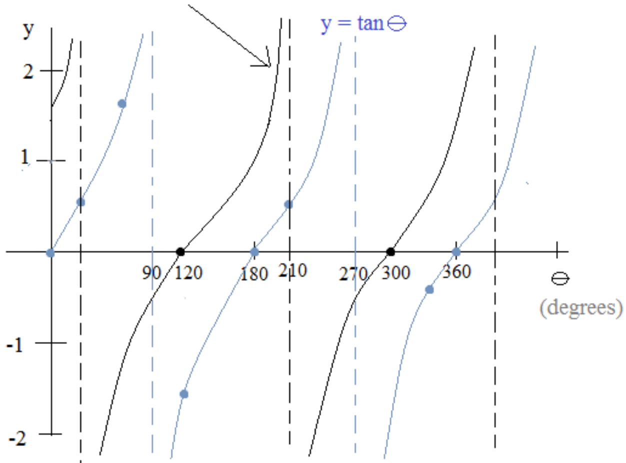
Horizontal ("Phase") Shift:

Question: If $\tan 180^\circ = 0$, then where does $\tan(\theta + 60^\circ) = 0$?

Answer: $\theta = 120^\circ$, because $\tan(120 + 60) = 0$.

Implication: $180^0 \Rightarrow 120^\circ$ (shift 60° <u>to the left)</u>

Example I: $y = \tan(\theta + 60^\circ)$



The curves and the asymptotes shift 60° to the left

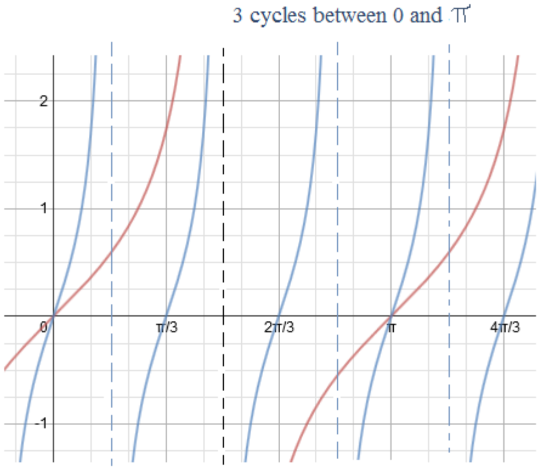
Note: The horizontal shift is the *opposite* direction of the sign.

Period: Horizontal distance required for a periodic function to complete one cycle.

$y = \tan Bx \longrightarrow \text{period} = \frac{\pi}{B}$

Example 2: $y = \tan 3x$ Period: $\frac{\pi}{3}$

3 cycles between 0 and π



$x = \pi/6$ $x = \pi/2$ $x = 5\pi/6$ $x = 7\pi/6$ ⇐ asymptotes

For the parent function $y = \tan x$ (where B = 1) the period is π

As B increases, the period decreases.
In other words, it takes less time to complete one cycle.

And, for $y = \tan Bx$, as B decreases, the period increases.
In other words, it takes more time to complete one cycle.

NOTE: sinx and cosx periods are 2π, but tanx is π

tanx has one cycle... (period is π)

tan3x has three cycles ... (period is $\pi/3$)
(so, 3 times as many asymptotes and intercepts)

Example: Sketch one period of the function $y = 2\tan\frac{1}{4}(x - \frac{\pi}{2}) + 3$

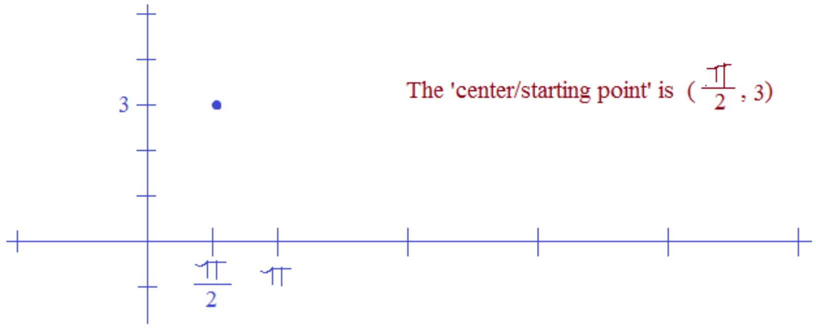
Graphing a Tangent Function: step-by-step

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

The 'D' value is 3, so the vertical shift is up 3 units

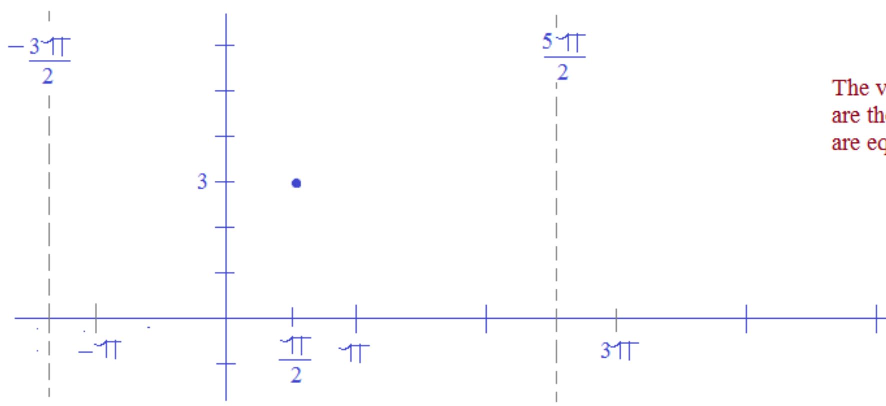
The 'C' value is $\frac{\pi}{2}$, so the horizontal shift is $\frac{\pi}{2}$ to the right



The 'center/starting point' is $(\frac{\pi}{2}, 3)$

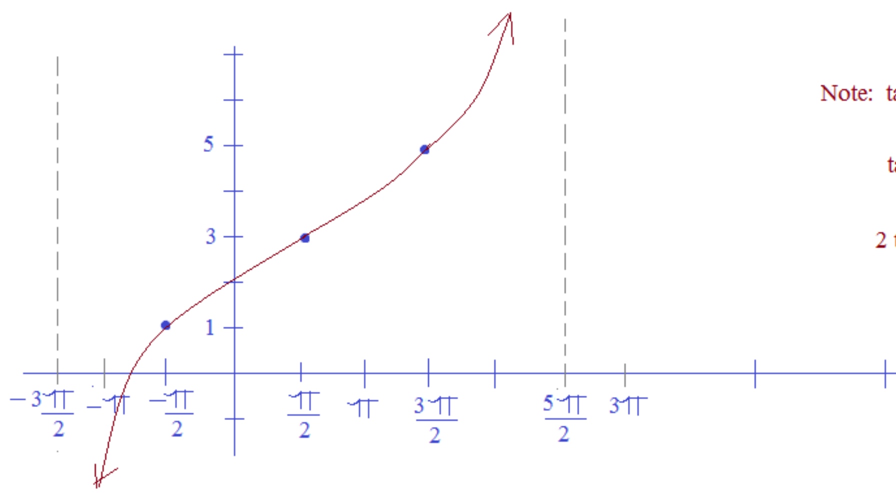
The 'B' value is $\frac{1}{4}$, so the period is $\frac{\pi}{\frac{1}{4}} = 4\pi$

The 'center point' is in the middle of the period... So, we can place vertical asymptotes 2π to the left... and, 2π to the right...



