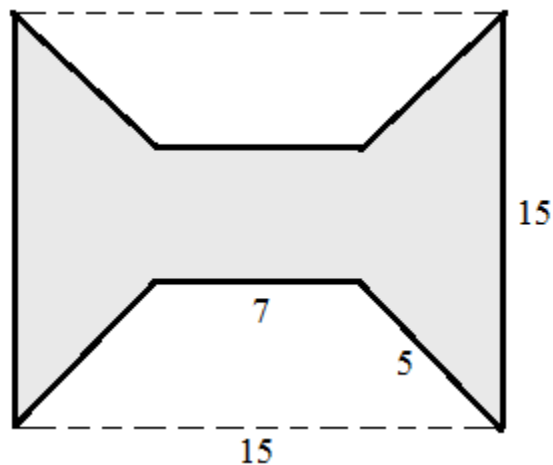


Geometry: Area and Perimeter of Complex Shapes

Notes, Examples, and Practice Quiz (with Solutions)

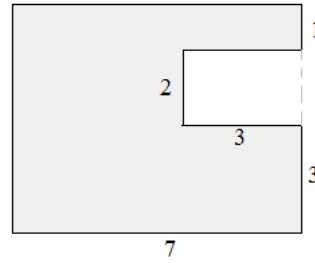


Topics include sector area, segments, special quadrilaterals, ratios, regular polygons, and more.

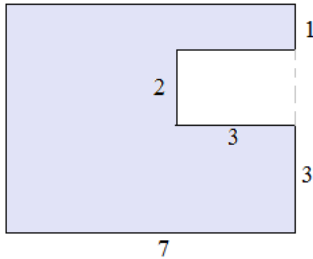
Area and Perimeter of Polygons

Example: Find the area and perimeter of the (concave) polygon.

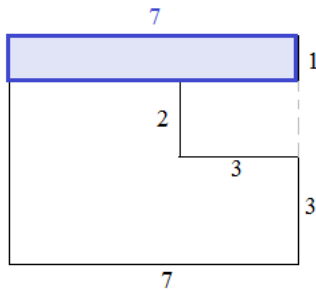
Since this is not an ordinary shape, a basic formula will not work.
There are 2 approaches to finding the area...



Method 1: "Add the parts"



Step 1: Divide into parts and label



Step 2: Find area of each part

area of rectangle: (length)(width)

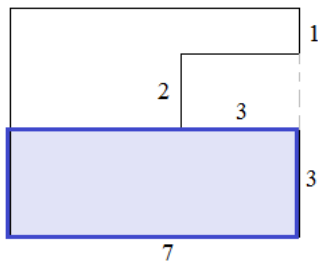
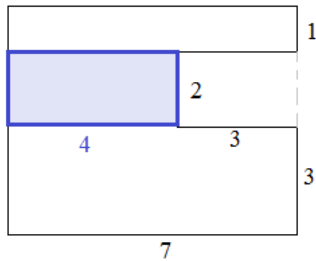
$$\text{areas: } 7 \times 1 = 7$$

$$4 \times 2 = 8$$

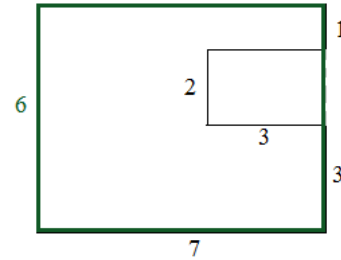
$$7 \times 3 = 21$$

Step 3: Add the areas

$$7 + 8 + 21 = \boxed{36 \text{ square units}}$$



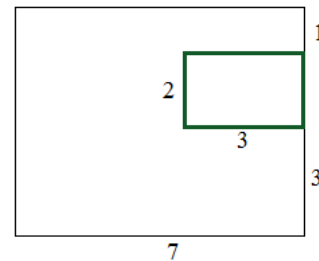
Method 2: "Cut out piece(s)"



Step 1: find the area of the entire rectangle

Area = length x width

$$42 = 7 \times 6$$



Step 2: cut out piece(s)

Area = length x width

$$6 = 3 \times 2$$

Step 3: subtract

$$42 \text{ sq. units} - 6 \text{ sq. units} = \boxed{36 \text{ square units}}$$

To find the perimeter, carefully add up all the sides:

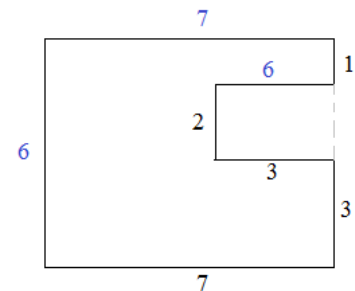
"outer" sides:

$$7 + 6 + 7 + 1 + 3 = 24$$

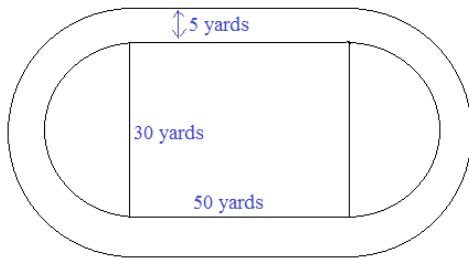
"inner" sides:

$$6 + 2 + 3 = 11$$

$$\boxed{\text{total perimeter: } 35 \text{ units}}$$



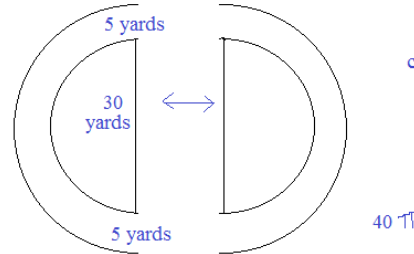
Example: Find the distance of the outer track:



The distance of the OUTER track is the lengths of 2 line segments and the circumference of a circle.

The 2 straight parts of the track: 50 yards each...
100 yards total

The combined turns form a circle with diameter 40 yards

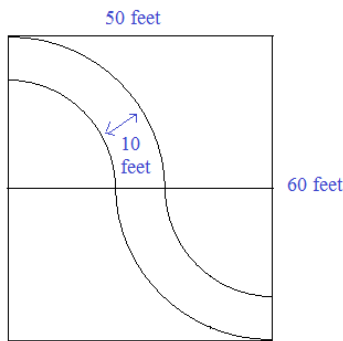


circumference is π (diameter)

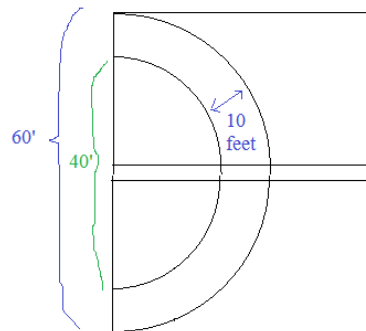
40π

Total circumference is
 $100 + 40\pi$ yards
approx. 225.7 yards

Example: The city wants to pave both sides of this winding road. If the road is 10 feet wide, how long will this stretch of side pavement be?



If you rearrange the arcs in the road, they form a semicircle!



"outer" sidewalk..

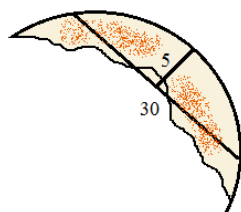
$$\frac{1}{2} (\text{circumference}) = 30\pi$$

"inner" sidewalk

$$\frac{1}{2} (\text{circumference}) = 20\pi$$

Total paved sides:
 50π

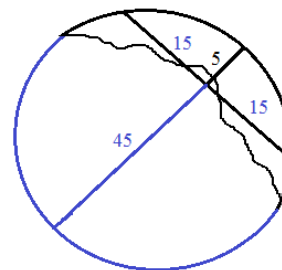
Example: At a pizzeria, a waiter and customer disagreed about a pizza order. Unfortunately, the customer had eaten most of the pizza. However, the math chef came out and measured the remaining portion and determined the size (diameter) of the original pizza.



What was the size of the original, full pizza?

Distance of midpoint of arc to the midpoint of the chord is 5 cm.

Chord measures 30 cm.



