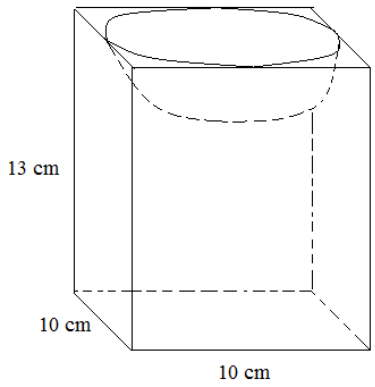
Geometry: Surface Area and Volume (Advanced)

Notes, Examples, and Practice Questions (with Solutions)

Mathplane.com

Example:



A hemisphere is "scooped out of a prism".
What is the surface area and volume?

Surface area: Base = $100\ cm^2$

Lateral Area = 4 x (10 cm x 13 cm) = $520\ cm^2$

Top: $100 - 25\pi$

Scooped out hemisphere: $\frac{4\pi \cdot 25}{2}$

Volume: Prism = 10 x 10 x 13 = $1300\ cm^3$

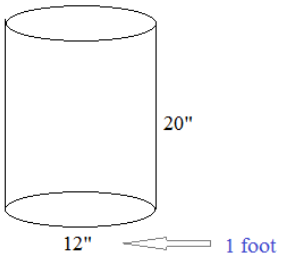
Hemisphere = $\frac{1}{2} \cdot \frac{4}{3}\pi$ (radius)^3

$= \frac{2}{3}(125)\pi\ cm^3$

TOTAL = $1300\ cm^3 - \frac{2}{3}(125)\pi\ cm^3$

Example: The density of a metal is 160 lbs/ft^3

What is the weight of the cylinder?



1.67' method 1: convert inches to feet...

$\pi(.5)^2(1.67) = 1.31$

209.75

method 2: convert feet to inches

160 lbs/ft^3

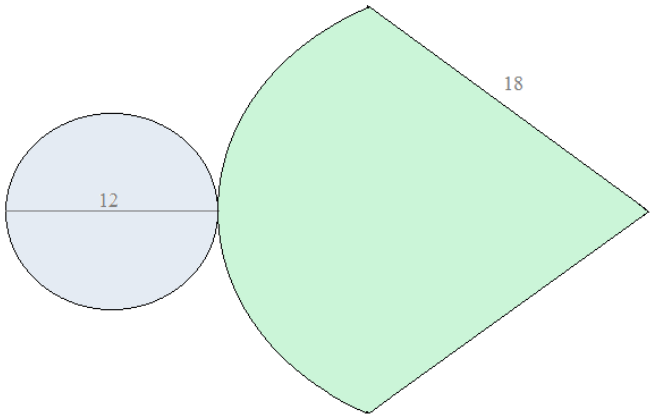
160 lbs/(12 x 12 x 12)

.0926 lbs/in^3

$\pi(6")^2(20") = 2260.8$

209.35

Example: Find the area of the shapes.



The blue circle is straightforward:

the area is 36π

Then, to find the green area, we'll consider the reconstructed shape: a cone

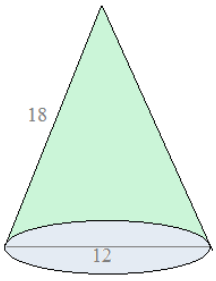
Surface area of the cone:

$36\pi + 108\pi = 144\pi$

Base Lateral Area

The green area is the lateral area..

108π



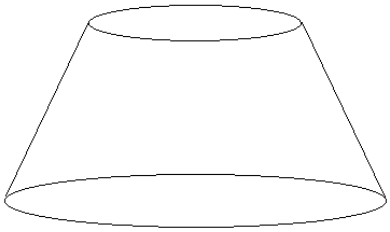
Frustum: Volume and Surface Area

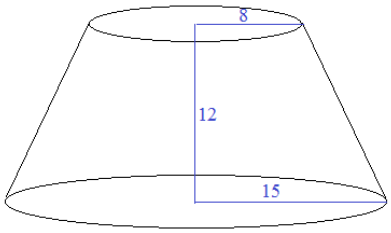
Example: Find the volume and surface area of the frustum...

radius of top: 8

radius of bottom: 15

height: 12

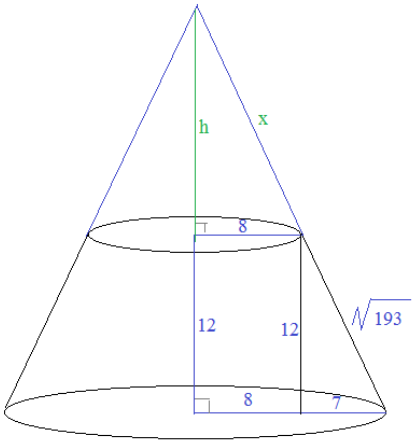




HINT: extend the frustrum top upward to create a cone...

