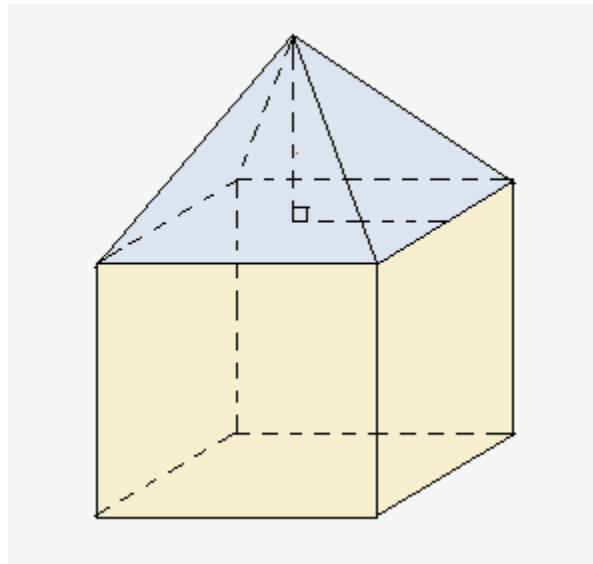# Lateral and Surface Area
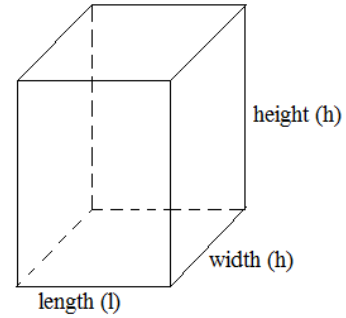
Formulas, Examples and Practice Questions (W/Solutions)

Topics include pyramids, prisms, triangles, cylinders, cones, slant height, and more.
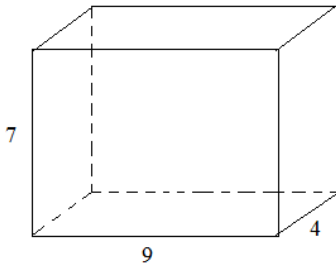
Right Rectangular Prism :   Lateral Area (LA) = 2(length)(height) + 2(width)(height)
(Box)
                            Base Area (BA) = (length)(width)

                            Surface Area = LA + 2BA

Observation: The surface area is the sum of all 6 areas

height (h)

width (h)

length (l)

Example: What is the surface area of a box with dimensions 4" x 7" x 9" ?

$LA = 2(9")(7") + 2(4")(7") = 126"^2 + 56"^2 = 182"^2$

$BA = 9" \times 4" = 36"^2$

$2BA = 72"^2$

$SA = 182 + 72 =$ | 254 square inches |

7

9   4

Area of front (and back) = 7" x 9" = 63 sq. inches	Total 126 sq. inches	Area of rectangle = *lw*
Area of left (and right) = 7" x 4" = 28 sq. inches	Total 56 sq. inches	
Area of bottom (and top) = 9" x 4" = 36 sq. inches	Total 72 sq. inches	Total area of 6 sides: 254 square inches

---

(Right Circular)
    Cylinder: Lateral Area (LA) = 2π (radius)(height)

                          = π (diameter)(height)

        LA = Circumference(Height)

        Base Area (BA) = π (radius)^2

        Surface Area (SA) = LA + 2BA

radius (r)

length

height (h)

radius (r)
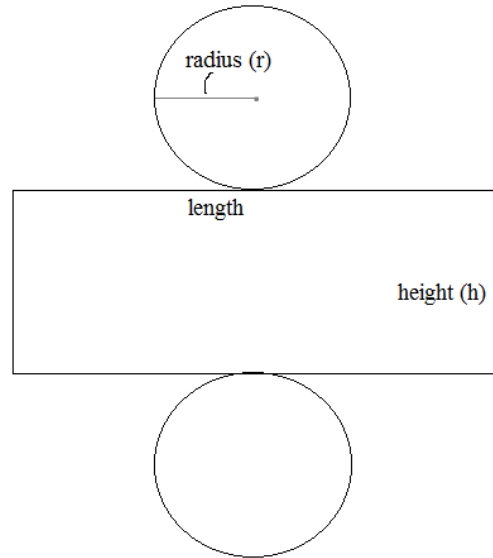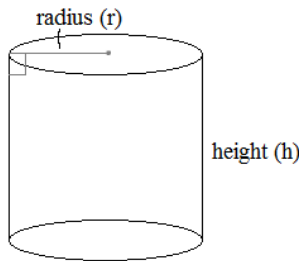
height (h)

Observation: The circumference of each base (circle) is
the same as the length of the rectangle.