The vertical asymptotes are 4π apart, are the boundaries of the period, and are equidistance from the 'center point'.

The 'A' Value is 2, so the "quarter values" will be up 2 and down 2...



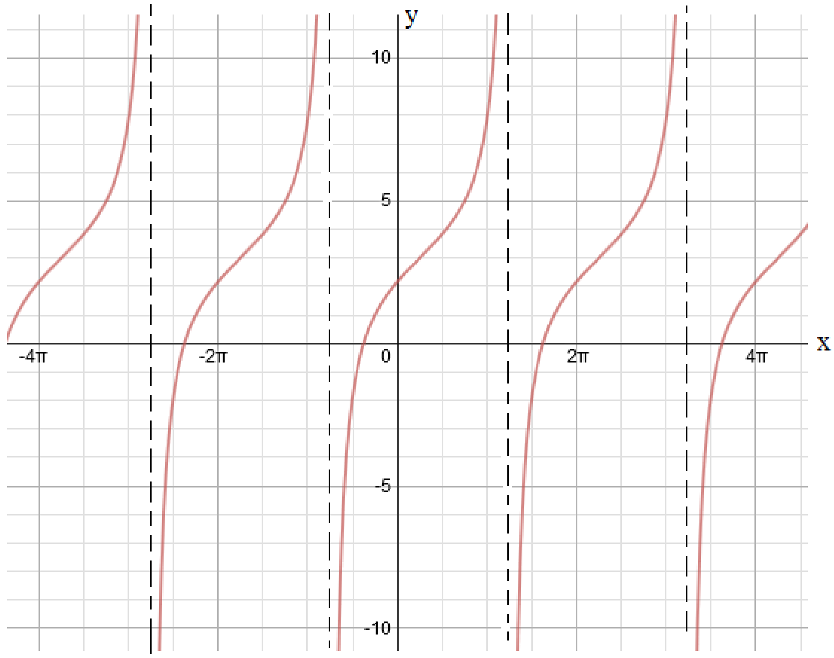
Note: $\tan\frac{\pi}{4} = 1$

$\tan\frac{1}{4}(\frac{3\pi}{2} - \frac{\pi}{2}) = 1$

$2\tan\frac{1}{4}(\frac{3\pi}{2} - \frac{\pi}{2}) + 3 = 5$

Finally, use the 3 points and asymptotes to guide your sketch....

Example: Identify the following tangent function:



$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

Since the midpoint (i.e. the curve's point of inflection) is at 3, the vertical shift is UP 3 — vertical shift (D): +3

One full cycle has a length of 2π. Since the period is 2π, $B = \frac{\pi}{2\pi} = \frac{1}{2}$ — period (B): $\frac{1}{2}$

If the period ("B" value) is 1/2, then the asymptotes would be at π 3π 5π etc..

But, in the graph, the asyptotes are at $\frac{5\pi}{4}$ $\frac{13\pi}{4}$ $\frac{21\pi}{4}$ ⟶ horizontal shift to the right (C): ("phase") $-\frac{\pi}{4}$

In a 1/4 cycle move, the value goes from 0 to 1... (i.e. $\tan(0) = 0$ $\tan(\frac{\pi}{4}) = 1$

In the above graph, 1/4 of a cycle is $\frac{\pi}{2}$ ... At $x = 0$, the output is 3

At $x = \frac{\pi}{2}$, the output is 5

This is an increase of 2 (instead of 1) — amplitude (A): 2

$$y = 2\tan\frac{1}{2}\left(x - \frac{\pi}{4}\right) + 3$$

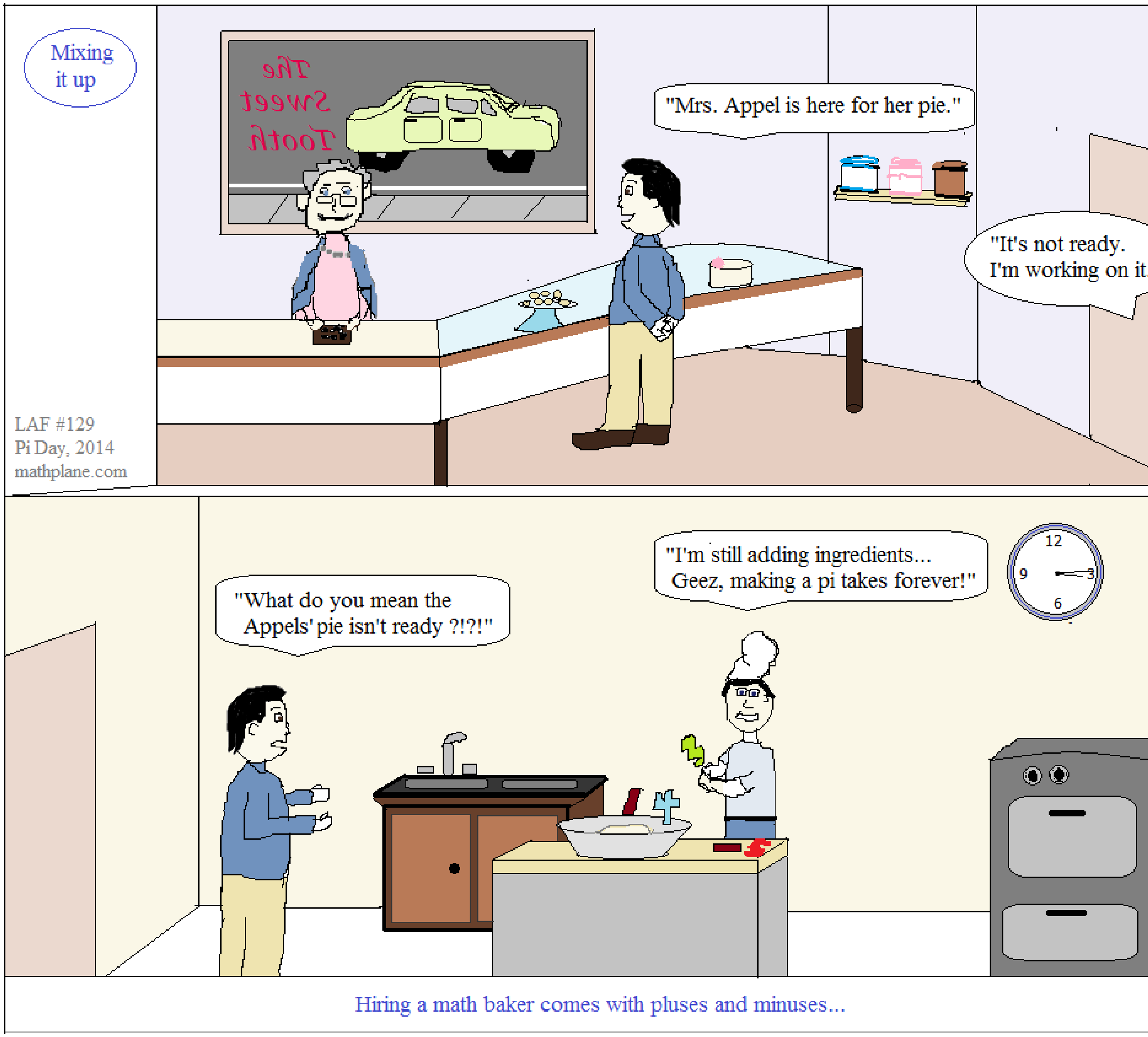
Test points to confirm your equation!

