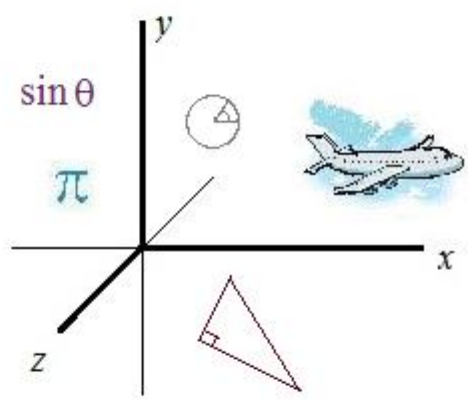
Trigonometry Review Test

(and, Solutions)

Questions include angle of elevation, law of cosines, radians/degrees, trig identities, graphing, navigation, and other trig concepts!



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Trigonometry Review Test

1) Find the 4 other trig functions if $\tan \theta = \frac{\sqrt{11}}{5}$ $\sec \theta = \frac{6}{5}$

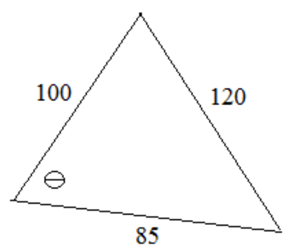
$\sin \theta =$

$\cos \theta =$

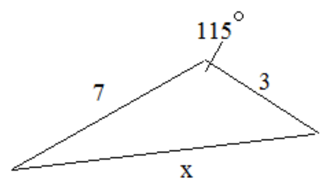
$\csc \theta =$

$\cot \theta =$

2) Find θ :



3) Find x :



4) Rewrite as an algebraic expression: $\sin(\tan^{-1} x)$

5) A tourist stands 500 feet from a tall statue. If the angle of elevation from his foot to the top of the statue is 31°, how high is the statue (to the nearest foot)?

Trigonometry Review Test

6) For $\frac{13\pi}{2}$, find the minimum positive coterminal angle and maximum negative coterminal angle.

7) Point P is on the unit circle.
If the x-coordinate is in Quadrant I and it is $\frac{1}{5}$, what is the point?

8) Graph $y = \tan(x - \frac{\pi}{2})$

9) A windmill blade is 16 feet long. If it goes around at a rate of 12 rotations/minute,

a) what is its *angular* speed? (in degrees or radians per minute)

b) what is its *linear* speed? (in feet per minute)

10) Verify the following.

A) $\tan\theta \sin\theta + \cos\theta = \sec\theta$

B) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

C) $\dfrac{\tan x}{1 - \cos x} = \csc x(1 + \sec x)$

11) Find the exact value.

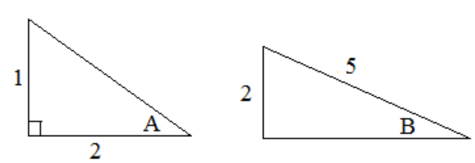
A) $\sin 12^\circ \cos 18^\circ + \cos 12^\circ \sin 18^\circ$

B) $\cos 75^\circ$

C) $\sin \dfrac{\pi}{12}$

12) Solve

A) For angles A and B, find $\sin(A + B)$



B) If $\sin y = \dfrac{-4}{5}$ and $\tan y > 0$, find $\tan\left(\dfrac{y}{2}\right)$

Trigonometry Review Test

13) If $\text{Sin}A = \frac{3}{5}$ and A is in the 1st quadrant, find:

a) Sin2A

b) Cos2A

c) $\text{Sin}\,\frac{1}{2}A$

d) $\text{Cos}\,\frac{1}{2}A$

14) If $\text{Sin}B = \frac{3}{5}$ and the terminal side of B is in the 2nd quadrant, find:

a) Sin2B

b) Cos2B

15) Solve for $0^\circ < \theta < 360^\circ$

$$2\cos^2\theta + 5\cos\theta + 2 = 0$$

16) Find <u>all</u> values of x:

$$2\cos^2 x + 3\sin x = 3$$

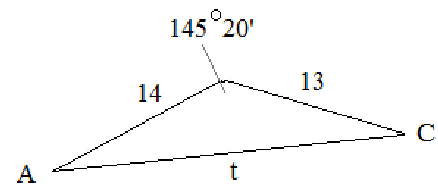
Trigonometry Review Test

17) Find the following:
(use degrees and minutes)

