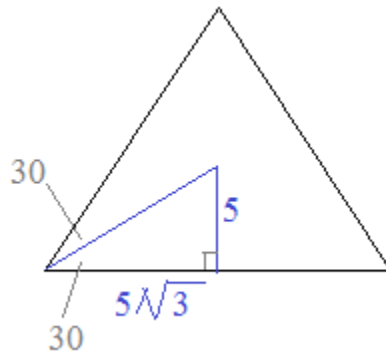
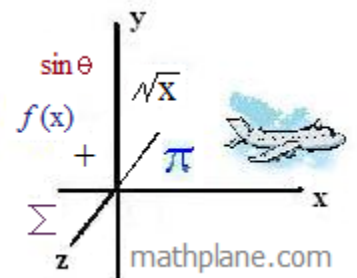


# Area and Perimeter of Polygons 2

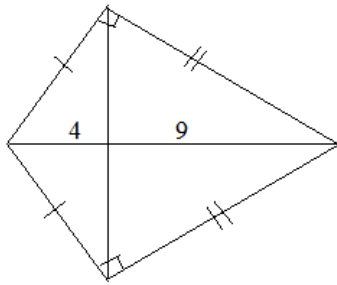


Examples and Practice Exercises (with solutions)



Area and Perimeter of Polygons

*Example:* Find the area and perimeter of the figure.



Since the quadrilateral has 2 pairs of adjacent sides that are congruent, it's a kite.

And, the diagonals of a kite are perpendicular.

We can find the geometric mean (altitude to hypotenuse) to find the length of the other diagonal!

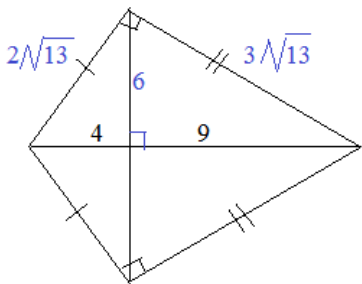
$$\frac{4}{x} = \frac{x}{9} \quad x = 6, \text{ so the diagonal is } 12$$

The area of the kite is  $\frac{1}{2}$  (diagonal 1)(diagonal 2) =

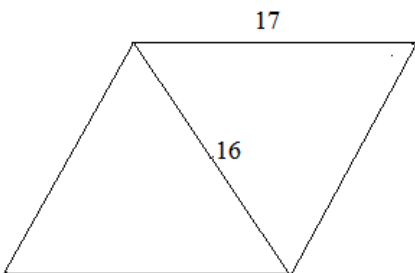
$$\frac{1}{2} (12)(13) = 78$$

Since the diagonals form 4 right triangle, we can use Pythagorean Theorem to find the sides of the kite...

$$\text{The perimeter is } 10\sqrt{13}$$



*Example:* Find the perimeter and area of the rhombus



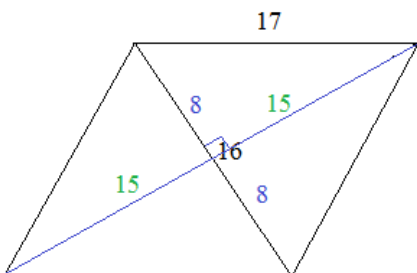
Since all sides of a rhombus are congruent, the perimeter =  $4 \times 17 = 68$  units

To find the area, the most direct approach is finding the length of the other diagonal.

\*\*\*The diagonals of a rhombus are perpendicular bisectors!

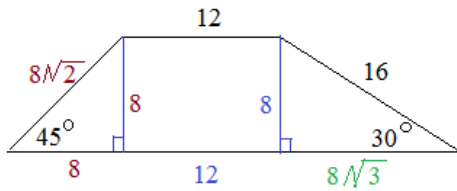
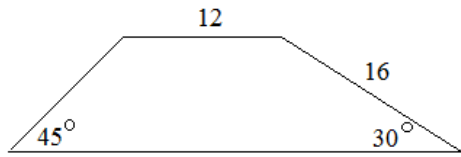
Then, recognizing the 8-15-17 Pythagorean Triple.. the other diagonal is 30

$$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2} (16)(30) = 240 \text{ sq. units}$$



Area and Perimeter of Polygons

*Example:* What is the area and perimeter of the trapezoid?



To find the area, we'll need to determine the altitude (height).

Fortunately, the altitudes form special right triangles.

30-60-90 right triangle: hypotenuse is 2x the small leg..

So, the height is 8...

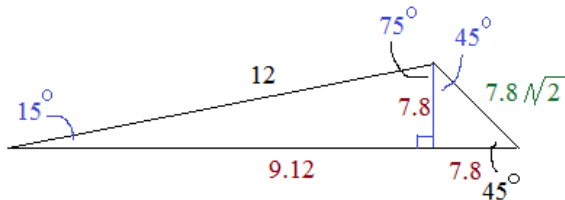
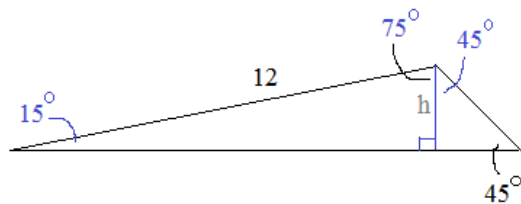
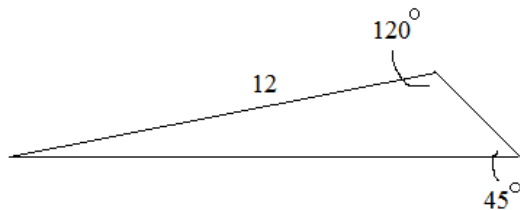
Then, the other leg is  $8\sqrt{3}$

And, since the height is 8, the other part of the base is 8... 45-45-90 right triangle...