If $x = \pi$: $2\tan\frac{1}{2}\left(\pi - \frac{\pi}{4}\right) + 3 = 2\tan\frac{3\pi}{8} + 3$

$= 2 \cdot 2.41 + 3 \approx 7.8$ ✓

If $x = \frac{-\pi}{2}$: $2\tan\frac{1}{2}\left(\frac{-\pi}{2} - \frac{\pi}{4}\right) + 3 = 2\tan\frac{-3\pi}{8} + 3$

$= 2 \cdot -2.41 + 3 \approx -1.8$ ✓

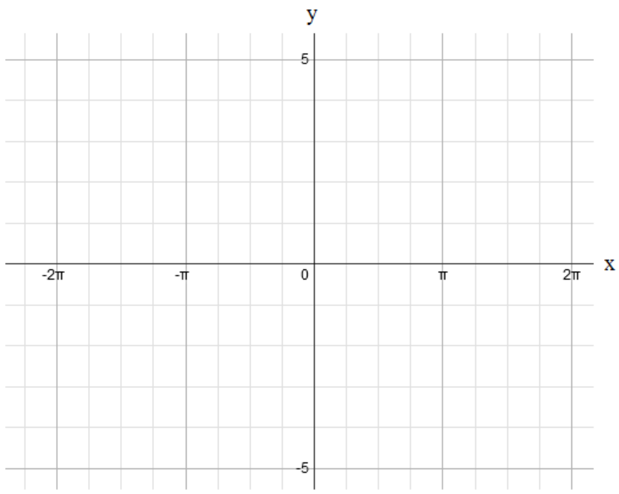

Mixing it up
The Sweet Tooth
"Mrs. Appel is here for her pie."
"It's not ready. I'm working on it."
LAF #129
Pi Day, 2014
mathplane.com
"What do you mean the Appels' pie isn't ready ?!?!"
"I'm still adding ingredients... Geez, making a pi takes forever!"
12
9
3
6
Hiring a math baker comes with pluses and minuses...

Practice ->

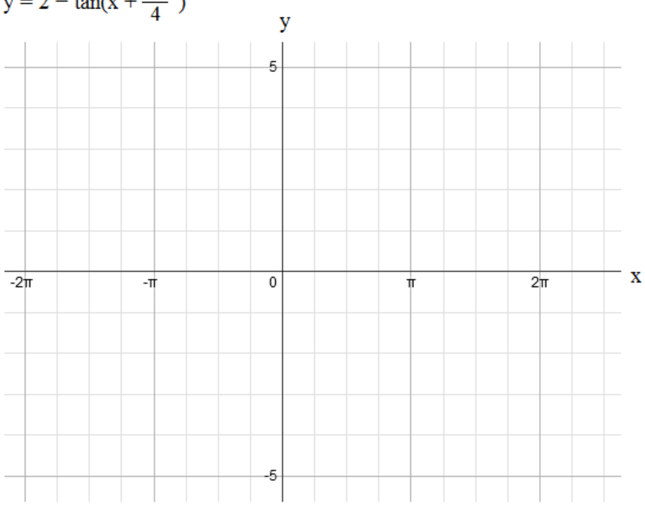
Tangent Function Practice

I. Graphing: Sketch each of the following equations. (Include at least 2 periods. And, label the asymptotes.)

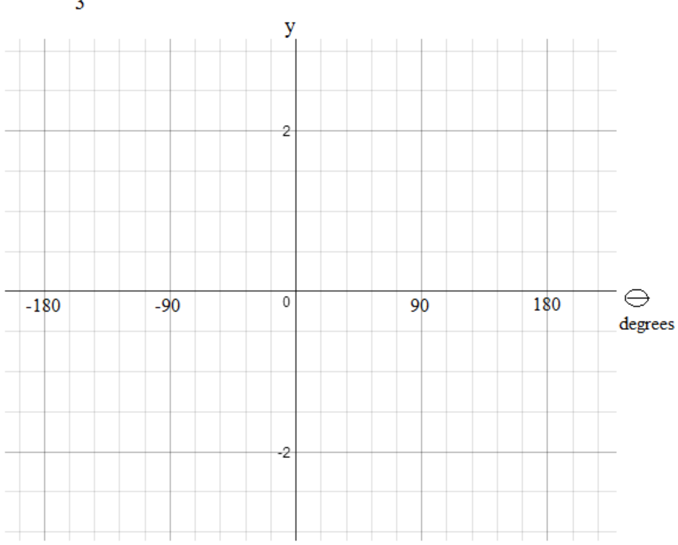
A) $y = 2\tan x - 3$



B) $y = 2 - \tan(x + \frac{\pi}{4})$

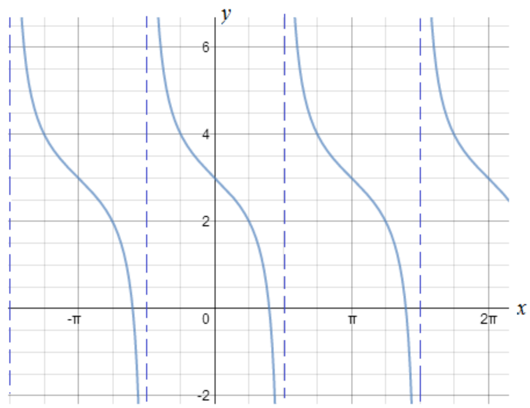


C) $y = \frac{\tan 2\theta}{3}$

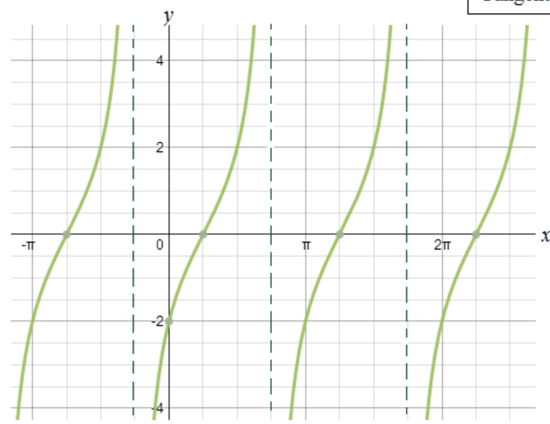


II. Identifying: Determine the equations of the following.

A)



B)



III. For the function $f(x) = \tan(x - \frac{\pi}{2})$, determine the

a) domain

b) range

c) maximum

d) minimum

e) x-intercepts (or, zeros)

f) y-intercept

Challenge Question: Solve algebraically. Then, graph to confirm your solution.