Then, find ratio of top cone to entire cone...

then, frustrum = entire cone − top cone..



ratio of top cone to entire cone

$$\frac{h}{h+12} = \frac{8}{15}$$

$$15h = 8h + 96$$

$$7h = 96$$

$$h = \frac{96}{7}$$

Pythagorean Theorem
for top cone slant height

$$64 + \left(\frac{96}{7}\right)^2 = (\text{slant height})^2$$

$$x = \frac{8\sqrt{193}}{7}$$

Volume of top cone:

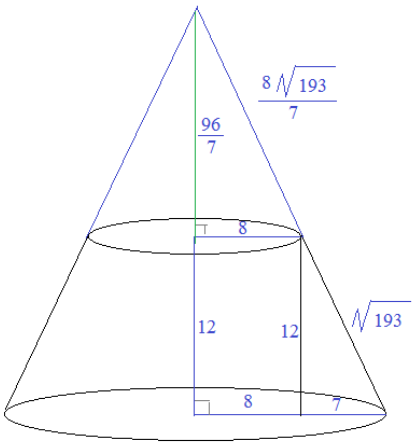
$$\frac{1}{3}(64\pi)\left(\frac{96}{7}\right) = \frac{2048\pi}{7}$$ approx. 919

Volume of entire cone:

$$\frac{1}{3}(225\pi)\left(\frac{180}{7}\right) = \frac{13500\pi}{7}$$ approx. 6059

919/6059 ≈ 512/3375

Volume of frustum: 5140 (approx.)



Note ratio of top cone to entire cone:

8:15

therefore, area will be 64:225

and volume will be 512:3375

Lateral area of top cone:

$$\frac{1}{2}(16\pi)(15.9) = 399.6$$

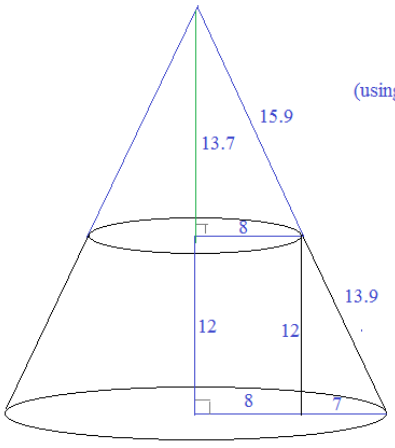
base circumference; slant height

Lateral area of entire cone:

$$\frac{1}{2}(30\pi)(29.8) = 1404.3$$

399.6/1404.3 ≈ 64/225

(using decimal approximations)



Lateral area of frustum: 1004.7

\+

Area of top circle: 64π

\+

Area of bottom circle: 225π

Surface area = $1004.7 + 289\pi$

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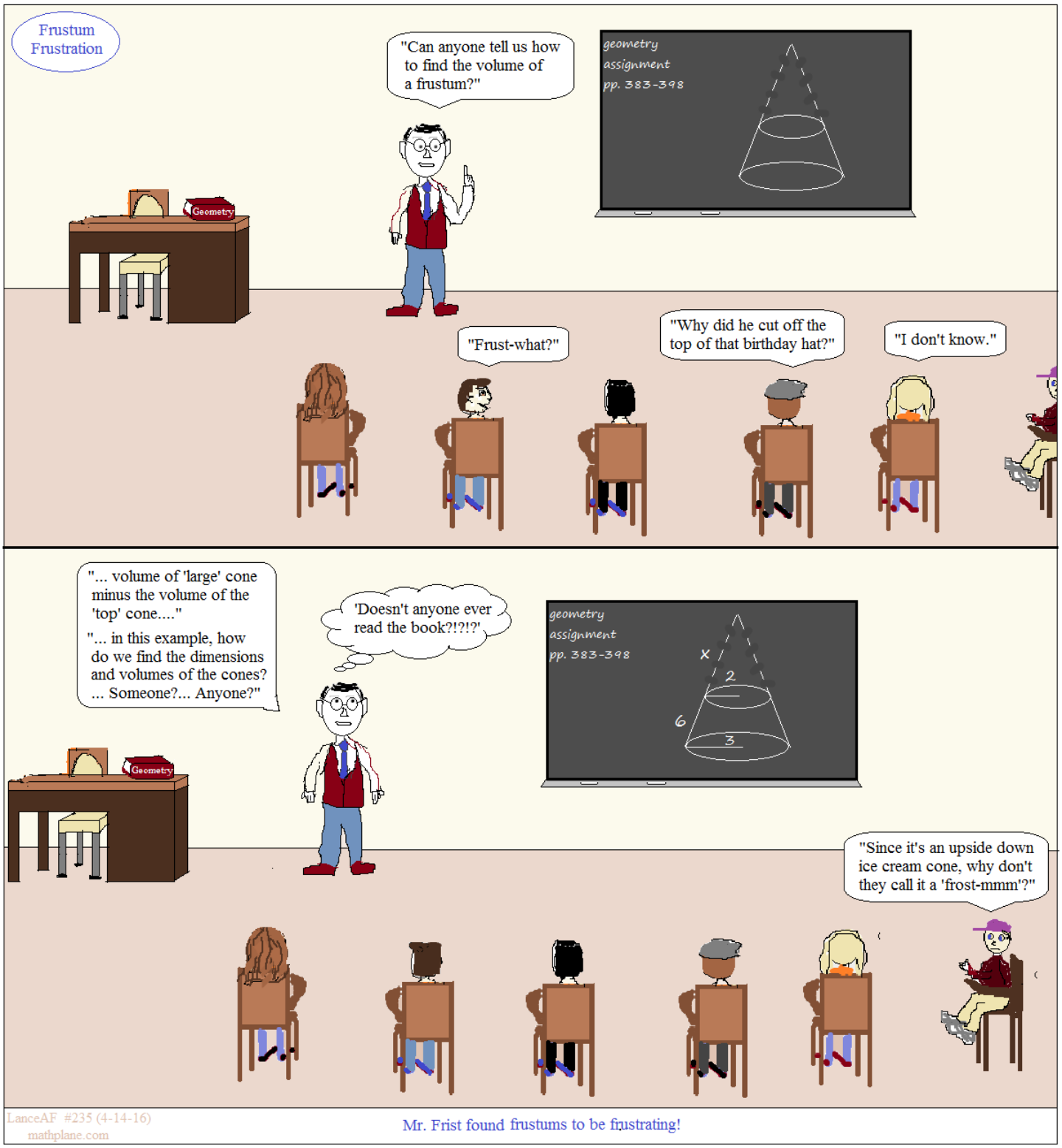
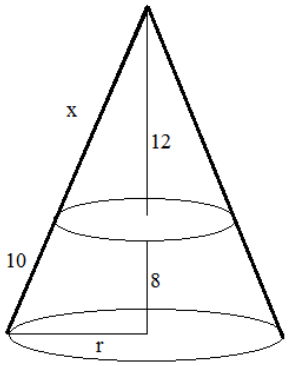

