Average, Midpoint, Distance

Practice Questions (and Solutions)

Content includes formulas, applications, graphing, Pythagorean Theorem, and more.

Part I: Find the Average

    a) 8, 24

    b) -2, 0, 14

    c) 12, 37, 51

    d) If the numbers in set S are -2, 8, 19, X,
       and, the average of set S is 12, what is X?

    e) After 3 tests, John has a 75 average. What score
   does he need to get on his 4th test to raise
       his average to 80?

Part II: Find the Midpoint

    a) between 6 and -2

    b) between (2, 7) and (12, 1)

    c) between (-4, 6) and (7, 6)

    d) between (-1, -6) and (-6, 4)

    e) (6, 9) is the midpoint of $\overline{AB}$.

          point A = (12, 12).. What is point B?

    f) (-4, -3) is the midpoint of $\overline{CD}$.

          point C = (X, -9)   point D = (0, Y)

             What is X?   What is Y?

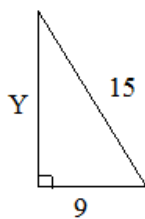Part III: Distance and Length   (Distance Formula/Pythagorean Theorem) | Average, Midpoint, and Distance Test |
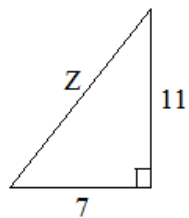
a)

X   5

12

What is X?

b)

15

Y

9

What is Y?

c)

Z   11

7
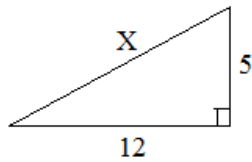
What is Z?

d)   A = (3, 6)   What is the length of $\overline{AB}$?
     B = (6, 10)

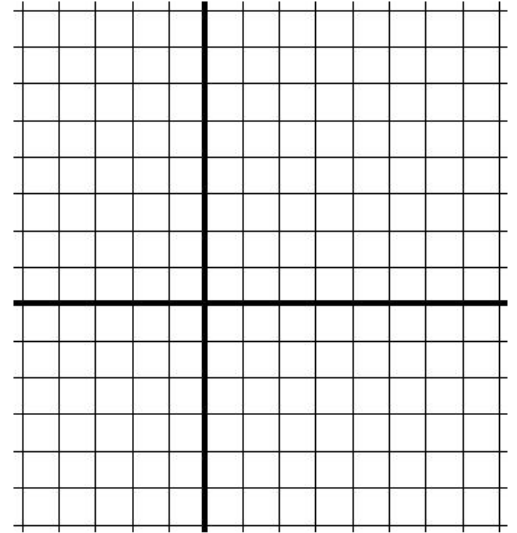e)   C = (-1, 4)   What is the distance between C and D?
     D = (5, 5)

f)   E = (8, 1)   What is the distance between E and F?
     F = (-2, 1)

Part IV: Graphing and Applications

For the points   A = (-1, 6)
                 B = (6, 6)
                 C = (6, -2)
                 D = (-1, -2)

a) plot the points A, B, C, and D on the graph.

b) What is the length of $\overline{AB}$? $\overline{AD}$?

c) What is the length of $\overline{AC}$? $\overline{BD}$?

d) What is the area of the figure ABCD?

e) What is the area inside the triangle   DCB?

Part V: Extra question

The distance between A and B is 10.

If A is (3, 11) and B is (x, 5),   what is x?

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Part I: Find the Average

a) 8, 24 $\qquad \dfrac{8 + 24}{2} = \boxed{16}$

b) -2, 0, 14 $\qquad \dfrac{-2 + 0 + 14}{3} = \boxed{4}$

c) 12, 37, 51 $\qquad \dfrac{12 + 37 + 51}{3} = \dfrac{100}{3} = \boxed{33.3\overline{3}}$ $\qquad$ (vinculum bar, indicates repeating indefinitely...)

d) If the numbers in set S are -2, 8, 19, X, and, the average of set S is 12, what is X?

$\dfrac{-2 + 8 + 19 + X}{4} = 12 \qquad 25 + X = 48$

$\boxed{X = 23}$

e) After 3 tests, John has a 75 average. What score does he need to get on his 4th test to raise his average to 80?