$\text{Cos}x = \text{Tan}x$ (in the interval $0^{\circ} < x < 360^{\circ}$)

LanceAF #111 11/7/13
mathplane.com

Skip got a disappointing C+ on the math assignment...
(but, he did see 2 awesome reruns of Magnum P.I.)

Answers-→

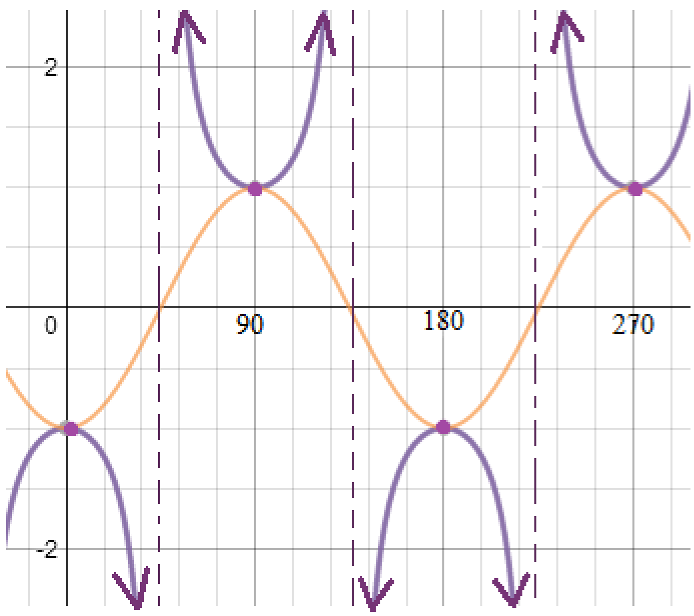
Thanks for checking out this preview.

You can find the solutions and other content posted at mathplane.com (in the trigonometry section).

Or, check out the mathplane stores at

TES.com and TeachersPayTeachers.

Reciprocals Functions



Topics include sketching secant, cotangent, & cosecant, solving systems, transformations, and more.

Periodic trig functions: "The reciprocals"

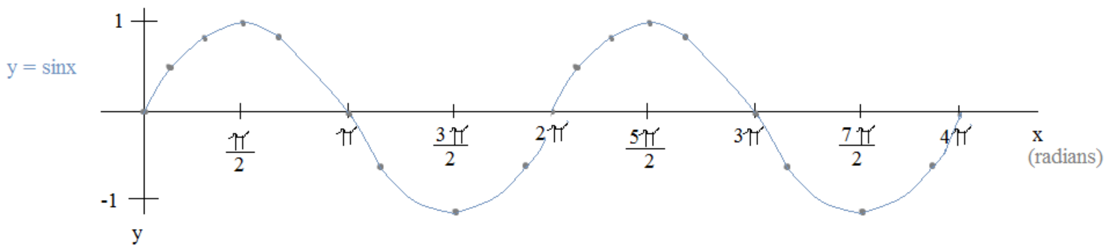
Review:

$y = \sin\theta$

The following is a table of chosen values:

θ	0	30	60	90	120	180	210	270	330	360
y	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	$\frac{-1}{2}$	-1	$\frac{-1}{2}$	0

Then, plot the points on a graph....



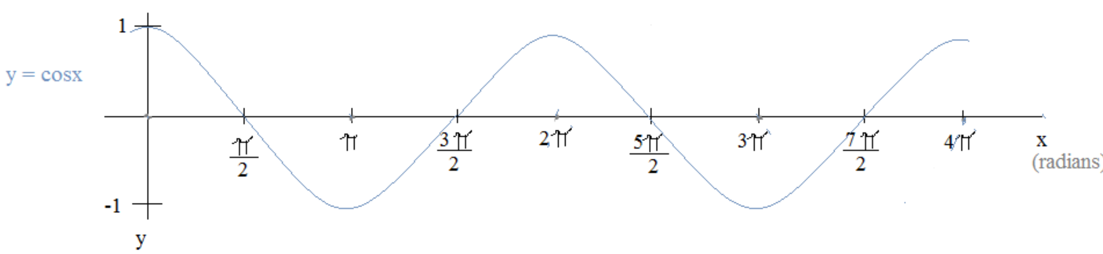
A cycle is a complete pattern repetition. This graph contains 2 cycles.

The period is 2π (Horizontal length of one cycle)

$y = \cos\theta$

The following is a table of chosen values:

θ	0	30	60	90	120	180	210	270	330	360
y	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	-1	$\frac{-\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$	1

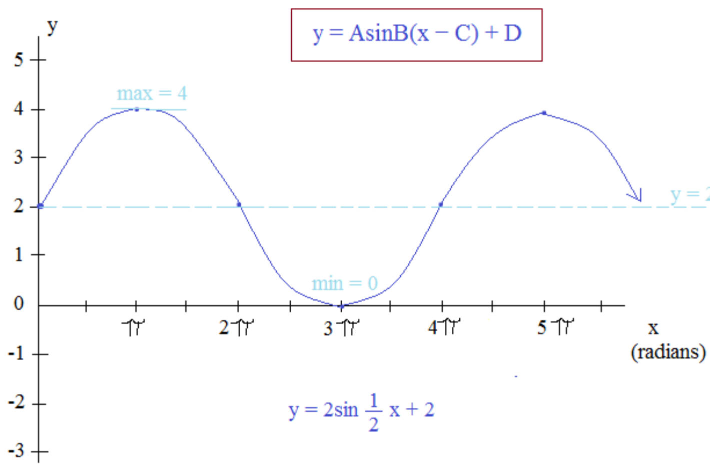


A cycle is a complete pattern repetition. This graph contains 2 cycles.

The period is 2π (Horizontal length of one cycle)

Example I:

$y = A\sin B(x - C) + D$



D (vertical shift) : The *center* that the sine wave is oscillating over is $y = 2$. Therefore, the vertical shift is up 2 units.

A (amplitude) : The maximum y-value is 4, and the minimum y-value is 0 --- a total span of 4 units. The amplitude is 1/2 of that amount: 2 units

C (horizontal shift) : Since $y = 2$ at 0 radians, there is no horizontal shift.

B (period) : The horizontal distance of one cycle is 4π.