$(15)(15) = (5)(X)$
chord-chord power theorem

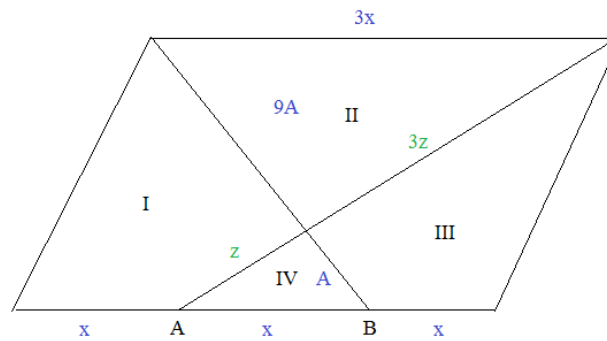
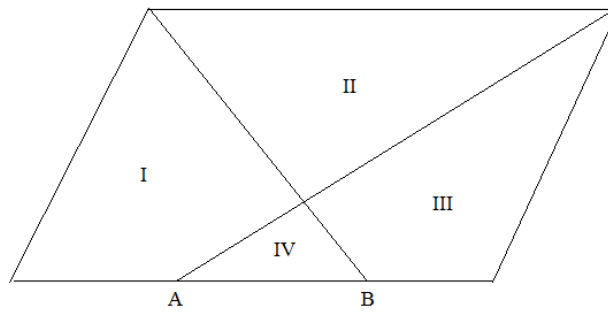
$$X = 45$$

So, the diameter of the pizza is 50 cm..

And, the area of the pizza is 625 sq cm..

Example: Given: Parallelogram and
trisection points A and B

Find the ratio of the areas of the
sections I : II : III : IV

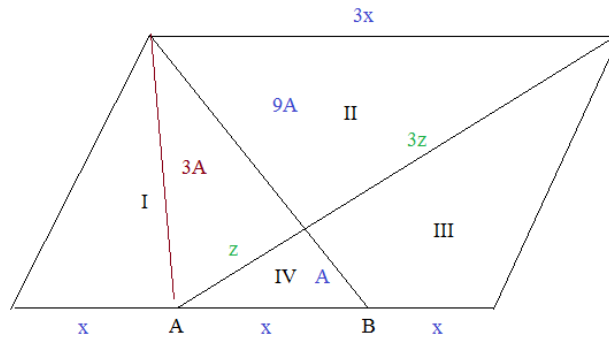


Since triangles II and IV are similar (angle-angle) and their sides are proportional with a ratio of $x:3x$, the ratio of the areas is $1:9$

Draw an auxiliary line bisecting triangle I/IV

Since triangles II and IV are proportional $3:1$, we know the sides z and $3z$...

Answer: I $7A$
II $9A$
III $7A$
IV A



since triangles have same height and bases of ratio $z:3z$,

the triangles are $3A:9A$

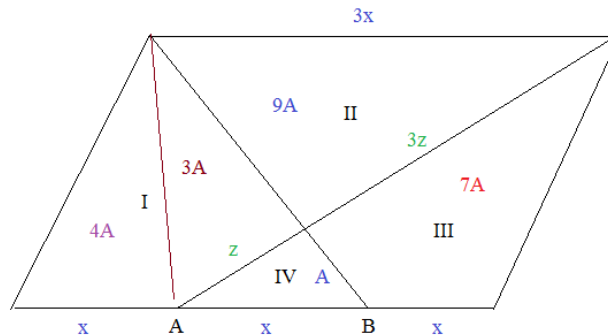
$$3A + A = 4A$$

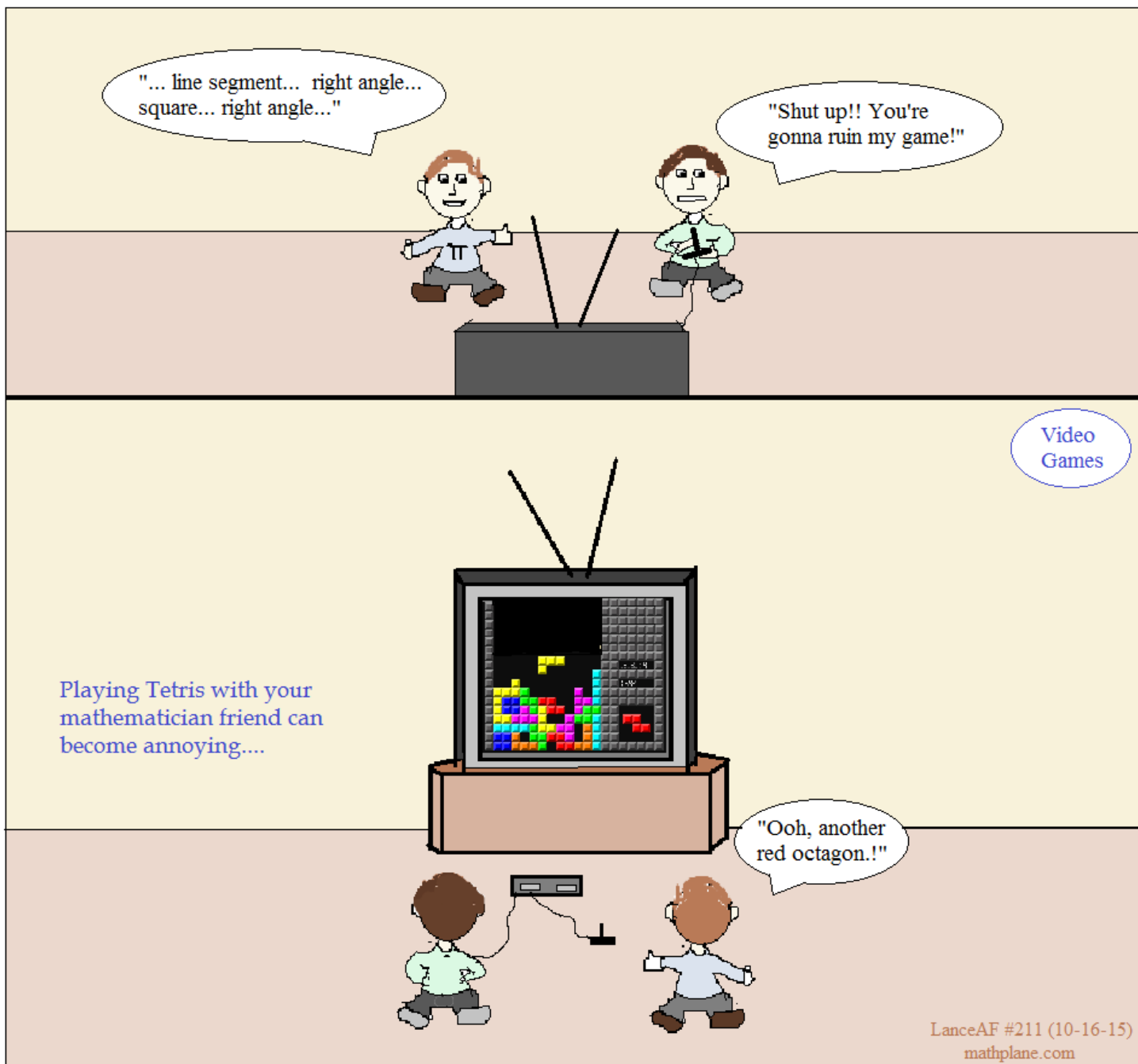
because the triangles have the same base x and x and the same height...

Section I has an area of $4A$ (with a base of x)

Sections IV and III has an area of $8A$ because the base is $2x$..

Both triangles have the same height...