3 (tests) x 75 (average) = 225 (total)

4 (tests) x 80 (average) = 320 (total)

$$\begin{array}{r} 320 \\ -\ 225 \\ \hline 95 \end{array}$$

$\boxed{\text{John needs a 95}}$

Part II: Find the Midpoint

a) between 6 and -2 $\qquad$ (midpoint is 'average' on the number line) $\qquad \dfrac{6 + (-2)}{2} = \boxed{2}$

b) between (2, 7) and (12, 1) $\qquad \left(\dfrac{2 + 12}{2}, \dfrac{7 + 1}{2} \right) = \boxed{(7, 4)}$

Midpoint formula

$$\left(\dfrac{X_1 + X_2}{2}, \dfrac{Y_1 + Y_2}{2} \right)$$

c) between (-4, 6) and (7, 6) $\qquad \left(\dfrac{-4 + 7}{2}, \dfrac{6 + 6}{2} \right) = \boxed{(3/2, 6)}$

d) between (-1, -6) and (-6, 4) $\qquad \left(\dfrac{-1 + (-6)}{2}, \dfrac{-6 + 4}{2} \right) = \boxed{(-7/2, -1) \text{ or } (-3.5, -1)}$

e) (6, 9) is the midpoint of $\overline{AB}$.

point A = (12, 12).. What is point B?

$(6, 9) = \left(\dfrac{12 + x}{2}, \dfrac{12 + y}{2} \right) \qquad \begin{array}{l} x = 0 \\ \\ y = 6 \end{array} \qquad \boxed{B = (0, 6)}$
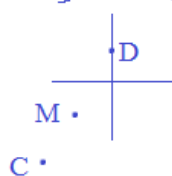
f) (-4, -3) is the midpoint of $\overline{CD}$.

point C = (X, -9) $\quad$ point D = (0, Y)
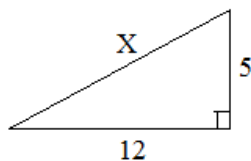
What is X? What is Y?

$(-4, -3) = \left(\dfrac{X + 0}{2}, \dfrac{-9 + Y}{2} \right) \qquad \boxed{\begin{array}{l} X = -8 \\ \\ Y = 3 \end{array}}$

(check points graphically)

Part III: Distance and Length   (Distance Formula/Pythagorean Theorem)

Average, Midpoint, and Distance Test

SOLUTIONS

a)

What is X?

$5^2 + 12^2 = X^2$

$25 + 144 = X^2$

$\boxed{X = 13}$

Pythagorean Theorem:
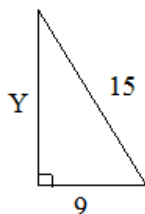
(right triangle) $A^2 + B^2 = C^2$

leg   leg   hypotenuse

Distance Formula:

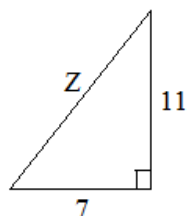$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

b)

What is Y?

$9^2 + Y^2 = 15^2$

$81 + Y^2 = 225$ $Y^2 = 144$ $\boxed{Y = 12}$

c)

What is Z?

$7^2 + 11^2 = Z^2$

$49 + 121 = Z^2$

$\boxed{Z = \sqrt{170}}$

Note: 5-12-13 and 3-4-5 special right triangles...

3x (3-4-5) is a 9-12-15

d) A = (3, 6)
    B = (6, 10)

What is the length of $\overline{AB}$?

$m\overline{AB} = \sqrt{(6-3)^2 + (10-6)^2}$

$= \sqrt{9 + 16} = \boxed{5}$

e) C = (-1, 4)
    D = (5, 5)

What is the distance between C and D?

$d\overline{CD} = \sqrt{(-1-5)^2 + (4-5)^2}$

$= \sqrt{36 + 1} = \boxed{\sqrt{37}}$

f) E = (8, 1)
    F = (-2, 1)

What is the distance between E and F?

$d\overline{EF} = \sqrt{100 + 0} = \boxed{10}$

(horizontal line segment connects these 2 points)

SOLUTIONS

For the points   A = (-1, 6)
B = (6, 6)
C = (6, -2)
D = (-1, -2)

a) plot the points A, B, C, and D on the graph.

b) What is the length of $\overline{AB}$? $\overline{AD}$?

> AB is 7 units (across)
>
> AD is 8 units (down)

c) What is the length of $\overline{AC}$? $\overline{BD}$?

AC and BD are diagonals of the rectangle...

$dAC = \sqrt{(6 - (-1))^2 + (-2 - 6)^2}$ $dBD = \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$ $= \sqrt{113}$ $= \sqrt{113}$
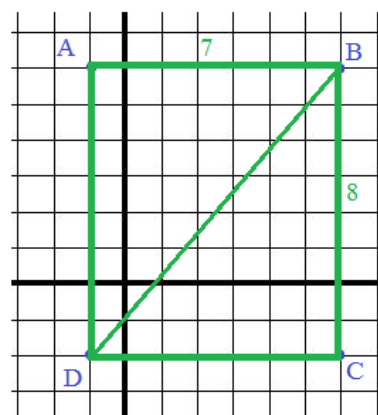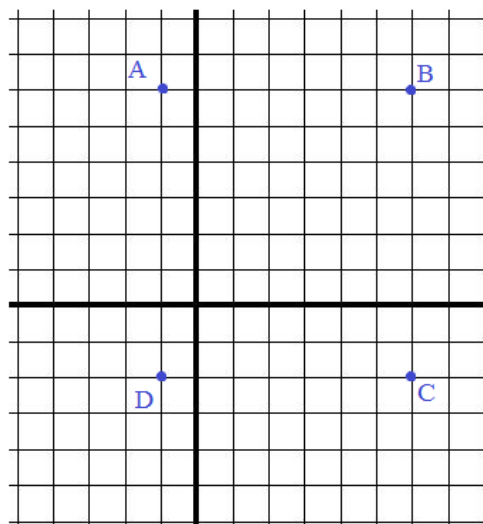
d) What is the area of the figure ABCD?