$\angle A =$

$\angle C =$

$t =$

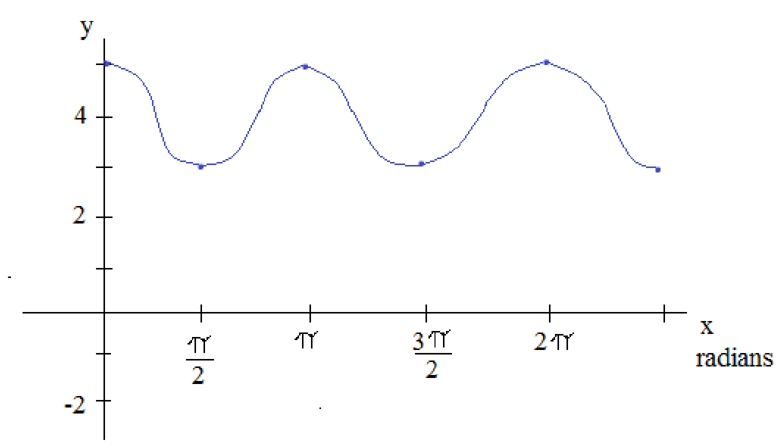


18) What is the exact value of $\sin(-210^\circ)$?

19) $f(x) = a\sin b(x - c) + d$

Write an equation where $a < 0$.

Write an equation where $a > 0$.



20) Graph $y = 3\cos(\theta - 40^\circ) - 4$

Amplitude:

Horizontal Shift:

Vertical Shift:

Period:

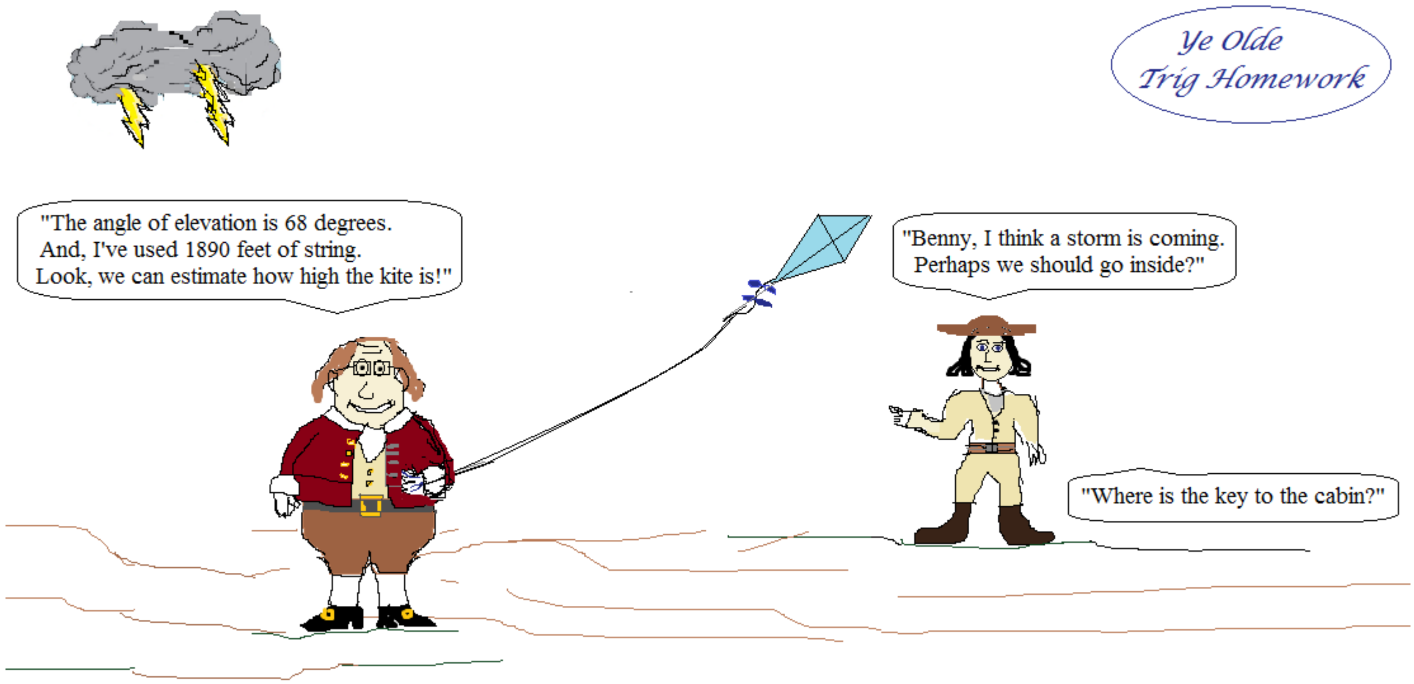
21) A ship leaves port and travels 20 miles at a bearing of N32E. Then, another ship leaves the port and travels 28 miles at a bearing S42E. What is the distance between the two ships?