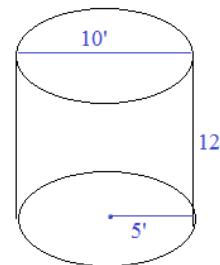*Example:* Find the surface area of a right cylinder with
        base diameter of 10 feet and height 12 feet.

Lateral Area = $2\pi r h = 2\pi (5 \text{ feet})(12 \text{ feet}) = 60\pi \text{ feet}^2$

Base = area of a circle = $\pi r^2 = \pi (5 \text{ feet})^2 = 25\pi \text{ feet}^2$

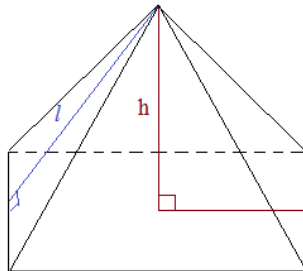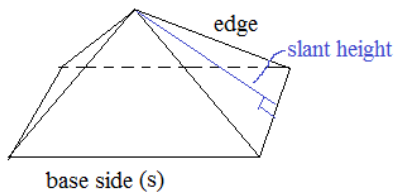Surface area = LA + 2B = $60\pi \text{ feet}^2 + 2 \cdot 25\pi \text{ feet}^2$

                = | 110π square feet |

10'

12'

5'

Lateral Area and Surface Area

$\text{Lateral Area (LA)} = \frac{1}{2} (\text{perimeter of base})(\textit{slant height})$

Square Right Pyramid:

$$= \frac{1}{2}(4s)(l)$$

4 congruent triangles and a square base

$\text{Base Area (BA)} = (\text{side})^2$

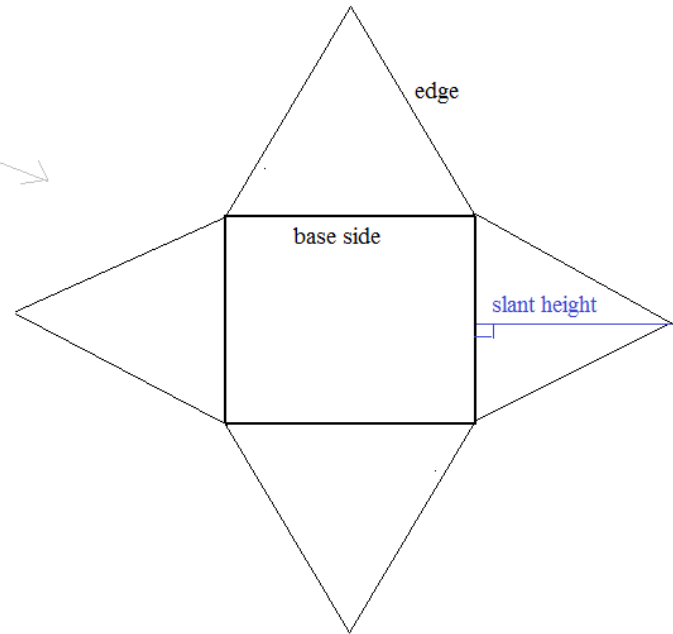$\text{Surface Area (SA)} = \text{LA} + \text{BA}$

edge
slant height
base side (s)

l h

height (h) extends from top to middle of the base

slant height (l) extends from top to middle of base side

edge
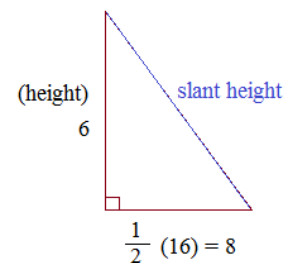base side
slant height

Example: Find the surface area of a square pyramid with base side length 16 and <u>height</u> 6.

