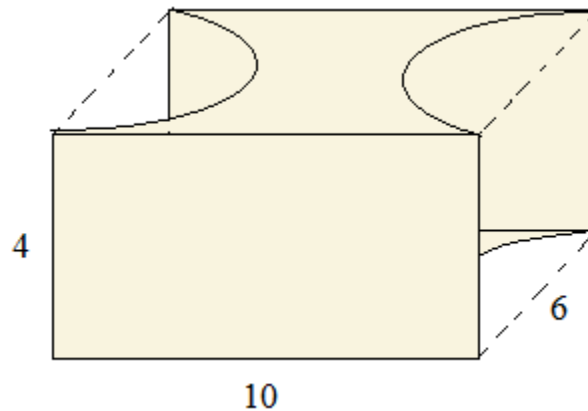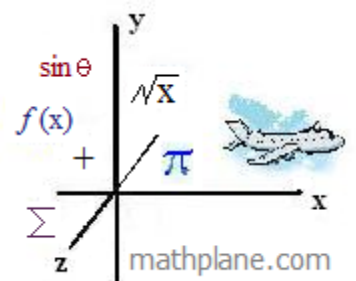# Geometry: Volume

Notes, Formulas, Examples, and Practice (with Solutions)
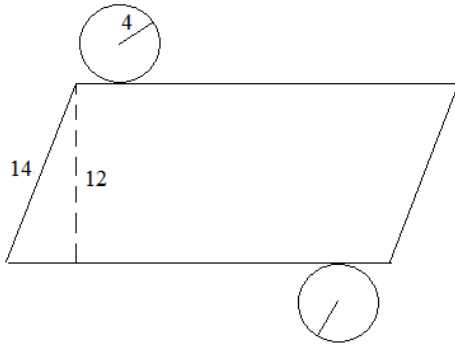

Topics include surface area, prisms, interpreting 2-D 'blueprints', spheres, cylinders, and more.
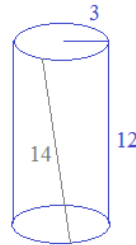
Example: If you fold up the parts of this figure, what is the volume?

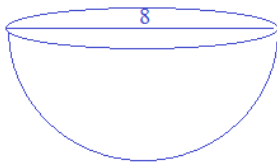The circles fold up (and down), and the parallelogram rolls up --- forming a cylinder..

$$\text{Volume of Cylinder} = \pi(\text{radius})^2(\text{height})$$

$$\text{Volume} = \pi(4)^2(12)$$

$$= 192\,\pi$$

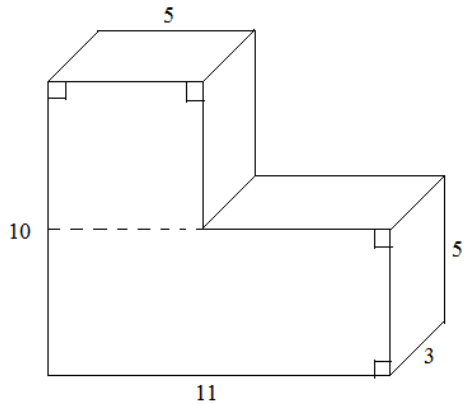*Example:* Cement is poured into a hemisphere that is 8" across. What is the volume of the cement that is used?

$$\text{Volume of sphere} = \frac{4}{3}\pi(\text{radius})^3$$

$$\text{Volume of hemisphere} = \frac{2}{3}\cdot\pi(\text{radius})^3$$

$$= \frac{2}{3}\cdot\pi(4\text{ inches})^3$$

$$= \boxed{\frac{128}{3}\cdot\pi\text{ cubic inches}}$$

Example: What is the volume and surface area of the figure?
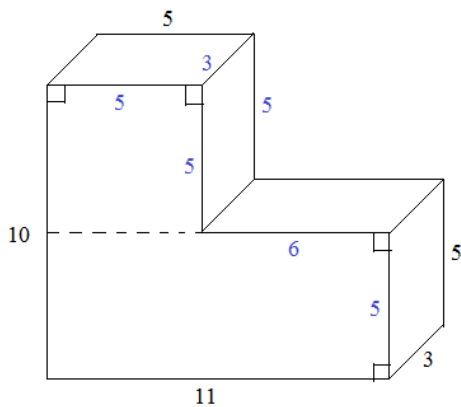
$$\text{Volume of rectangular prism} = (\text{length})(\text{width})(\text{height})$$
$$(\text{area of base})(\text{height})$$