22) Convert each radian measure into degrees

a) $\frac{5\pi}{6}$

b) 7π

c) 4



LanceAF #33 5-20-12
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During his math assignment,
Franklin makes another shocking discovery!

SOLUTIONS......

Trigonometry Review Test

SOLUTIONS

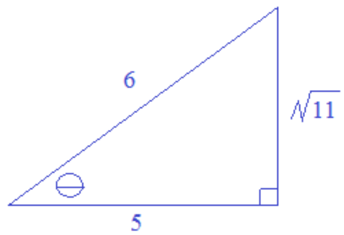
1) Find the 4 other trig functions if $\tan\theta = \frac{\sqrt{11}}{5}$ $\frac{\text{opp}}{\text{adj}}$ $\sec\theta = \frac{6}{5}$ $\frac{\text{hyp}}{\text{adj}}$

$\sin\theta = \frac{\sqrt{11}}{6}$

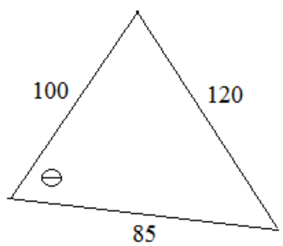
$\cos\theta = \frac{5}{6}$

$\csc\theta = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$

$\cot\theta = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$



2) Find θ :



Use Law of Cosines:

$c^2 = a^2 + b^2 - 2ab(\cos C)$

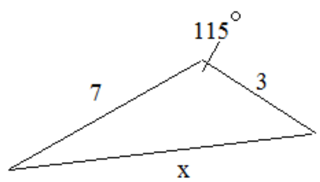
$120^2 = 100^2 + 85^2 - 2(85)(100)\cos\theta$

$14400 = 10000 + 7225 - 17000\cos\theta$

$-2825 = -17000\cos\theta$

$\theta = 80.43^\circ$

3) Find x :



Use Law of Cosines:

$c^2 = a^2 + b^2 - 2ab(\cos C)$

$x^2 = 49 + 9 - 2(7)(3)(\cos 115)$

$x^2 = 58 - 42(\cos 115)$

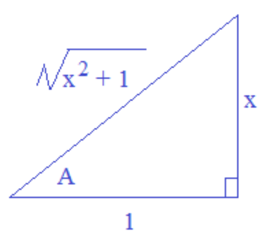
$x^2 = 75.75$

$x = 8.7$ approximate

4) Rewrite as an algebraic expression: $\sin(\tan^{-1}x)$

Draw a triangle and label:

tan is $\frac{\text{opposite}}{\text{adjacent}} = \frac{x}{1}$



$\tan A = x/1 = x$

then,

$\tan^{-1} x = A$

and, $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{x^2+1}}$

5) A tourist stands 500 feet from a tall statue. If the angle of elevation from his foot to the top of the statue is 31°, how high is the statue (to the nearest foot)?

Draw a picture;
then, identify and label
the right triangle...

$\tan 31^\circ = \frac{h}{500}$

300 feet

$(.601)(500) \cong 300.4$

SOLUTIONS

Trigonometry Review Test

6) For $\frac{13\pi}{2}$, find the minimum positive coterminal angle and maximum negative coterminal angle.