Perimeter is the sum of all the sides:  $48 + 8\sqrt{2} + 8\sqrt{3}$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(\text{base1} + \text{base2})(\text{height}) = \frac{1}{2}(32 + 8\sqrt{3})(8) \\ &= 128 + 32\sqrt{3} \end{aligned}$$

*Example:* Find the area and perimeter of the triangle.



First, we drop an altitude and see that we can use trigonometry and geometry tools to find the other parts...

$$\sin(15^\circ) = \frac{\text{height}}{12} \quad \text{height} = 7.8 \quad (\text{approximately})$$

Since the right part is 45-45-90 right triangle, the right part of the base is 7.8

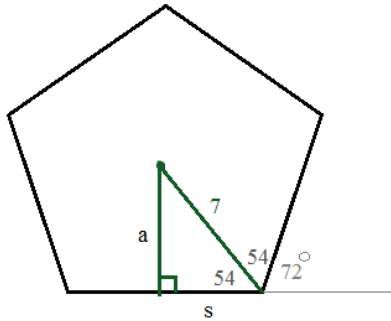
Then, using the Pythagorean Theorem the left part of the base is 9.12

$$9.12^2 + 7.8^2 = 12^2$$

Perimeter of triangle =  $12 + 16.92 + 11.03 = 39.95$  approximately

$$\text{Area} = \frac{1}{2}(16.92)(7.8) = 66 \quad \text{approximately}$$

Example: What is the perimeter and area of a regular pentagon with radius of 7?



Method 1:  $\text{Area} = \frac{1}{2} (\text{apothem})(\text{perimeter})$

From the diagram,  $\text{Exterior angle} = \frac{360}{5} = 72^\circ$

Therefore, each interior angle is  $180 - 72 = 108^\circ$

$$\sin(54) = \frac{a}{7}$$

$$\text{apothem } a = 7\sin(54) = 5.66$$

$$\cos(54) = \frac{s}{7}$$

$$1/2 \text{ of each side} = s = 7\cos(54) = 4.11$$

$$\text{apothem} = 5.66$$

$$\text{perimeter} = 10 \times 4.11 = 41.1$$

$$\text{Area} = \frac{1}{2} (5.66)(41.1) = 116.3 \text{ (approximately)}$$

Method 2: Find area of triangle then,

$$\text{Area}_{\text{pentagon}} = \left( \frac{1}{2} (\text{base})(\text{height}) \right) \times 10 \text{ triangles}$$

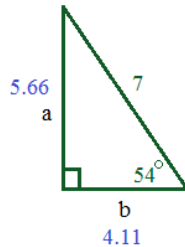
$$\sin(54) = \frac{a}{7}$$

$$a = 7\sin(54) = 5.66$$

$$b = 7\cos(54) = 4.11$$

$$\text{Area of triangle: } \frac{1}{2} (4.11)(5.66) = 11.63$$

$$\text{Area of pentagon: } 10 \times 11.63 = 116.3$$



note: to check, use pythagorean theorem:

$$4.11^2 + 5.66^2 = 7^2 \text{ approximately}$$

Example: Find the area of 15 sided regular n-gon with perimeter 180 feet.

To find the area, we want the apothem and perimeter...

The perimeter is given: 180 feet...

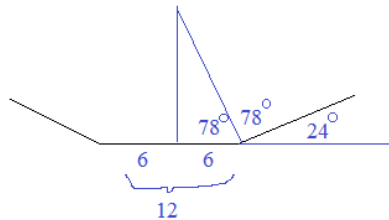
$$\text{Area} = \frac{1}{2} (\text{apothem})(\text{perimeter})$$

To find the measure of the apothem, draw a diagram:

Sum of the exterior angles is  $360^\circ$ .  
So, each exterior angle measure is  $360/15 = 24$  degrees...

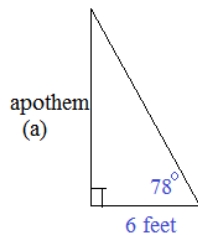
therefore, each interior angle is  $180 - 24 = 156$  degrees....

And, the radius bisects the angle into 78 degree angles..



$$\text{Area} = \frac{1}{2} (28.23 \text{ feet})(180 \text{ feet}) = 2540 \text{ square feet}$$

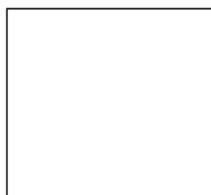
$$\frac{180}{15} = 12 \text{ feet/side}$$



$$\tan(78) = \frac{a}{6}$$

$$\text{apothem } a = 6\tan(78) = 28.23$$

Part I: Answer the following:



Square; apothem = 5 feet

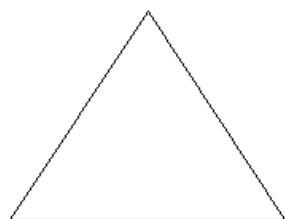
What is the area?