Frustum Frustration
"Can anyone tell us how to find the volume of a frustum?"
geometry assignment pp. 383-398
Geometry
"Frust-what?"
"Why did he cut off the top of that birthday hat?"
"I don't know."
"... volume of 'large' cone minus the volume of the 'top' cone...."
"... in this example, how do we find the dimensions and volumes of the cones? ... Someone?... Anyone?"
'Doesn't anyone ever read the book?!?!?'
geometry assignment pp. 383-398
x
2
6
3
Geometry
"Since it's an upside down ice cream cone, why don't they call it a 'frost-mmm'?"
LanceAF #235 (4-14-16)
mathplane.com
Mr. Frist found frustums to be frustrating!

Questions-→

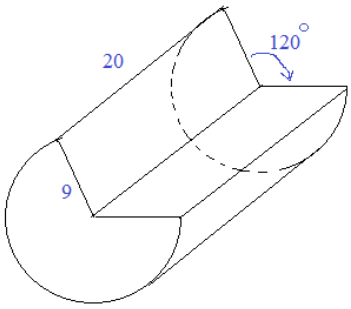
Surface Area and Volume (Advanced)

1) Find the volume and surface area of the entire cone (with radius r).

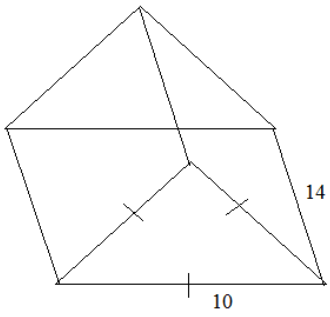


2) A 20" log with a radius of 9" has a uniform part carved out of it. (See diagram)

What is the volume of the carved log?
If you wanted to paint the log, what is the surface area that will be painted?

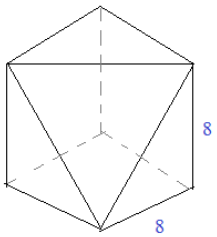


3) What is the volume of the triangle prism? And, surface area?

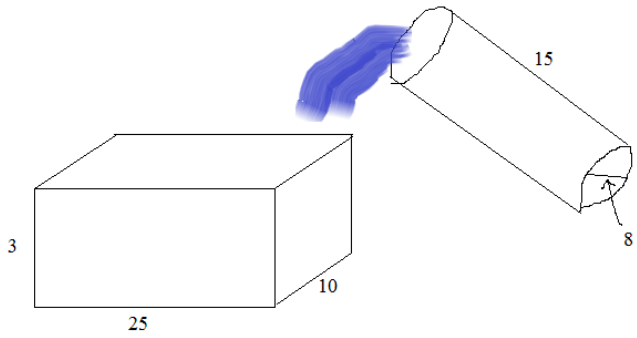


4) A cone has dimensions in which the ratio of the radius to height is 3:4. If the surface area is 384π, what is the length of the height?

