Graphing II

Translation, Reflection, & Rotation

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Changing Positions: Translations ("Shifts"), Reflection, & Rotation

Note: Translation ⟶ moving the entire figure
Transformation ⟶ changing the figure

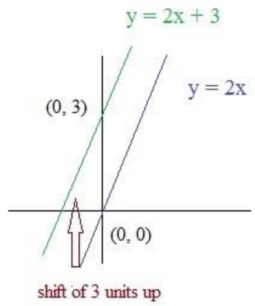
SHIFTS

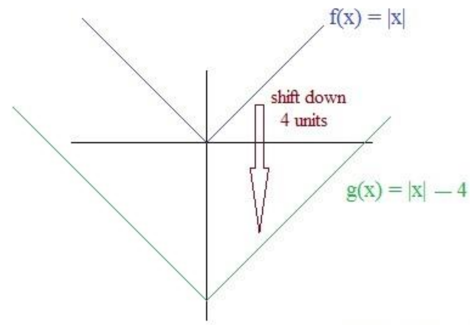
Vertical Shift: $f(x) + d$

d units along the y-axis

If $d > 0$, shift up

If $d < 0$, shift down





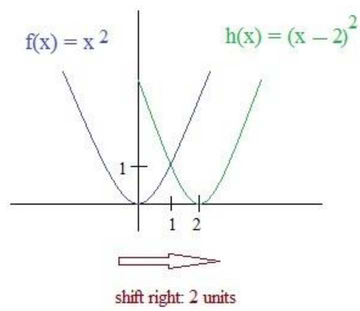
$g(x) = f(x) - 4$

Horizontal Shift: $f(x - b)$

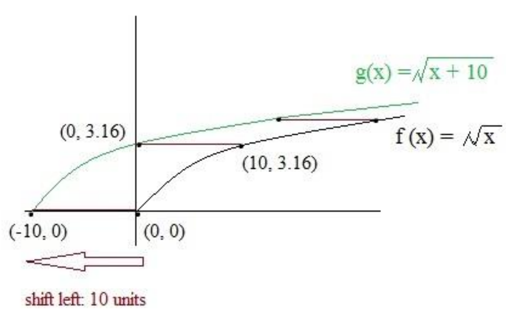
b units along the x-axis

If $b > 0$, shift left

If $b < 0$, shift right



x	f (x)	h (x)
-2	4	16
-1	1	9
0	0	4
1	1	1
2	4	0
3	9	1
4	16	4



Note: When the shift is vertical, a value is outside the parent function. When the shift is horizontal, a value is within the parent function!

⟶ $f(x + 5) \neq f(x) + 5$

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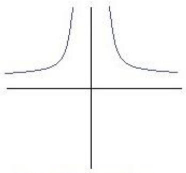
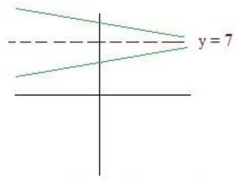
REFLECTION

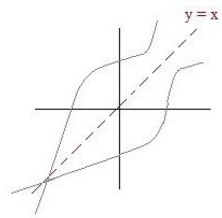
Symmetry Illustrations:



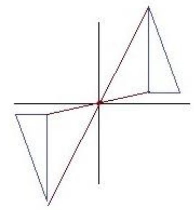
line of symmetry:
the y-axis (or $x = 0$)



horizontal line of symmetry:
$y = 7$



line of symmetry:
$y = x$
(note: the two curves are inverses of each other)



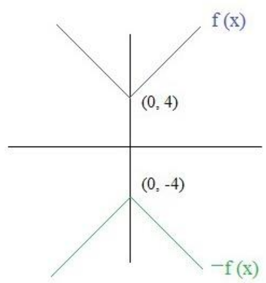
point of symmetry:
(0, 0)

Reflection creates an axis of symmetry (or line of symmetry) between the original function and translated function

Reflection over the x - axis:

$f(x) \Longrightarrow -f(x)$

(every output is turned negative)