Square; diagonal = 16cm

What is the area?

What is the length of the apothem?



*Equilateral* triangle; sides = 7 inches

What is the area?



Equilateral triangle; apothem = 5

What is the area?

What is the length of each side?



Rectangle; diagonal = 26  
width = 10

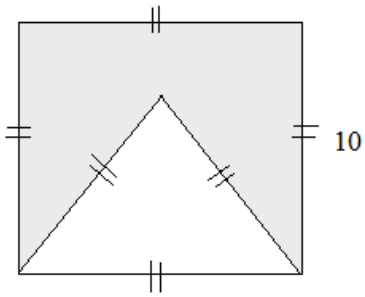
What is the perimeter?

What is the area?

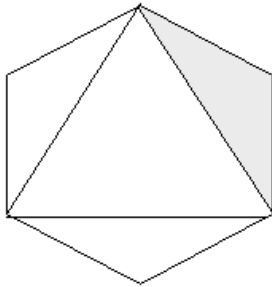
Part II: Find the area of each shaded region:

Area of shapes and regular polygons

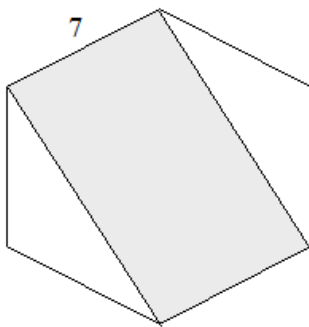
1) Triangle inside a square:



2) A regular hexagon with sides measuring 6 feet:



3) Rectangle inscribed in a regular hexagon:



Part III: Find the area of a *regular* hexagon, if

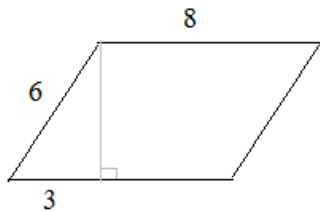
Area of shapes and regular polygons

a) the length of a side is 8

b) the length of an apothem is 8

c) the length of a radius is 8

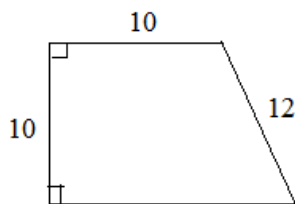
Part IV:



Parallelogram:

What is the perimeter?

What is the area?



Trapezoid:

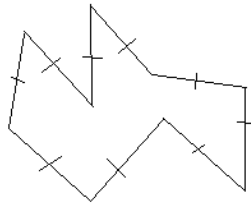
What is the perimeter?

What is the area?

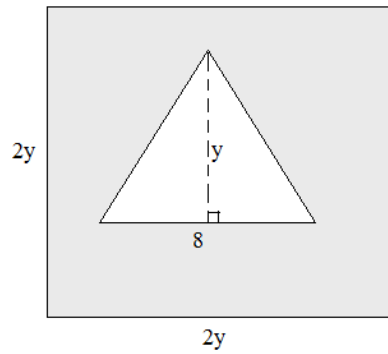
Part V:

- 1) Find the angles formed by
  - a) 2 consecutive radii
  - b) the radius and adjoining side
 in a regular
  - 1) pentagon
    - a)
    - b)
  - 2) hexagon
    - a)
    - b)
  - 3) octagon
    - a)
    - b)
  - 4) decagon
    - a)
    - b)

- 2) Is this a regular 9 sided polygon?