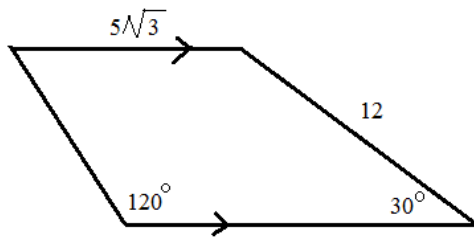




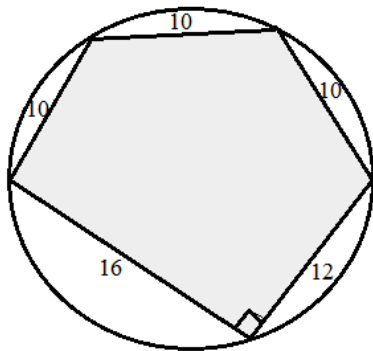
Practice Quiz-→

Area and Perimeter of
Shapes Exercise

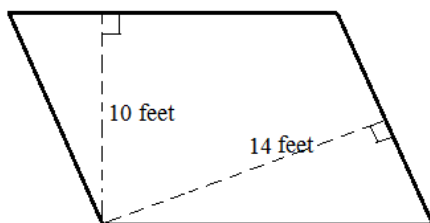
- 1) Find the perimeter and area of the quadrilateral:



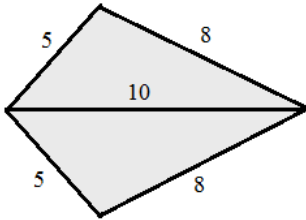
- 2) The pentagon is inscribed in the circle:
What is the area?



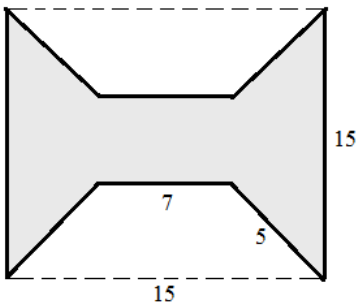
- 3) If the perimeter of the parallelogram is 120 feet,
what is the area?



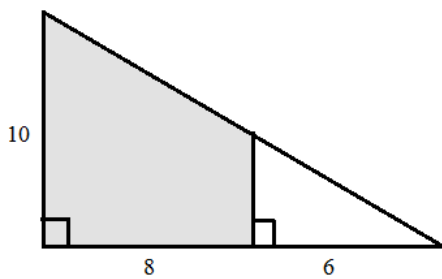
4) Find the area of the kite



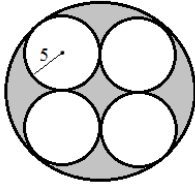
5) The shaded region is a square with 2 congruent isosceles trapezoids removed.
What is the perimeter and area of the shaded figure?



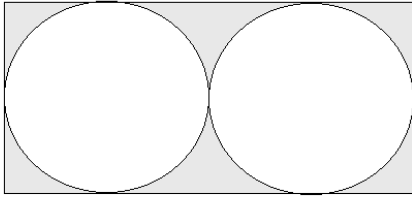
6) Determine the area and perimeter of the shaded region.



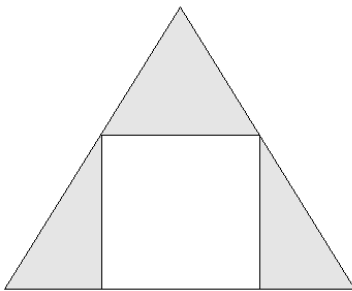
- 7) 4 congruent, tangent circles are inscribed in a large circle.
If each small circle has radius 5, what is the shaded area?



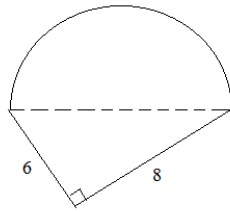
- 8) The radius of each tangent circle is 8.
What is the shaded area?



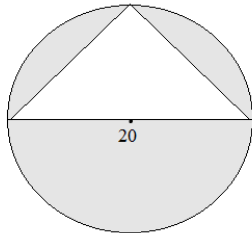
- 9) The figure is a square 'inscribed' in an equilateral triangle with perimeter 24.
Find the shaded area...



10) Find the perimeter and area of the semicircle and right triangle figure:



11) Find the shaded area:



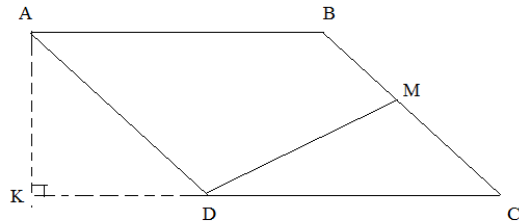
12) Parallelogram ABCD has an area of 60

$\overline{BC} = 14$

M is the midpoint of \overline{BC}

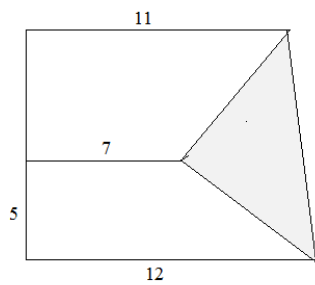
$DM \perp BC$

What is the length of \overline{AK} ?



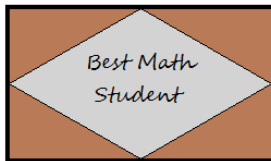
13) The 2 trapezoids have the same areas.

What is the area of the triangle?

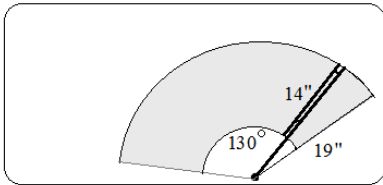


- 14) The height of a triangle is 8 inches greater than its base.
If the area of the triangle is 90 square inches, what is the base and the height?

- 15) The plaque is a rhombus inscribed in a rectangle.
If the diagonals are 12" and 8", how much wood area is available for engraving?

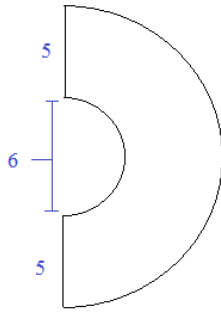


- 16) A windshield wiper extends 130 degrees.
Using the diagram, what area of glass is cleared by the wiper?

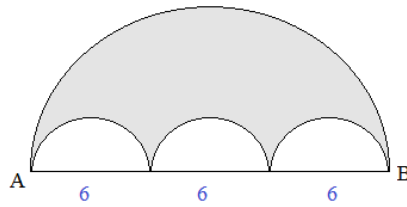


- 17) A trapezoid has diagonals measuring 30 and height 18.
What is the area?

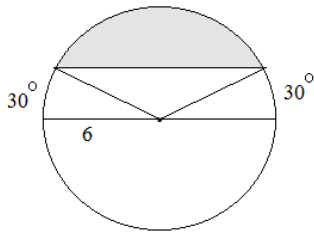
18) Find the perimeter.



19) Length of \overline{AB} is 18.
The 3 semicircles are congruent.
What is the shaded area?

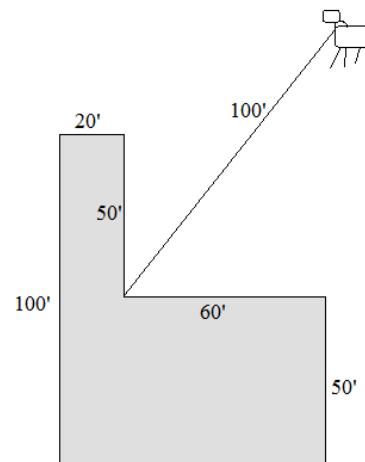


20) Find the measure of the shaded area:



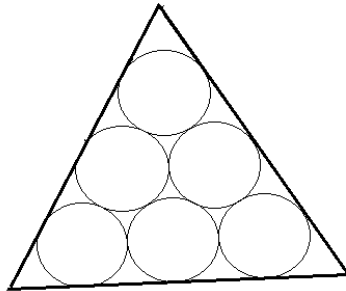
21) A cow is tethered to a 100-foot rope, attached to the inside corner of an L-shaped building (as shown in the diagram).

Find the grazing area of the cow.

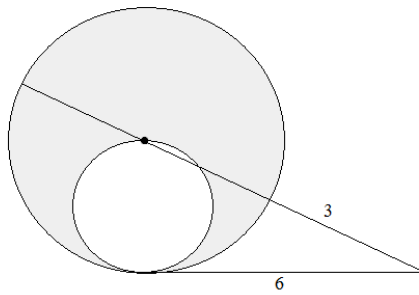


22) A regular hexagon is created by connecting the midpoints of a larger regular hexagon. What is the ratio of the area of the small hexagon to the area of the large hexagon?