Since $\frac{2\pi}{B} = 4\pi$, $B = \frac{1}{2}$

Example II:

$y = A\cos B(x - C) + D$

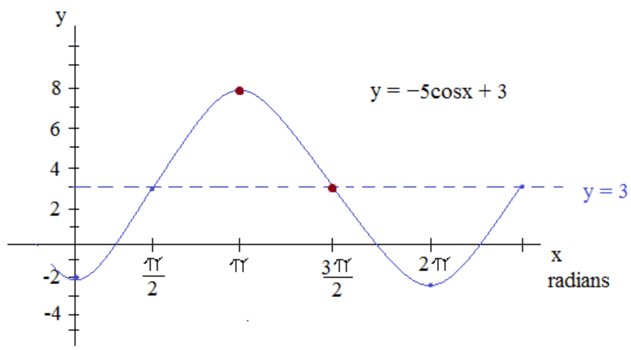
amplitude: A = -5 (negative, so "faces down")

period: 2π

Vertical shift: D = 3
Up 3

Horizontal shift: C
none..

max 8 ; min -2



Check: $x = \pi$

$x = \frac{3\pi}{2}$

At $x = \pi$,

$y = -5\cos(\pi) + 3$

$= -5(-1) + 3 = 8$ ✓

At $x = \frac{3\pi}{2}$

$y = -5\cos(\frac{3\pi}{2}) + 3$

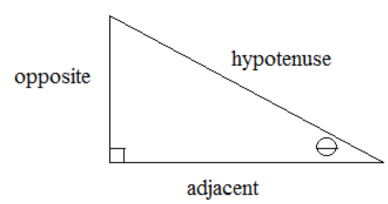
$= -5(0) + 3 = 3$ ✓

---

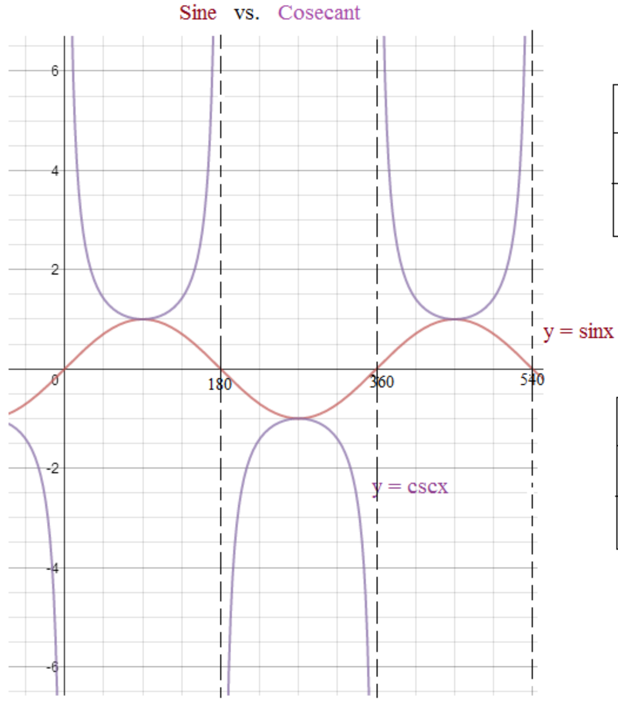
sine = $\frac{\text{opposite}}{\text{hypotenuse}}$ ⇨ cosecant = $\frac{1}{\text{sine}} = \frac{\text{hypotenuse}}{\text{opposite}}$

cosine = $\frac{\text{adjacent}}{\text{hypotenuse}}$ ⇨ secant = $\frac{1}{\text{cosine}} = \frac{\text{hypotenuse}}{\text{adjacent}}$

tangent = $\frac{\text{opposite}}{\text{adjacent}}$ ⇨ cotangent = $\frac{1}{\text{tangent}} = \frac{\text{adjacent}}{\text{opposite}}$



Sine vs. Cosecant



θ degrees	0	30	60	90	100	120	150	170	175	180
Sine	$\frac{0}{1}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	.985	.866	.5	.174	.087	0
Cosecant	undefined	$\frac{2}{1}$	$\frac{2}{\sqrt{3}}$	1	1.015	1.155	2	5.759	11.47	undefined

$\text{Sin}\,\theta = \frac{1}{\text{Csc}\,\theta}$

θ degrees	180	210	240	270	300	315	330	340	350	360
Sine	$\frac{0}{1}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	-1	$-.866$	$-.707$	$-.5$	$-.342$	$-.174$	0
Cosecant	undefined	$\frac{-2}{1}$	$\frac{-2}{\sqrt{3}}$	-1	-1.155	-1.414	-2	-2.92	-5.759	undefined

NOTE: Where there is a *zero for sinx*, there is an *asymptote for cscx*

Observation: The intersections of sinx and cscx are at y = 1 and y = -1

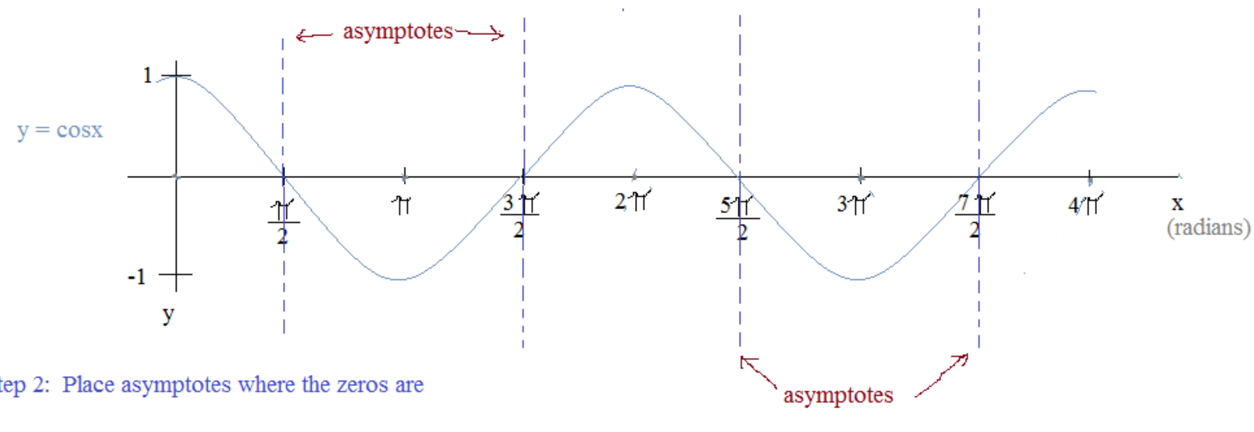
Sketching a reciprocal trig function

Sketch the 'original' trig function.
Place asymptotes where the zeros are.
Identify the maximums/minimums
Plot easy points and extend.

Example: Sketch y = secx on the interval [0, 4 π]

Step 1: Sketch the 'original' function

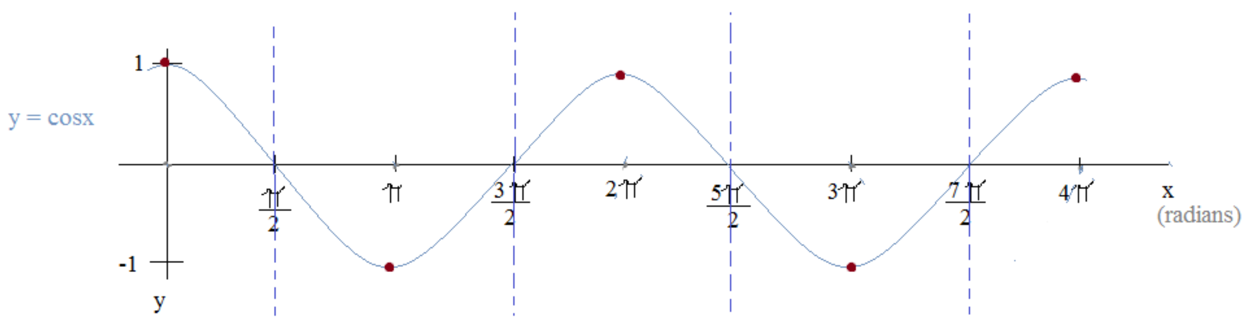
The reciprocal of secx is cosx:



Step 2: Place asymptotes where the zeros are

Step 3: Maximums and Minimums are the same

cos(0) = sec(0) = 1   cos(π) = sec(π) = -1   cos(2 π) = sec(2 π) = 1   etc...

