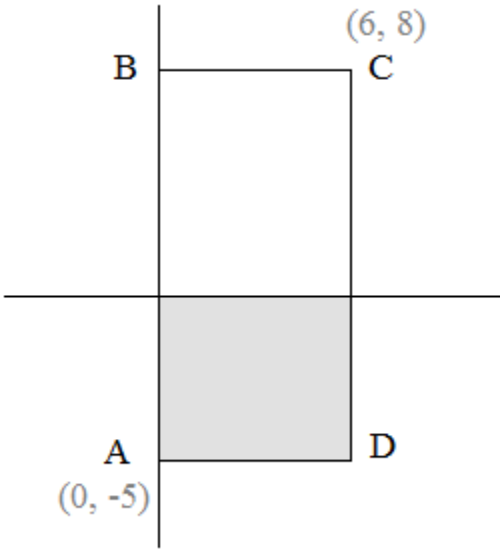
Coordinate Geometry 2

Translation and Transformation Practice

Exercises (w/solutions)



Topics included ordered pairs, quadrilaterals, probability, area, perimeter, symmetry, reflection, and more.

Rotating points, segments and figures around a point

3 things to know:

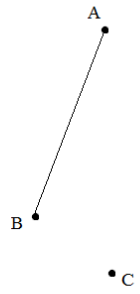
a) Point of rotation ("pivot" point)

b) Direction (clockwise or counterclockwise?)

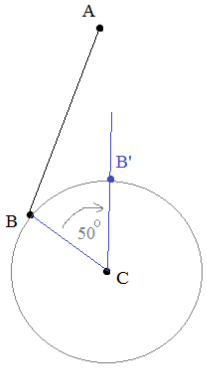
c) angle/degree amount

Example: Rotate the segment $\overline{AB}$ 50 degrees, clockwise, around the point C

Method 1: Using a compass and protractor

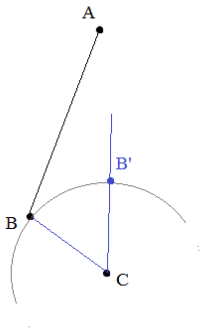


To rotate point B: Using your compass, construct a circle (or arc) with center C and radius of length $\overline{BC}$

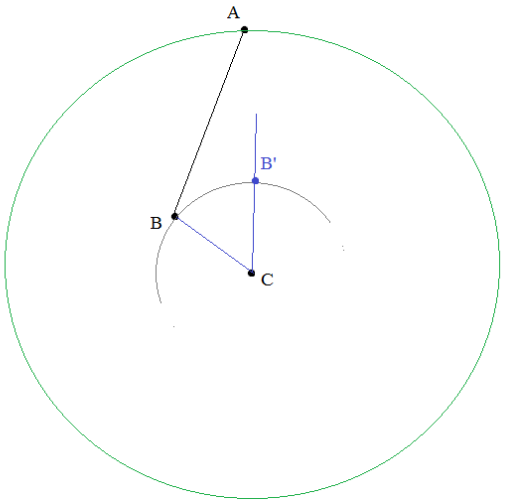


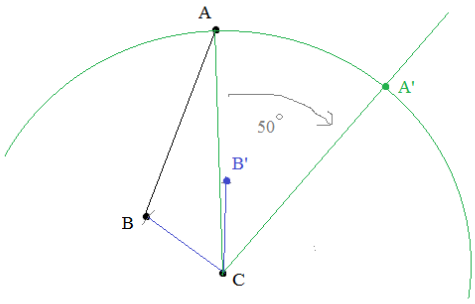
Then, using your compass, draw a 50 degree angle where C is the vertex and B moves clockwise to B'

(the intersection of the 'terminal' side and the circle/arc is B')



To rotate point A: Using your compass, construct a circle (or arc) with center C and radius of length $\overline{AC}$

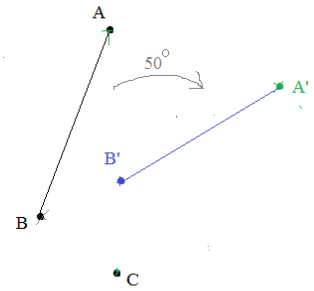




Then, using your compass, draw another 50 degree angle where C is the vertex and A moves clockwise to A'

(again, the intersection of the terminal side and the arc is A')

Finally, draw a segment from A' to B'



Rotating points, segments and figures around a point

3 things to know:

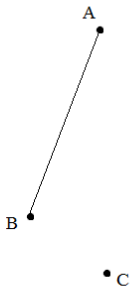
a) Point of rotation ("pivot" point)

b) Direction (clockwise or counterclockwise?)

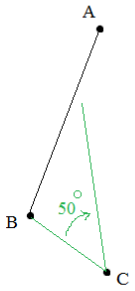
c) angle/degree amount

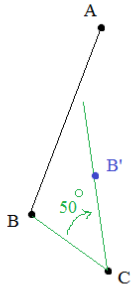
Example: Rotate the segment $\overline{AB}$ 50 degrees, clockwise, around the point C

Method 2: Using a ruler and protractor



To rotate B clockwise around C: Draw a segment connecting points B and C.
then, using your protractor, draw a 50 degree angle
with C as the vertex.

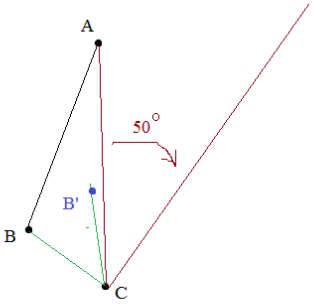


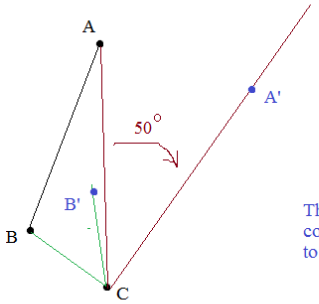


Then, use your ruler
to copy the length of BC
to CB'...

$\overline{BC} \cong \overline{B'C}$

To rotate A clockwise around C: Draw a segment connecting points A and C.
Then, use your protractor to draw a 50 degree angle
with C as the vertex and A' "to the clockwise right" of A.

