Trigonometry Review 1

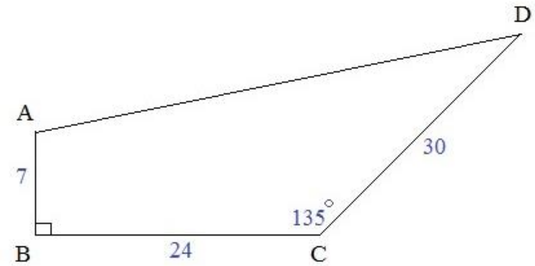
Questions and Examples

Topics include graphing, angular vs linear speed, law of sines, area, geometry theorems, word problems, and more.

Trigonometry Review

Find the area of quadrilateral ABCD.

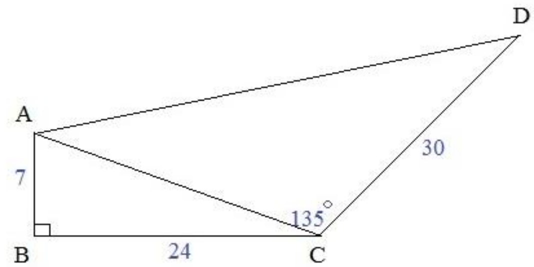
Find the perimeter of quadrilateral ABCD.



Step 1: Divide ABCD into two triangles.

(Take advantage of right angle $\angle ABC$)

Draw diagonal $\overline{AC}$.

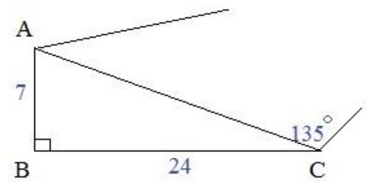


Step 2: Find area of right triangle.

Area of triangle: $\frac{1}{2}bh$

Area $\triangle ABC = 1/2(24)(7)$

$= 84$



Now, let's find the area of $\triangle$ ACD.

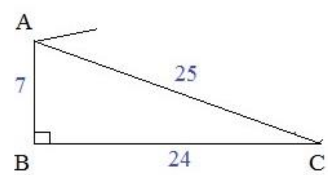
Step 3: Find the length of $\overline{AC}$.

Pythagorean Theorem: $a^2 + b^2 = c^2$

$(7)^2 + (24)^2 = (AC)^2$

$625 = (AC)^2$

$\overline{AC} = 25$



Special Right Triangles include: 3-4-5 5-12-13 8-15-17 7-24-25

Step 4: Find $\angle ACD$

$\angle ACD = \angle BCD - \angle BCA$

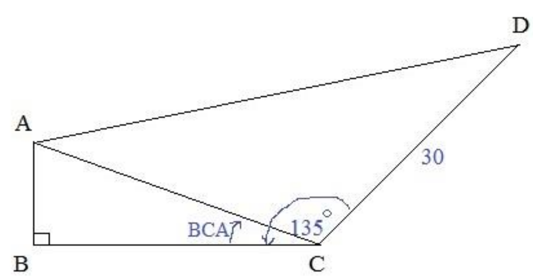
Tangent $\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

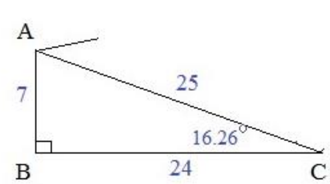
$\tan \angle BCA = 7/24$

$\angle BCA \cong 16.26^\circ$

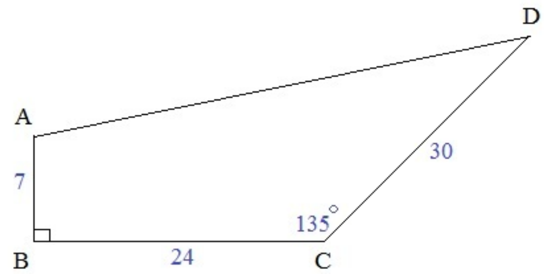
$\angle BCD = 135^\circ$

$\angle ACD = 135^\circ - 16.26^\circ = 118.74^\circ$





Trigonometry Review

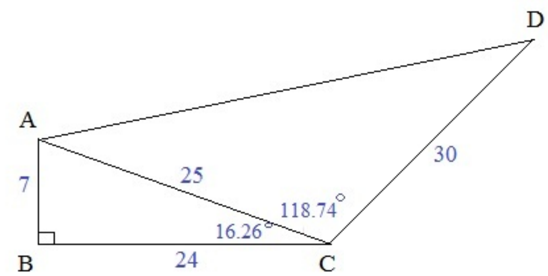


Find the area of quadrilateral ABCD.

Find the perimeter of quadrilateral ABCD.