Top block: volume = (5)(5)(3) = 75

Bottom block: volume = (11)(3)(5) = 165

$$\boxed{\text{total volume} = 240\text{ cubic units}}$$

Since this is a prism, we can use Lateral Area = (perimeter)(height)

$$= (10 + 11 + 5 + 6 + 5 + 5)(3)$$

$$= (42)(3) = 126$$
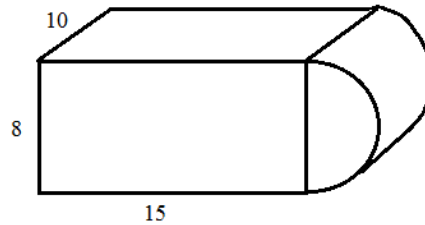
then, the area of each base is top: (5)(5) = 25
bottom: (11)(5) = 55
total: 80...

$$\boxed{\text{total surface area} = 126 + 2(80) = 286\text{ square units...}}$$

Also, the surface area can be found by adding up each face of the figure..

Example: Find the volume:

10

8

15

Step 1: Cut into sections   The figure is a rectangular prism and 1/2 cylinder.

Step 2: Find total area of prism base (main face)   The base of the rectangular prism is a rectangle:

$$\text{Area} = \text{length} \times \text{width} = (15)(8) = 120$$

The base of the 1/2 cylinder is a <u>semicircle</u>:

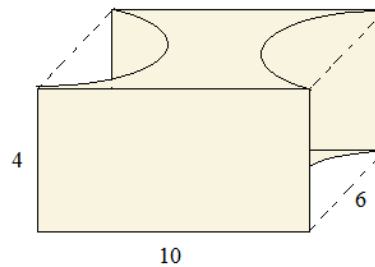$$\text{Area} = \frac{1}{2}\pi(\text{radius})^2 = \frac{1}{2}\pi(4)^2 = 8\pi$$

Step 3: Multiply height (depth) to get volume

Since the area of the entire base of the prism is $120 + 8\pi$

$$\text{the volume is } 10(120 + 8\pi) = \boxed{1200 + 80\pi \approx 1451.3 \text{ cubic units}}$$

Height (depth)   area

Example: Find the volume and surface area:

The volume is the entire box minus the two cut-outs.
**Since each cut-out is a semi-circle, they can be combined to create a cylinder (with diameter 6)!

Volume of box = (length)(width)(height)

$$= (10)(6)(4) = 240 \text{ cubic units}$$

Volume of cylinder = (area of base)(height)
(2 cut-outs)

$$= \pi(3)^2(4) = 36\pi \text{ cubic units}$$

Total volume = $240 - 36\pi \approx \boxed{126.9 \text{ units}^3}$

The surface area is the total area that 'would be painted'....

Front (and back) area: $2 \times (4)(10) = 80$ sq. units

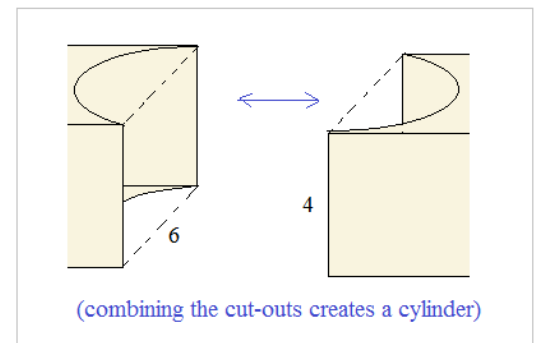Top (and bottom) area: $2 \times \left[(10)(6) - \pi(3)^2 \right] = 120 - 18\pi$ sq. units

entire top   cut out semi-circles
(again, the 2 semi-circles combine to a circle)

4

6

10

(combining the cut-outs creates a cylinder)