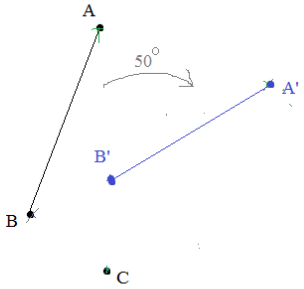


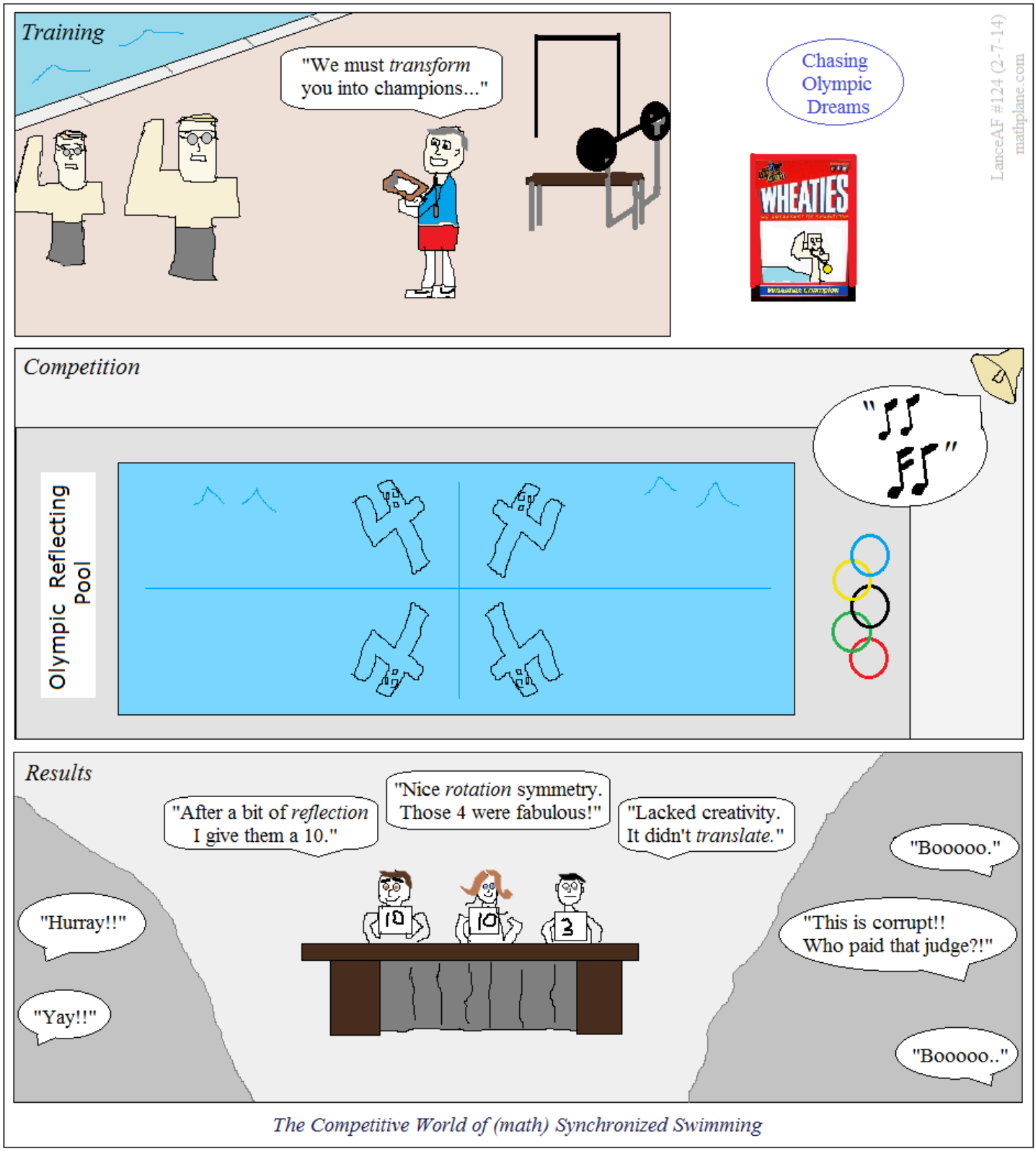


Then, use your ruler to
copy the length of AC
to A'C...

$\overline{AC} \cong \overline{A'C}$

Finally, draw a segment from A' to B'





Practice Exercises-→

Coordinate Geometry, Translation, and Transformation

I. Introduction

1) A = (2, 2)
B = (7, 2)
C = (2, -1)
D = (7, -1)

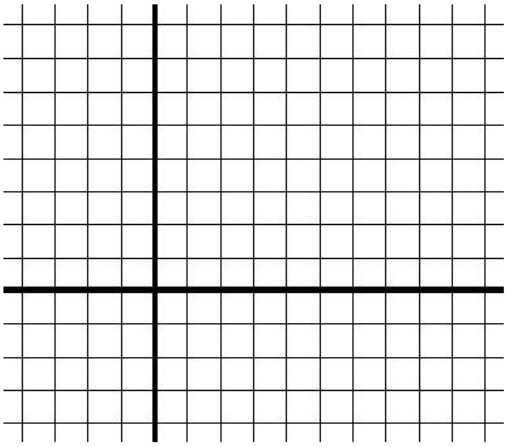
(Plot each point on the graph)

a) What is the perimeter of ABDC?

b) What is the area of ABDC?

c) Describe the figure..
(what type of quadrilateral?)

d) What are the lengths of the diagonals?



2) E = (1, 1)
F = (1, 6)
G = (4, 5)
H = (4, 2)

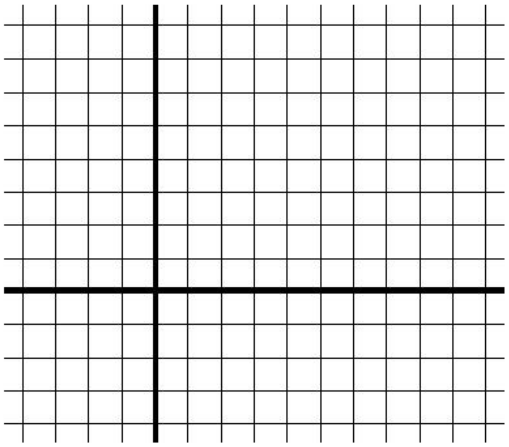
(plot each point on the graph)

a) Describe the figure EFGH..

b) What is the perimeter of EFGH?

c) What is the area of EFGH?

**d) Extra: What is the line of symmetry of figure EFGH?



II. Quadrilaterals, Shapes, and Polygons

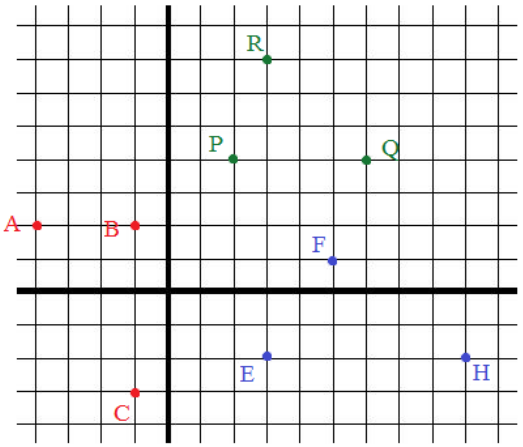
1) Identify the coordinates A, B, and C.

a) What point (D) would form a rectangle?

b) What is the area of ABCD?

2) If EFGH is an isosceles trapezoid, what is the coordinate of G?