Area of rectangle = (length)(width)

$= 7 \times 8 = $ 56 square units

e) What is the area inside the triangle DCB?

area of triangle = $\frac{1}{2}$ (base)(height)

$= \frac{1}{2} (7)(8) = $ 28 square units

Part V: Extra question

The distance between A and B is 10.

If A is (3, 11) and B is (x, 5),   what is x?

distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

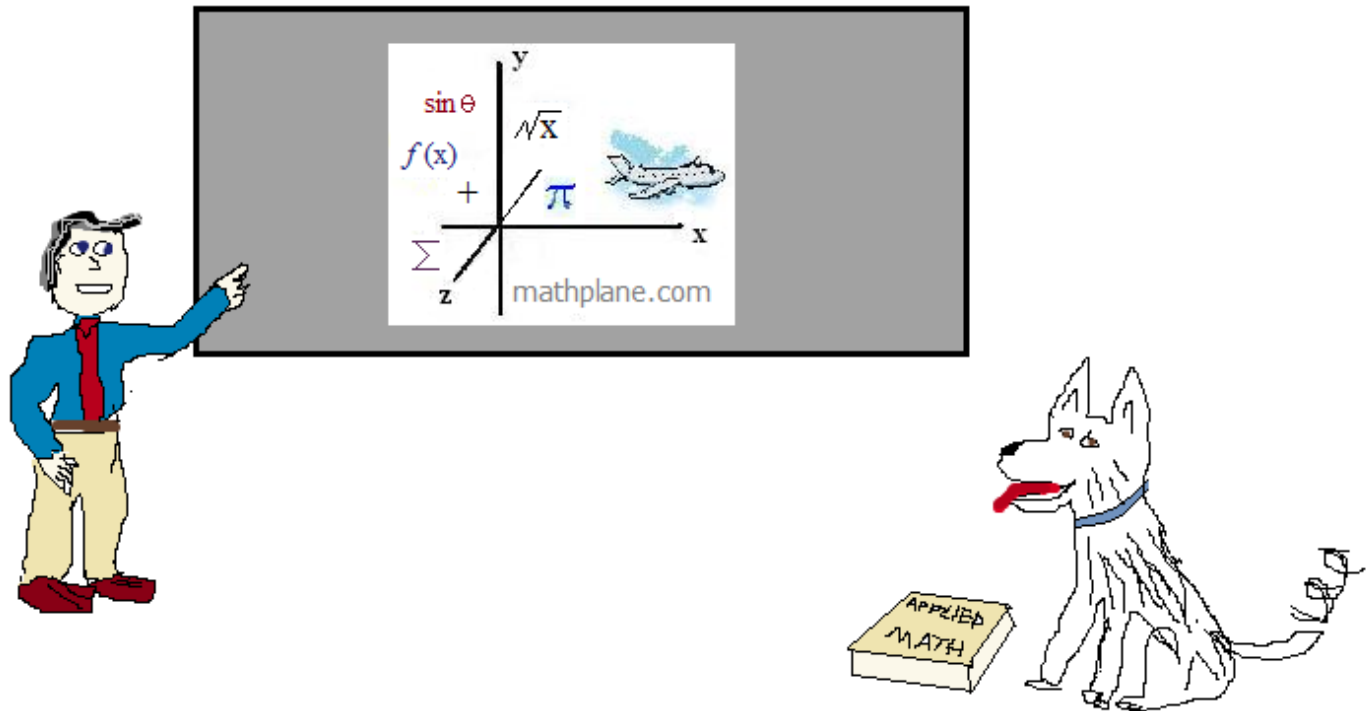$10 = \sqrt{(3 - x)^2 + (11 - 5)^2}$

$100 = (3 - x)^2 + 36$

$64 = (3 - x)^2$

$\pm 8 = 3 - x$

$x = -5$ or 11

Thanks for visiting. Hope it helps!

If you have questions, suggestions, or requests, let us know.

Cheers

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