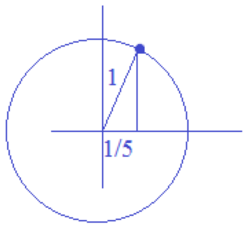
all coterminal angles are $\frac{13\pi}{2} + 2\pi n$

$\frac{13\pi}{2} - 6\pi n = \frac{\pi}{2}$ least *positive* coterminal

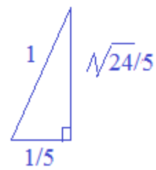
$\frac{13\pi}{2} - 8\pi n = -\frac{3\pi}{2}$ greatest *negative* coterminal

7) Point P is on the unit circle.
If the x-coordinate is in Quadrant I and it is $\frac{1}{5}$, what is the point?

(find the y-coordinate)



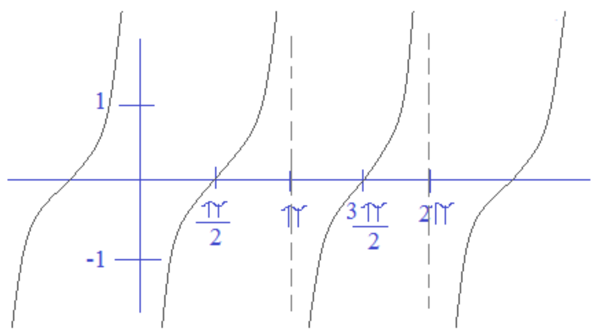
pythagorean theorem



$(1/5,\ 2\sqrt{6}/5)$

8) Graph $y = \tan(x - \frac{\pi}{2})$

(tangent function shifted $\frac{\pi}{2}$ to the right)



x	y
0	undefined
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined
$\frac{3\pi}{2}$	0

9) A windmill blade is 16 feet long. If it goes around at a rate of 12 rotations/minute,

a) what is its *angular* speed? (in degrees or radians per minute)

b) what is its *linear* speed? (in feet per minute)

a) angular speed is the amount of distance covered *measured in degrees.*

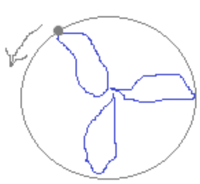
Every rotation will be 360 degrees covered. (or 2π radians)

The windmill goes at 12 rotations/minute ---- $12\ \frac{\text{rotations}}{\text{minute}} \cdot 360\ \frac{\text{degrees}}{\text{rotation}} =$ 4320 degrees/minute

or 24π radians/minute

$\cong 75.4$ radians/minute

b) linear speed is the distance covered from a spot *on the tip of the blade.*



The spot will travel the circumference of the circle during one rotation...

circumference $= 2\pi r = 2\pi 16$ feet

$= 32\pi$ feet per rotation

Therefore, the linear speed is 384π feet/minute

approx. 1206 feet/minute

Trigonometry Review Test

10) Verify the following.

A) $\tan\theta\sin\theta + \cos\theta = \sec\theta$

$$\frac{\sin\theta}{\cos\theta}\sin\theta + \cos\theta = \sec\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \cos\theta\,\frac{\cos\theta}{\cos\theta} = \sec\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \sec\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\sec\theta = \sec\theta$$

B) $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$$\frac{2\tan x}{\sec^2 x} = \sin 2x$$

$$2\tan x\,(\cos^2 x) = \sin 2x$$

$$2\frac{\sin x}{\cos x}(\cos^2 x) = \sin 2x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin 2x = \sin 2x$$

C) $\dfrac{\tan x}{1-\cos x} = \csc x(1+\sec x)$

$$\frac{\tan x(1+\cos x)}{(1-\cos x)(1+\cos x)} = \csc x(1+\sec x)$$

$$\frac{\frac{\sin x}{\cos x}(1+\cos x)}{(1-\cos^2 x)} = \csc x(1+\sec x)$$

$$\frac{\frac{\sin x}{\cos x} + \sin x}{\sin^2 x} = \csc x(1+\sec x)$$

$$\frac{1}{\sin x\cos x} + \frac{1}{\sin x} = \csc x(1+\sec x)$$

$$\csc x\sec x + \csc x = \csc x + \csc x\sec x$$

11) Find the exact value.