**Challenge: 3) If P, Q, and R are vertices of a parallelogram, what is the coordinate(s) of the 4th vertex?



III. Reflection and Rotation

Determine the new coordinates:

1) Reflect over the x-axis:

A' = B' = C' =

2) Reflect over the y-axis:

A' = B' = C' =

3) Shift up 3 units, left 4 units:

A' = B' = C' =

4) Rotate clockwise 90° :

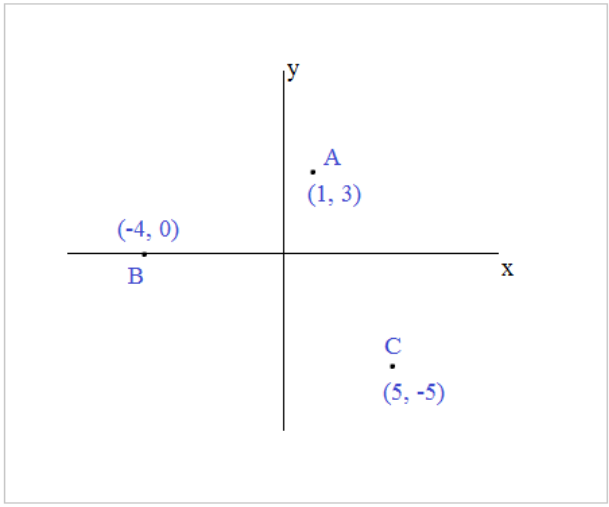
A' = B' = C' =

5) Rotate counter-clockwise 90° :

A' = B' = C' =

6) Reflect over the origin:
(rotate 180°)

A' = B' = C' =



**Challenge:

7) Reflect over $y = 4$:

A' = B' = C' =

8) Rotate clockwise 90° around the point (3, 3):

A' = B' = C' =

IV: Transformation

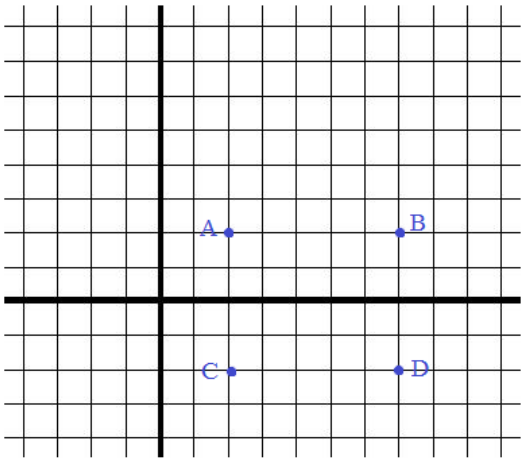
A) What are the coordinates of A, B, C, and D?
Describe ABDC. (what is the quadrilateral?)

B) Shift A and B up 4 units.
Describe A'B'DC.
What is the length of CB' ?

C) Shift A and B to the right 3 units.
What is the length of CA'?
Describe A'B'DC.

**D) Extra: In figure ABDC, where do the diagonals cross?

What is the area of triangle ABC ?



V. General Concepts

A) For the coordinate (x, y), match the operation with the output:

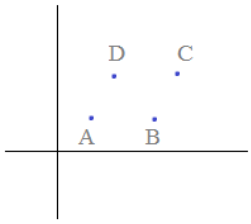
1) shift up 4 units; shift left 3 units _____
2) rotate clockwise 90° _____
3) reflect over the x-axis _____
4) shift down 3 units; shift right 4 units _____
5) reflect over the y-axis _____
6) reflect over the origin _____

a) (x, -y)
b) (-x, -y)
c) (x + 4, y - 3)
d) (x - 3, y + 4)
e) (x + 3, y - 4)
f) (y, -x)
g) (-x, y)

(Need help? Pick a random point and try each operation)

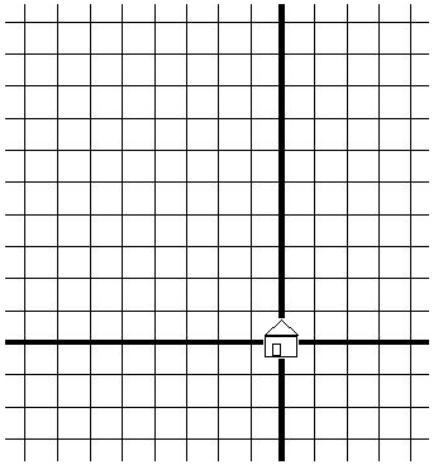
B) The base vertices of an isosceles triangle are (1, 2) and (7, 2).
If the altitude (height) is 6 units, what is the coordinate of the 3rd vertex?
(**Bonus: identify the other possible vertex!)

C) The vertices of rhombus ABCD are A (3, 3) B (8, 3) D (6, 7).
What is the coordinate of C?
What is the length AD ?
What is the slope of AD ? CD ?



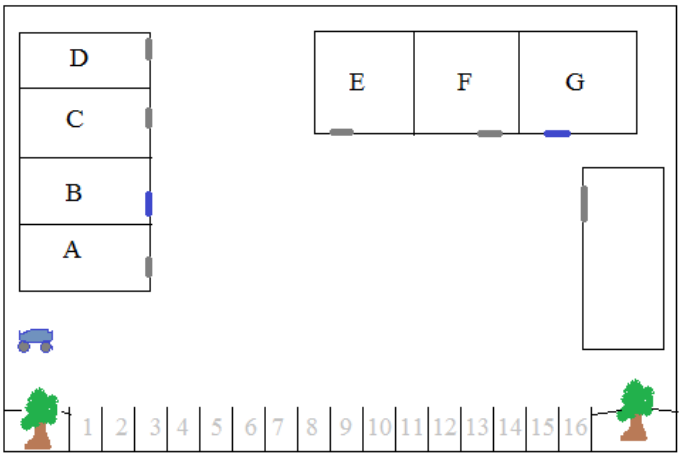
D) Natalie leaves for a long hike. She walks 6 miles north. Then, she hikes 4 miles west.
And, then she turns and goes 2 miles due south.

How far is she from her place of origin?

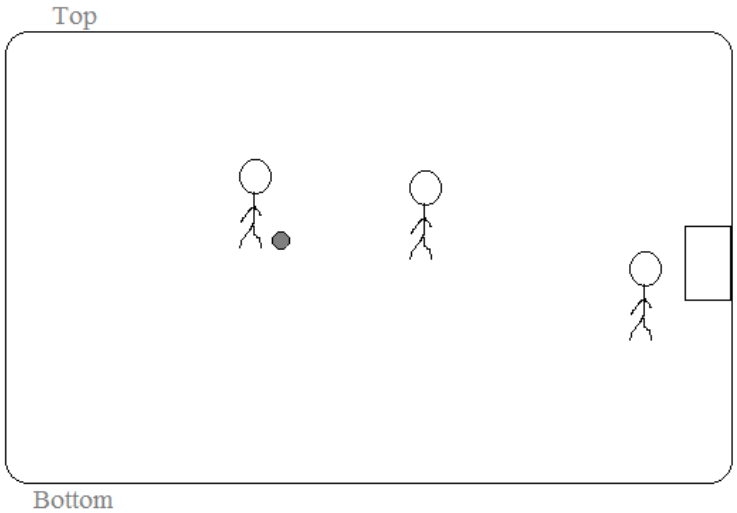


Geometry Reflection Application

At the shopping plaza (see diagram), I need to pick up a large package (in shop B) and running shoes (in shop G). Suppose I need to put the package in the car before buying the running shoes. Which parking space would minimize my walking distance between the shops and the car?