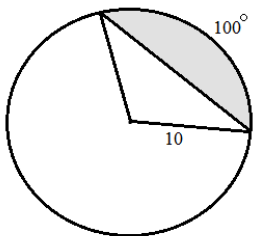
23) Each of these 6 inscribed/tangent circles are congruent with radius 8. What is the perimeter of the triangle?



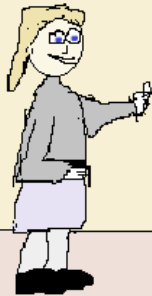
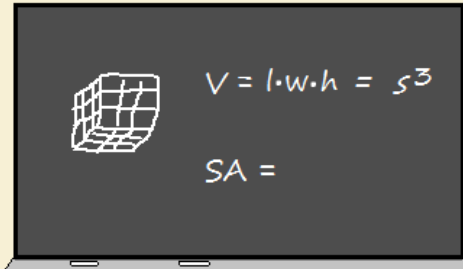
24) Can you find the shaded area?



25) What is the area of the shaded segment?



"... if we wanted to paint this object, ---- "



"--- the surface area is the total area of the six sides. Since each side is 9 square units, the surface area is 54!"

"Can we use different colors?"



Rubik's
Cube

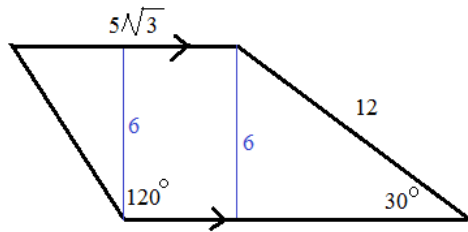
Thinking about the box....

... and, *outside the box*....

ANSWERS-→

SOLUTIONS

1) Find the perimeter and area of the quadrilateral:

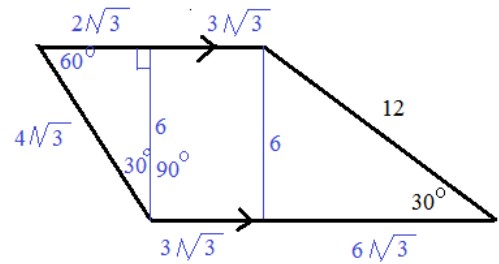


Since upper and lower base are parallel, it's a trapezoid.

Perimeter of trapezoid: $12 + 18\sqrt{3}$
 area of trapezoid: $42\sqrt{3}$

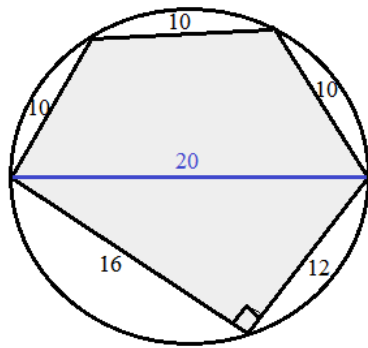
Draw 2 (congruent) altitudes, dividing figure into 3 measurable shapes...

Since the 30-60-90 right triangle has hypotenuse of 12, the opposite side is 6...



2) The pentagon is inscribed in the circle:

What is the area?



Since the bottom of the pentagon is a right angle, we can divide the pentagon into 2 parts!

(triangle inscribed in a semicircle is a right triangle)

Part 1: right triangle area --- $1/2(12)(16) = 96$

Part 2: inscribed quadrilateral --- (Brahmagupta's Formula)

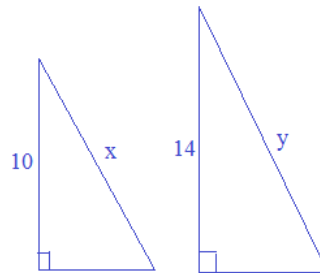
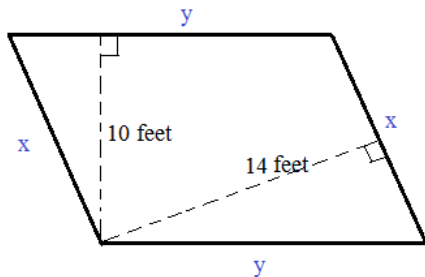
semi-perimeter (s) = $\frac{10 + 10 + 10 + 20}{2} = 25$

area = $\sqrt{(25 - 10)(25 - 10)(25 - 10)(25 - 20)} = \sqrt{16875}$

129.9

Total area of pentagon = 154.9 square units (approximately)

3) If the perimeter of the parallelogram is 120 feet, what is the area?



According to Angle-Angle similarity theorem, the 2 right triangles are similar!

$\frac{10}{14} = \frac{x}{y}$

$y = \frac{7x}{5}$

Parallelogram: opposite angles are congruent
 opposite sides are congruent

$2x + 2y = 120$ feet

area = (base)(height)

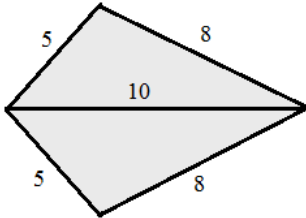
= (10 feet)(35 feet) = 350 square feet

$2x + 2(\frac{7}{5}x) = 120$ feet

$\frac{10}{5}x + \frac{14}{5}x = 120$ feet

$x = 25$ feet and, $y = 35$ feet..

4) Find the area of the kite



SOLUTIONS

The area of a kite is $\frac{1}{2}(\text{diagonal 1})(\text{diagonal 2})$
 However, we don't know the length of the other diagonal...

So, view this figure as 2 triangles, and use Hero's formula to solve...

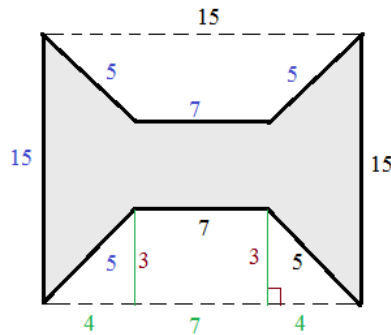
$$\text{semiperimeter } (s) = \frac{5 + 8 + 10}{2} = 11.5$$

$$\text{area} = \sqrt{(11.5)(11.5 - 10)(11.5 - 8)(11.5 - 5)}$$

$$= \sqrt{(11.5)(1.5)(3.5)(6.5)} = 19.81 \text{ (approximately)}$$

So, area of kite is **39.62 square units**

5) The shaded region is a square with 2 congruent isosceles trapezoids removed.
 What is the perimeter and area of the shaded figure?



The perimeter is the sum of all the segments:

$$5 + 7 + 5 + 15 + 5 + 7 + 5 + 15 = \mathbf{64 \text{ units}}$$

The area of the shaded figure = area of the square - area of 2 trapezoids

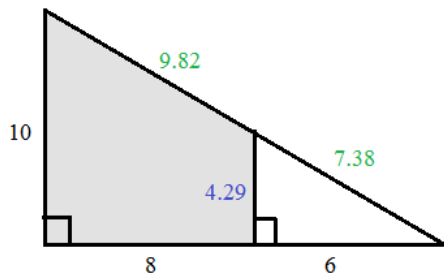
$$\text{area of square} = 225 \text{ sq. units}$$

$$\begin{aligned} \text{area of trapezoid} &= \frac{1}{2} (\text{base1} + \text{base2})(\text{height}) \\ &= \frac{1}{2} (15 + 7)(3) = 33 \end{aligned}$$

using pythagorean theorem and fact that it's isosceles, we determine the height is 3

$$\text{shaded area} = 225 - 2(33) = \mathbf{159 \text{ square units}}$$

6) Determine the area and perimeter of the shaded region.



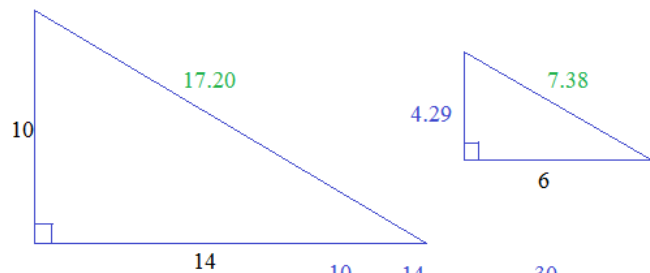
$$\text{Perimeter} = 9.82 + 4.29 + 8 + 10 = \mathbf{32.11}$$

$$\begin{aligned} \text{Area} &= \text{big triangle area} - \text{small triangle area} \\ &= 70 - 12.87 = \mathbf{57.13} \end{aligned}$$

(also, area is lower rectangle + upper triangle...