Step 1: Draw a sketch, and label known parts
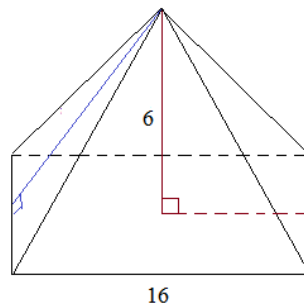
Step 2: Find missing parts

We need the <u>slant height</u> to find the lateral area:

6

16

(height)
6
slant height
$\frac{1}{2}(16) = 8$

Pythagorean Theorem reveals slant height (l) = 10

Step 3: Use formulas to find surface area

$\text{LA} = \frac{1}{2}(\text{perimeter})(\text{slant height})$

$= \frac{1}{2}(64)(10) = 320$

$\text{BA} = (\text{side})^2 = 256$

$\boxed{\text{SA} = \text{LA} + \text{BA} = 576 \text{ square units}}$

$\text{Area of triangle} = \frac{1}{2}bh$

Alternate approach: Find area of each side individually

bottom (square) area is 256
4 triangle sides: 80, 80, 80, 80
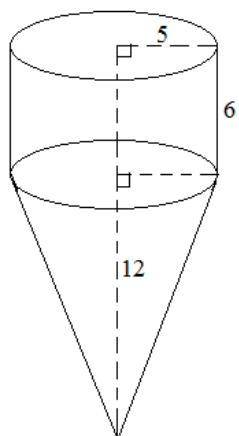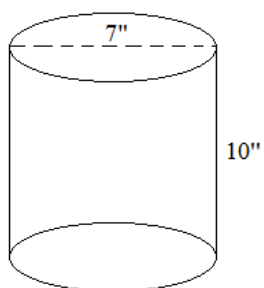total of all 5 faces = 576

1) What is the surface area?

2) Find the total surface area of the right cylinder:
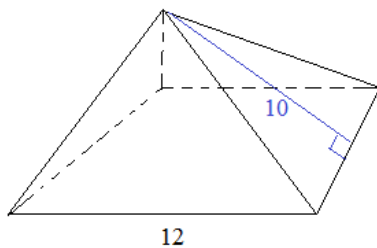

3) The surface area of a cube is 900 square feet.
   What is the length of each edge?

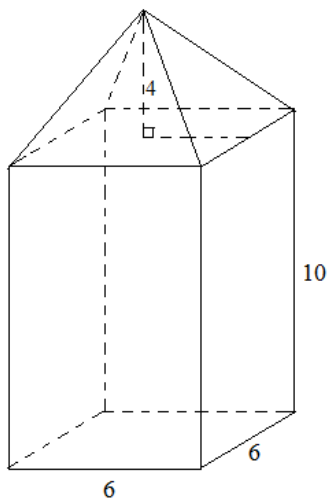4) Find the lateral area of a *Great Pyramid* with height 470 feet and each side of its square base is 710 feet.

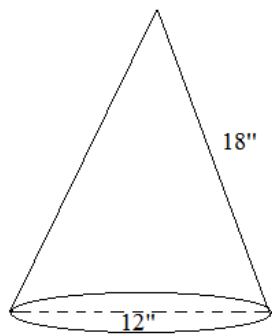5) Find the surface area of the figure:

6) The roof of a house is a square pyramid with side length 10 yards and height 5 yards.
   What is the slant height?
   How much roofing material is needed to cover the roof?
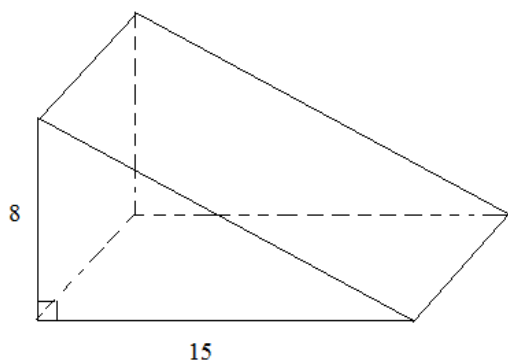
7) Find the surface area of the figure:

8) Find the surface area of the cone:

18"

12"
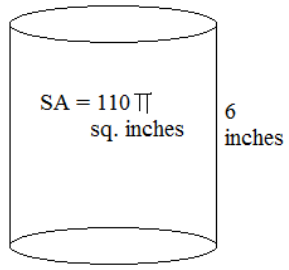
9) The lateral area of a square pyramid is 240 square feet.
The base edges are 12 feet each.
What is the height of the pyramid?