4

6

Left (and right) area: $6\pi(4) = 24\pi$ sq. units

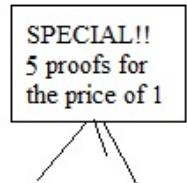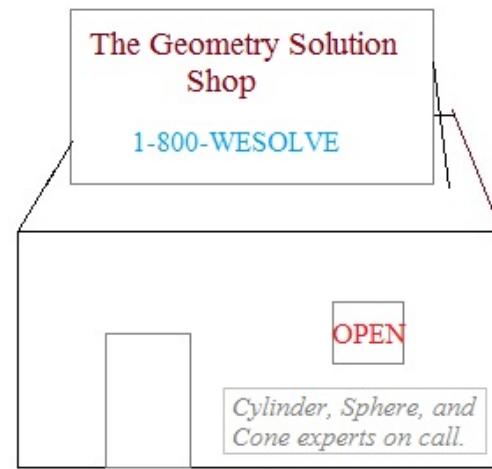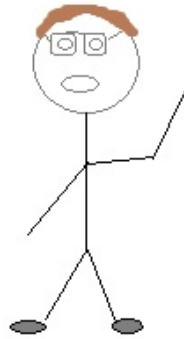Total surface area = $200 + 6\pi \approx \boxed{218.8 \text{ sq. units}}$

<u>combined</u>, the two sides form a cylinder...
So, we need to find the lateral area of the cylinder...
(circumference of base x height)
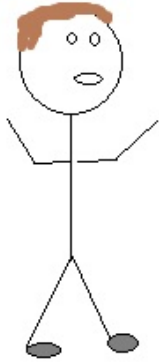
Practice Questions -→

1) What is the volume of a 3' x 6' x 9' rectangular prism?

2) A grain silo has a cylindrical base and dome top (i.e. 1/2 sphere).
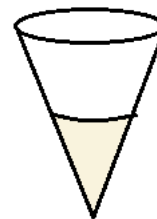   What is the interior volume?
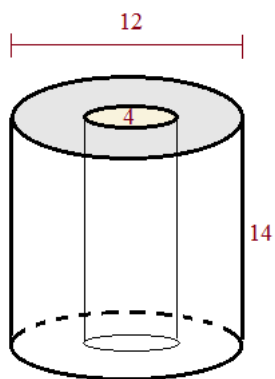

60'

100'

3) A cone has volume 900π cubic inches.

What are 2 possible measures of the radius and height?

4) A water cooler uses cone shaped paper cups.
   The opening of the cup has a diameter of 3 inches, and the side edge is 5 inches.

   a) How much water can a full cup hold?

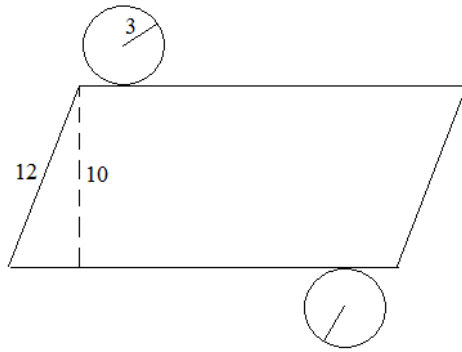   b) How much water is in a cup where the water level is halfway up?
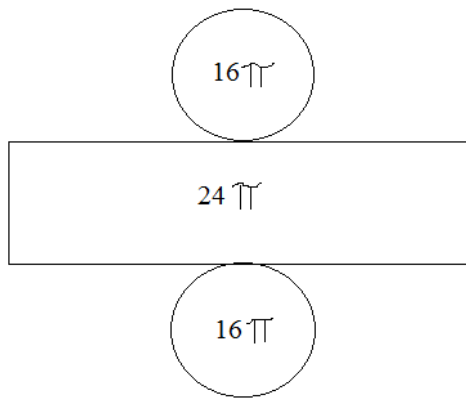

5) Find the volume of the hollowed out cylinder:

12

4

14

6) If you fold up the parts of this figure, what is the volume?
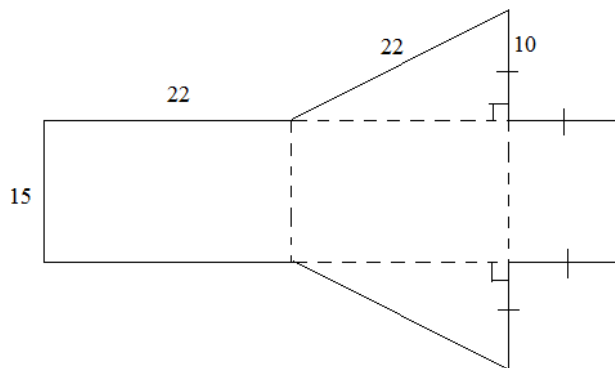
7) The areas of each part are labeled. What is the volume of the constructed figure?
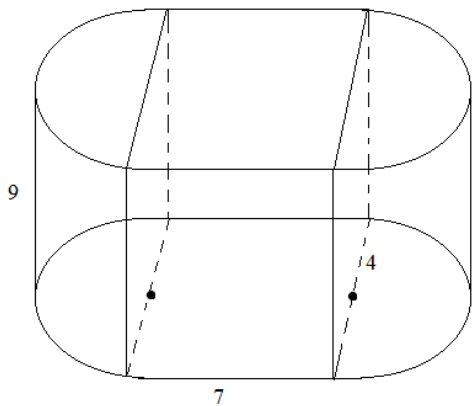

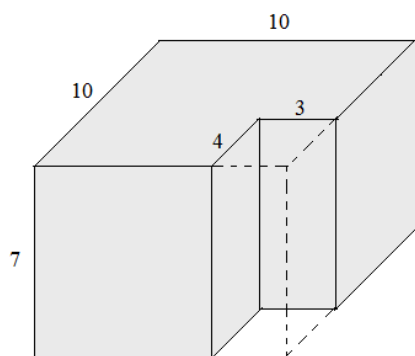8) Describe the constructed 3-d figure. Then, find its volume.

9) Find the volume and surface area of the figure.

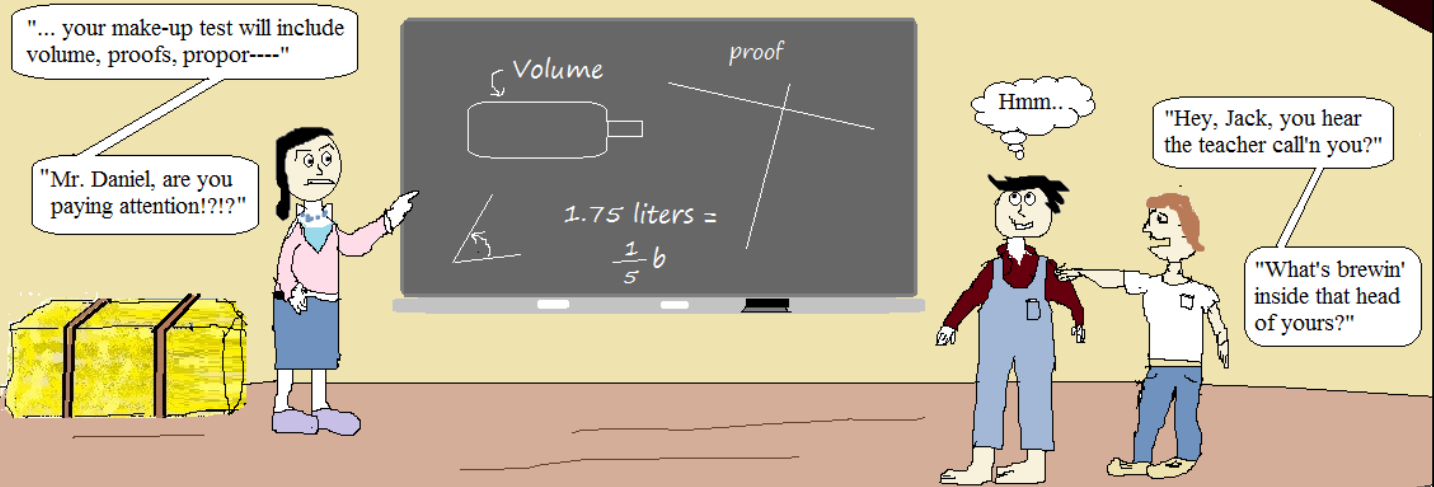(Note: the 'radius' is 4 units)

9

4

7

10) The height of a cylinder is 6". If the volume of the cylinder is 1527 cubic inches, what is the approximate radius?