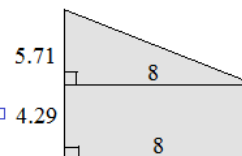
$$(8 \times 4.29) + \frac{1}{2}(8)(5.71) = 34.32 + 22.84 = \mathbf{57.16}$$

First, we need to find the lengths of the remaining segments.
 (Notice, 2 similar right triangles...)

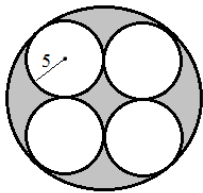


$$\frac{10}{x} = \frac{14}{6} \quad x = \frac{30}{7} = 4.29$$

Then, use Pythagorean Theorem to find hypotenuses..



7) 4 congruent, tangent circles are inscribed in a large circle. If each small circle has radius 5, what is the shaded area?



Finding the area of each small circle is straightforward:

$$\text{area of circle} = \pi(\text{radius})^2 = 25\pi$$

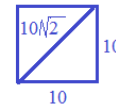
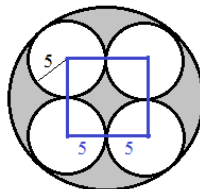
$$\text{Therefore, total "area that will be cut out"} = 100\pi$$

$$\text{Total shaded area} = (50\sqrt{2} - 25)\pi$$

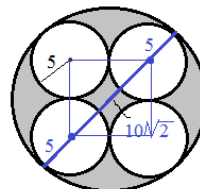
approximately 143.6 square units

How do we find the area of the large circle?

If we connect the 4 centers to each other, we construct a square with sides 10!



SOLUTIONS



Diameter of large circle is

$$10 + 10\sqrt{2}$$

then, radius of large circle is

$$5 + 5\sqrt{2}$$

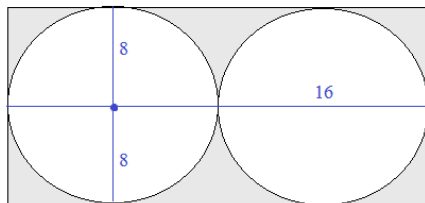
$$\text{area of circle} = \pi(\text{radius})^2 = (5 + 5\sqrt{2})^2 \pi$$

$$\begin{aligned} \text{area of large circle} &= (25 + 50\sqrt{2} + 50)\pi \\ &= (75 + 50\sqrt{2})\pi \end{aligned}$$

8) The radius of each tangent circle is 8.

What is the shaded area?

The area of the rectangle is
length x width =
 $32 \times 16 = 512$



Then, the area of each circle is

$$\pi(\text{radius})^2 =$$

$$\pi(8) = 64\pi$$

So, the shaded area is

$$\text{rectangle} - 2(\text{each circle}) =$$

$$512 - 128\pi \quad (\text{approximately } 110)$$

9) The figure is a square 'inscribed' in an equilateral triangle with perimeter 24

Find the shaded area...

$$\text{area of triangle is } \frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3}$$

$$x + \frac{x}{\sqrt{3}} + \frac{x}{\sqrt{3}} = 8$$

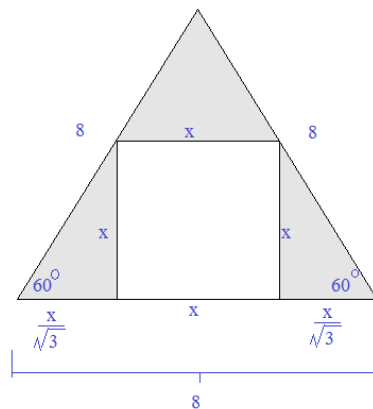
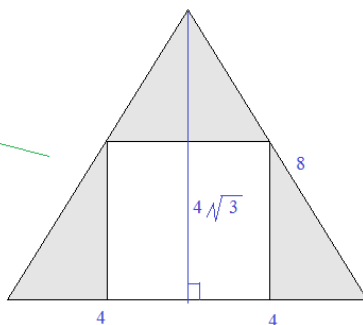
$$x(1 + \frac{2}{\sqrt{3}}) = 8$$

$$x = \frac{8}{(1 + \frac{2}{\sqrt{3}})} = 3.71$$

so, the area of the square is approx. 13.8

shaded area = triangle - square

$$16\sqrt{3} - 13.8 = 13.9$$



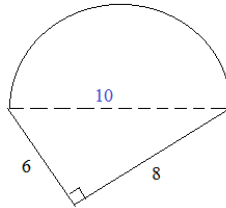
SOLUTIONS

10) Find the perimeter and area of the semicircle and right triangle figure:

diameter is 10 (because of Pythagorean Theorem)

Arc length of semicircle = $(1/2)10\pi$

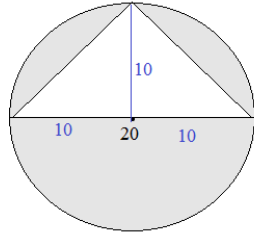
Area of semicircle = $(1/2)25\pi$



Area of figure = $24 + 12.5\pi$

Perimeter of figure = $14 + 5\pi$

11) Find the shaded area:



circle area is 100π

triangle area: $1/2(\text{base})(\text{height})$

$1/2(20)(10) = 100$

height is 10, because all radii are congruent

Shaded area = $100\pi - 100$

approx. 214 sq units

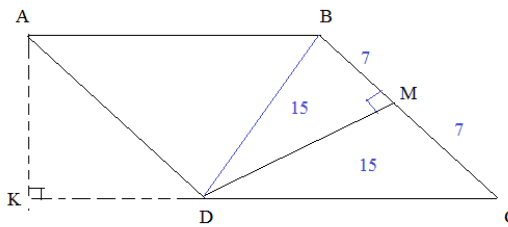
12) Parallelogram ABCD has an area of 60

$\overline{BC} = 14$

M is the midpoint of \overline{BC}

$DM \perp BC$

What is the length of \overline{AK} ?



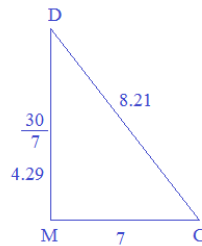
Since M is midpoint, we can divide the segments..

Then, since area of parallelogram is 60, we can draw segments to partition it..

The area of triangle CMD is 15 and the base is 7

$\frac{1}{2} (7)(\text{height}) = 15$

height = $30/7$



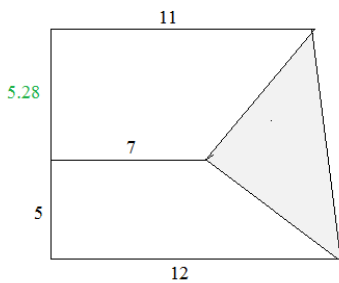
$DC = 8.21...$ This is the base of parallelogram ABCD !!

$60 = 8.21(\text{height})...$

height = 7.31

13) The 2 trapezoids have the same areas.

What is the area of the triangle?



Area of top trapezoid = $\frac{1}{2} h(7 + 11)$

Since trapezoids have same areas, $\frac{1}{2}(5)(7 + 12) = \frac{1}{2} h(7 + 11)$

Area of bottom trapezoid = $\frac{1}{2}(5)(7 + 12)$

$95 = 18h$

$h = 5.28$

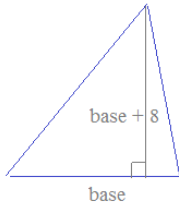
Area of ENTIRE trapezoid = $\frac{1}{2} (10.28)(12 + 11) = 118.22$

Area of each small trapezoid = 47.5

Therefore, area of triangle = $118.22 - 47.5 - 47.5 = 23.22$

- 14) The height of a triangle is 8 inches greater than its base.
If the area of the triangle is 90 square inches, what is the base and the height?

let b = base
 h = height
 $h = b + 8$



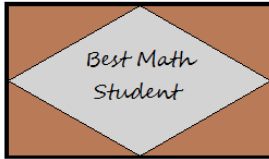
$$\begin{aligned} \text{Area} &= \frac{1}{2} (\text{base})(\text{height}) \\ &= \frac{1}{2} (b)(b+8) = 90 \\ &= b^2 + 8b = 180 \\ b^2 + 8b - 180 &= 0 \\ (b+18)(b-10) &= 0 \end{aligned}$$

$$b = 10, -18$$

base = 10 inches (cannot be negative!)
then, height = 18 inches

SOLUTIONS

- 15) The plaque is a rhombus inscribed in a rectangle.
If the diagonals are 12" and 8", how much wood area is available for engraving?