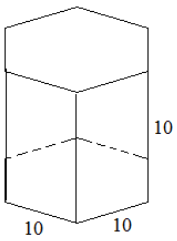
5) A cube with edges of length 8 cm has been sliced at 3 vertices as shown in the figure. What is the surface area?



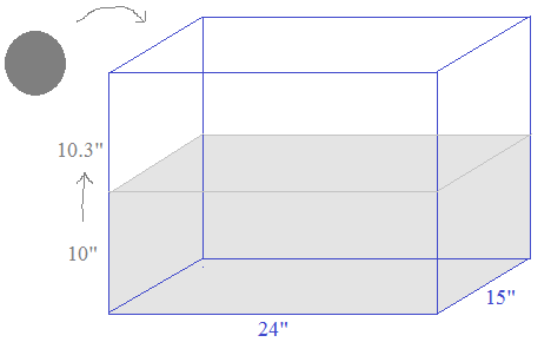
6) A cylinder with these dimensions is full of water. If it is poured into the open rectangular prism, will it overflow?



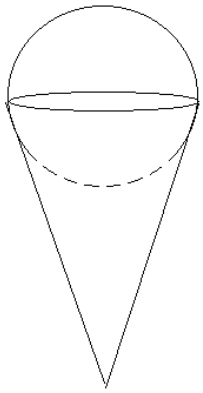
7) Find the volume and surface area of the hexagonal prism:



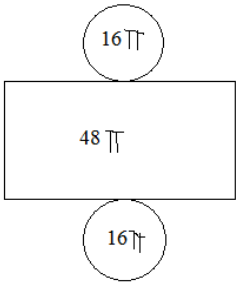
8) A rectangular tank has length 24 inches, height 20 inches, and depth 15 inches.
After filling half the tank with water, a solid steel ball is placed in the tank,
raising the water level from 10 inches to 10.3 inches.
What is the diameter of the solid ball?



9) An ice cream cone is 7" deep and 3" across.
If a round scoop of ice cream that is 3" wide sits on the cone,
would the ice cream overflow after it melts?

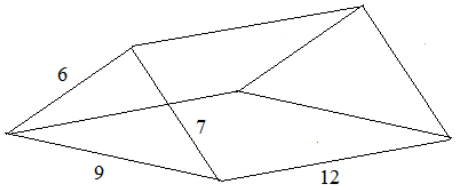


10) The areas of each shape within the "flattened cylinder" are shown in the diagram.
What is the volume of the constructed cylinder?



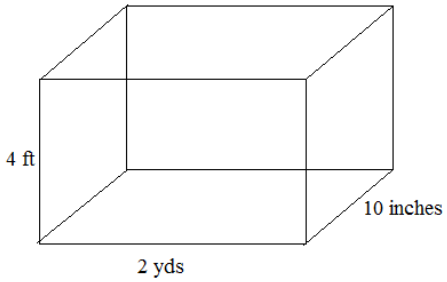
Surface Area and Volume (Advanced)

11) Find the volume and surface area of the triangular prism:

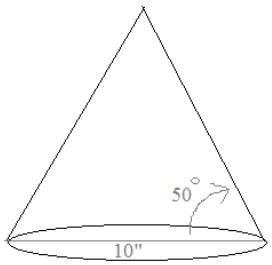


12) Find the surface area:

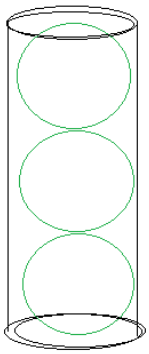
(diagram not drawn to scale!)



13) The diameter of a cone's base is 10 inches.
The slant height makes a 50 degree angle with the diameter of the base.
What is the volume and surface area of the cone?



14) Three tennis balls come in a cannister.
Is there more space or more tennis balls occupying the cylinder container?
What is the ratio?