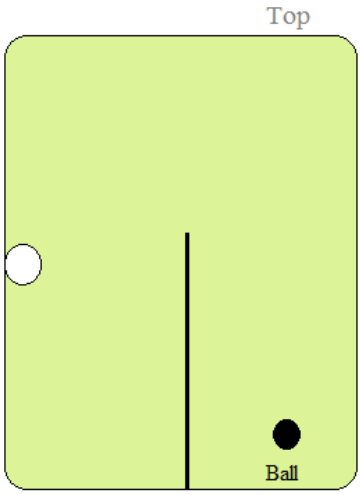


A soccer player can score if he kicks the ball off the top wall and into the upper part of the goal (past both defenders). In order to make this narrow shot, where must the player aim the soccer ball?

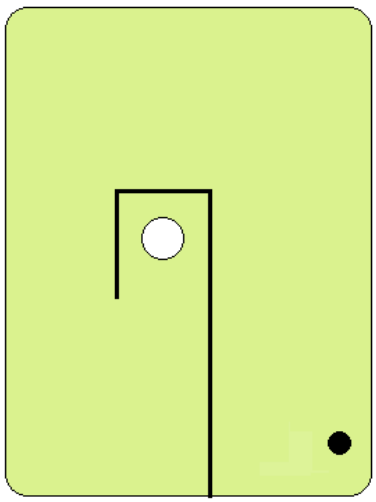


Geometry Reflection Application: Bankshots

A barrier stands between the black ball and the white hole.
However, a bankshot off the top of the table can result
in a successful shot.
Where should the player aim?



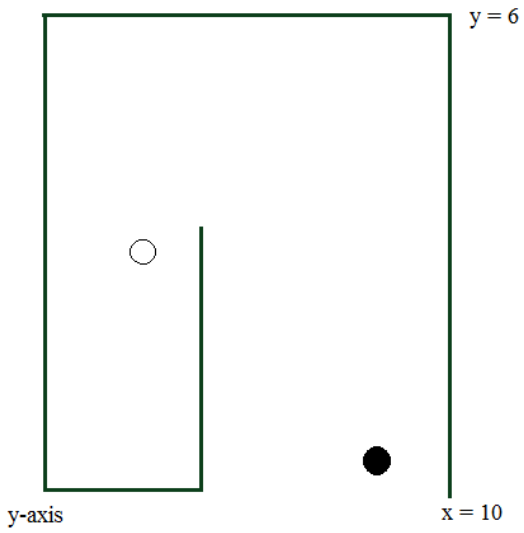
Additional barriers are added to the table.
Where should the player aim the black ball to
successfully sink the shot into the white hole?



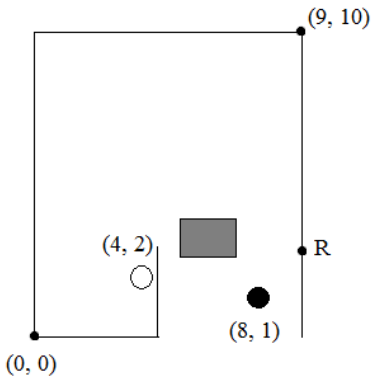
The diagram is a "pool table" and coordinate plane,
where the hole is at (3, 2) and the ball is positioned at (8, -2).

a) If you reflect the image of the hole over the (upper) cushion, what is the coordinate?

b) What is the coordinate (on the cushion) a player must hit in order to sink the bank shot?



The grid models a mini-golf course.
R is spot needed to hit in order to sink the shot.
What is the coordinate of R?



Coordinate Geometry, Translation, and Transformation

SOLUTIONS

I. Introduction

1) A = (2, 2)
B = (7, 2)
C = (2, -1)
D = (7, -1)

(Plot each point on the graph)

a) What is the perimeter of ABDC?

16 units

b) What is the area of ABDC?

15 square units

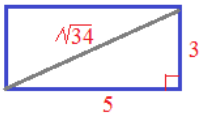
c) Describe the figure..
(what type of quadrilateral?)

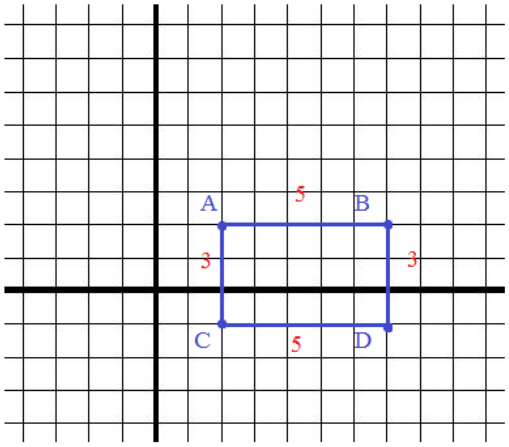
rectangle

d) What are the lengths of the diagonals?

$\sqrt{34}$

(use pythagorean theorem)





2) E = (1, 1)
F = (1, 6)
G = (4, 5)
H = (4, 2)

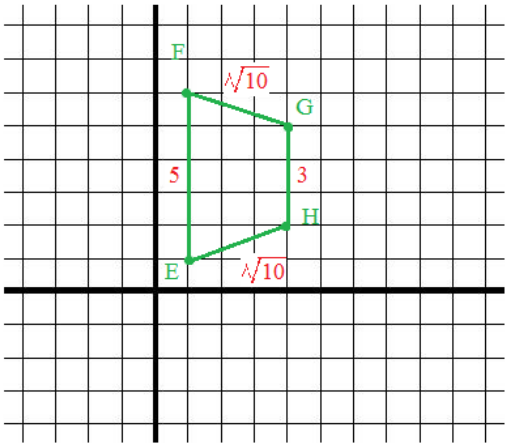
(plot each point on the graph)

a) Describe the figure EFGH..

Isosceles Trapezoid

b) What is the perimeter of EFGH?

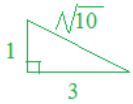
$8 + 2\sqrt{10} \cong 14.3$



c) What is the area of EFGH?

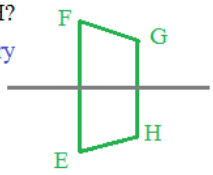
(viewing the trapezoid with $\overline{EF}$ as the bottom)
base 1 = 3 height = 3
base 2 = 5

A = 1/2 (3 + 5)(3)
= 12 square units



**d) Extra: What is the line of symmetry of figure EFGH?

y = 7/2 is the line of symmetry



II. Quadrilaterals, Shapes, and Polygons

1) Identify the coordinates A, B, and C. A (-4, 2) B (-1, 2) C (-1, -3)

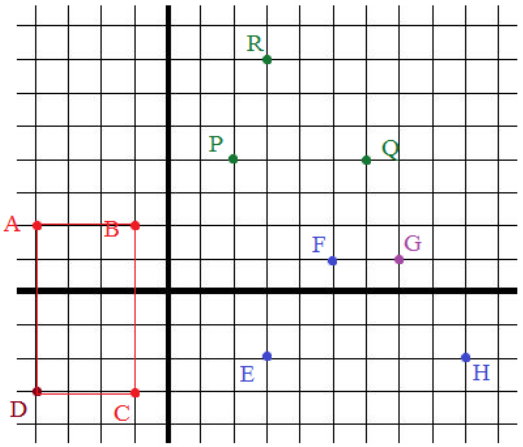
a) What point (D) would form a rectangle? (-4, -3)

b) What is the area of ABCD? 3 x 5 = 15 square units

2) If EFGH is an isosceles trapezoid, what is the coordinate of G?