$f(x) = |x| + 4$

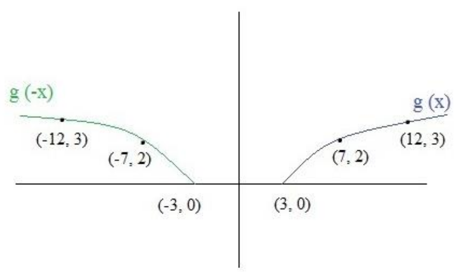
$-f(x) = -(|x| + 4)$

x	f (x)	- f (x)
-2	6	-6
-1	5	-5
0	4	-4
1	5	-5
2	6	-6
3	7	-7
4	8	-8

Reflection over the y-axis

$f(x) \Longrightarrow f(-x)$

(Every input becomes negative before calculating the output)



$g(x) = \sqrt{x - 3}$

$g(-x) = \sqrt{(-x) - 3}$

Note: Only the input (i.e. the x value) turned negative. The horizontal shift remains - 3

Reflection over y = x

$f(x) \Longrightarrow f^{-1}(x)$

To confirm inverses, you can switch the x and y values

"one-to-one functions"

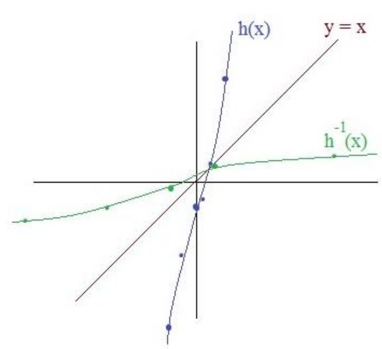
Also, if $(f \circ g)(x) = x$
$(g \circ f)(x) = x$
then f (x) and g (x) are inverses.

$h(x) = x^3 - 6$

$h^{-1}(x) = \sqrt[3]{x + 6}$

x	h (x)
-2	-14
-1	-7
0	-6
1	-5
2	2
3	21

x	$h^{-1}(x)$
21	3
2	2
0	$\sqrt[3]{6}$
-7	-1
-14	-2
-33	-3



Changing Positions: Translations ("Shifts"), Reflection, & Rotation

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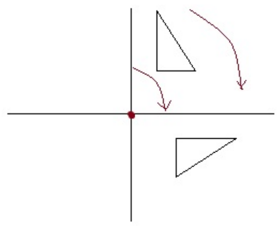
Transformation ⟶ changing the figure

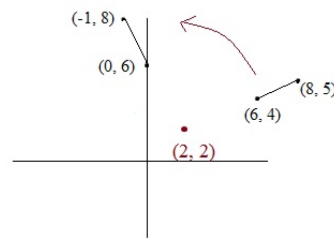
ROTATION

Every rotation has an "angle", "direction", and "center point".

90° clockwise around the origin



90° counter-clockwise around (2, 2)



Note: Rotating a function or image

180° ⟹ reflection over the origin

270° clockwise ⟹ 90° counter-clockwise

270° counter-clockwise ⟹ 90° clockwise

Rotating Functions: Examples

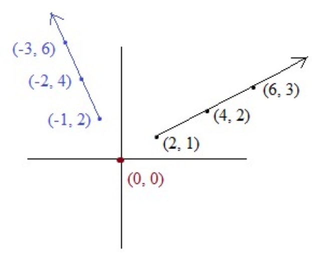
Rotate 90° Counter-Clockwise (around the origin)

(a, b) ⟶ (−b, a)

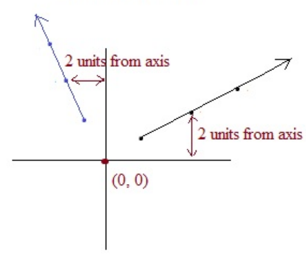
(2, 1) ⟶ (-1, 2)

(4, 2) ⟶ (-2, 4)

(6, 3) ⟶ (-3, 6)



(Rotation symmetry)



Rotate 90° Clockwise (around the origin)

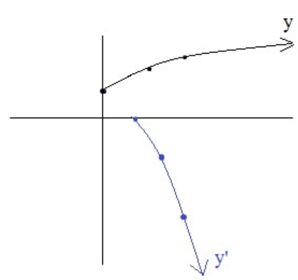
(a, b) ⟶ (b, −a)

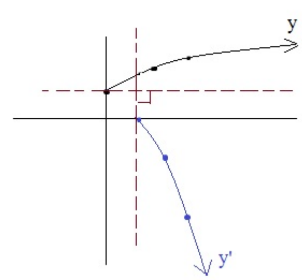
$y = \sqrt{x} + 3$

(0, 3) ⟶ (3, 0)