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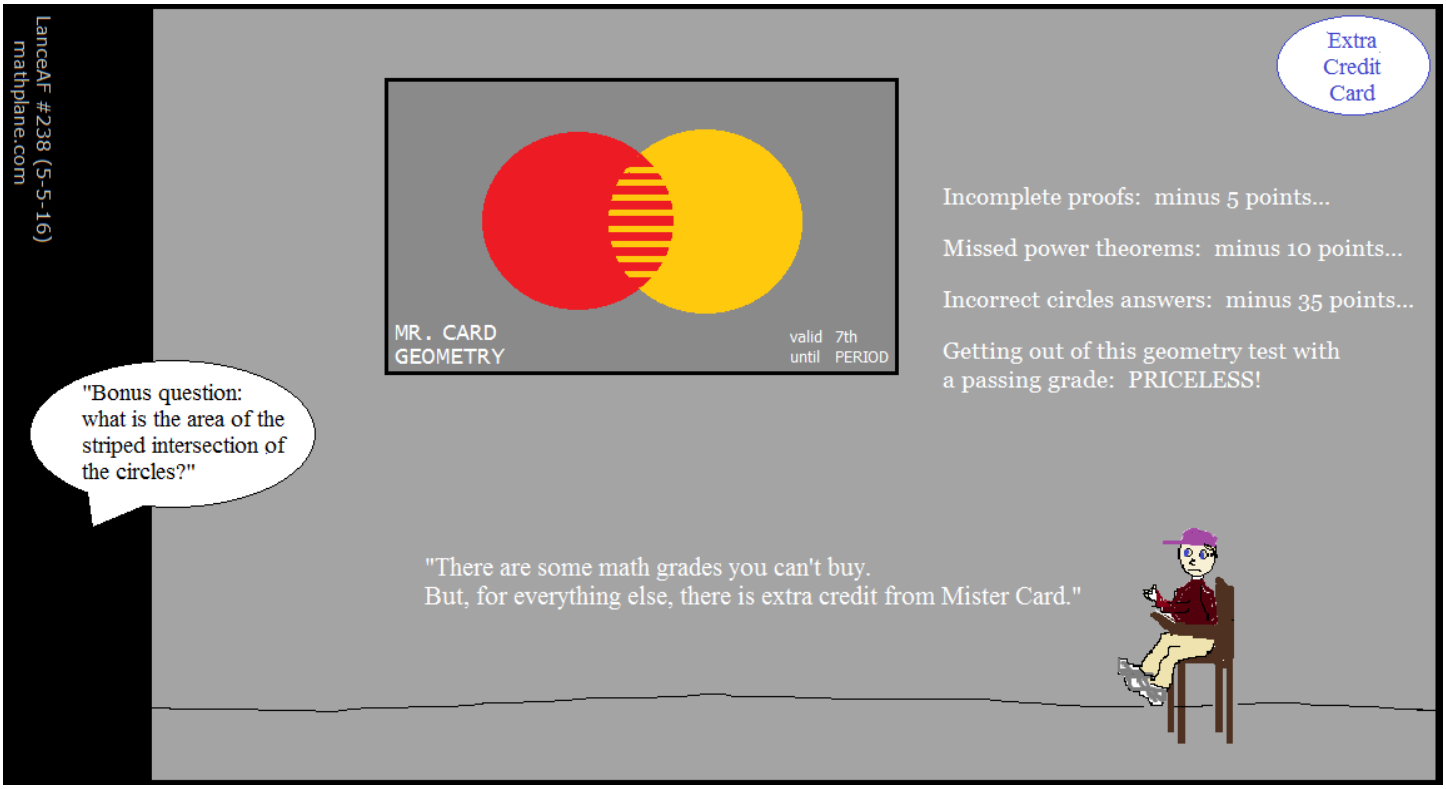
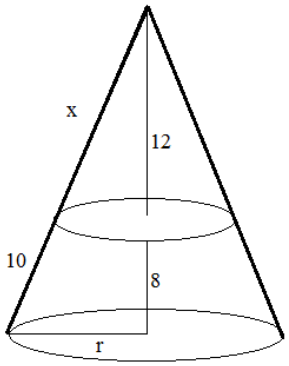

LanceAF #238 (5-5-16)
mathplane.com
Extra Credit Card
MR. CARD
GEOMETRY
valid 7th
until PERIOD
Incomplete proofs: minus 5 points...
Missed power theorems: minus 10 points...
Incorrect circles answers: minus 35 points...
Getting out of this geometry test with a passing grade: PRICELESS!
"Bonus question: what is the area of the striped intersection of the circles?"
"There are some math grades you can't buy.
But, for everything else, there is extra credit from Mister Card."

Solutions-→

1) Find the volume and surface area of the entire cone (with radius r).

SOLUTIONS

Surface Area and Volume (Advanced)



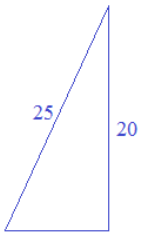
similar triangles

$$\frac{x}{(x+10)} = \frac{12}{20}$$

$$20x = 12x + 120$$

$$x = 15$$

Pythagorean Theorem



$r = 15$

$$\text{Volume} = \frac{1}{3}(\text{Base})(\text{Height})$$

$$= \frac{1}{3}(225\pi)(20)$$

$$\boxed{1500\pi}$$

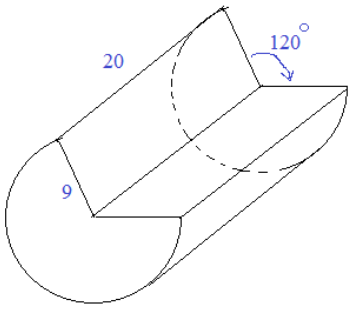
$$\text{Surface Area} = \text{Base} + \text{Lateral Area}$$

$$225\pi + \frac{1}{2}(30\pi)(25)$$

$$= \boxed{600\pi}$$

2) A 20" log with a radius of 9" has a uniform part carved out of it. (See diagram)

What is the volume of the carved log?
If you wanted to paint the log, what is the surface area that will be painted?