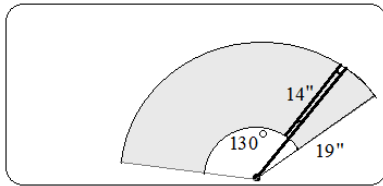
Since the rhombus is inscribed in the rectangle, the diagonals are the same lengths as the sides of the rectangle...

$$\text{Area of rectangle} = (\text{length})(\text{width}) = 12 \times 8 = 96 \text{ sq. inches}$$

$$\text{Area of the rhombus} = \frac{1}{2} d_1 d_2 = \frac{1}{2} (12)(8) = 48 \text{ sq. inches}$$

$$\text{The remaining wood area is } 96 - 48 = 48 \text{ sq. inches...}$$

- 16) A windshield wiper extends 130 degrees.
Using the diagram, what area of glass is cleared by the wiper?



The "outer" sector has a radius of 19 inches.

$$\text{outer area} = \frac{130}{360} \pi (19)^2 = 409.5$$

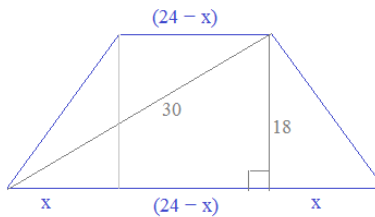
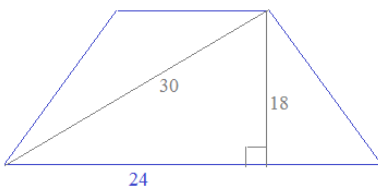
The "inner" sector has a radius of 5 inches.

$$\text{inner area} = \frac{130}{360} \pi (5)^2 = 28.4$$

shaded area is approximately 381.1 sq inches

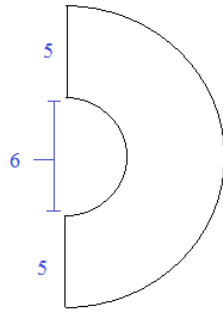
- 17) A trapezoid has diagonals measuring 30 and height 18.
What is the area?

Since the diagonals are 30, it must be an isosceles trapezoid...
Here is a sketch..



$$\begin{aligned} \text{Area} &= \frac{1}{2} (\text{base1} + \text{base2})(\text{height}) \\ &= \frac{1}{2} ((24-x) + (x + (24-x) + x))(18) \\ &= \frac{1}{2} (48)(18) = 432 \end{aligned}$$

18) Find the perimeter.



SOLUTIONS

Area and Perimeter of Shapes Exercise

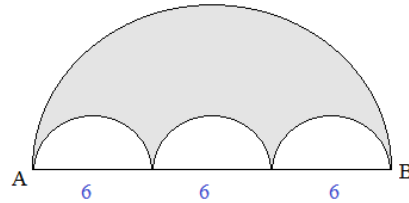
arc length of semicircle: $(1/2)\pi(\text{diameter})$

diameter of inner semicircle is 6
diameter of outer semicircle is 16

$$\left(\frac{1}{2}\right)16\pi + \left(\frac{1}{2}\right)6\pi + 5 + 5$$

$$11\pi + 10$$

19) Length of \overline{AB} is 18.
The 3 semicircles are congruent.
What is the shaded area?



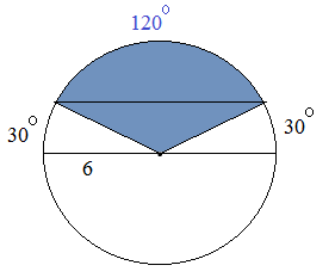
Area of semicircle = $(1/2)\pi(\text{radius})^2$

Large semicircle: $\left(\frac{1}{2}\right)81\pi = 40.5\pi$

Small semicircle: $\left(\frac{1}{2}\right)9\pi = 4.5\pi$

Shaded area: $40.5\pi - 3(4.5\pi) = 27\pi$

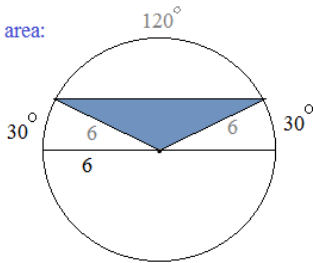
20) Find the measure of the shaded area:



"piece of pie" sector area:

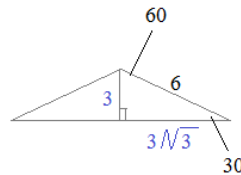
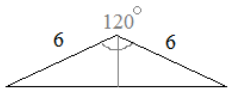
$$\frac{120^\circ}{360^\circ}\pi(6)^2 = 12\pi$$

"middle triangle" area:

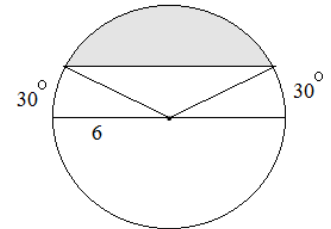


(sides are 6, because all radii are congruent)

obtuse angle is 120, because $30 + 30 + 120 = 180$



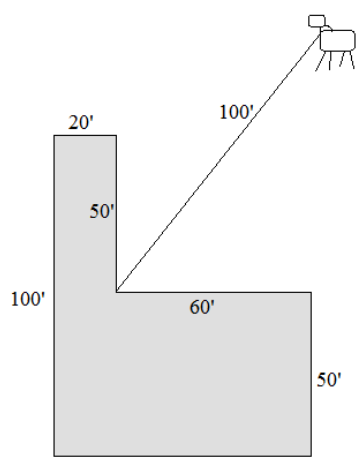
$$12\pi - 9\sqrt{3}$$



$$\frac{1}{2}(6\sqrt{3})(3) = 9\sqrt{3}$$

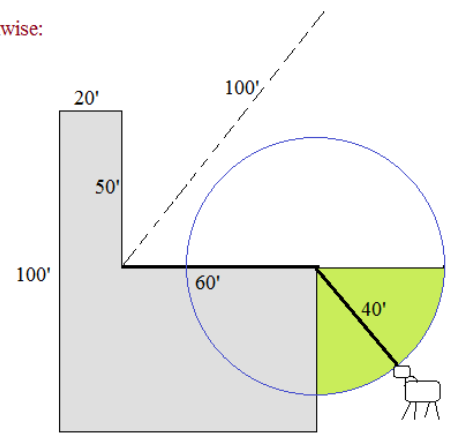
21) A cow is tethered to a 100-foot rope, attached to the inside corner of an L-shaped building (as shown in the diagram).