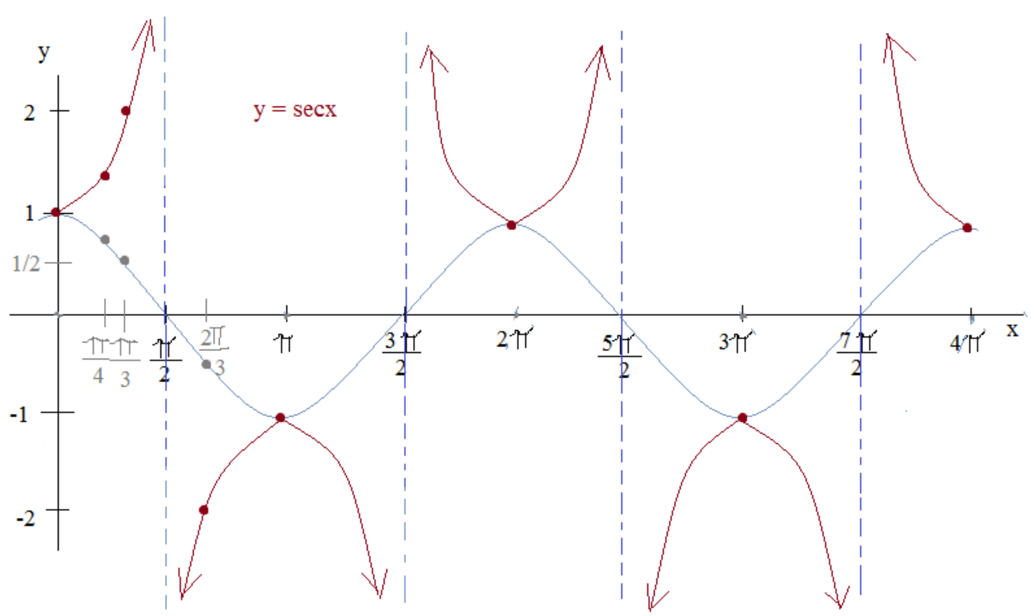


Step 4: Plot easy points and extend

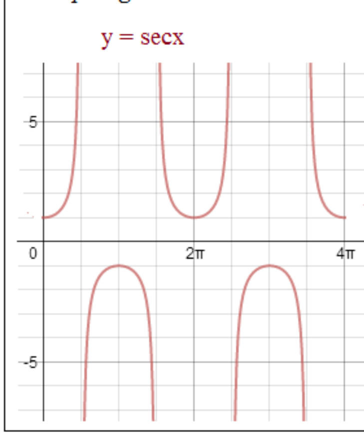
We know (from the unit circle) that cos(π/3) = 1/2 And, so sec (π/3) = 2/1

cos(2π/3) = -1/2 And, so sec (2π/3) = −2/1

cos(π/4) = .707 (approx.) then, sec (π/4) = 1.41 (approx)



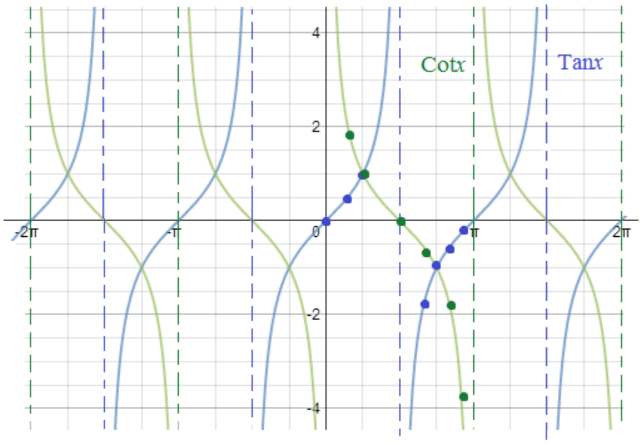
Graphing Calculator:

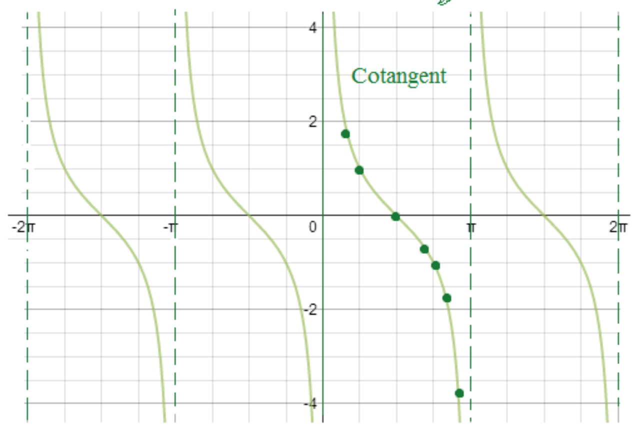


Sketching Cotangent Functions

Using a sample of points, we can discover a pattern...

x radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
Tan$x$	0	$\frac{1}{\sqrt{3}}$	1	undefined	-1.732	-1	-.577	-.268	0
Cot$x$	undefined	$\frac{\sqrt{3}}{1}$	1	0	-.577	-1	-1.732	-3.732	undefined





Observation: If you take the graph of Tan$x$, reflect it over the x-axis and shift $\frac{\pi}{2}$ to the left (or right), it becomes Cot$x$

$$\text{Cot}x = -\text{Tan}(x + \frac{\pi}{2})$$

Example: Sketch the function $f(x) = \cot(\frac{1}{4}x) + 2$

Recognizing the general periodic equation: $y = A\cot B(x - C) + D$

A: Amplitude is 1 (the shape of the function remains)

B: $\frac{1}{4}$ so, it takes 4π to move one period

period $= \frac{\pi}{B}$

C: Horizontal shift is 0

D: Vertical shift is UP 2 units

$\text{Cot}(0) = \frac{1}{0}$ and, since there is no horizontal shift, asymptotes are placed at the beginning (or end) of each period...

asymptotes at .... -4π, 0, 4π .....

The 'center' of each period is the midpoint between the asymptotes and, because there is a vertical shift of +2

$y = 2$ at ..., -2π, 2π, ...