Step 5: Find area of $\triangle ACD$

Area of triangle = $\frac{1}{2}ab\text{Sin}C$

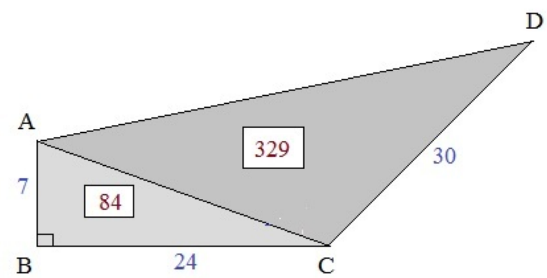


Area of $\triangle$ ACD = 1/2(25)(30)Sin118.74

= 375 * Sin118.74

= 375 x .877 = 328.875

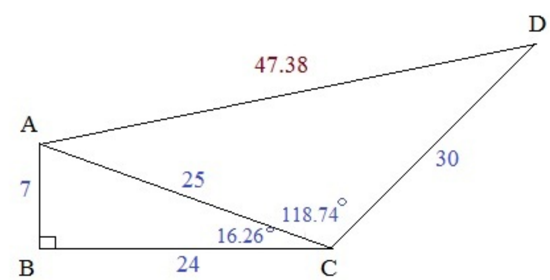
Step 6: Combine the triangles!



$\text{Area}_{ABC} + \text{Area}_{ACD} = \text{Area}_{ABCD}$

84 + 329 = 413

To find the Perimeter, use law of Cosines:



Law of Cosines $c^2 = a^2 + b^2 - 2ab\text{Cos}C$

$\overline{AD}^2 = (25)^2 + (30)^2 - 2(25)(30)\text{Cos } 118.74$

= 625 + 900 - 1500(-.48)

= 1525 + 720

$\overline{AD}^2 = 2245$

$\overline{AD} = 47.38$

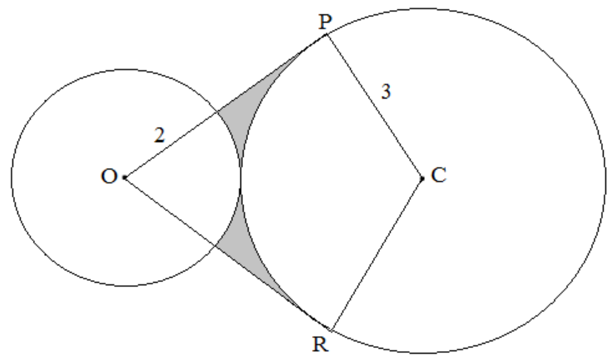
Perimeter = 7 + 24 + 30 + 47.38 = 108.38

Example: OP and OR are external tangents

a) find m∠POR

m∠PCR

b) find the shaded area

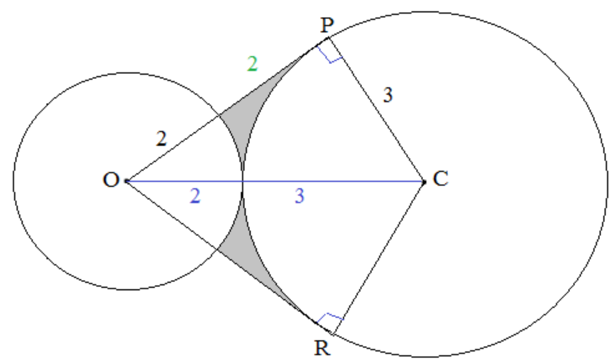


Tangents and radii form right angles...

OP and OR are congruent (because they are external tangents that meet at a common point)

△POC and △ROC are congruent right triangles...

Pythagorean Theorem: 3-4-5 right triangles



a) $\sin(\angle POC) = \frac{3}{5}$

$\angle POC = 36.87^\circ$

then, $\angle POR = 2 \times \angle POC = \boxed{73.74^\circ}$

$\cos(\angle PCO) = \frac{3}{5}$

$\angle PCO = 53.13^\circ$

then, $\angle PCR = 2 \times \angle PCO = \boxed{106.26^\circ}$

b) To find the shaded area:

1) area of each right triangle: $\frac{1}{2}$(base)(height) $= \frac{1}{2}(4)(3) = 6$

so, area of the triangles is 12

2) area of each sector

Sector in circle O: $\frac{\theta}{360}\pi(\text{radius})^2$

$\frac{73.74}{360}\pi(2)^2 = 2.57$

Sector in circle C: $\frac{\theta}{360}\pi(\text{radius})^2$

$\frac{106.26}{360}\pi(3)^2 = 8.35$

3) Calculate the shaded area

Shaded = total triangles − sectors

$= (12) - (2.57 + 8.35) = \boxed{1.08 \text{ square units}}$

Trigonometry Review 2

Solve the following algebraically. Then, verify graphically.

$$\cos\theta = \sin 2\theta \quad \text{for} \quad 0^\circ \le \theta < 360^\circ$$

Algebraically:

$\cos\theta = \sin 2\theta$

$\cos\theta = 2\sin\theta\cos\theta$ (Trig identity)