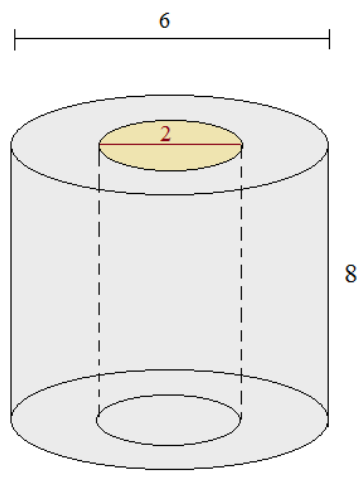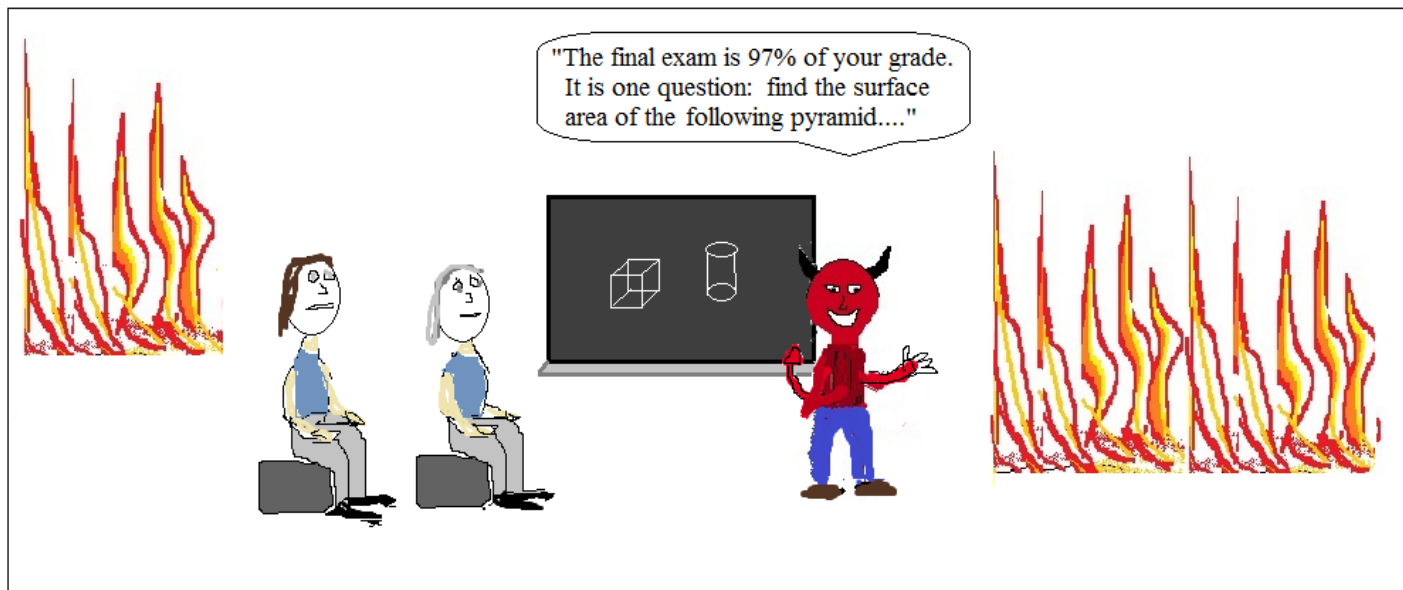10) If the surface area of the triangular prism is 560 square units,
what is the height?

8

15

11) If the height of a cylinder is 6" and the surface area is $110\,\pi$ inches^2, what is the radius of the cylinder?