11) Determine the volume and surface area of the figure.

10

10

3

4

7

Solutions -→

1) What is the volume of a 3' x 6' x 9' rectangular prism?
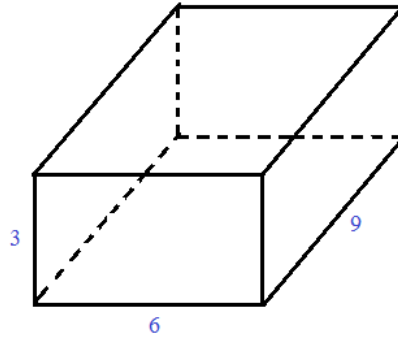
The volume of a rectangular prism is

(area of base) x (height/depth)

Area of base = length x width
rectangle

= 6' x 3' = 18 sq. feet

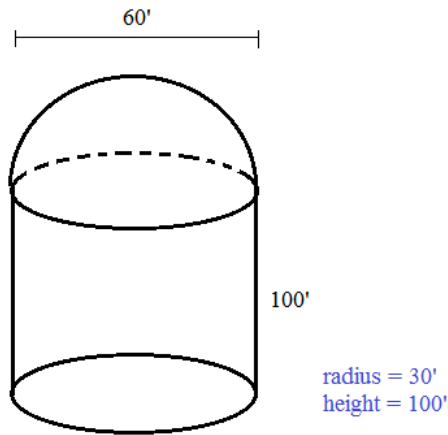then, the volume is

18 sq. feet x 9 feet = 162 cubic feet

3

9

6

2) A grain silo has a cylindrical base and dome top (i.e. 1/2 sphere).
What is the interior volume?

Divide silo into two parts: cylinder and 1/2 sphere...

60'

100'

radius = 30'
height = 100'

Volume of
cylinder = (Area of base) x (height)

Base of cylinder is a circle ---> Area is $\pi(radius)^2$

$V = \pi (30')^2 (100') = 90,000 \, \pi$ cubic feet

Volume of
sphere = $\frac{4}{3} \pi (radius)^3$

Volume of dome = $\frac{1}{2}$(volume of sphere)

$\frac{1}{2} \cdot \frac{4}{3} \pi (30')^3 = 18,000 \, \pi$ cubic feet

Total Volume: $108,000 \, \pi \, feet^3$ approx. 339,292 cubic feet

3) A cone has volume $900 \, \pi$ cubic inches.
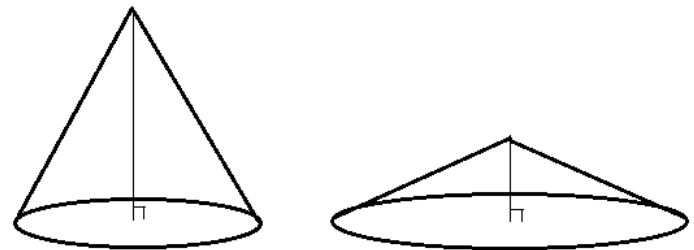
What are 2 possible measures of the radius and height?

Volume = $\frac{1}{3} \pi (radius)^2 (height)$

$900 \, \pi$ cubic inches = $\frac{1}{3} \pi (radius)^2 (height)$

2700 cubic inches = $(radius)^2 (height)$

two
possibilities

If radius were 10, then height is 27.

If radius were 15, then height is 12

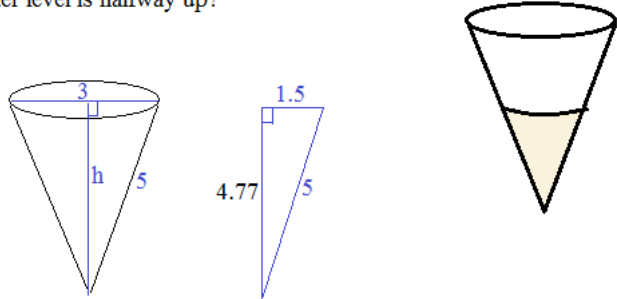4) A water cooler uses cone shaped paper cups.
   The opening of the cup has a diameter of 3 inches, and the side edge is 5 inches.

   a) How much water can a full cup hold?

   b) How much water is in a cup where the water level is halfway up?