Find the grazing area of the cow.



$$\text{Sector Area} = \frac{\text{angle measure}}{360 \text{ degrees}} \pi r^2$$

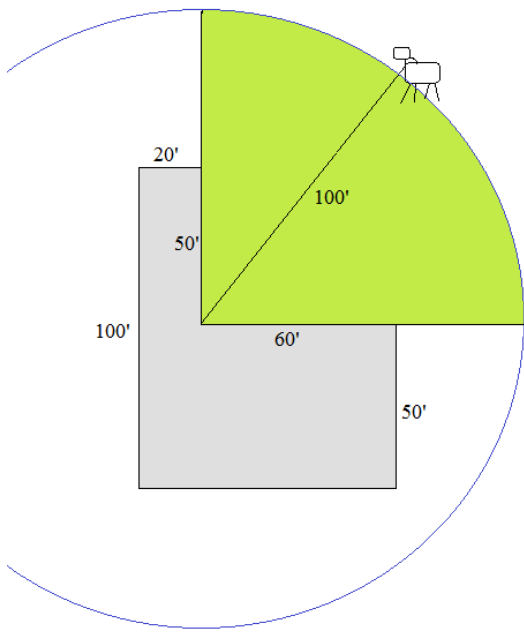
Going clockwise:



sector area: radius 40'
 $\ominus = 90 \text{ degrees}$

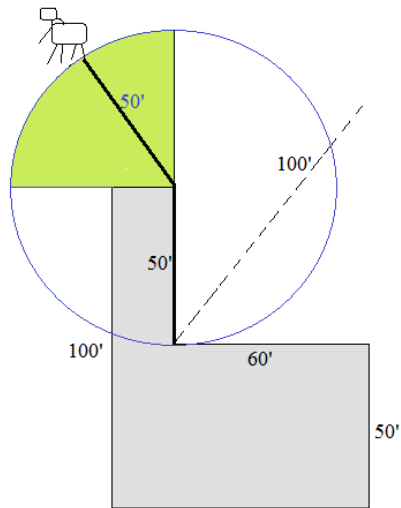
$$\frac{90 \text{ degrees}}{360 \text{ degrees}} \pi (40')^2 = 400\pi \text{ square feet}$$

Going Counterclockwise:



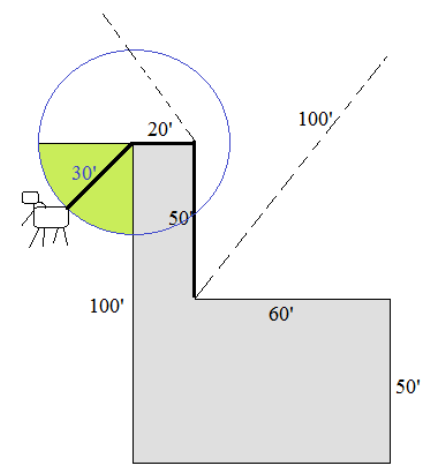
First sector area: radius 100'
 $\ominus = 90 \text{ degrees}$

$$\frac{90 \text{ degrees}}{360 \text{ degrees}} \pi (100')^2 = 2500\pi \text{ square feet}$$



Second sector area: radius 50'
 $\ominus = 90 \text{ degrees}$

$$\frac{90 \text{ degrees}}{360 \text{ degrees}} \pi (50')^2 = 625\pi \text{ square feet}$$

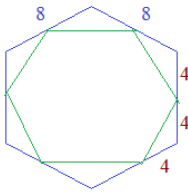


Third sector area: radius 30'
 $\ominus = 90 \text{ degrees}$

$$\frac{90 \text{ degrees}}{360 \text{ degrees}} \pi (30')^2 = 225\pi \text{ square feet}$$

Total Grazing area: 3750π square feet
 (approximately 11,781 sq. feet)

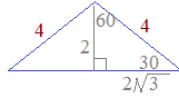
- 22) A regular hexagon is created by connecting the midpoints of a larger regular hexagon. What is the ratio of the area of the small hexagon to the area of the large hexagon?



We could use variables to describe the sides...
But, for ease, we'll assign easy numbers...

Let each side of the large hexagon be 8 units

Since each side is bisected by the inner hexagon, the lengths are 4 and 4



So, each side of outer hexagon is 8
and, each side of inner hexagon is $4\sqrt{3}$

since the ratio of the sides is

$$4\sqrt{3} : 8$$

the ratio of the areas is

$$48:64 \text{ or } 3:4$$

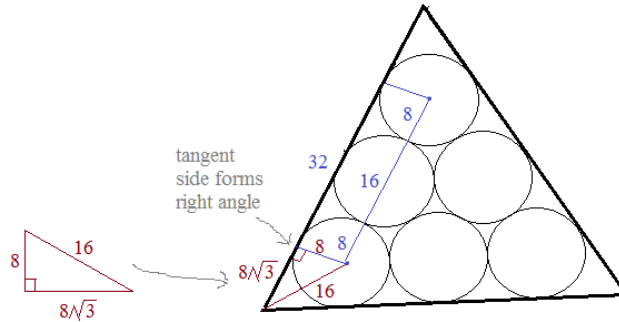
SOLUTIONS

- 23) Each of these 6 inscribed/tangent circles are congruent with radius 8. What is the perimeter of the triangle?

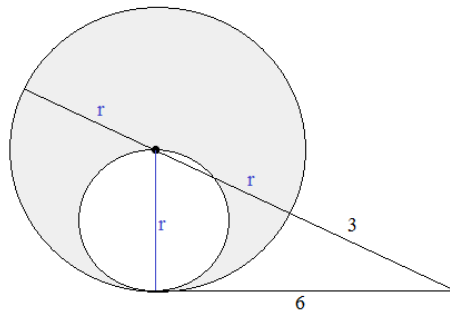
What is the perimeter of the triangle?

$$3(32) + 6(8\sqrt{3})$$

$$96 + 48\sqrt{3}$$



- 24) Can you find the shaded area?



Using power theorem:

$$6 \times 6 = 3 \times (3 + r + r)$$

$$12 = 2r + 3$$

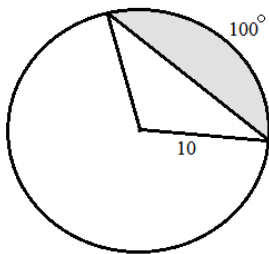
$$r = 4.5$$

So, $r = 4.5$ is the radius of the big circle..

And, 2.25 is the radius of the little circle..

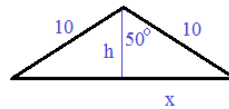
$$20.25 \pi - 5.0625 \pi = 15.1875 \pi$$

- 25) What is the area of the shaded segment?



$$\text{Area of full sector: } \pi (10)^2 \cdot \frac{100}{360} = 87.27 \text{ (approx)}$$

Area of triangle:



Using trigonometry...

$$\cos(50) = \frac{h}{10} \quad h = 6.43$$

$$\sin(50) = \frac{x}{10} \quad x = 7.66$$

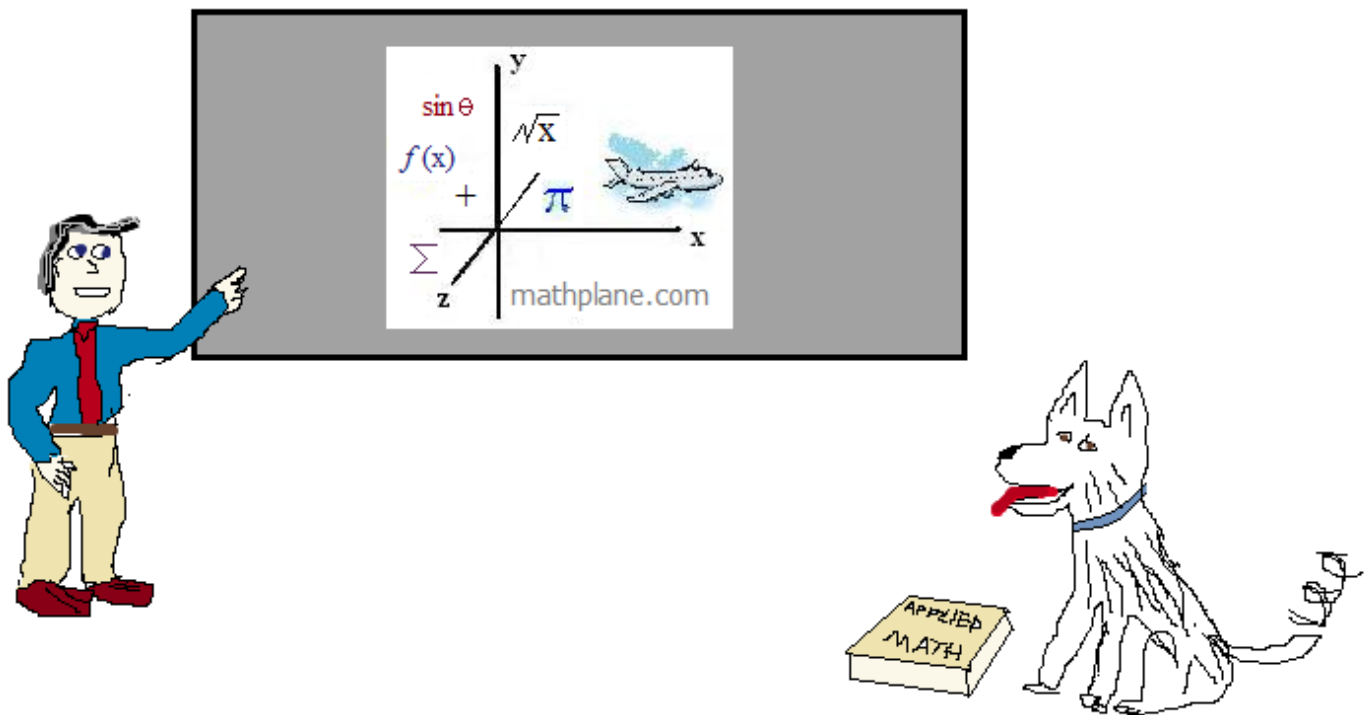
$$87.27 + 49.25 = 38 \text{ (approx)}$$

$$\text{Area} = \frac{1}{2} (15.32)(6.43) = 49.25 \text{ (approx)}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