$2\sin\theta\cos\theta - \cos\theta = 0$

(Important Note: If you divide both sides by Cos ⊖ , you may eliminate one of the solutions! Instead, move all terms to one side and factor out the Cos ⊖)

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ → $\theta = 90^\circ$, $\theta = 270^\circ$

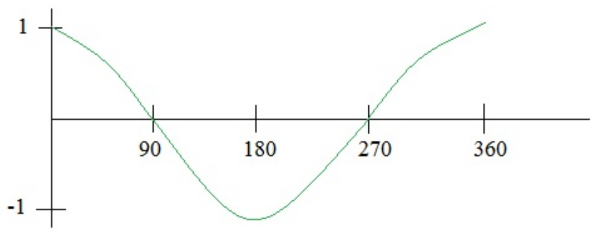
or

$2\sin\theta - 1 = 0$

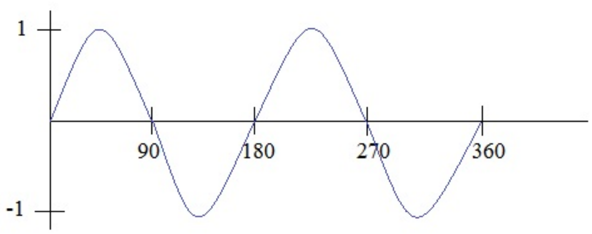
$\sin\theta = \frac{1}{2}$ → $\theta = 30^\circ$, $\theta = 150^\circ$

Graphically:

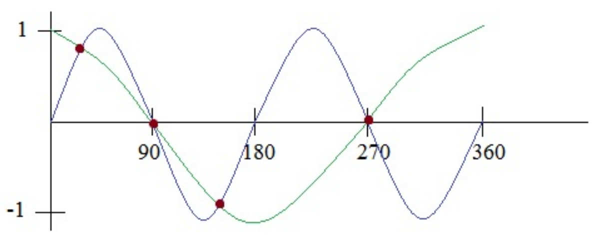
Cos ⊖



Sin 2⊖



Cos ⊖ and Sin 2⊖



Points of intersection are the solutions!

Trigonometry Review: Special Periodic Functions

Graph the following Trig Functions:

1) $f(x) = |\sin x|$ $\quad 0 \leq x < 2\pi$

2) $y = |2\sin\theta + 1|$ $\quad 0 \leq \theta < 360$

3) $y = \sin^2 x + \cos^2 x$

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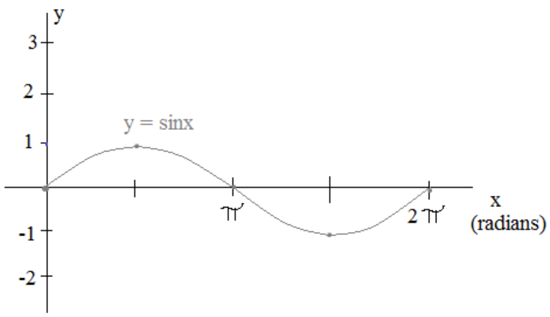
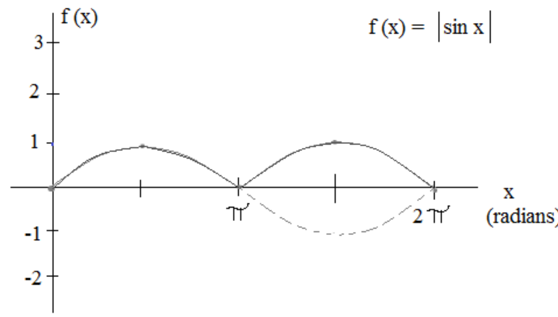
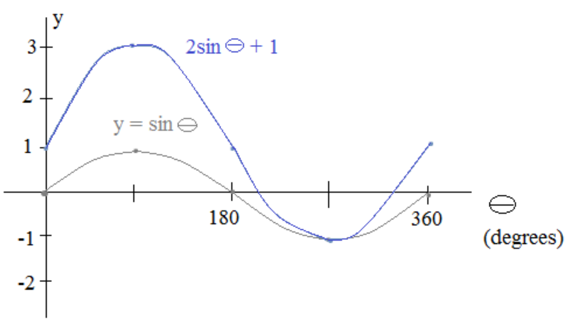
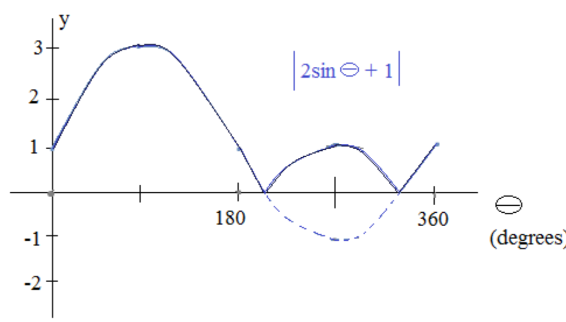
1) First graph the parent function sinx. Then, reflect all the negative outputs (y-values) over the x-axis.