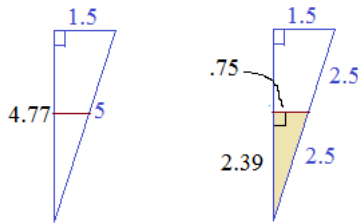$$\text{Volume (cone)} = \frac{1}{3}(\text{Base area})(\text{height})$$

a)   Base area: $\pi(\text{radius})^2$

   $= 2.25\,\pi$

Height: 4.77

Volume $= \frac{1}{3}(4.77)(7.07) \approx$ **11.24 cubic inches**

b) If water level is halfway up, we'll use proportional tirangles...

Notice the difference in volume
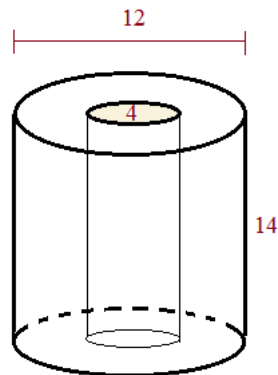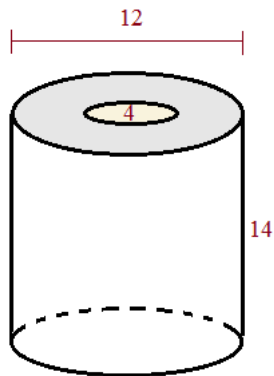of the upper "half" vs. lower "half"!

Base area: $\pi(\text{radius})^2$

   $= .5625\,\pi$

Height: 2.39

Volume $= \frac{1}{3}(1.77)(2.39) \approx$ **1.41 cubic inches**

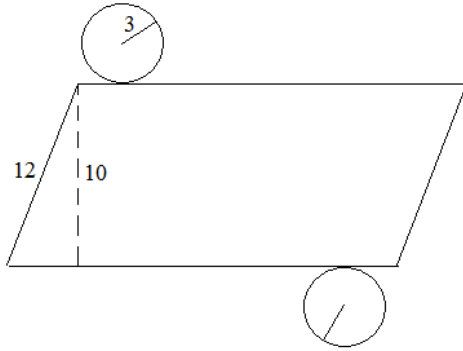5) Find the volume of the hollowed out cylinder:

$$\text{Volume (cylinder)} = \text{Area base(height)}$$
$$= \pi(\text{radius})^2(\text{height})$$

Entire cylinder volume $= \pi(6)^2(14)$ — Hallowed out volume $= \pi(2)^2(14)$
(inside space)

$= 504\,\pi$ $= 56\,\pi$

Total Volume $= 448\,\pi \approx$ 1407 cubic units
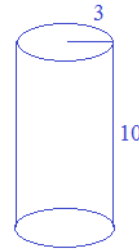
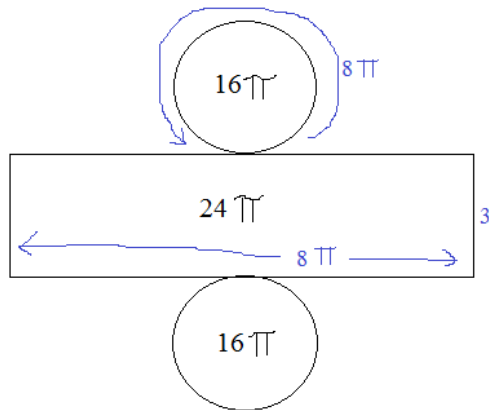6) If you fold up the parts of this figure, what is the volume?

The circles fold up (and down), and the parallelogram rolls up --- forming a cylinder..

Volume = $\pi(3)^2(10)$

$= 90\,\pi$

7) The areas of each part are labeled. What is the volume of the constructed figure?