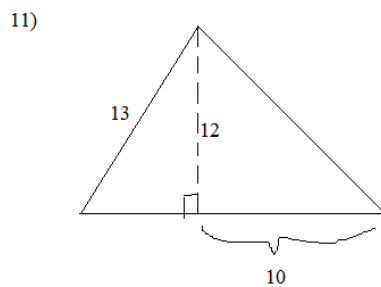
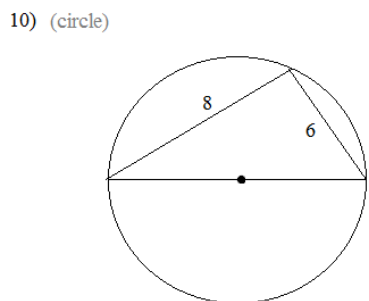
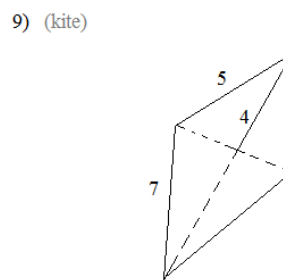
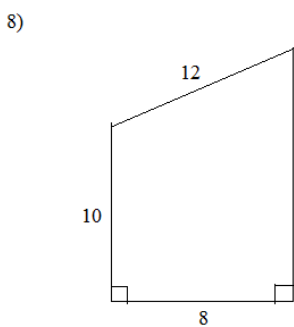
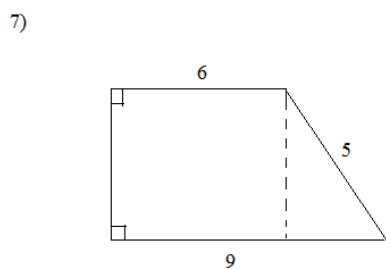
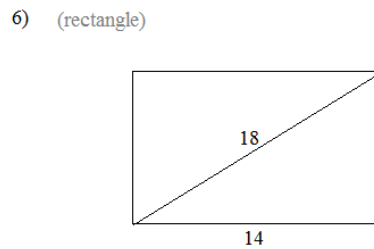
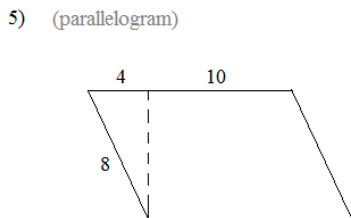
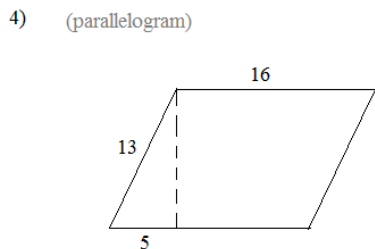
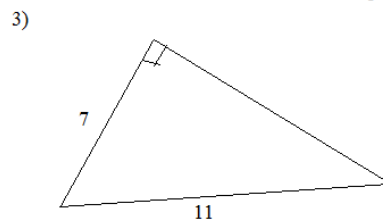
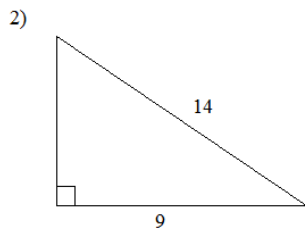
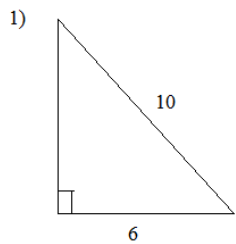


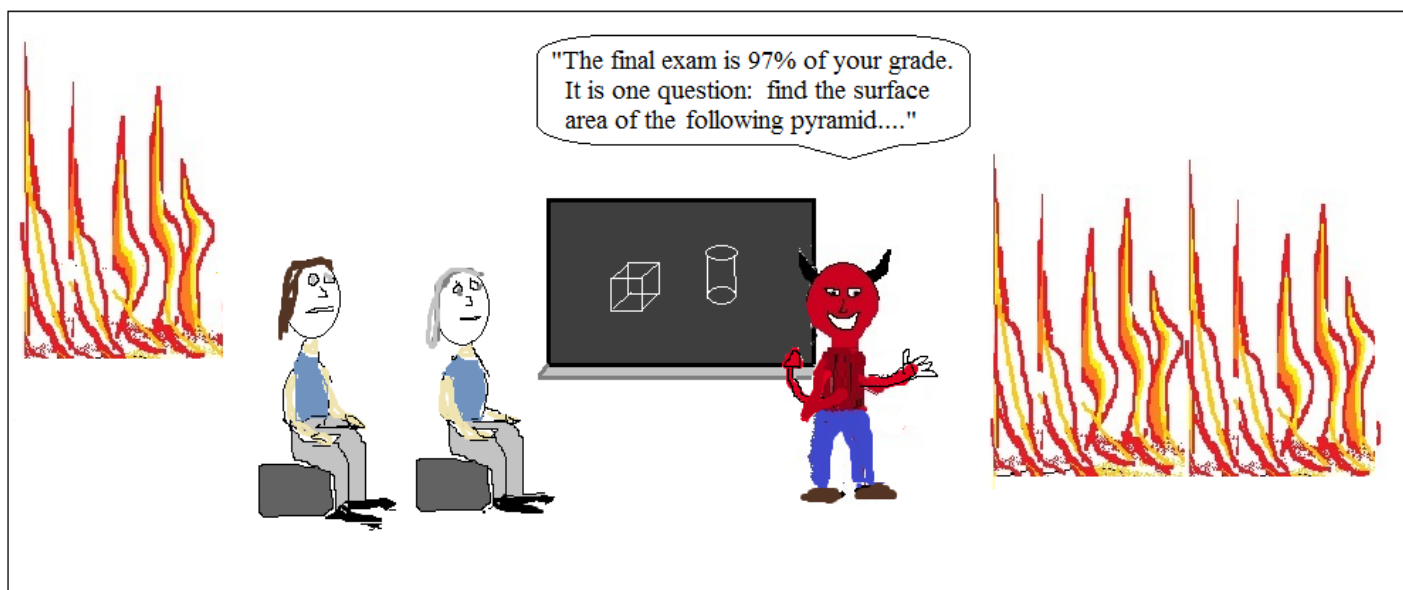
- 3) If the area of the shaded region is 168 square units, what is  $y$ ?