2) Graph the function $y = 2\sin\theta + 1$ Then, reflect all the negative outputs (y-values) over the x-axis.

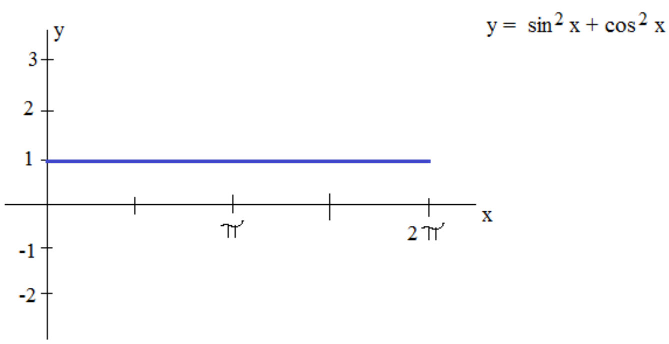




3) (Trigonometry/Pythagorean Identity) $\sin^2 x + \cos^2 x = 1$

Therefore, $y = \sin^2 x + \cos^2 x = 1$

Or, for every input x, the outcome y = 1



Trigonometry Review: Inverse Cosine

$$y = 3 + 4\cos 2(\theta + 10^\circ)$$

1) Transform the above into an equation where θ is expressed in terms of y.

2) Find the first 3 positive values of θ for $y = 5$

3) Sketch a graph to verify the answers are reasonable.

1)

$$y = 3 + 4\cos 2(\theta + 10^\circ) \quad \text{(subtract 3)}$$

$$y - 3 = 4\cos 2(\theta + 10^\circ) \quad \text{(divide by 4)}$$

$$\frac{y-3}{4} = \cos 2(\theta + 10^\circ) \quad \text{(multiply by arccos)}$$

$$\arccos \frac{1}{4}(y-3) = 2(\theta + 10^\circ) \quad \text{(multiply by } \tfrac{1}{2}\text{)}$$

$$\frac{1}{2}\arccos \frac{1}{4}(y-3) = (\theta + 10^\circ) \quad \text{(subtract } 10^\circ\text{)}$$

$$\boxed{\frac{1}{2}\arccos \frac{1}{4}(y-3) - 10^\circ = \theta}$$

2)

$$\theta = \frac{1}{2}\arccos \frac{1}{4}(y-3) - 10^\circ$$

(for $y = 5$)

$$\theta = \frac{1}{2}\arccos \frac{1}{4}(5-3) - 10^\circ$$

$$\theta = \frac{1}{2}\arccos\left(\frac{1}{2}\right) - 10^\circ$$

$$\arccos\left(\frac{1}{2}\right) = 60^\circ + n360^\circ$$
$$300^\circ + n360^\circ$$
(n is any integer)

For $\arccos\left(\frac{1}{2}\right) = 60^\circ$: $\frac{1}{2}(60^\circ) - 10^\circ = \boxed{20^\circ}$

For $\arccos\left(\frac{1}{2}\right) = 300^\circ$: $\frac{1}{2}(300^\circ) - 10^\circ = \boxed{140^\circ}$

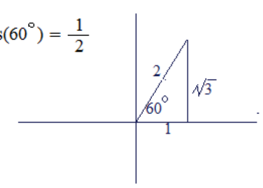
For $\arccos\left(\frac{1}{2}\right) = 420^\circ$: $\frac{1}{2}(420^\circ) - 10^\circ = \boxed{200^\circ}$

1st three positive values for y = 5

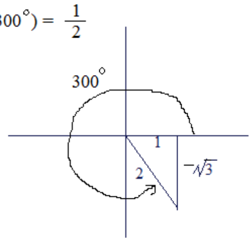
NOTES:

1) The cosine of 60°, 300°, and all the coterminal angles is $\frac{1}{2}$

$\cos(60^\circ) = \frac{1}{2}$



$\cos(300^\circ) = \frac{1}{2}$



2) Multiplying Trig Inverses -- Examples:

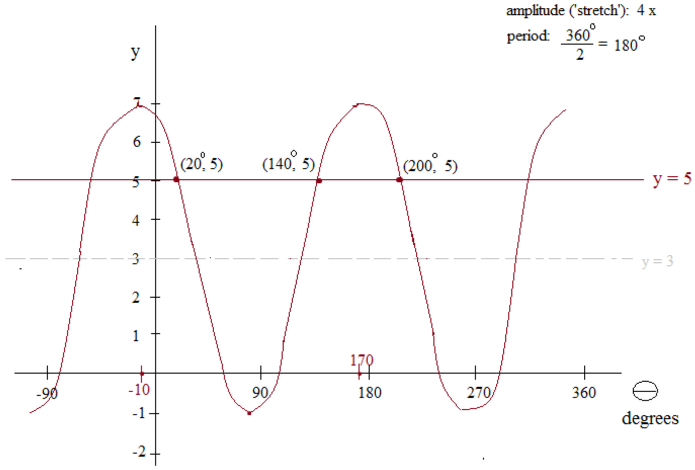
$\text{Arccos}(\cos\theta) = \theta$

or

$\sin^{-1}(\sin x) = x$

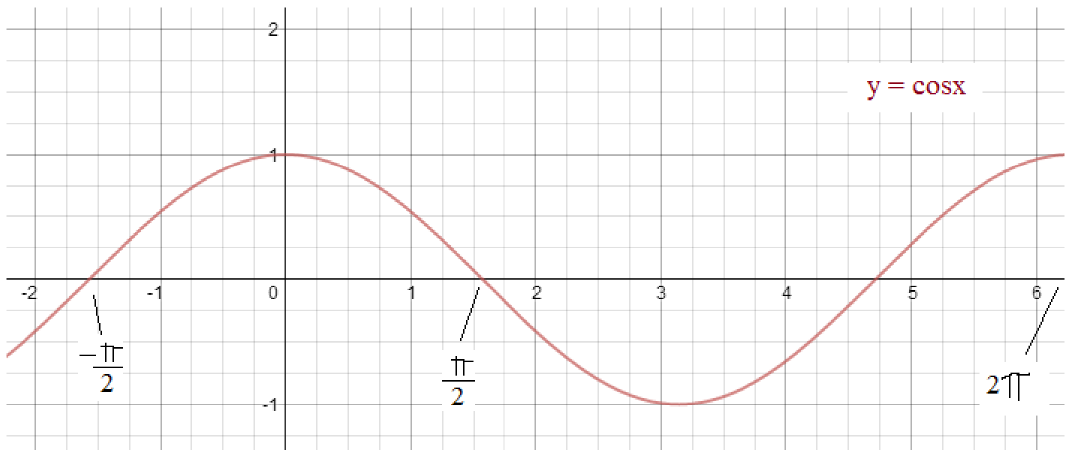
3) Graph $y = 3 + 4\cos 2(\theta + 10^\circ)$

Vertical shift: up 3
horizontal shift: left 10°
amplitude ('stretch'): 4 x
period: $\frac{360^\circ}{2} = 180^\circ$



Example: Rewrite the equation $y = \cos x$ in terms of sine

Method 1: Graph interpretation



Since cosx and sinx are similar (i.e. identical if one of them is shifted) we can rewrite the above graph in terms of sine!

$y = \sin(x + \frac{\pi}{2})$ OR $y = -\sin(x - \frac{\pi}{2})$

Then, we can also rewrite by adding/subtracting $2\pi k$ to the shifts

such as $y = \sin(x + \frac{5\pi}{2})$ or $y = -\sin(x - \frac{3\pi}{2})$ etc...

Method 2: Using the (cofunction) identities

Recognizing the cofunction identities, use simple substitution...

$\sin(90^\circ - x) = \cos x$
or
$\cos(90^\circ - x) = \sin x$

So, $y = \cos x$ -----> $y = \sin(90^\circ - x)$

or $y = \sin(\frac{\pi}{2} - x)$

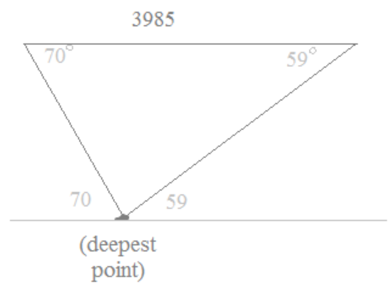
Trigonometry Review Word Problem

A bridge across a canyon is 3985 feet long.
From the deepest point in the canyon, the angles of elevation to the ends of the bridge are 59° and 70°.

How deep in the canyon?

SOLUTION

Step 1: Draw a picture

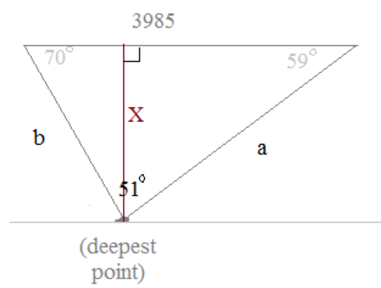


Step 2: Establish variables, formulas, and strategy

X is the depth of the canyon
(It forms a right angle)

The bottom angle is 51°
(because sum of angles is 180°)

Strategy: Use Law of Sines to find a and b
then, use trig values to get X

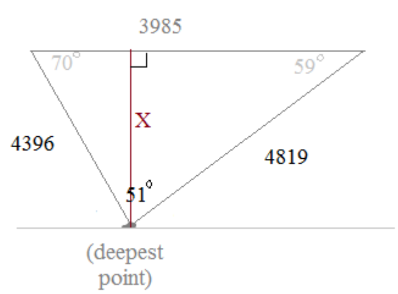


Step 3: Solve

LAW OF SINES: $\dfrac{\text{Sin }(51)}{3985} = \dfrac{\text{Sin }(70)}{a} = \dfrac{\text{Sin }(59)}{b}$