The circles fold up/down, and the rectangle gets 'rolled up' to form a cylinder...
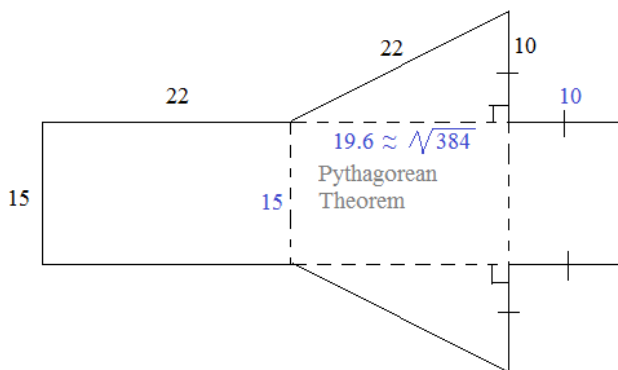
**Since the area of each circle is 16π, the radius is 4 units... Then, if the radius is 4 units, the circumference is 8π ....

Therefore, if the circumference is 4π, and the area of the rectangle is 24π, then the width is 3 units!

Volume of the cylinder = $16\pi \times 3 = 48\pi$ cubic units

8) Describe the constructed 3-d figure. Then, find its volume.

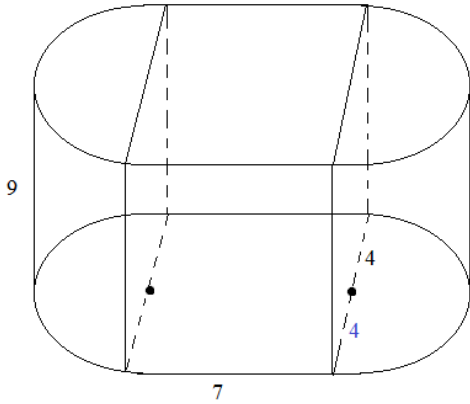$19.6 \approx \sqrt{384}$
Pythagorean Theorem

If you fold the triangles up (and down), then fold the right rectangle up... and, finally, fold the left rectangle over the top, you form a triangular prism!
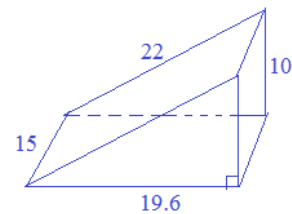
Volume = $\frac{1}{2}$ (length)(width)(depth)

area of base

$= \frac{1}{2}(19.6)(10)(15) = 1470$

mathplane.com

9) Find the volume and surface area of the figure.

(Note: the 'radius' is 4 units)

9

4

4

7

The figure is a cylinder split in half and a rectangular prism..

Volume of the cylinder = $\pi \,(radius)^2\,(height)$

$= \pi\,(16)(9) \; = \; 144\,\pi$

Volume of prism $= $ (length)(width)(height)

$= (7)(8)(9) \; = \; 504$

$504 + 144\,\pi \; units^3$

Surface area: front/back of prism = 2 x (9)(7) = 126
top/bottom of prism = 2 x (7)(8) = 112

two halves of cylinder = 8π x (9) = $72\,\pi$
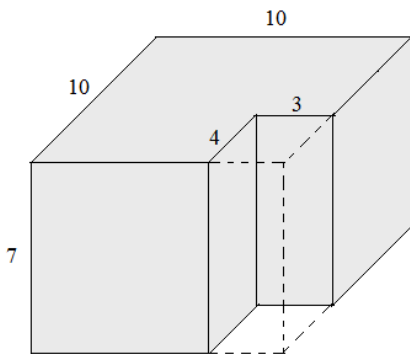top/bottom of cylinder = 2 x $16\,\pi$ = $32\,\pi$

Total SA = $238 + 104\,\pi$

10) The height of a cylinder is 6". If the volume of the cylinder is 1527 cubic inches, what is
the approximate radius?

Volume = $\pi\,(radius)^2(height)$

$1527 \; = \; \pi\,r^2\,(6)$

$254.5 \; = \; \pi\,r^2$

$81.00 = r^2$ radius = 9

r

6"

11) Determine the volume and surface area of the figure.