A) $\sin 12^\circ\cos 18^\circ + \cos 12^\circ\sin 18^\circ$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$\sin 12\cos 18 + \cos 12\sin 18 = \sin(12+18)$

$\sin 30 = \dfrac{1}{2}$

B) $\cos 75^\circ$

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\cos(30+45) = \cos 30\cos 45 - \sin 30\sin 45$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

note: $\cos 75 = \sin 15$

C) $\sin\dfrac{\pi}{12}$

$\sin 15^\circ = \sin(45-30)$

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

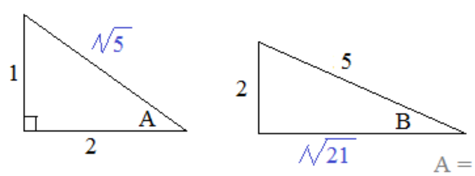
$\sin(45-30) = \sin 45\cos 30 - \cos 45\sin 30$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

12) Solve

A) For angles A and B, find $\sin(A+B)$



$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$= \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{21}}{5} + \frac{2}{\sqrt{5}}\cdot\frac{2}{5} = \frac{4+\sqrt{21}}{5\sqrt{5}}$$

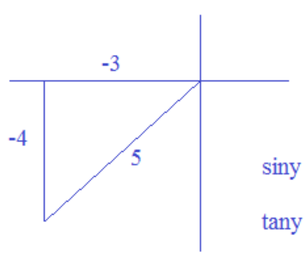
$A = 26.56 \quad B = 23.58$

$\sin 50.14 = .768$ ✓

$$\boxed{\frac{4\sqrt{5}+\sqrt{105}}{25}}$$

approximately .768 ✓

B) If $\sin y = \dfrac{-4}{5}$ and $\tan y > 0$, find $\tan\left(\dfrac{y}{2}\right)$



$\sin y = -4/5$

$\tan y = 3/4 > 0$

$$\tan\frac{1}{2}x = \frac{\sin x}{1+\cos x} \quad \text{or} \quad \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{-4/5}{1+(-3/5)} \qquad \sqrt{\frac{1-(-3/5)}{1+(-3/5)}}$$

$$= \frac{-4/5}{2/5} = \boxed{-2} \qquad \sqrt{\frac{8/5}{2/5}} = -2$$

note: the half angle ends up in quadrant II, so tangent of y/2 is negative

Trigonometry Review Test

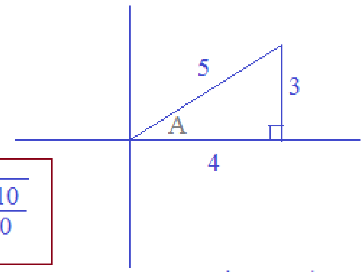
13) If $\sin A = \frac{3}{5}$ and A is in the 1st quadrant, find:

a) Sin2A $\quad 2\sin A\cos A = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \boxed{\frac{24}{25}}$

b) Cos2A $\quad \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \boxed{\frac{7}{25}}$

c) $\sin \frac{1}{2}A \quad \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-4/5}{2}} = \boxed{\frac{\sqrt{10}}{10}}$

d) $\cos \frac{1}{2}A \quad \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+4/5}{2}} = \boxed{\frac{3\sqrt{10}}{10}}$



SinA = 3/5
CosA = 4/5

$A = 36.87^\circ$ (approx)

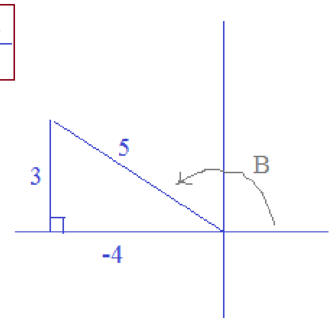
14) If $\sin B = \frac{3}{5}$ and the terminal side of B is in the 2nd quadrant, find:

a) Sin2B $\quad 2\sin B\cos B = 2 \cdot \frac{3}{5} \cdot \frac{-4}{5} = \boxed{\frac{-24}{25}}$