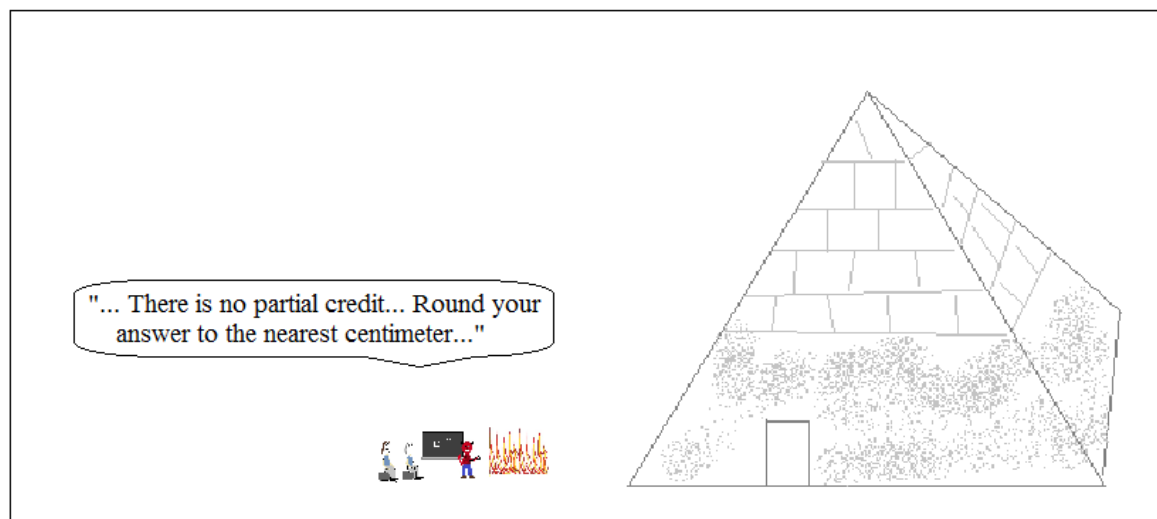SA = 110 π
sq. inches

6
inches

12) A hole is drilled through the solid.
What is the surface area (inside and out)?

6

2

8

Math in Hell

"The final exam is 97% of your grade. It is one question: find the surface area of the following pyramid...."

"... There is no partial credit... Round your answer to the nearest centimeter..."

In its 1000 year history, no one ever passed Mr. Devlin's Geometry class.
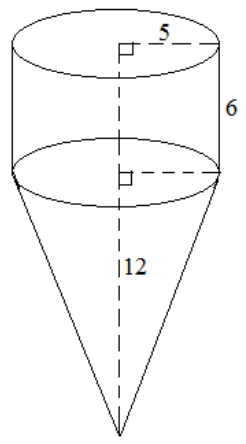
LanceAF #39 7-1-12
www.mathplane.com

SOLUTIONS-→

1) What is the surface area?

Note: We ignore the middle circle (i.e. the top of the cone), because it is not exposed

Also, the *slant height* of the cone is 13 (5-12-13 right triangle)

Step 1: Area of the top

The top surface (base) is a circle:

$$\text{Area of circle} = \pi(\text{radius})^2$$
$$= 25\pi$$

Step 2: Area of middle

The middle shape is a cylinder:

$$\textit{Lateral Area of cylinder} = 2\pi(\text{radius})(\text{height})$$
$$= 2\pi(5)(6) = 60\pi$$
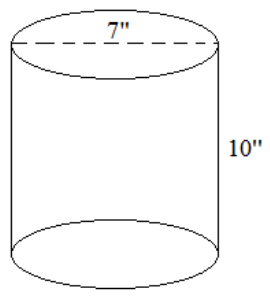
Step 3: Area of bottom

The bottom shape is a cone:

$$\textit{Lateral Area of a cone} = \frac{1}{2}\,2\pi(\text{radius})(\text{slant height})$$
$$= \pi(5)(13) = 65\pi$$

Total surface area = 150π

---

2) Find the total surface area of the right cylinder:

$$\textit{Lateral Area of cylinder} = 2\pi(\text{radius})(\text{height})$$
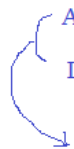$$\pi(\text{diameter})(\text{height})$$