10

10

3

4

7
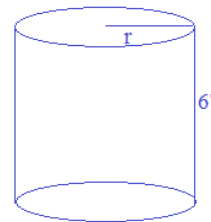
Volume of 'entire block' — volume of 'cut out block' = volume of figure

(10)(10)(7)   —   (3)(4)(7)   =   616 cubic units

Surface Area: top = 100 - 12 = 88
bottom = 88
left = 70
back = 70
right = 6 x 7 = 42
front = 7 x 7 = 49

total: 456 square units

indentation: left = 28
right = 21

Thanks for visiting! (Hope it helped)

If you have questions, suggestions, or requests, let us know.

Enjoy

_____
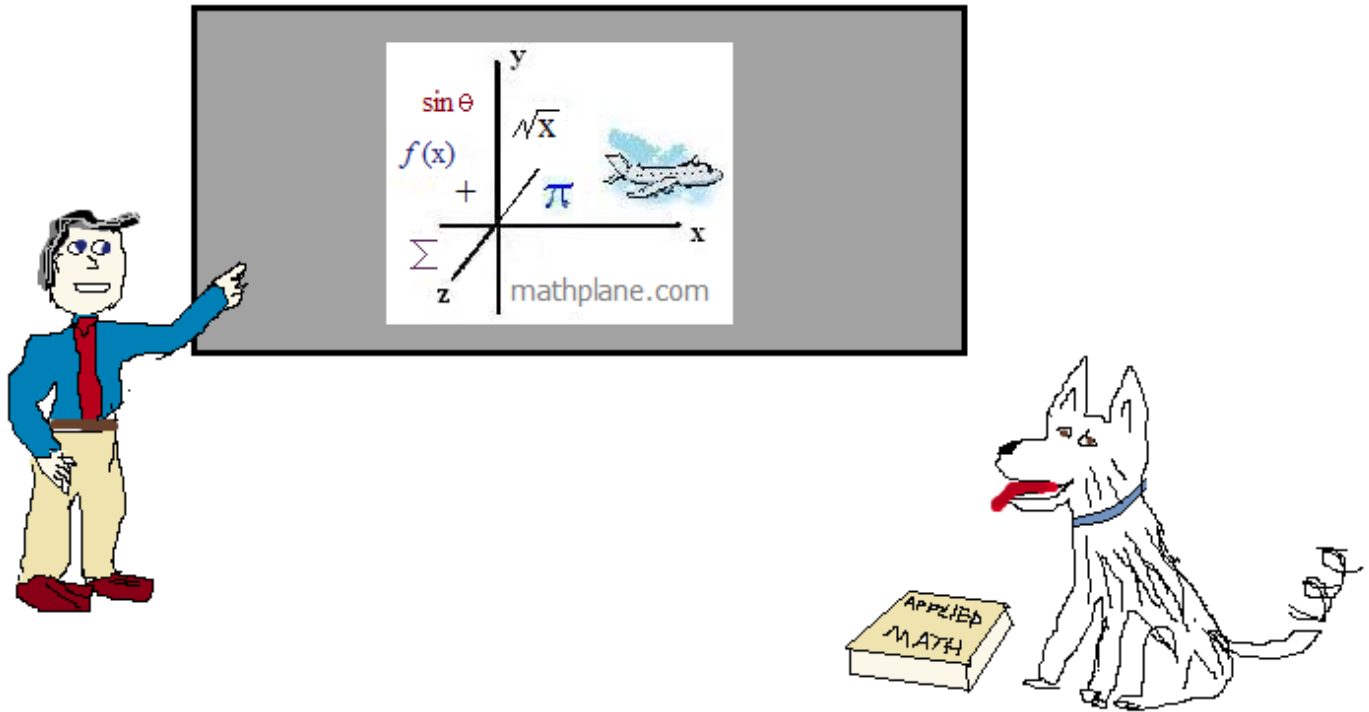
One more volume question?

(measured in cubic feet,) how much dirt is in a rectangular shaped hole that is 4 feet by 6 feet and 18 inches deep?

ANSWER-→

Volume Question:

(Measured in cubic feet), how much dirt is in
a rectangular shaped hole that is 4 feet by 6 feet
and 18 inches deep?

Answer: 0   (there is no dirt in a hole!)

(If the hole were filled, it would have a volume of
4' x 6' x 1.5' = 36 cubic feet...)