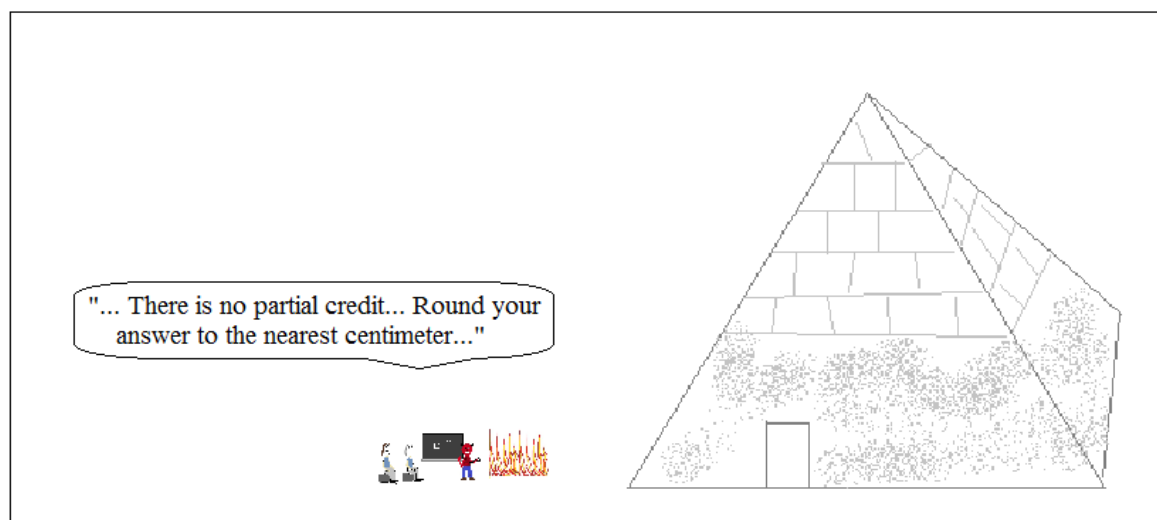


Find the area and perimeter of each figure.





*Math in Hell*



In its 1000 year history, no one ever passed Mr. Devlin's Geometry class.

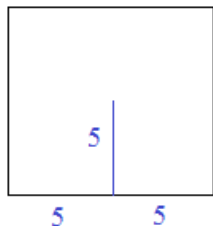
LanceAF #39 7-1-12  
[www.mathplane.com](http://www.mathplane.com)

**Solutions ->**

Part I: Answer the following:

SOLUTIONS

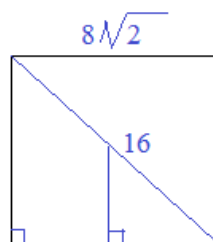
Area of shapes and regular polygons



Square; apothem = 5 feet  
What is the area?

The apothem connects the center to the midpoint of a side..  
(If the apothem is 5', then 1/2 of each side is 5')

$$\text{area} = (\text{side})(\text{side}) = (10\text{ft})(10\text{ft}) = 100 \text{ square feet}$$



Square; diagonal = 16cm

Since the diagonal divides the square into two 45-45-90 triangles, the sides are

What is the area?

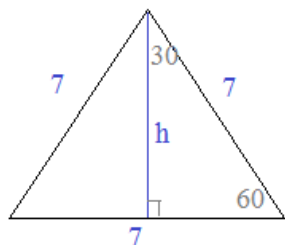
$$\frac{16}{\sqrt{2}} = 8\sqrt{2}$$

$$\text{Area} = (\text{side})^2 = 128 \text{ square cm}$$

What is the length of the apothem?

Then, the apothem is 1/2 of each side:

$$4\sqrt{2} \text{ cm}$$



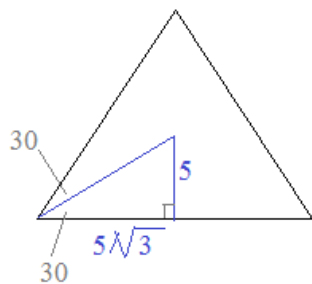
Equilateral triangle; sides = 7 inches

What is the area?

$$\text{area} = \frac{1}{2} (\text{base})(\text{height})$$

Since it is an equilateral  $\triangle$  the altitude forms a 30-60-90 right triangle...

$$\text{area} = \frac{1}{2} (7)\left(\frac{7}{2}\sqrt{3}\right) = \frac{49\sqrt{3}}{4} \text{ sq. units}$$



Equilateral triangle; apothem = 5

What is the area?  $75\sqrt{3}$

Since it is an equilateral  $\triangle$ , the apothem and radius will form a 30-60-90 right  $\triangle$  with small side = 5

What is the length of each side?

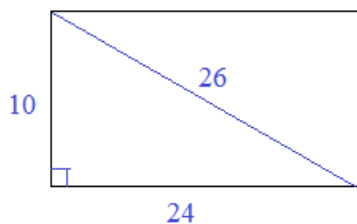
$$\text{each side is } 10\sqrt{3}$$

the area of the right triangle

$$\text{is } \frac{1}{2} (\text{base})(\text{height}) = \frac{1}{2} \cdot 5\sqrt{3} \cdot 5$$

and, since there are six right triangles inside the entire triangle, the area

$$\text{is } 75\sqrt{3}$$



Rectangle; diagonal = 26  
width = 10

What is the perimeter?  $68$

The diagonal is the hypotenuse of a right triangle.. Using pythagorean theorem (or recognizing 5-12-13), the length of the rectangle is 24