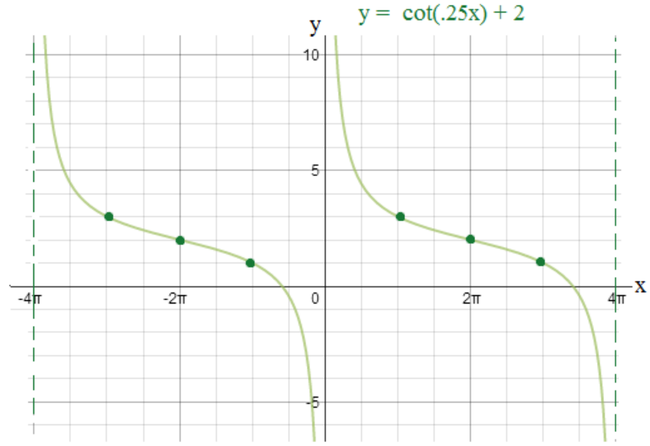
Then, since the amplitude is 1, the 'quarter' points will be up 1 unit and down 1 unit....

$y = 3$ at ..., -3π, π, ...

$y = 1$ at ..., $-\pi$, 3π, ...

Sketching Cotangent and Tangent Functions

1) Identify the parts
2) Draw the vertical asymptotes
3) Plot the points of inflection (midpoints of each period)
4) Plot the 'quarter points'
5) Extend the curves



Graphing a cosecant function with transformations

Sketch the 'original' trig function.
Place asymptotes where the zeros are.
Identify the maximums/minimums
Plot easy points and extend.

Example: Sketch $y = 3\csc\frac{1}{2}(x - 60^{\circ}) + 4$

Step 1: Lightly sketch the 'original' function (i.e. reciprocal of this reciprocal)

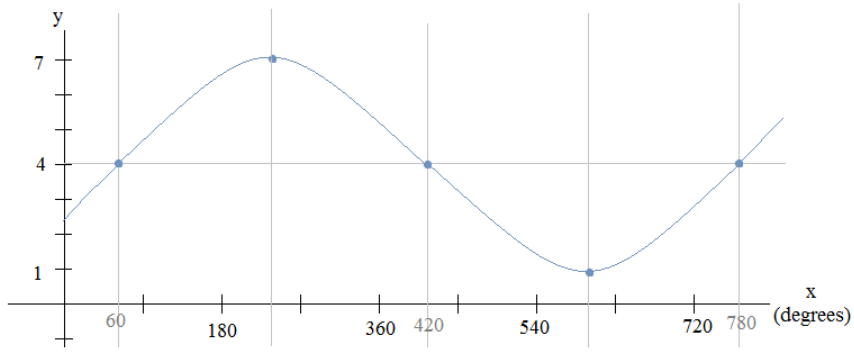
$$y = 3\sin\frac{1}{2}(x - 60^{\circ}) + 4$$

Amplitude: 3

Period: $\dfrac{2(180)}{\frac{1}{2}} = 720$ degrees

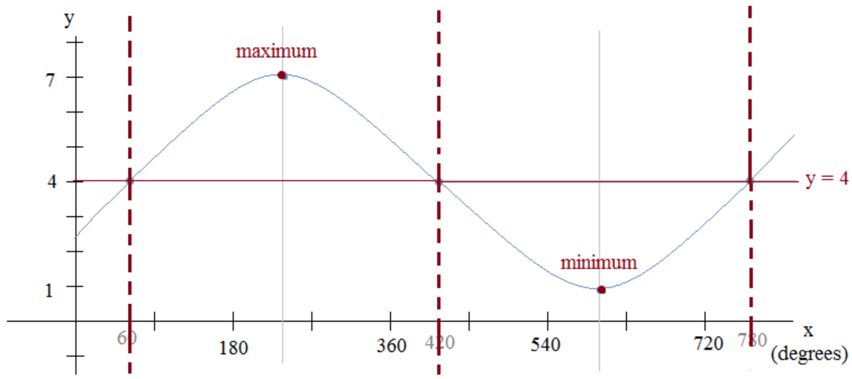
Horizontal shift: 60 degrees to the right ("phase")

Vertical shift: UP 4 units



Step 2: Place asymptotes where the zeros are...

Since the equation is shifted up 4 units, the "zeros" are at y = 4



Step 3: Identify the maximums and minimums

Since the amplitude is 3, the maximums are 3 above the 'wave center' and the minimums are 3 below the center.

maximums: where $y = 7$

minimums: where $y = 1$

Step 4: Plot a few points and extend

$$y = 3\csc\frac{1}{2}(x - 60^{\circ}) + 4$$

If x = 360 degrees

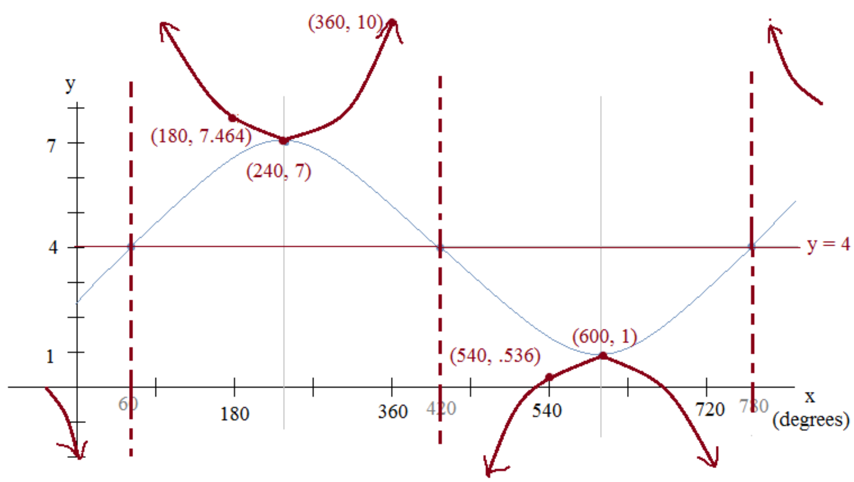
$= 3\csc(150^{\circ}) + 4 = 3(2) + 4 = 10$

If x = 540 degrees

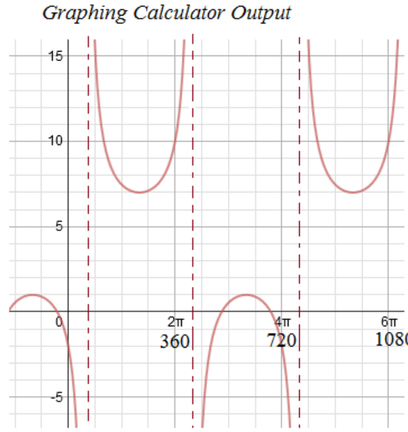
$= 3\csc(240^{\circ}) + 4 = 3(\frac{2}{-\sqrt{3}}) + 4 = .536$