"circumference times the height"

$$\textit{Surface Area of cylinder} = \text{Lateral Area} + \text{Area of 2 bases}$$

$$\text{Base area} = \pi(\text{radius})^2$$

Area of 1 base = $\pi(3.5)^2 = 12.25\pi$

Area of 2 bases = 24.5π

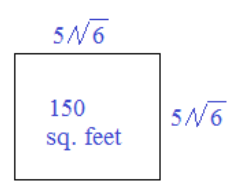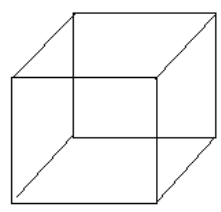Lateral Area = $2\pi(3.5)(10) = 70\pi$

Total surface area = 94.5π

---

3) The surface area of a cube is 900 square feet. What is the length of each edge?

The cube has six congruent square sides.

The area of each side is $\dfrac{900}{6} = 150$ square feet

Length of each edge = $\sqrt{150} = 5\sqrt{6}$
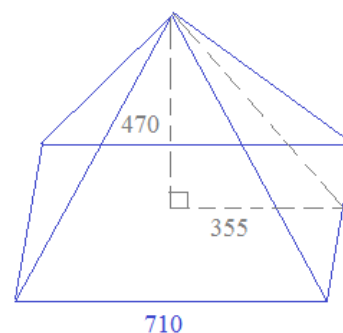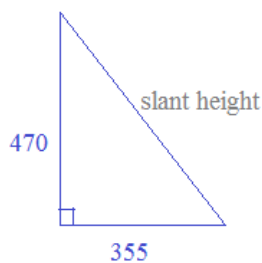
4) Find the lateral area of a *Great Pyramid* with height 470 feet and each side of its square base is 710 feet.

Lateral Area = $\frac{1}{2}$ (perimeter)(slant height)

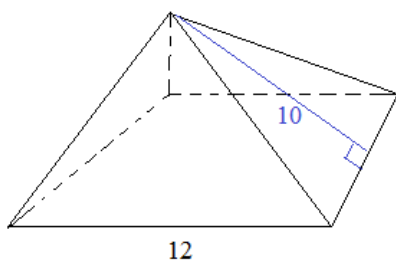Use Pythagorean Theorem:

Slant height = $\sqrt{(470)^2 + (355)^2}$ = 589

LA = $\frac{1}{2}$ (2840)(589) = $\boxed{836,380 \text{ sq. feet}}$

5) Find the surface area of the figure:

Method 1: Use the formulas

LA = $\frac{1}{2}$(perimeter)(slant height)

= $\frac{1}{2}$ (48)(10)   = 240 square units

Base = (length)(width)

= (12)(12)   = 144 square units

SA = LA + Base

= 240 + 144 = $\boxed{384 \text{ square units}}$

Method 2: Find area of each side

Base (square) = 12 x 12 = 144 square units

Side (triangle) = $\frac{1}{2}$ (base)(height)

$\frac{1}{2}$(12)(10) = 60 square units
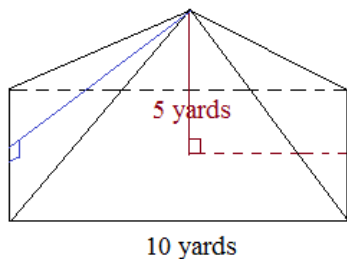
4 sides: 240 square units

Total of 5 sides: $\boxed{384 \text{ square units}}$

6) The roof of a house is a square pyramid with side length 10 yards and height 5 yards.
   What is the slant height?
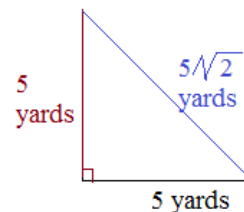   How much roofing material is needed to cover the roof?