$a \text{ Sin }(51) = 3985 \text{ Sin }(70)$
$a\ (.777) = 3985\ (.940)$
$a = 4819$ feet

$b \text{ Sin }(51) = 3985 \text{ Sin }(59)$
$b\ (.777) = 3985\ (.857)$
$b = 4396$ feet



(note: a quick check shows shortest side of triangle is across from smallest angle. And, longest side is across from largest angle.)

TRIG VALUES: Sine $= \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\text{Sin }(59) = \dfrac{X}{4819}$

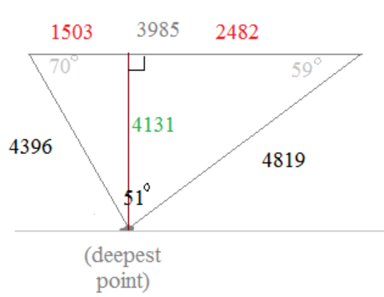
X = 4131 feet

To check solution, find X using the other side!

$\text{Sin }(70) = \dfrac{X}{4396}$

X = 4131 feet

Step 4: Check answer



Further check:
(Pythagorean Theorem)

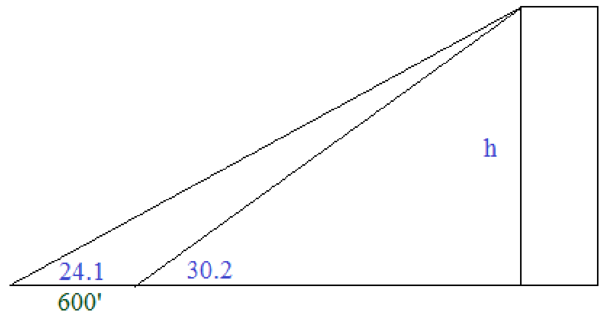
$(4819)^2 - (4131)^2 = 2481.5$ ✓

$(4396)^2 - (4131)^2 = 1503$ ✓

3984.5

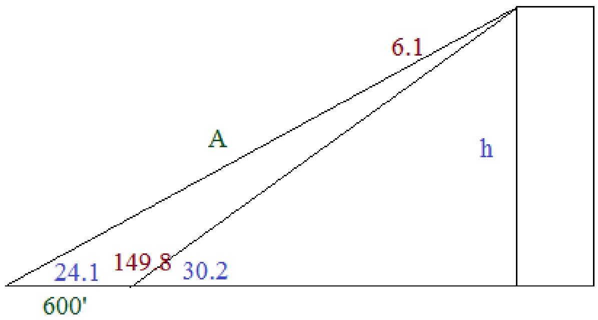
Trigonometry: Finding height of a building

Two people are standing 600 feet apart. If the angles of elevation from each person to the top of a building are 24.1° and 30.2°, what is the height of the building?



Method 1: Using geometry and law of sines

supplementary angles are 180°
interior angles of triangel are 180°



Use law of sines to find SIDE A: $\frac{\sin(6.1)}{600'} = \frac{\sin(149.8)}{A}$ $\quad A = \frac{\sin(149.8)600}{\sin(6.1)} = 2840$ feet

Use trig functions to find height (h): $\sin(24.1) = \frac{h}{2840'}$

height is (approx) 1159.7 feet

Method 2: Using trig and algebra

$\text{Tan}(24.1) = \frac{h}{(600 + x)}$ $\quad \text{Tan}(30.2) = \frac{h}{x}$

$\text{Tan}(24.1)(600 + x) = h$ $\quad \text{Tan}(30.2)(x) = h$

substitute h:

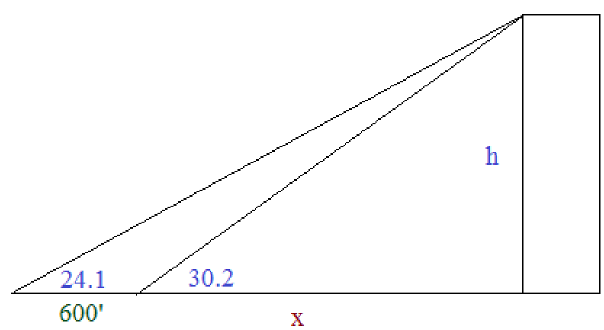
$\text{Tan}(30.2)(x) = \text{Tan}(24.1)(600 + x)$

$.582x = .447(600 + x)$

$.582x = 268.4 + .447x$

$.135x = 268.4$

$x = 1988$



Since $x = 1988$, $\tan(30.2) = \frac{h}{1988}$

h is (approximately) 1157 feet

**Note: rounding errors account for the variation between the two answers

Angular vs. Linear Speed

Example: A bicycle wheel spins at a rate of 400 rotations/minute.
If the diameter of the wheel is 26",

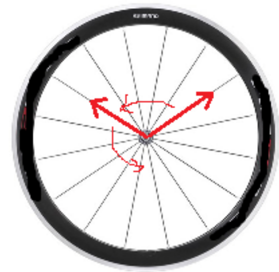
a) what is the *angular* speed?

b) what is the *linear* speed?

a) Angular speed describes the amount of distance covered in terms of *angles* and time.