SinB = 3/5
CosB = -4/5

b) Cos2B $\quad \cos^2 B - \sin^2 B =$

$\left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \boxed{\frac{7}{25}}$



$B = 143.13^\circ$ (approx)

15) Solve for $0^\circ < \theta < 360^\circ$

$2\cos^2\theta + 5\cos\theta + 2 = 0$

$(2\cos\theta + 1)(\cos\theta + 2) = 0$

~~$\cos\theta + 2 = 0$~~

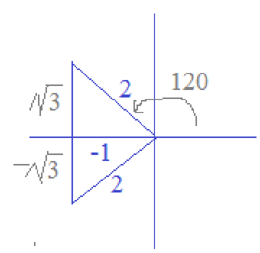
~~$\cos\theta = -2$~~

Extraneous solution

$2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\boxed{\theta = 120^\circ, 240^\circ}$



16) Find <u>all</u> values of x:

$2\cos^2 x + 3\sin x = 3$

"change the cos to sin"

$2(1 - \sin^2 x) + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

(collect terms and multiply by -1)

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6} + 2\pi k, \quad \frac{5\pi}{6} + 2\pi k}$

$\sin x - 1 = 0$

$\sin x = 1$

$\boxed{x = \frac{\pi}{2} + 2\pi k}$

Trigonometry Review Test

SOLUTIONS

17) Find the following:
(use degrees and minutes)

$\angle A = 16^\circ 40'$

$\angle C = 18^\circ$

$t = 25.78$

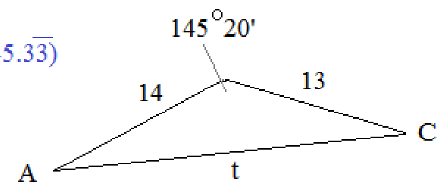
Quick check: angles A, C, T add up to 180 degrees
A is smallest, C is medium, and T is largest angle; each is opposite the corresponding side ✓

Since we know 2 sides and the included angle, we can use law of cosines:

$t^2 = (14)^2 + (13)^2 - 2(14)(13)(\text{Cos}145.\overline{33})$

$t^2 = 365 - 364(-.8225) = 664.38$

$t = 25.78$



Now, we can use law of sines:

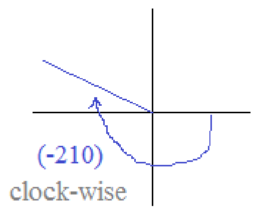
$\frac{\text{Sin}145.\overline{33}}{25.78} = \frac{\text{Sin}C}{14} \rightarrow \text{Sin}C = \frac{\text{Sin}145.\overline{33}\,(14)}{25.78} = \frac{7.963}{25.78} = .3089$

$C = 17.99^\circ$

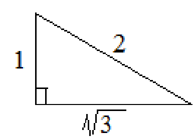
$\rightarrow \text{Sin}A = \frac{\text{Sin}145.\overline{33}\,(13)}{25.78} = \frac{7.394}{25.78} = .2868$

$16^\circ 40' \leftarrow A = 16.67^\circ$

18) What is the exact value of $\sin(-210^\circ)$?



(30 degree reference angle)



$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

$= \frac{1}{2}$

19) $f(x) = a\sin b(x - c) + d$

Write an equation where $a < 0$.

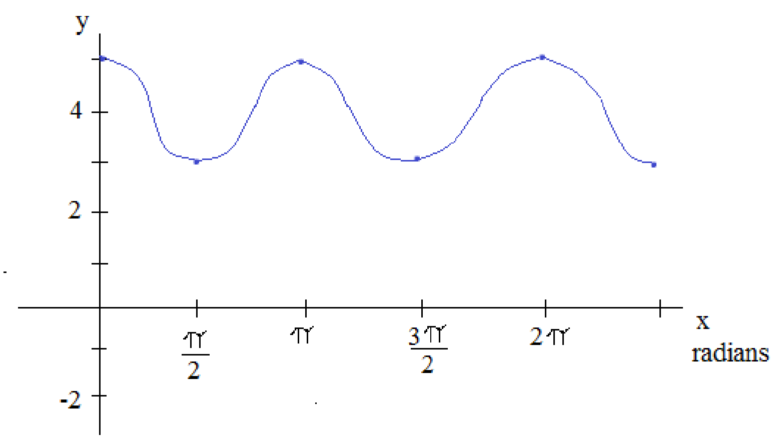
amplitude a: 1
vertical shift d: up 4
horizontal shift c: $3\pi/4$ to the right
period b: $2\pi/\pi$