Use Pythagorean Theorem to find slant height:

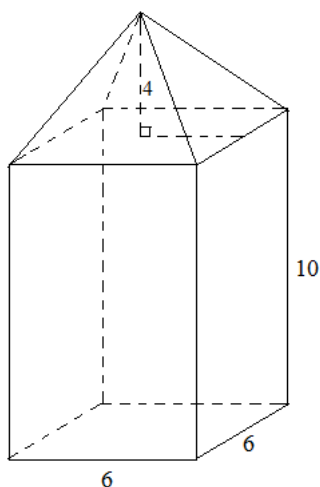slant height = $5\sqrt{2}$ yards

LA = $\frac{1}{2}$(perimeter)(slant height)

= $\frac{1}{2}$ (40 yards)(5 $\sqrt{2}$ yards) = 100 $\sqrt{2}$ sq. yards

At least 141.2 square yards of material
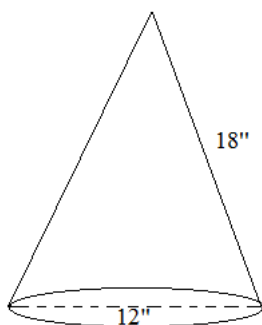are necessary to cover the roof.

SOLUTIONS

7) Find the surface area of the figure:

(note: the upper square is ignored, because it is 'hidden' between the pyramid and prism)

Surface area = 60 + 240 + 36 = 336 square units

Base:

The base is a square, so the area is 6 x 6 = 36

Middle:

method 1: Lateral area = (perimeter)(height)

$$= (24)(10) = 240$$

method 2: Add up the 4 sides' areas!

$$60 + 60 + 60 + 60 = 240$$

Top:

Lateral area of the pyramid = $\frac{1}{2}$ (perimeter)(slant height)

slant height is 5

(1/2 of 6)

$$LA = \frac{1}{2} (24)(5) = 60$$

8) Find the surface area of the cone:

Surface area = LA + Base Area

$$= 144\ \pi\ \text{square inches}$$

Base:

Base is a circle

Base is a circle, so the area is $\pi r^2 = 36\pi$ square inches

Lateral Area:

LA = $\frac{1}{2}$ (Perimeter of base)(slant height)

(The perimeter of the base is the circumference of the circle)

LA = $\frac{1}{2} \cdot 12\pi$ " $\cdot$ 18" = 108π square inches

9) The lateral area of a square pyramid is 240 square feet.
The base edges are 12 feet each.
What is the height of the pyramid?

Lateral Area (LA) = 240 sq. feet
Base edge = 12 feet
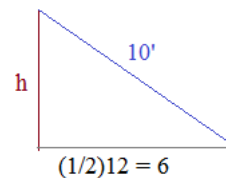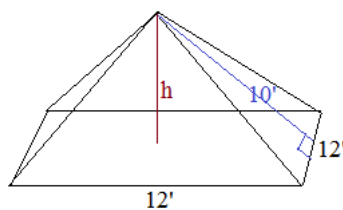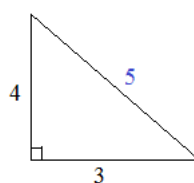
Since base edge is 12 feet, the perimeter of base is 4(12) = 48

$$LA = \frac{1}{2} p_{base} \, l$$

$$240 \text{ sq. ft} = \frac{1}{2} (48') \, l$$

slant height (l) = 10 feet

height is 8 feet

10) If the surface area of the triangular prism is 560 square units, what is the height?

Surface Area (SA) = Lateral Area (LA) + 2(Base)

The base area is the area of a triangle:

$$\frac{1}{2}(8)(15) = 60$$

The lateral area is:

(perimeter of base)(height) = $(8 + 15 + 17)$(height) = $40h$

OR   add up the 3 area sides: 'left' = 8(height)
                                   'top' = 17(height)
                                   'bottom' = 15(height)

$$8h + 17h + 15h = 40h$$

17

8

height

15

8-15-17 Pythagorean Triple (right triangle)

Substitute: $560 = 40h + 2(60)$

$$440 = 40h$$

height = 11 units

11)   If the height of a cylinder is 6" and the surface area is 110π inches^2, what is the radius of the cylinder?

Surface area = Lateral Area + 2(Base)
(cylinder)

LA = Perimeter(height)   Base = Area of circle
or Circumference(height) π(radius)^2

110π inches^2 = 2π(radius)(6 inches)   + $2(\pi)$(radius)^2

= $(12\pi$ inches)(radius)   + $2(\pi)$(radius)^2

(divide out π and the inches squared)

$$110 = 12r + 2r^2$$

$$2(r^2 + 6r - 55) = 0$$

$$2(r + 11)(r - 5) = 0$$

SA = 110π
sq. inches

6 inches

radius = 5 or -11

But, since length cannot be negative, the radius must be 5 inches

12)   A hole is drilled through the solid.
What is the surface area (inside and out)?