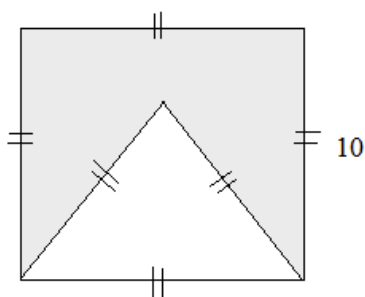
What is the area?  $120$

Part II: Find the area of each shaded region:

**SOLUTIONS**

Area of shapes and regular polygons

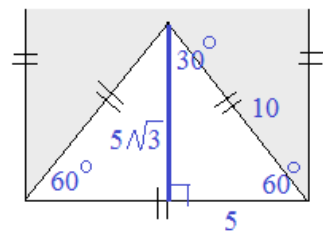
1) Triangle inside a square:



The figure is an equilateral triangle inside a square.

$$\text{Area of square} = s^2 = (10)^2 = 100$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} (\text{base})(\text{height}) \\ &= \frac{1}{2} (10)(5\sqrt{3}) \\ &= 25\sqrt{3} \end{aligned}$$

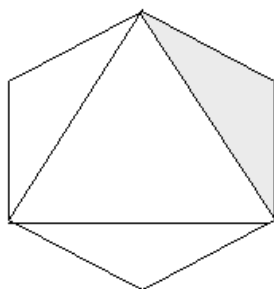


30-60-90 right  $\triangle$

$$\text{shaded area} = A_S - A_T$$

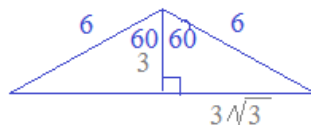
$$= 100 - 25\sqrt{3}$$

2) A regular hexagon with sides measuring 6 feet:



Each interior angle measures 120 degrees:

$$\frac{(6 - 2) \cdot 180}{6} = 120$$

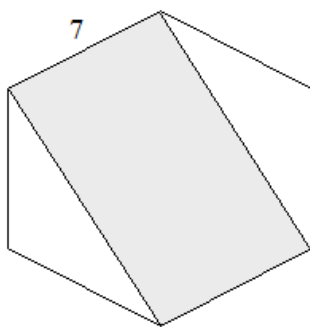


Base of the triangle =  $6\sqrt{3}$

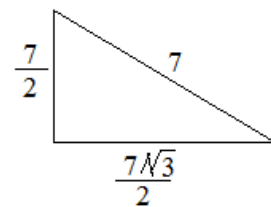
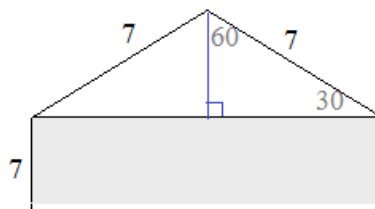
Height of the triangle = 3

$$\text{Area} = \frac{1}{2}bh = 9\sqrt{3} \text{ square feet}$$

3) Rectangle inscribed in a regular hexagon:



Area of rectangle = length x width...  
So, we need to find the length:



So, the length of the entire triangle base (i.e. the length of the rectangle) =  $7\sqrt{3}$

$$\text{Area rectangle} = 7 \times 7\sqrt{3} = 49\sqrt{3}$$

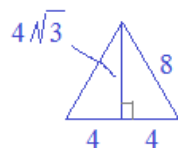
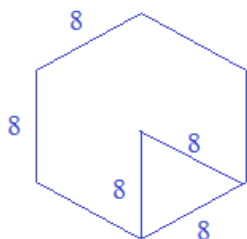
Part III: Find the area of a *regular* hexagon, if

**SOLUTIONS**

**Area of shapes and regular polygons**

a) the length of a side is 8

Each interior angle of a regular hexagon is 120 degrees...  
So, there are 6 equilateral triangles inside the hexagon...



Area of equilateral triangle =  $16\sqrt{3}$

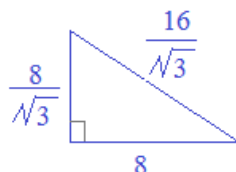
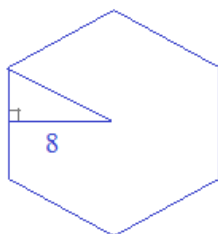
therefore, the area of the hexagon is

$$6 \times (\text{area of triangle}) = \boxed{96\sqrt{3}} \quad \boxed{166.3}$$

b) the length of an apothem is 8

Again, we have a 30-60-90 triangle..  
The big length is 8... So, the small length is

$$\frac{8}{\sqrt{3}}$$



The area of this right triangle is

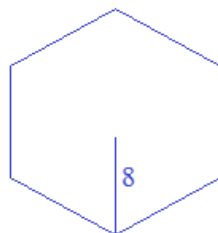
$$\frac{1}{2} (8) \left( \frac{8}{\sqrt{3}} \right) = \frac{32}{\sqrt{3}}$$

therefore, the area of the hexagon is

$$12 \times (\text{area of triangle}) = \boxed{\frac{384}{\sqrt{3}}} \quad \boxed{221.7}$$

c) the length of a radius is 8

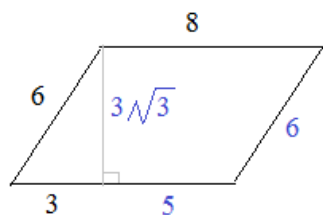
Since this is a (regular) hexagon,  
the length of a radius is the same  
as the length of a side...