$\sin 2(x - \frac{3\pi}{4}) + 4$

Write an equation where $a > 0$.

same vertical shift and period....
amplitude: 1 ('negative')
horizontal shift: $\pi/4$ to the right

$-\sin 2(x - \frac{\pi}{4}) + 4$



SOLUTIONS

20) Graph $y = 3\cos(\theta - 40^\circ) - 4$

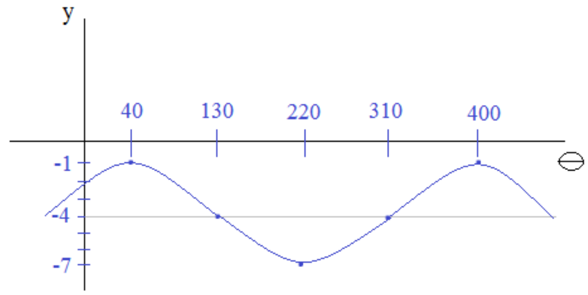
Amplitude: 3

Horizontal Shift: 40° to the right

Vertical Shift: 4 units down

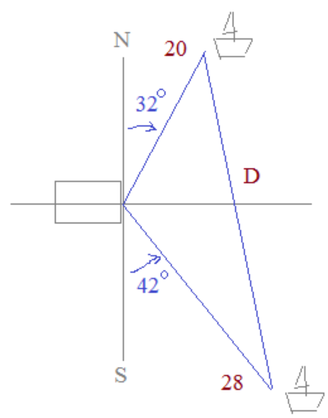
Period: $\frac{360}{1} = 360^\circ$

(Note: to check answers, plug in points)

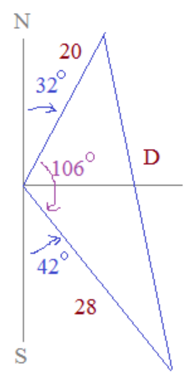


21) A ship leaves port and travels 20 miles at a bearing of N32E. Then, another ship leaves the port and travels 28 miles at a bearing S42E. What is the distance between the two ships?

Step 1: Draw a picture



Step 2: Identify triangle and formula



**Use law of cosines to find the distance!

Step 3: Solve

$D^2 = 20^2 + 28^2 - 2(20)(28)\cos 106$

$D^2 = 1184 - 1120(\cos 106)$

$D^2 = 1184 - 1120(-.2756)$

$D^2 = 1492.7$

distance D = 38.6 miles

Step 4: Quick check..

38 miles seems reasonable (compared to given distances)

law of sines: $\frac{\sin 106^\circ}{38.6} = \frac{\sin A}{20} = \frac{\sin B}{28}$

$A = 29.87^\circ$

$B = 44.21^\circ$

$$\begin{array}{r} 29.87 \\ 44.21 \\ +\ 106 \\ \hline \text{approx. } 180 \end{array}$$

22) Convert each radian measure into degrees

a) $\frac{5\pi}{6} \cdot \frac{180 \text{ degrees}}{\pi \text{ radians}} = $ 150 degrees

b) $7\pi \cdot \frac{180 \text{ degrees}}{\pi \text{ radians}} = $ 1260 degrees

c) $4 \cdot \frac{180 \text{ degrees}}{\pi \text{ radians}} = \frac{720}{\pi} \text{ degrees} \approx$ 229 degrees

π radians = 180 degrees

$1 = \frac{180 \text{ degrees}}{\pi \text{ radians}}$

Thanks for checking out this review test.   (Hope it helped!)

If you have questions, suggestions, or feedback, let me know…

Cheers,

Lance