G = (7, 1)

$\overline{FG} \parallel \overline{EH}$



**Challenge: 3) If P, Q, and R are vertices of a parallelogram, what is the coordinate(s) of the 4th vertex?

There are 3 possibilities!! (7, 7) (5, 1) or (-1, 7)

PRXQ PRQX PXRQ

III. Reflection and Rotation SOLUTIONS

Determine the new coordinates:

1) Reflect over the x-axis:

A' = (1, -3) B' = (-4, 0) C' = (5, 5)

2) Reflect over the y-axis:

A' = (-1, 3) B' = (4, 0) C' = (-5, -5)

3) Shift up 3 units, left 4 units:

A' = (-3, 6) B' = (-8, 3) C' = (1, -2)

4) Rotate clockwise 90° :

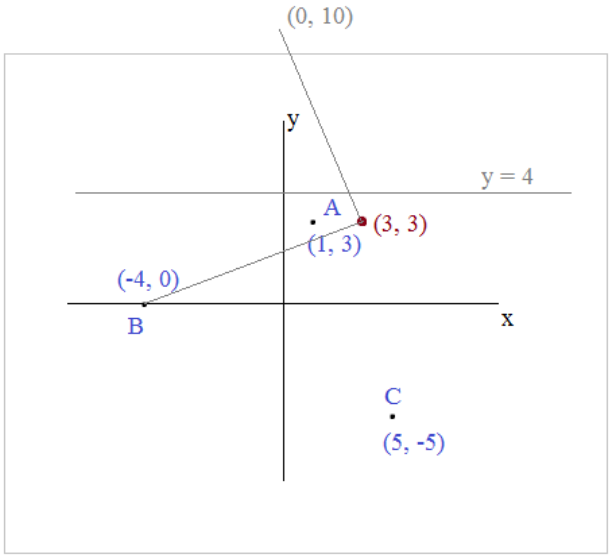
A' = (3, -1) B' = (0, 4) C' = (-5, -5)

5) Rotate counter-clockwise 90° :

A' = (-3, 1) B' = (0, -4) C' = (5, 5)

6) Reflect over the origin:
(rotate 180°)

A' = (-1, -3) B' = (4, 0) C' = (-5, 5)



**Challenge:

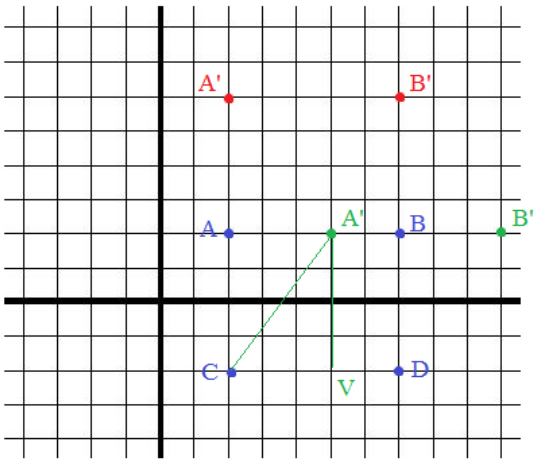
7) Reflect over y = 4:

A' = (1, 5) B' = (-4, 8) C' = (5, 13)

8) Rotate clockwise 90° around the point (3, 3):

A' = (3, 5) B' = (0, 10) C' = (-5, 1)

notice: (3, 3) to (-4, 0)
is left 7 and down 3...
So, to rotate around (3, 3), we
found the point up 7, left 3....
(0, 10)



IV: Transformation

A) What are the coordinates of A, B, C, and D?
Describe ABDC. (what is the quadrilateral?)
A (2, 2) B (7, 2) C (2, -2) D (7, -2) RECTANGLE

B) Shift A and B up 4 units.
Describe A'B'DC.
What is the length of CB' ?

Use Pythagorean theorem: CD = 5 B'D = 8

so, CB' = $\sqrt{89}$

C) Shift A and B to the right 3 units.
What is the length of CA'?
Describe A'B'DC.

Use Pythagorean theorem: CV = 3 VA' = 4

so, CA' = 5

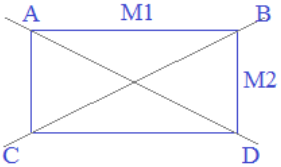
RHOMBUS (all sides are 5!)

**D) Extra: In figure ABDC, where do the diagonals cross?

at the horizontal and vertical midpoint: (4 1/2, 0)

What is the area of triangle ABC ?

1/2(bh) = 1/2(5)(4) = 10 square units



V. General Concepts

SOLUTIONS

A) For the coordinate (x, y), match the operation with the output:

example:
if (x, y) = (3, 6)

(0, 10) 1) shift up 4 units; shift left 3 units d

(6, -3) 2) rotate clockwise 90° f

(3, -6) 3) reflect over the x-axis a

(7, 3) 4) shift down 3 units; shift right 4 units c

(-3, 6) 5) reflect over the y-axis g

(-3, -6) 6) reflect over the origin b

a) (x, -y)

b) (-x, -y)

c) (x + 4, y - 3)

d) (x - 3, y + 4)

e) (x + 3, y - 4)

f) (y, -x)

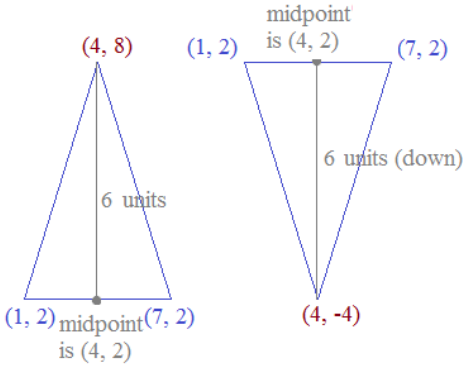
g) (-x, y)

(Need help? Pick a random point and try each operation)

B) The base vertices of an isosceles triangle are (1, 2) and (7, 2).
If the altitude (height) is 6 units, what is the coordinate of the 3rd vertex?
(**Bonus: identify the other possible vertex!)

(4, 8) is the top vertex of the isosceles triangle

***If the triangle were "upside down" then
the other vertex would be (4, -4)



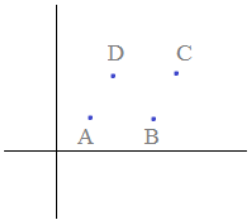
C) The vertices of rhombus ABCD are A (3, 3) B (8, 3) D (6, 7).
What is the coordinate of C?
What is the length AD ?
What is the slope of AD ? CD ?

rhombus: length of sides are all the same; opposite sides are parallel

C = (11, 7) therefore, DC || AB and DC = 5

Length of AD is 5

slope of AD is ("rise over run") = 4/3 slope of CD is 0.. (horizontal line)



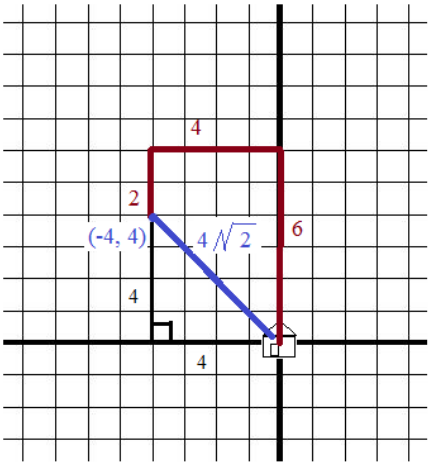
D) Natalie leaves for a long hike. She walks 6 miles north. Then, she hikes 4 miles west.
And, then she turns and goes 2 miles due south.