Therefore, the area is the same  
as the hexagon in a)

$$\boxed{96\sqrt{3}}$$

Part IV:



Parallelogram:

What is the perimeter?

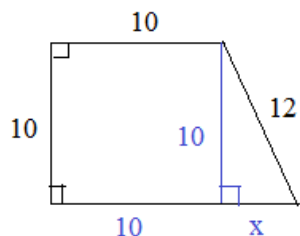
opposite sides of parallelogram are congruent,  
so perimeter =  $6 + 8 + 6 + 8 = \boxed{28}$

What is the area?

$$\text{area} = (\text{base})(\text{height}) = (8)(3\sqrt{3})$$

$$\boxed{24\sqrt{3}}$$

since the right triangle has hypotenuse 6 and base 3, it must be a 30-60-90 right triangle..  
(the hypotenuse is 2x the length).. Therefore,  
the height of the parallelogram is  $3\sqrt{3}$



Trapezoid:

What is the perimeter?

$$10 + 10 + 10 + 12 + 2\sqrt{11} =$$

Pythagorean Theorem:

$$(10)^2 + (x)^2 = (12)^2$$

$$x = \sqrt{44} = 2\sqrt{11}$$

What is the area?

$$\boxed{42 + 2\sqrt{11}}$$

$$\text{Area} = \frac{1}{2} (\text{base1} + \text{base2})(\text{height})$$

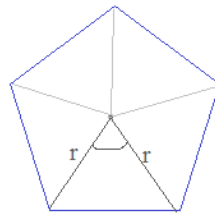
$$= \frac{1}{2} (20 + 2\sqrt{11})(10) = \boxed{100 + 10\sqrt{11}}$$

Part V:

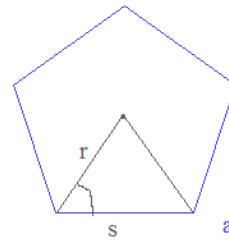
Area of shapes and regular polygons

SOLUTIONS

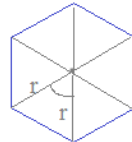
- 1) Find the angles formed by
- a) 2 consecutive radii
  - b) the radius and adjoining side
- in a regular
- 1) pentagon
    - a)  $72^\circ$
    - b)  $54^\circ$
  - 2) hexagon
    - a)  $60^\circ$
    - b)  $60^\circ$
  - 3) octagon
    - a)  $45^\circ$
    - b)  $62.5^\circ$
  - 4) decagon
    - a)  $36^\circ$
    - b)  $72^\circ$



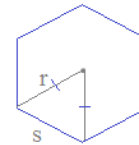
$$360 \div 5 = 72^\circ$$



Since the 5 triangles are isosceles...  
 $\text{angle} + \text{angle} + 72 = 180$   
 $\text{angle} = 54^\circ$



$$360 \div 6 = 60^\circ$$



Since the radii angle is 60, the other two angles add up to 180..

Therefore, the other angles are 60 and 60..

An octagon has 8 congruent radii angles...

$$360 \div 8 = 45^\circ$$

Since the radii angle is 45 degrees.. the 2 base angles must add up to 135 degrees...

$$62.5 \text{ and } 62.5$$

Decagon has 10 sides.. 10 congruent radii angles.. 36 degrees

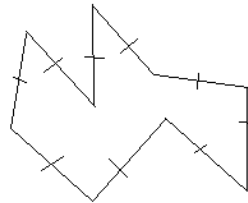
The radius/side angles are 72 degrees...

- 2) Is this a regular 9 sided polygon?

No, it is not...

It is 'equilateral'...

but, it is not 'equiangular'...



- 3) If the area of the shaded region is 168 square units, what is y?

$$(2y)^2 - \frac{1}{2}(8)(y) = 168$$

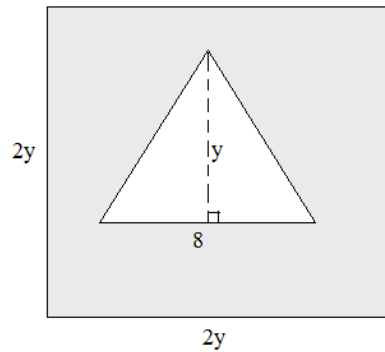
$$4y^2 - 4y = 168$$

$$4(y^2 - y - 42) = 0$$

$$4(y + 6)(y - 7) = 0$$

$y = -6$  or  $7$ ... (but, it cannot be negative...)

$$y = 7$$



Check:

Area of square:  $14 \times 14 = 196$

Area of triangle:  $\frac{1}{2}(8)(7) = 28$

Shaded area =  $196 - 28 = 168$  ✓

Find the area and perimeter of each figure.