If x = 180 degrees

$= 3\csc(60^{\circ}) + 4 = 3(\frac{2}{\sqrt{3}}) + 4 = 7.464$

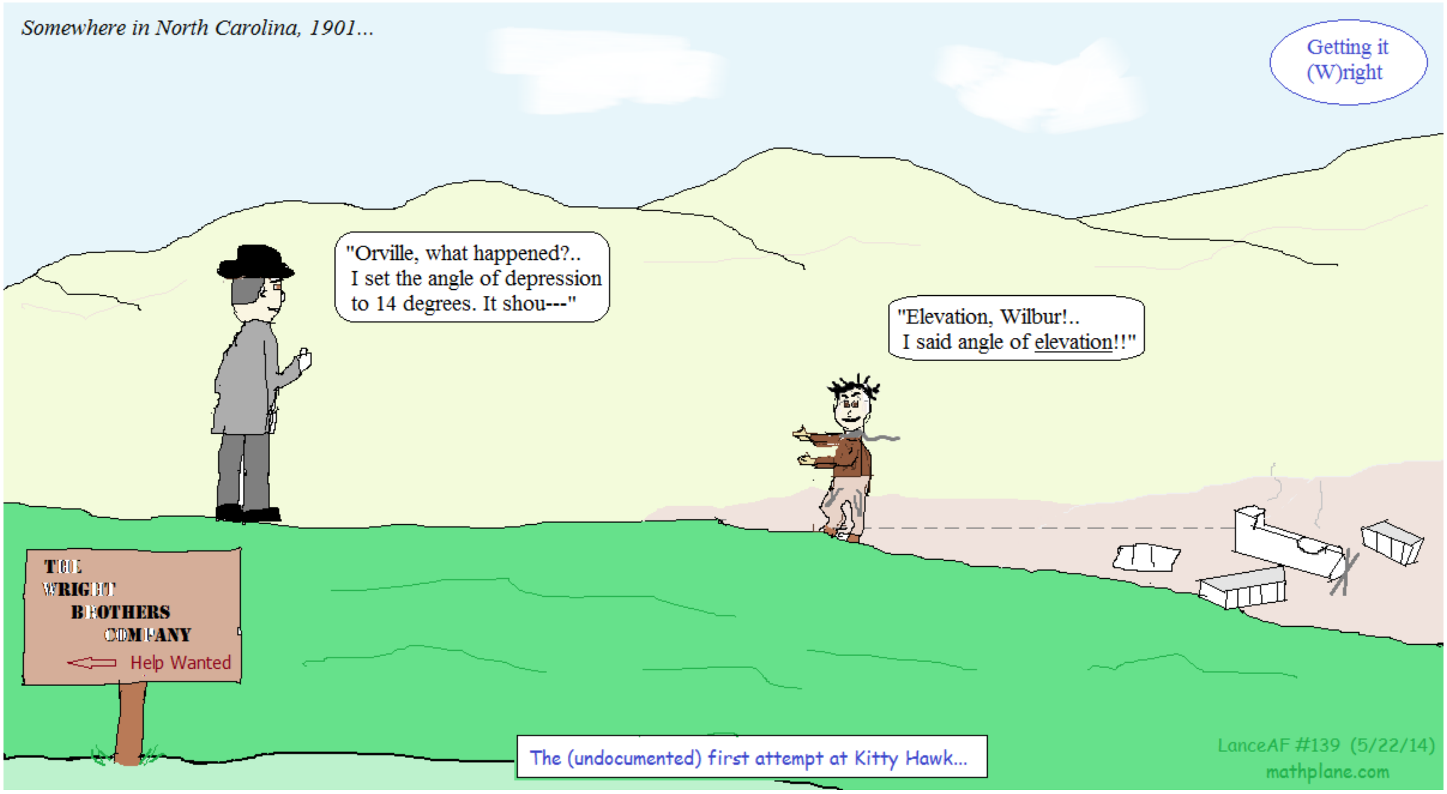


Graphing Calculator Output



Note: the maximum and minimum points of sine are also points in cosecant.

(240, 7) and (600, 1)



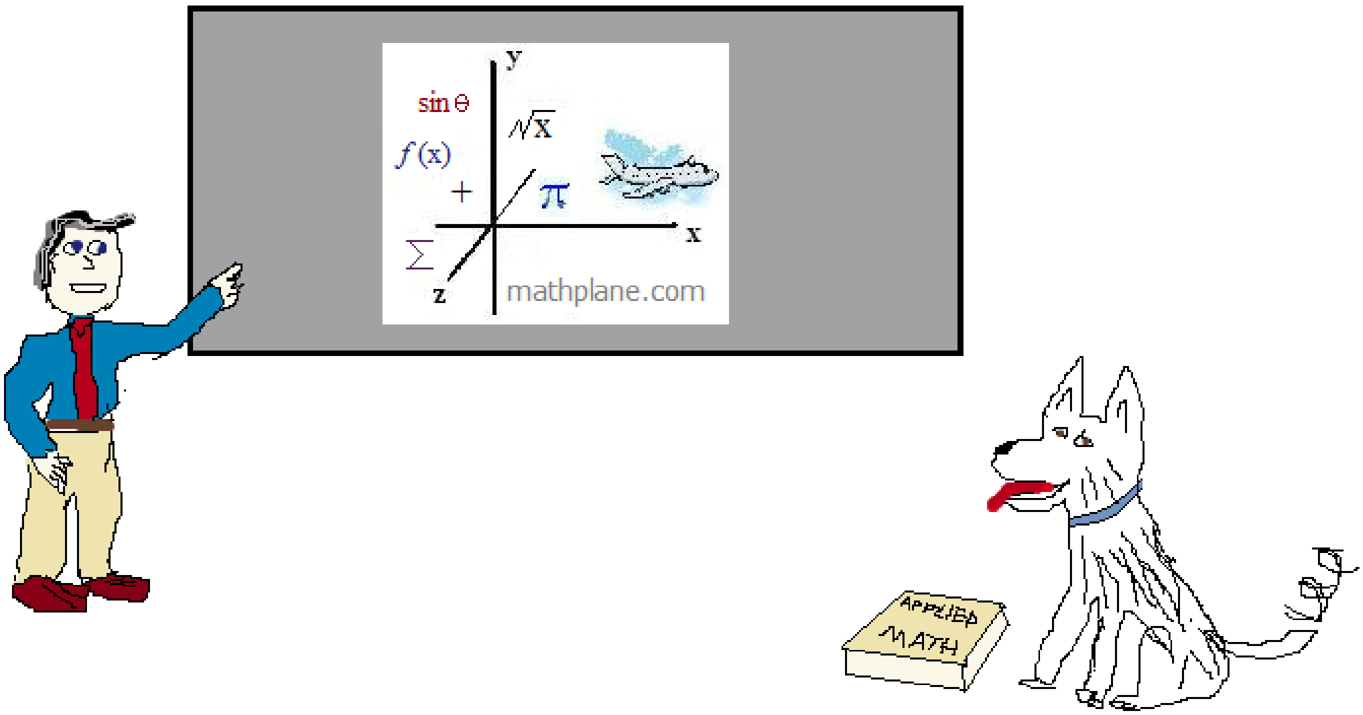
This is a preview file.

If you'd like to view the reciprocals practice test (with solutions), download the product file from our stores at TES.com and TeachersPayTeachers.com. (We appreciate the support!)

***Or, find the material at mathplane.com (in the Trigonometry section).

If you have questions, suggestions, or requests, let us know.

Cheers



All proceeds go to site maintenance and improvement. (Plus, treats for Oscar the Dog!)

Also, at Facebook, Google+, and Pinterest