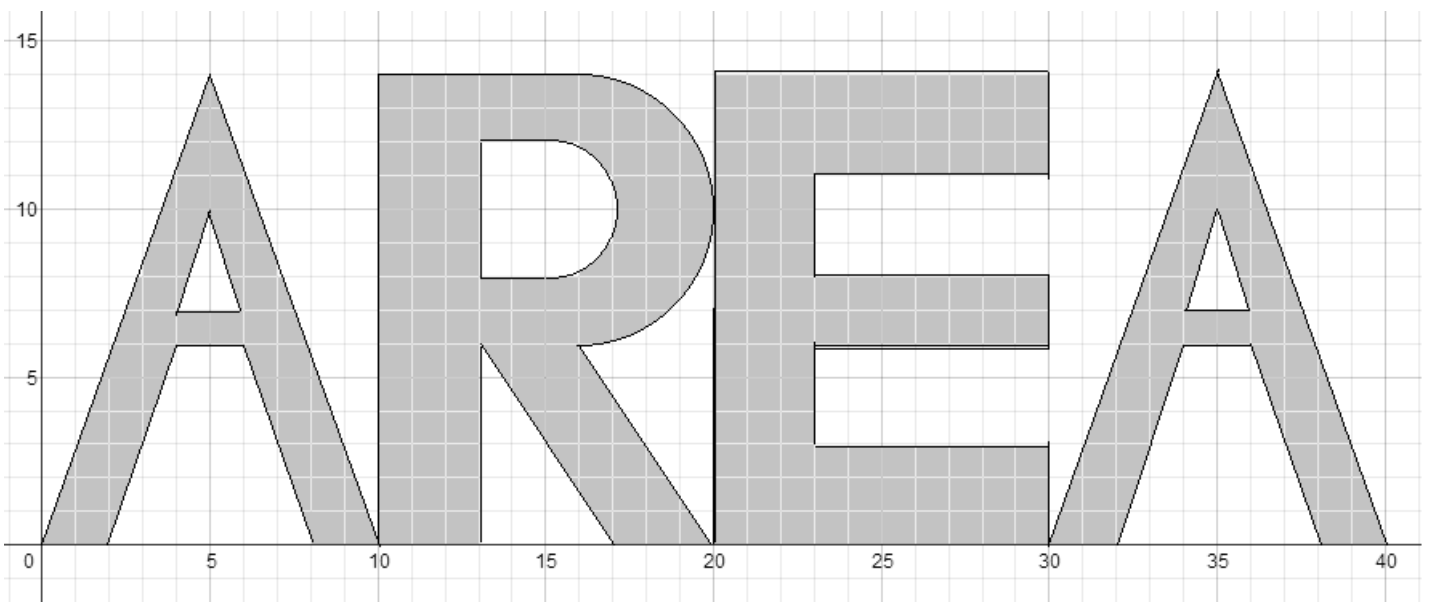
Cheers.



Also, Mathplane Express for mobile at mathplane.ORG

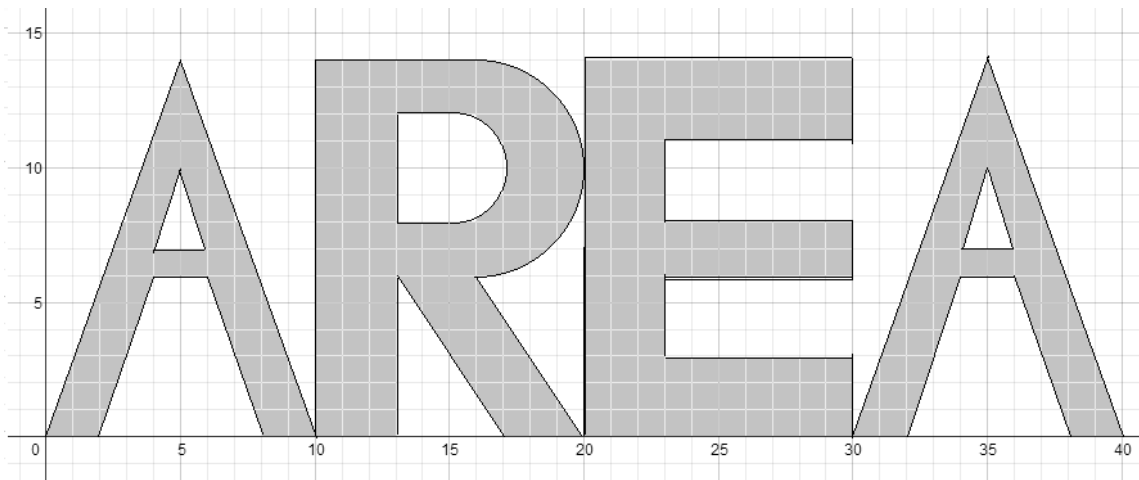
And, we're at Facebook, Google+, Pinterest, and TES and TeachersPayTeachers.com

ONE MORE QUESTION:



What is the measure of the shaded AREA ?

ANSWER-→



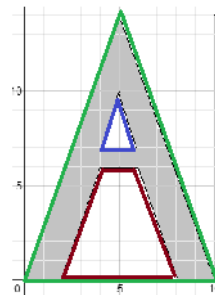
SOLUTION

What is the measure of the shaded AREA ?

Area of each "A":

Entire isosceles triangle - little triangle - trapezoid = Area of A

$$\frac{1}{2}(10 \times 14) - \frac{1}{2}(2 \times 3) - \frac{1}{2}(6 + 2)(6) = 43$$

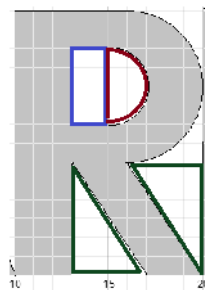


Area of the "R":

Area of bottom part:

Entire rectangle - right triangle - upside down right triangle =

$$(10 \times 6) - \frac{1}{2}(4 \times 6) - \frac{1}{2}(6 \times 4) = 36$$



Total area of the "R":

$$76 + 6\pi$$

Area of top part:

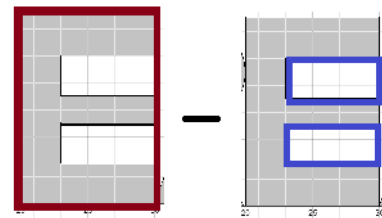
Entire "rectangle/semicircle" - Interior "rectangle/semicircle"

$$(8 \times 6) + \frac{1}{2}\pi(4)^2 - \left((2 \times 4) + \frac{1}{2}\pi(2)^2 \right) = 48 + 8\pi - 8 - 2\pi = 40 + 6\pi$$

Area of the "E":

Entire rectangle - cut out rectangles = Area of E

$$(14 \times 10) - 2(3 \times 7) = 98$$



OR, add the parts...

Bottom + Middle + Top + Left = Total

$$(3 \times 7) + (2 \times 7) + (3 \times 7) + (14 \times 3) = 98$$



TOTAL AREA: A R E A
 $43 + (76 + 6\pi) + 98 + 43$
 $260 + 6\pi$ or approximately 278.85