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SOLUTIONS

Pythagorean Theorem

1)  $10^2 - 6^2 = b^2 \quad b = 8$

area =  $\frac{1}{2}$  (base)(height)  
 $= \frac{1}{2} (6)(8)$   
 $= 24$

perimeter: 24

2)  $14^2 - 9^2 = b^2 \quad b = \sqrt{115}$

area =  $\frac{9}{2} \sqrt{115}$   
 approx 48.26

perimeter:  $23 + \sqrt{115}$

3) area =  $21\sqrt{2}$

perimeter:  $18 + 6\sqrt{2}$

4) (parallelogram)

area = (base)(height)  
 $= (16)(12) = 192$

perimeter: 58

5) (parallelogram)

area =  $56\sqrt{3}$

perimeter: 44

6) (rectangle)

area =  $112\sqrt{2}$

perimeter =  $28 + 16\sqrt{2}$

7)

perimeter: 24

area =  $\frac{1}{2}$  (base1 + base2)(height)  
 $= \frac{1}{2} (9 + 6)(4) = 30$

8)

area =  $80 + 16\sqrt{5}$

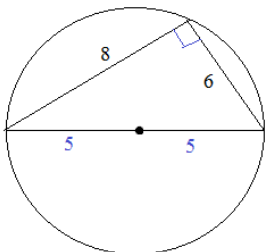
perimeter =  $40 + 4\sqrt{5}$

9) (kite)

perimeter = 24

area =  $\frac{1}{2}$  (diagonal 1)(diagonal 2)  
 $= \frac{1}{2} (6)(4 + 2\sqrt{10}) \approx 40.97$

10) (circle)



Note: triangle inscribed in semicircle is right triangle..

perimeter/circumference:  $10\pi$

area =  $25\pi$

11)

area =  $(1/2)(10)(12) = 60$

perimeter =  $28 + 2\sqrt{61}$

Pythagorean Theorem:  $a^2 + b^2 = c^2$

rectangle: area = (length)(width)  
 perimeter = 2(length) + 2(width)

triangle: area =  $(1/2)$ (base)(height)  
 perimeter = (side) + (side) + (side)

circle: area =  $\pi$ (radius)<sup>2</sup>  
 circumference =  $2\pi$ (radius)

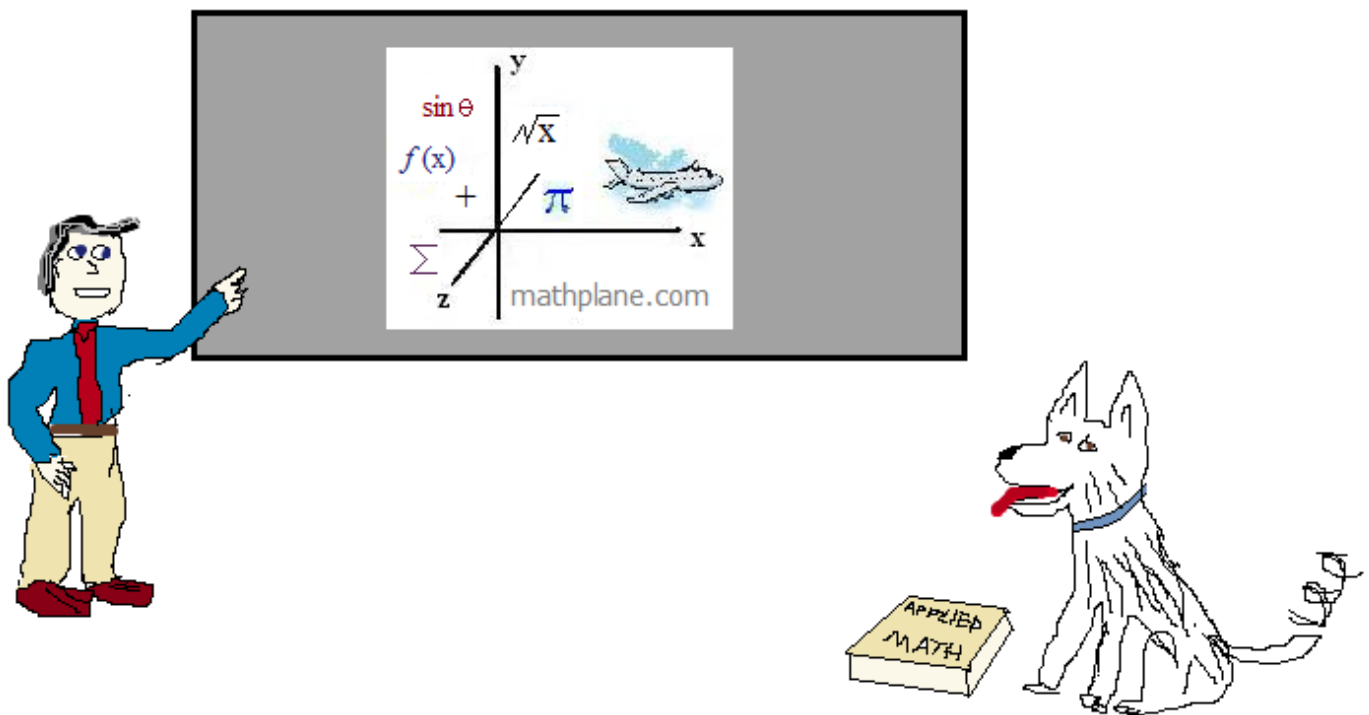
kite: area =  $(1/2)$ (diagonal 1)(diagonal 2)  
 (or, find area of each triangle)

trapezoid: area =  $(1/2)$ (base1 + base2)(height)

Thanks for visiting. (Hope it helped!)

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