Find the area of each part:

top (and bottom):   area of circle: π(radius)^2

entire circle: $\pi(3)^2$

drilled out circle: $\pi(1)^2$

area of each base: 8π

6

2

8

Lateral area (exterior): LA = (circumference)(height)

$$LA = \pi(6)(8) = 48\pi$$

Lateral area (interior):   LA = $\pi(2)(8)$ = 16π

Surface Area = 2(area of base) + LA (exterior) + LA (interior)

= 16π + 48π + 16π = 80π square units

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy
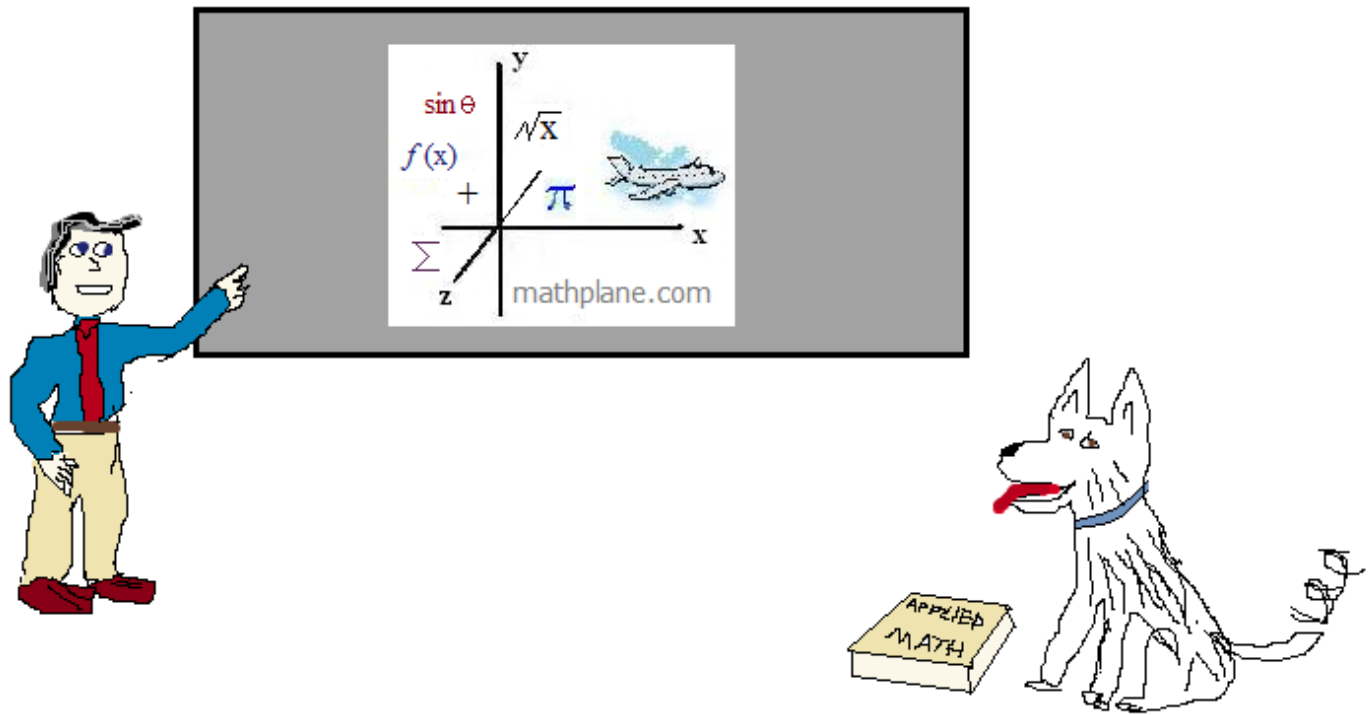
Also, find us at Facebook, Google+, Pinterest, and TeachersPayTeachers