Volume of entire log (cylinder)

$$\frac{240}{360}(81\pi)(20)$$

$$= \boxed{1080\pi}$$

Surface Area:

Top and bottom:

$$\frac{240}{360}(81\pi) = 54\pi \times 2$$

"Curved Lateral Area"

$$\frac{240}{360}(20)(18\pi)$$

240π

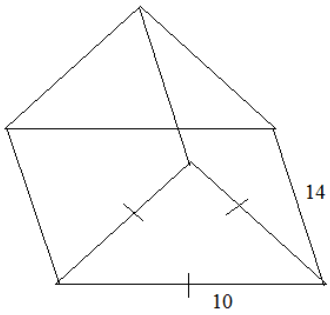
"rectangles"

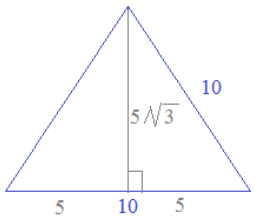
$(9)(20) = 180 \times 2$

108π
240π
360

$$\boxed{360 + 348\pi}$$

3) What is the volume of the triangle prism? And, surface area?





Area of Base triangle is $25\sqrt{3}$

so, volume is $14 \times 25\sqrt{3} = \boxed{350\sqrt{3} \text{ units}^3}$

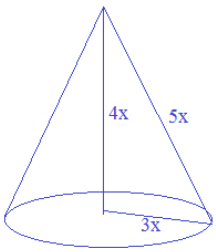
Surface Area: Bases ---> $50\sqrt{3}$

3 sides ----> $3 \times 140 = 420$

$$\boxed{420 + 50\sqrt{3} \text{ units}^2}$$

SOLUTIONS

4) A cone has dimensions in which the ratio of the radius to height is 3:4. If the surface area is 384π, what is the length of the height?



$384\pi = \pi(3x)^2 + \frac{1}{2}\pi(6x)(5x)$

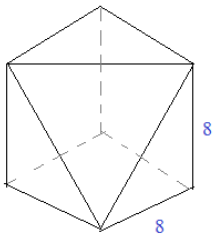
base lateral area

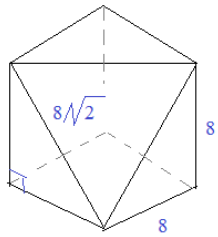
$384\pi = 9x^2\pi + 15x^2\pi$

$x = 16$

the height is 64

5) A cube with edges of length 8 cm has been sliced at 3 vertices as shown in the figure. What is the surface area?





Area of bottom and backs.. (3 full sides)

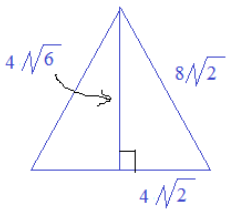
64 x 3...

Area of top and fronts... (3 half sides)

32 x 3...

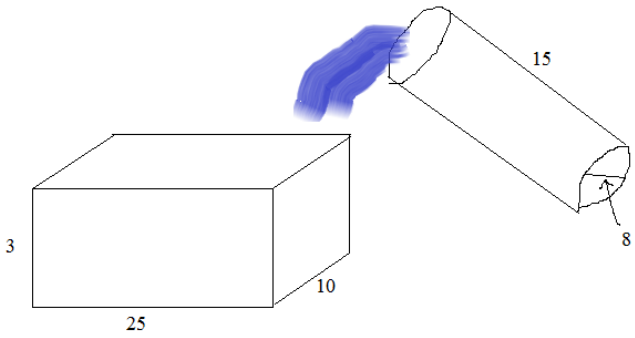
$288 + 32\sqrt{3}$

Area of front sliced equilateral triangle..



$16\sqrt{12} = 32\sqrt{3}$

6) A cylinder with these dimensions is full of water. If it is poured into the open rectangular prism, will it overflow?



Volume of prism:

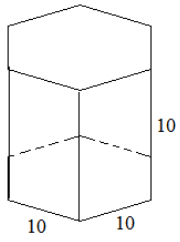
3 x 25 x 10 = 750

Volume of cylinder:

16π x 15 = 753.6 Approx.

YES, it will overflow!