How far is she from her place of origin?

Distance from (4, 4) to the origin:

$\sqrt{4^2 + 4^2} = 4\sqrt{2}$

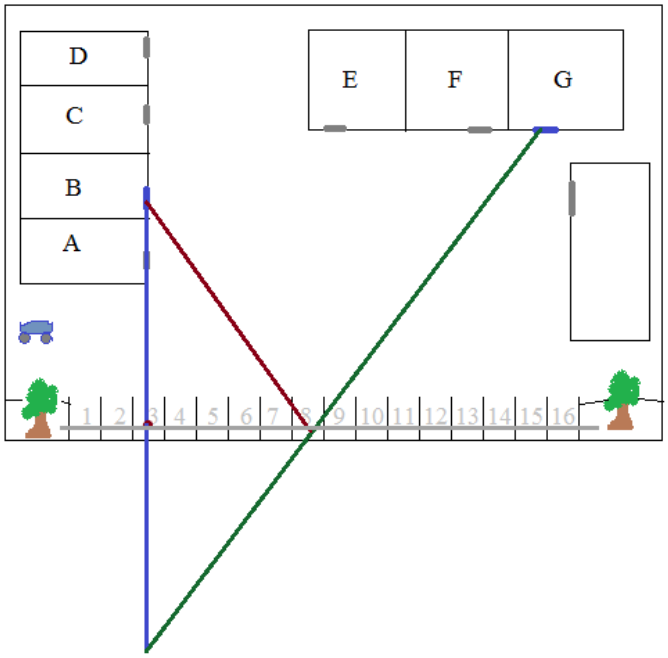
(Pythagorean theorem or
distance formula)



Geometry Reflection Application

SOLUTIONS

At the shopping plaza (see diagram), I need to pick up a large package (in shop B) and running shoes (in shop G). Suppose I need to put the package in the car before buying the running shoes. Which parking space would minimize my walking distance between the shops and the car?



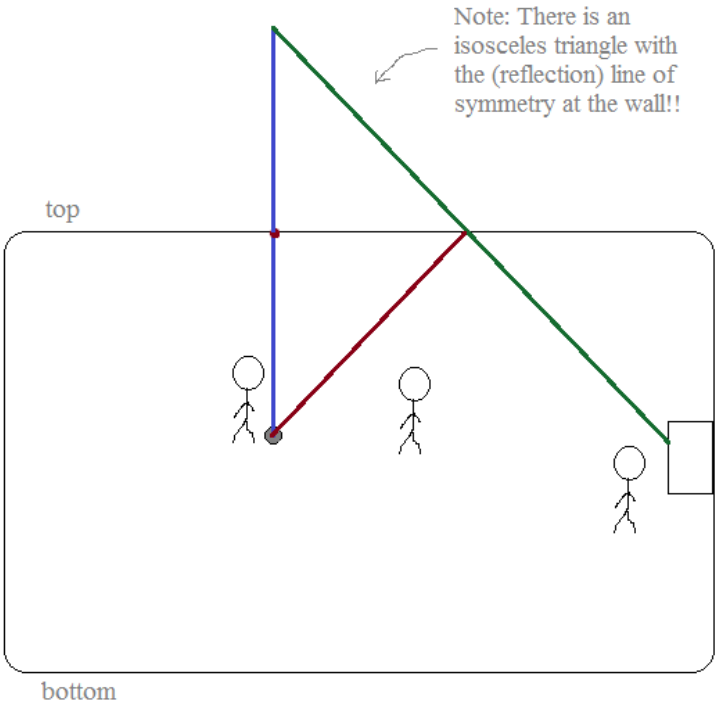
Start at entrance B...
Draw a perpendicular line segment to the parking spaces..
Then, reflect the segment over the parking spaces..

Then, go to entrance G...
Draw a segment to the end of the perpendicular segment.

Finally, draw a segment from entrance B to the intersection of the green segment and the line of symmetry.

From the sketch, parking space 8 appears to be the optimal parking spot (to minimize walking distance).

A soccer player can score if he kicks the ball off the top wall and into the upper part of the goal (past both defenders). In order to make this narrow shot, where must the player aim the soccer ball?



Draw a perpendicular line segment from the soccer ball to the top wall.
Then, *reflect* the segment over the top wall.

Draw a line segment from the top of the blue line segment to the goal.

Finally, draw a line segment from the soccer ball to the intersection of the wall and the green line segment.

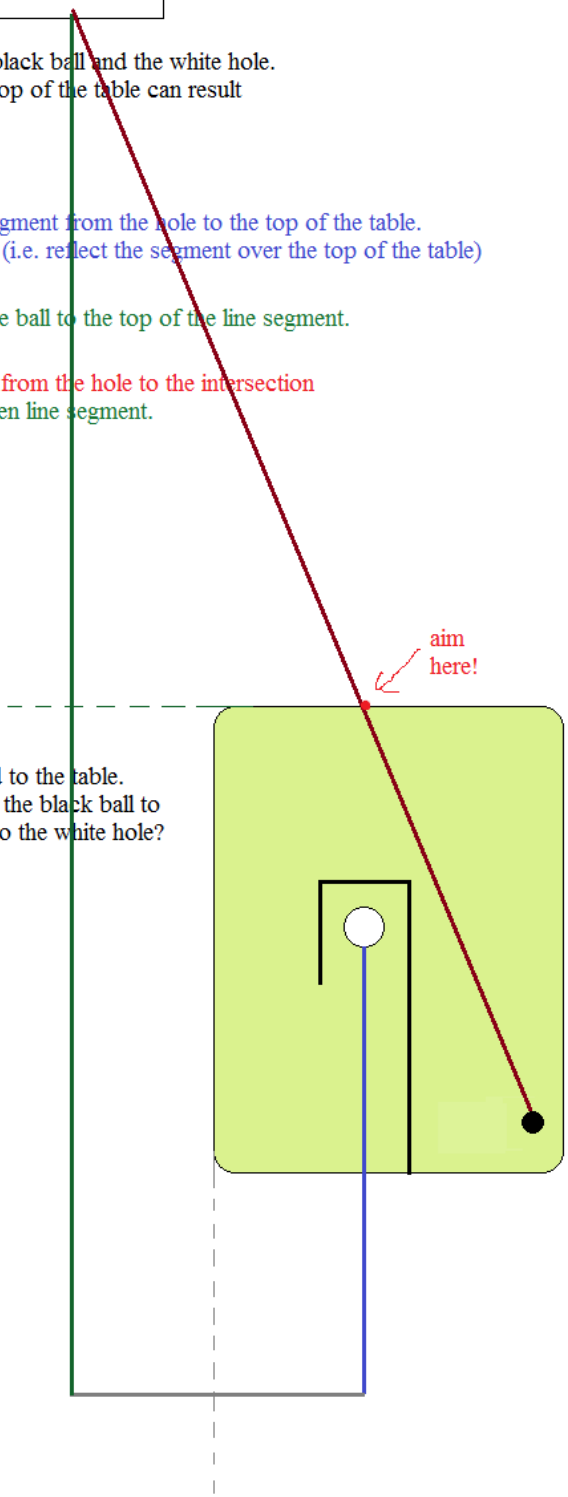
Geometry Reflection Application: Bankshots

A barrier stands between the black ball and the white hole.
However, a bankshot off the top of the table can result
in a successful shot.
Where should the player aim?