If a bicycle wheel (or any circle) goes around once, the angular distance is 360° or 2π radians

So, if the bicycle wheel rotates 400 times, the angular distance is $400 \cdot 360^\circ$



= 144,000 degrees/minute

or

= 800π radians/minute

approx. 2513 radians/minute

b) Linear speed describes the distance covered by a point on the circumference path of the rotating item.

Suppose a little person went around the bicycle wheel one time. He would travel the circumference of the wheel:

circumference = 2π radius or π diameter

Since the wheel's circumference is π x 26 inches = 81.68 inches,

the linear distance of 400 trips around would be 400(π x 26 inches) = 32,672 inches

Therefore, the linear speed of the wheel is approximately 32,672 inches/minute
or 2723 feet/minute

Now, suppose we measure the angular and linear speed of the bicycle rim.
Again, the wheel spins at a rate of 400 rotations/minute.
If the radius of the rim is 11 inches (diameter is 22 inches), then

a) what is the angular speed?

b) what is the linear speed?

a) Since the number of rotations/minute is the same, the angular speed is the same!

$$360\ \frac{\text{degrees}}{\text{rotation}} \times 400\ \frac{\text{rotations}}{\text{minute}} = 144{,}000\ \frac{\text{degrees}}{\text{minute}}$$

b) $$\text{linear speed} = \frac{\text{distance traveled}}{\text{time}}$$

$$= \frac{400 \text{ rotations} \cdot 22\pi \text{ inches/rotation}}{1 \text{ minute}} = 27{,}645 \text{ inches/minute} \quad \text{or } 2304 \text{ feet/minute}$$

(approximately)

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (*t*), **or any beverage...**

Also, look for Honey Graham Squares
in the geometry section of your local store...

Practice Quiz-→

Algebra II/Trig Exercises

1) Solve algebraically for all possible values:

$$(2\cos^2 x - 1) = \cos x + 2$$

2) Solve algebraically for $0 \leq \theta < 360^\circ$:

$$4\sin\theta\cos\theta + 2\sin\theta - 2\cos\theta - 1 = 0$$

3) Solve for $0 \leq \theta < 360^\circ$:

$$2\sin\theta\tan\theta - \tan\theta = 1 - 2\sin\theta$$

4) Solve for $0 \leq x < 2\pi$:

$$\sin 2x = \cos x$$

5) Solve for all possible values of θ:

$$2\cos^2\theta + 5\cos\theta + 2 = 0$$

6) CALCULATOR

Solve for $0 \leq x < 2\pi$:

$$5\cos 2x = 2$$

Algebra II/Trig Exercises

1) Solve algebraically for all possible values:

$(2\cos^2 x - 1) = \cos x + 2$

$2\cos^2 x - 1 - \cos x - 2 = 0$

$2\cos^2 x - \cos x - 3 = 0$

$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0$ | $\cos x + 1 = 0$

$\cos x = \frac{3}{2}$ | $\cos x = -1$

no solution! | $\pi, 3\pi, 5\pi \ldots$

$\pi + 2\pi n$ where n is an integer

2) Solve algebraically for $0 \le \theta < 360^\circ$:

$4\sin\theta\cos\theta + 2\sin\theta - 2\cos\theta - 1 = 0$

Factor by grouping:

group: $4\sin\theta\cos\theta + 2\sin\theta \quad - 2\cos\theta - 1 = 0$

GCF: $2\sin\theta(\cos\theta + 1) \quad -1(\cos\theta + 1) = 0$

re-group: $(2\sin\theta - 1)(\cos\theta + 1) = 0$

$2\sin\theta - 1 = 0$ | $\cos\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ | $\cos\theta = -1$

$\theta = 30^\circ, 150^\circ$ | $\theta = 180^\circ$

3) Solve for $0 \le \theta < 360^\circ$:

$2\sin\theta\tan\theta - \tan\theta = 1 - 2\sin\theta$

$2\sin\theta\tan\theta - \tan\theta + 2\sin\theta - 1 = 0$

factor by grouping:

$2\sin\theta\tan\theta + 2\sin\theta \quad - \tan\theta - 1 = 0$

$2\sin\theta(\tan\theta + 1) \quad -1(\tan\theta + 1) = 0$

$(2\sin\theta - 1)(\tan\theta + 1) = 0$

$2\sin\theta - 1 = 0$ | $\tan\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ | $\tan\theta = -1$