7) Find the volume and surface area of the hexagonal prism:



surface area: 2 hexagon bases + 6 rectangle sides

volume: hexagon base x (height)

height = 10

base area = $\frac{1}{2}$ (apothem)(perimeter)

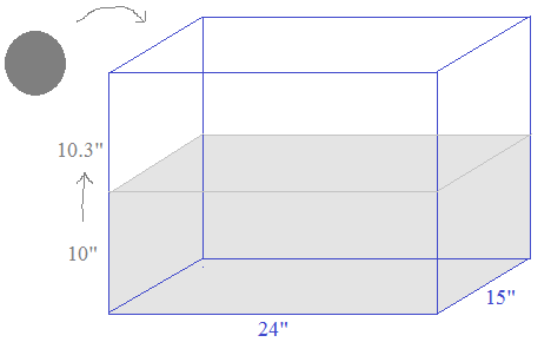
$= \frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$

Volume: $1500\sqrt{3}$

Surface Area: $600 + 300\sqrt{3}$

SOLUTIONS

8) A rectangular tank has length 24 inches, height 20 inches, and depth 15 inches. After filling half the tank with water, a solid steel ball is placed in the tank, raising the water level from 10 inches to 10.3 inches. What is the diameter of the solid ball?



Step 1: Find volume of water in tank

24" x 10" x 15" = 3600 cubic inches

(The actual height of tank is irrelevant. The water level height is what matters)

Step 2: Find volume of water AND solid ball in tank

24" x 10.3" x 15" = 3708 cubic inches

Step 3: Find dimensions of solid ball...

3708 + 3600 = 108 cubic inches is the volume of the ball...

Volume of sphere $= \frac{4}{3}\pi(\text{radius})^3$

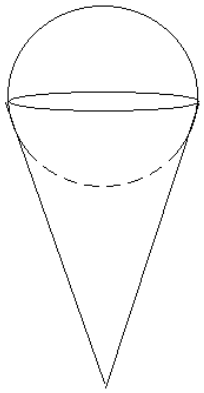
$108 = \frac{4}{3}\pi(r)^3$

$81 = \pi(r)^3$

$25.78 = r^3$

radius = 2.95 so, diameter = 5.9 (approximately)

9) An ice cream cone is 7" deep and 3" across. If a round scoop of ice cream that is 3" wide sits on the cone, would the ice cream overflow after it melts?



When the ice cream melts, the volume of the scoop will fill the cone...

So, is the volume of the scoop greater than the volume of the cone?

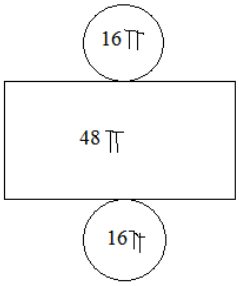
Volume of sphere $= \frac{4}{3}\pi(\text{radius})^3 = \frac{4}{3}\pi(\frac{3}{2}\text{inch})^3 = 14.137$

Volume of cone $= \frac{1}{3}$ (base area)(height)

$\frac{1}{3}\pi(\text{radius})^2(\text{height}) = \frac{1}{3}\pi(\frac{3}{2}\text{inch})^2(7\text{ inches}) = 16.49$

The ice cream would melt into the cone WITHOUT overflowing!

10) The areas of each shape within the "flattened cylinder" are shown in the diagram. What is the volume of the constructed cylinder?



radius of each circle: 4

diameter of each circle: 8

circumference: 8π

therefore, height is 6

so, the volume is 96π

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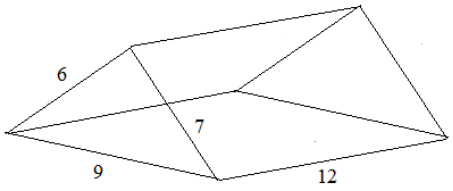
Surface Area and Volume (Advanced)

11) Find the volume and surface area of the triangular prism:

SOLUTIONS



Since it is a prism, the volume is the (area of the base) x (depth)

area of base (triangle) $= \sqrt{(11)(11-6)(11-7)(11-9)} = \sqrt{440}$

Using Heron's formula where semiperimeter is 1/2(perimeter) of triangle = 11

Volume $= \sqrt{440} \cdot (12) =$ **251.7 cubic units**

Surface Area = 2(base area) + lateral area

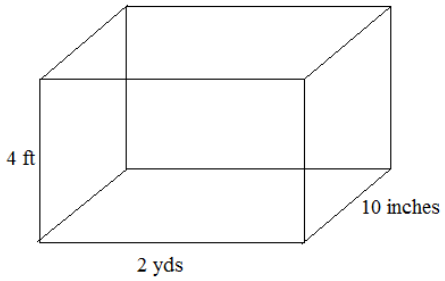
LA = perimeter(height)

$= 2 \cdot \sqrt{440} + (22)(12)$

$=$ **306 square units**

12) Find the surface area:

(diagram not drawn to scale!)



Step 1: Convert to common units... For ease, we'll use inches...

length x width x height

72" x 10" x 48"

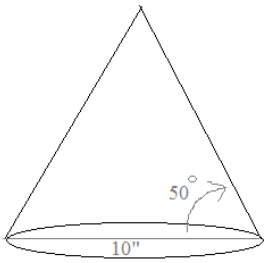
Step 2: Find area of each side...