Draw a perpendicular line segment from the hole to the top of the table.
Then, "double the segment" (i.e. reflect the segment over the top of the table)

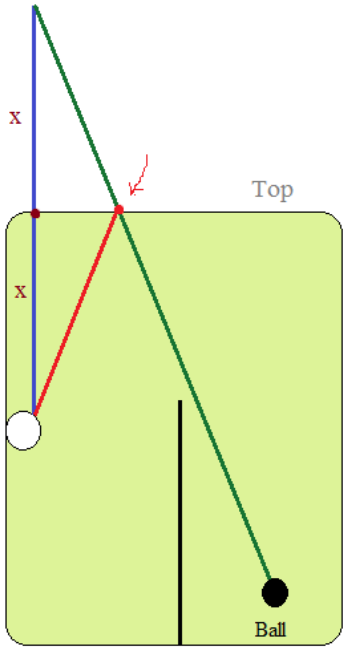
Draw a line segment from the ball to the top of the line segment.

Finally, draw a line segment from the hole to the intersection
of the table Top and the green line segment.



Additional barriers are added to the table.
Where should the player aim the black ball to
successfully sink the shot into the white hole?

solutions



1) Start with the white hole. Draw a perpendicular line segment to the bottom border. Then, reflect the segment over the bottom.

2) Extend the left border down. then, draw a perpendicular line segment from the end of the first one to the left border. then, reflect it...

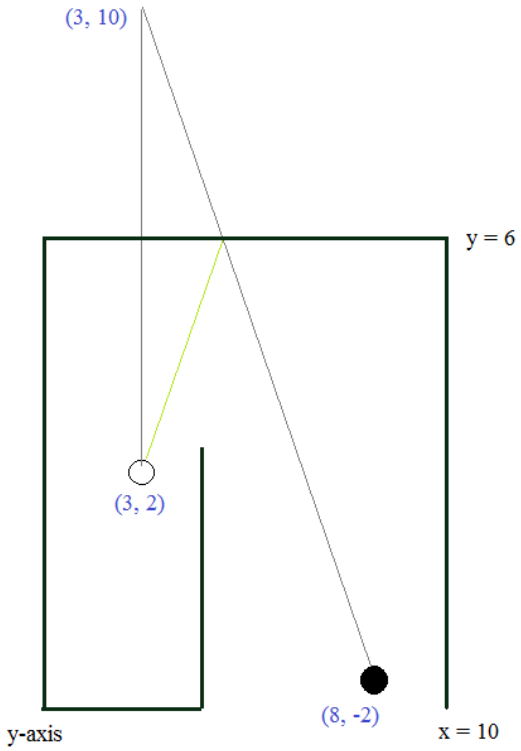
3) Extend the top border to the left. Then, continue with another perpendicular line segment to the top. Then, reflect it...

4) Draw line segment from endpoint to the black ball. (The intersection of the line segment and the top boundary is the solution!)

The diagram is a "pool table" and coordinate plane,
where the hole is at (3, 2) and the ball is positioned at (8, -2).

a) If you reflect the image of the hole over the (upper) cushion, what is the coordinate?

the hole (3, 2) is 4 units from y = 6...
then, the image is 4 units on the other side....
so, the image is (3, 10)



b) What is the coordinate (on the cushion) a player must hit in order to sink the bank shot?

We need to find where the line (from the image to the ball) intersects the cushion:

equation of line (from image to cushion):

$$\text{slope} = \frac{10 - (-2)}{3 - 8} = \frac{-12}{5}$$

point: (3, 10)

$$(y - 10) = \frac{-12}{5}(x - 3)$$

$$y = \frac{-12}{5}x + \frac{86}{5}$$

And, it intersects the cushion (y = 6)

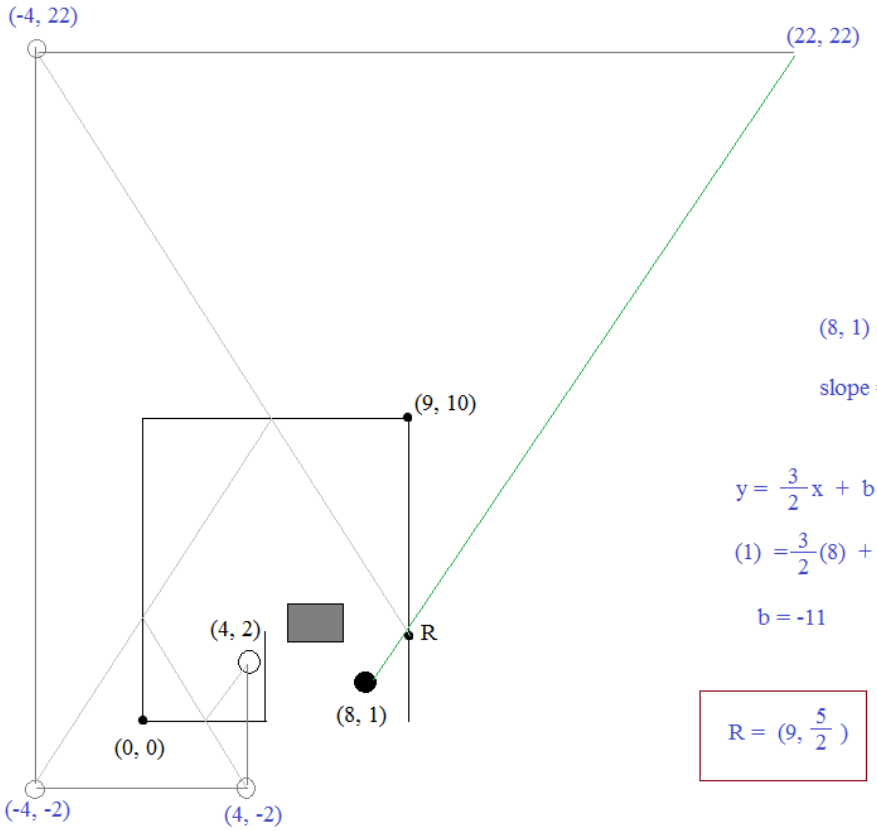
at $6 = \frac{-12}{5}x + \frac{86}{5}$

$$\frac{-56}{5} = \frac{-12}{5}x$$

$$x = \frac{56}{12} = 4\frac{2}{3}$$

$\left(4\frac{2}{3}, 6\right)$

The grid models a mini-golf course.
R is spot needed to hit in order to sink the shot.
What is the coordinate of R?