$\theta = 30^\circ, 150^\circ$ | $\theta = 135^\circ, 315^\circ$

4) Solve for $0 \le x < 2\pi$:

$\sin 2x = \cos x$

$\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$ — double angle trig identity

$\cos x(2\sin x - 1) = 0$ — GCF

$\cos x = 0$ | $\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $x = \frac{\pi}{6}, \frac{5\pi}{6}$

5) Solve for all possible values of θ:

$2\cos^2\theta + 5\cos\theta + 2 = 0$

$(2\cos\theta + 1)(\cos\theta + 2) = 0$

$2\cos\theta + 1 = 0$ | $\cos\theta + 2 = 0$

$\cos\theta = \frac{-1}{2}$ quad II and III | $\cos\theta = -2$

$\theta = 120^\circ, 240^\circ, 480^\circ, 600^\circ, \ldots$ | no solution!

$\theta = 120^\circ + 360^\circ n$

$240^\circ + 360^\circ n$

6) CALCULATOR

Solve for $0 \le x < 2\pi$:

$5\cos 2x = 2$

$\cos 2x = \frac{2}{5}$

Let $U = 2x$

$\cos U = \frac{2}{5}$

$U = \arccos\frac{2}{5} = \arccos(.4)$

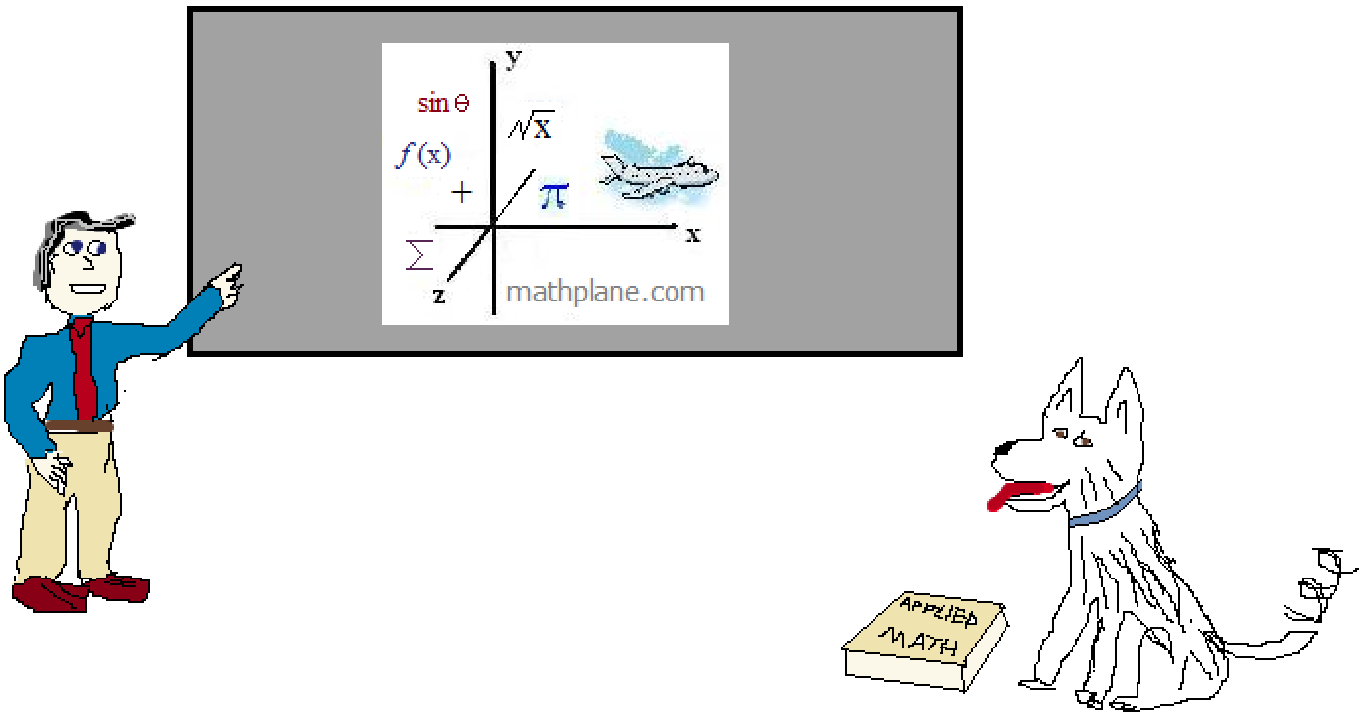
$= 1.16$ radians, 5.12 radians, 7.44 rad, 11.4 rad, ...

since $U = 2x$, then
$x = .58, 2.56, 3.72, 5.7$ radians

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at TeachersPayTeachers, Facebook, Google+, and Pinterest.