"left and right" 480 sq inches x 2 = 960
"top and bottom" 720 sq inches x 2 = 1440
"front and back" 3456 sq inches x 2 = 6912

Step 3: Add them up

9312 square inches
64.667 square feet
7.18 square yards

13) The diameter of a cone's base is 10 inches.
The slant height makes a 50 degree angle with the diameter of the base.
What is the volume and surface area of the cone?



$\cos(50^\circ) = \frac{5}{\text{slant height}}$

slant height $l = 7.78$

$\tan(50^\circ) = \frac{\text{height}}{5}$

height $h = 5.96$

Volume $= \frac{1}{3}(25\pi)(5.96)$

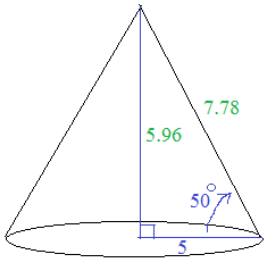
$=$ **156 cubic inches**

Lateral Area $= \frac{1}{2}(10\pi)(7.78)$

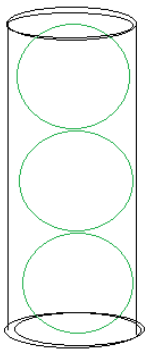
122.2 square inches

Surface Area $= 25\pi + 122.2$

$=$ **200.7 square inches**



14) Three tennis balls come in a cannister.
Is there more space or more tennis balls occupying the cylinder container?
What is the ratio?



Assume the radius of a tennis ball is 2
diameter is 4

height of cylinder is 12...

Volume of cylinder: 48π

Volume of one tennis ball: (sphere) $\frac{4}{3}\pi(\text{radius})^3 = \frac{4}{3}\pi(8) = \frac{32}{3}\pi$

3 tennis balls: 32π remaining space: 16π

tennis balls occupy more space at a ratio of 2:1

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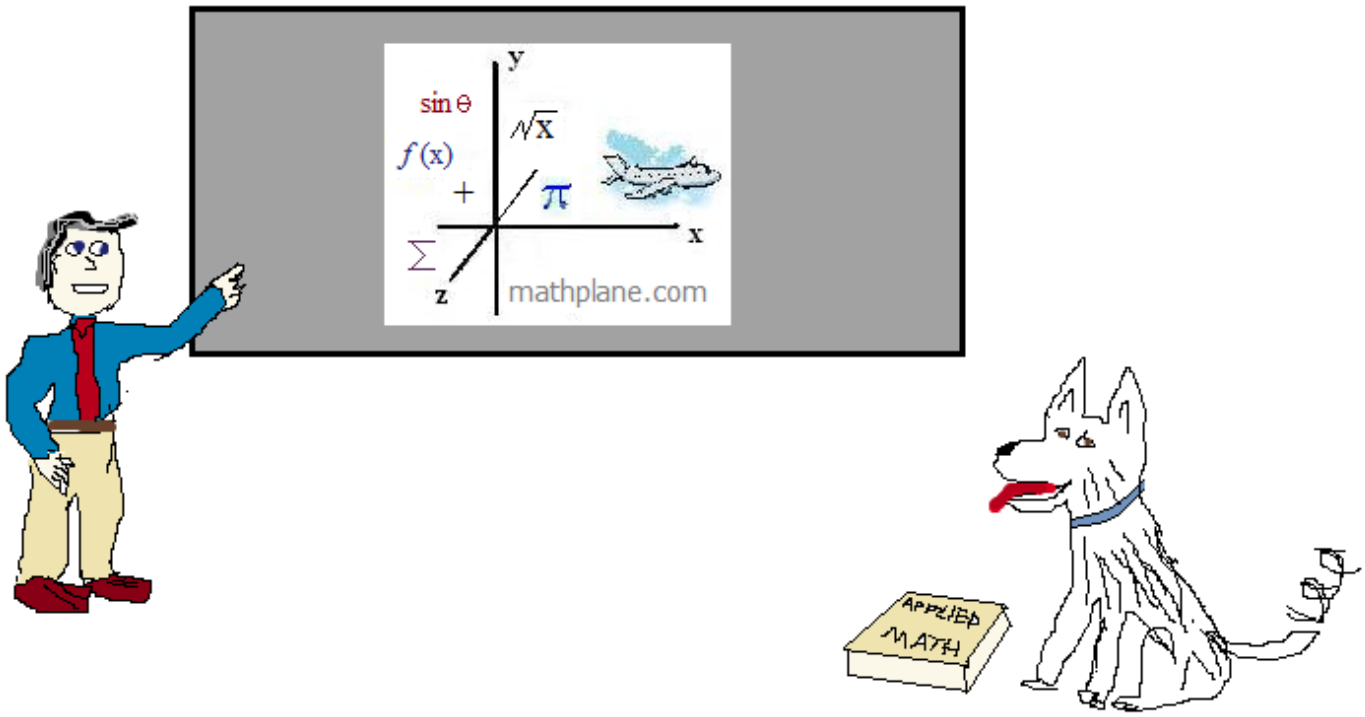

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