(8, 1) and (22, 22)

$$\text{slope} = \frac{21}{14} = \frac{3}{2}$$

$$y = \frac{3}{2}x + b$$

$$(1) = \frac{3}{2}(8) + b$$

$$b = -11$$

$$y = \frac{3}{2}x - 11$$

then, find where it intersects the right cushion (i.e. x = 9)

$$y = \frac{3}{2}(9) - 11$$

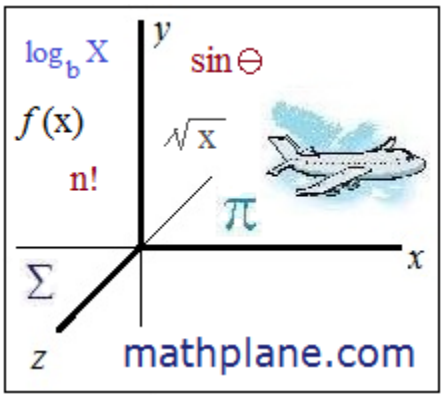
$$y = 5/2$$

$R = \left(9, \frac{5}{2}\right)$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or feedback, let us know.

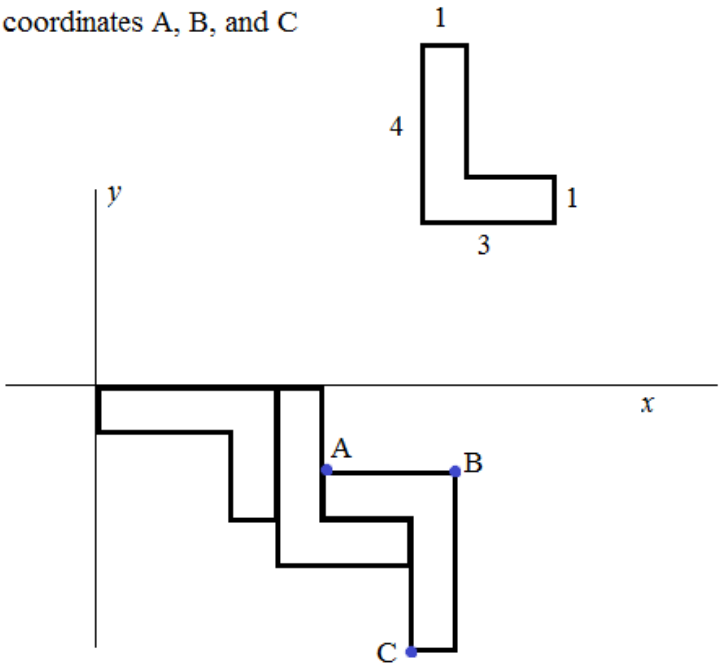
Cheers!



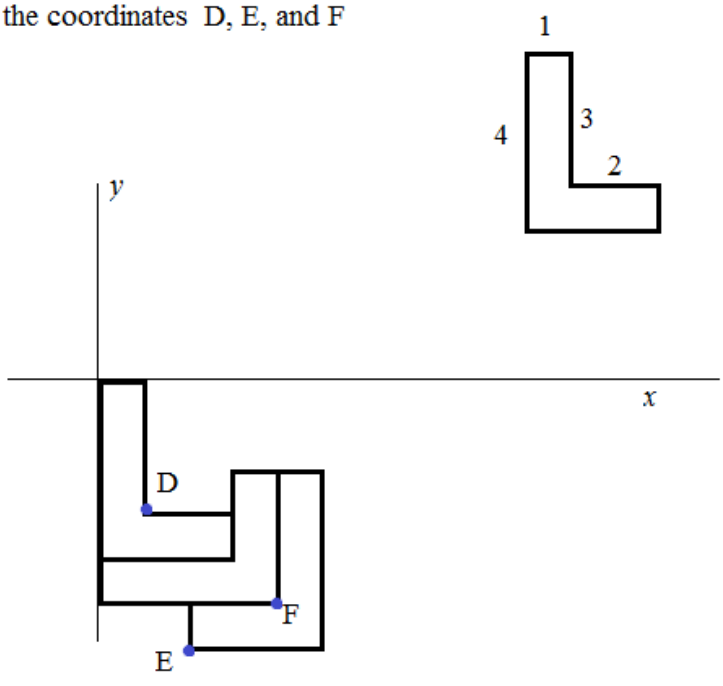
Also, at Mathplane *Express* for mobile and tablet at Mathplane.ORG

Extra Questions:

Find coordinates A, B, and C



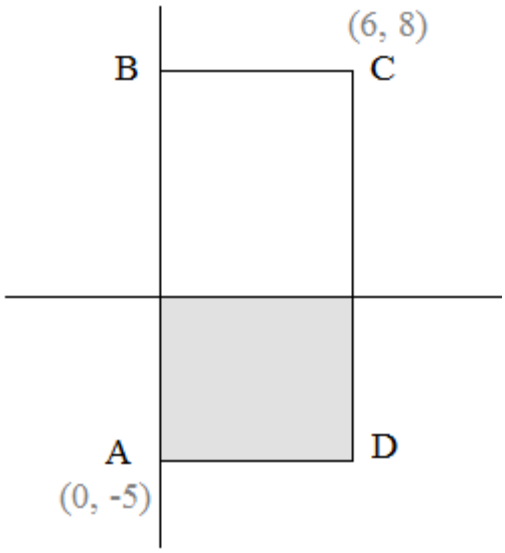
Find the coordinates D, E, and F



Coordinate Geometry Probability Question

1) Find coordinates B and D

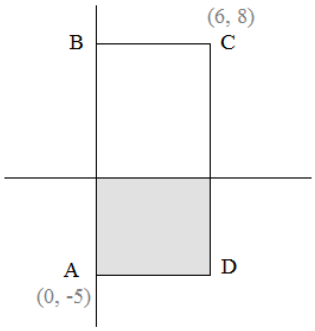
2) If you randomly choose a point within ABCD, what is the probability that this point is within the shaded area?



Extra Question SOLUTIONS

1) Find coordinates B and D

2) If you randomly choose a point inside ABCD, what is the probability that it is a point inside the shaded area?



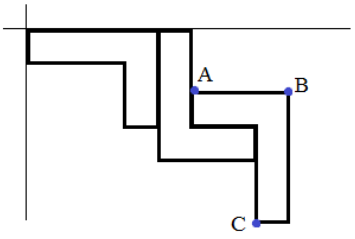
Answers:

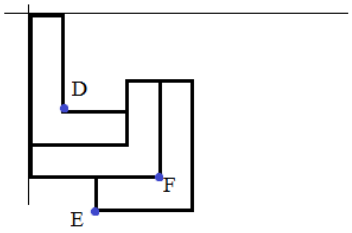
1) B (0, 8)
D (6, -5)

2) Area of ABCD
6 x 13 = 78
(length) (width)

Area of shade
6 x 5 = 30
(length) (width)

Therefore, p(point in shade) = $\frac{30}{78} = \frac{5}{13}$ (approx. .3846)





SOLUTIONS:

A = (5, -2)
B = (8, -2)
C = (7, -6)

D = (1, -3)
E = (2, -6)
F = (4, -5)