(4, 5) ⟶ (5, -4)

(9, 6) ⟶ (6, -9)





Rotate 180° Clockwise (around the origin)

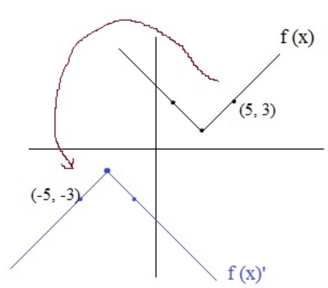
(a, b) ⟶ (− a, − b)

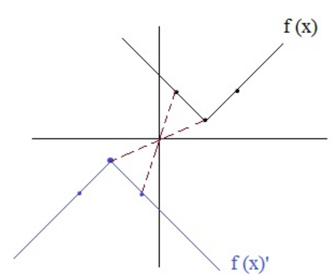
$f(x) = |x - 3| + 1$

(1, 3) ⟶ (-1, -3)

(3, 1) ⟶ (-3, -1)

(5, 3) ⟶ (-5, -3)





(rotate 180° is the same as reflection over the center)

Domain, Range, Symmetry, and graphing functions

Graph the following functions; Label three points. Then, identify the parent function, domain, range, and any symmetry (if it exists).

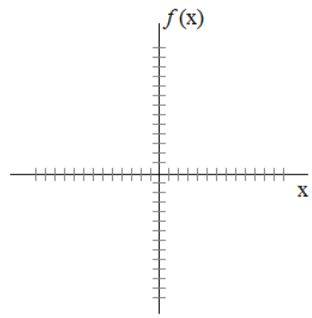
1) $f(x) = x^2 + 3$

Parent Function:

Domain:

Range:

Symmetry:



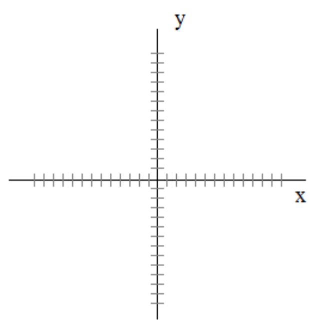
2) $y = -(x + 4)^2 + 2$

Parent Function:

Domain:

Range:

Symmetry:



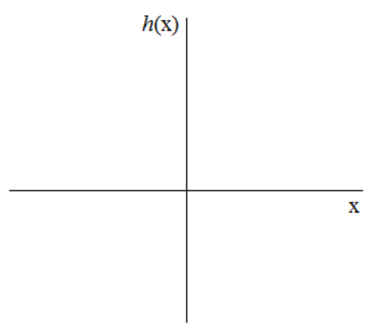
3) $h(x) = 3\sqrt{x} - 1$

Parent Function:

Domain:

Range:

Symmetry:



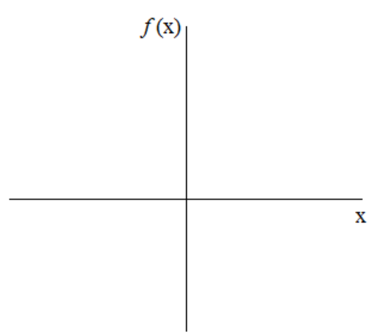
4) $f(x) = -\sqrt{-x}$

Parent Function:

Domain:

Range:

Symmetry:



Domain, Range, Symmetry, and graphing functions

Graph the following functions; Label three points. Then, identify the parent function, domain, range, and any symmetry (if it exists).

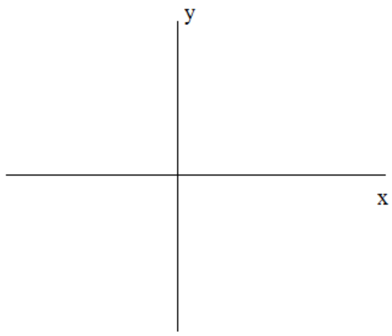
5) $y = 3|x + 4| - 1$

Parent Function:

Domain:

Range:

Symmetry:



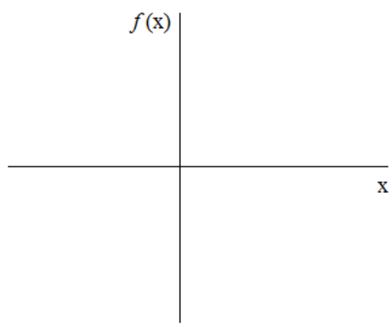
6) $f(x) = \frac{1}{x - 2}$

Parent Function:

Domain:

Range:

Symmetry:



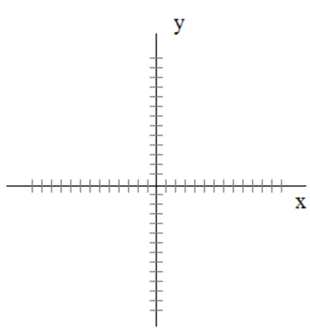
7) $y = -(x + 2)^3$

Parent Function:

Domain:

Range:

Symmetry:



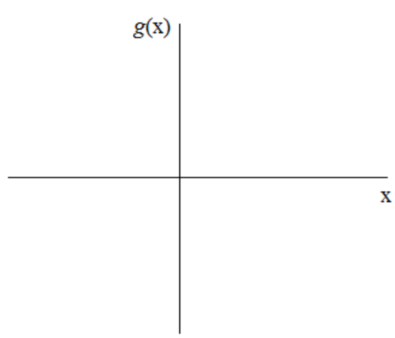
8) $g(x) = \sqrt[3]{x - 3} + 3$

Parent Function:

Domain:

Range:

Symmetry:



Domain, Range, Symmetry, and graphing functions

SOLUTIONS

Graph the following functions; Label three points. Then, identify the parent function, domain, range, and any symmetry (if it exists).

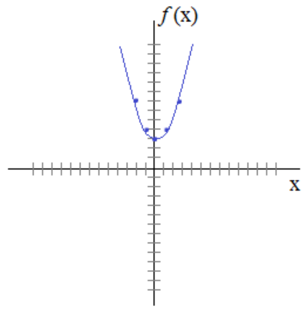
1) $f(x) = x^2 + 3$

Parent Function: x^2 (quadratic)

Domain: All real numbers

Range: $[3, \infty)$

Symmetry: y-axis



$f(0) = 3$ (0, 3)
$f(-1) = 4$ (-1, 4)
$f(1) = 4$ (1, 4)

$f(2) = 7$
$f(-2) = 7$

(quadratic that is shifted up 3 units)

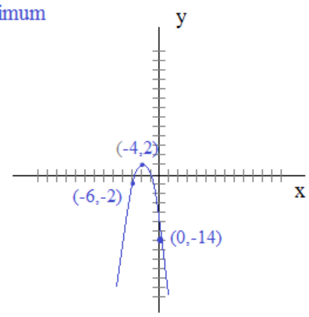
2) $y = -(x + 4)^2 + 2$ (-4, 2) is the vertex; maximum

Parent Function: x^2 (quadratic)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

Symmetry: $x = -4$



$x = 0;\ y = -(0 + 4)^2 + 2 = -14$ (0, -14)

$x = -4;\ y = -(-4 + 4)^2 + 2 = 2$ (-4, 2)

$x = -6;\ y = -(-6 + 4)^2 + 2 = -2$ (-6, -2)

(quadratic facing down; shifted up 2 units and to the left 4 units)

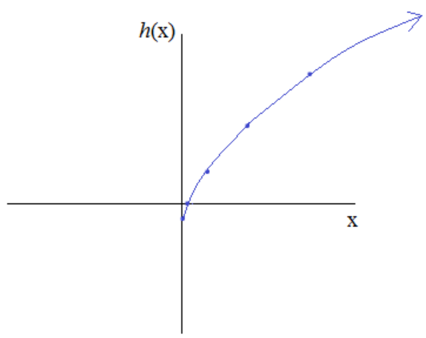
3) $h(x) = 3\sqrt{x} - 1$

Parent Function: $\sqrt{x}$ (square root)

Domain: $[0, \infty)$ non-negative numbers

Range: $[-1, \infty)$

Symmetry: NONE



$h(0) = -1$ (0, -1)

$h(4) = 3\sqrt{4} - 1 = 5$ (4, 5)

$h(9) = 3\sqrt{9} - 1 = 8$ (9, 8)

$h(1/9) = 0$ (1/9, 0)

(square root function shifted down 1 unit and "stretched" by a factor of 3)

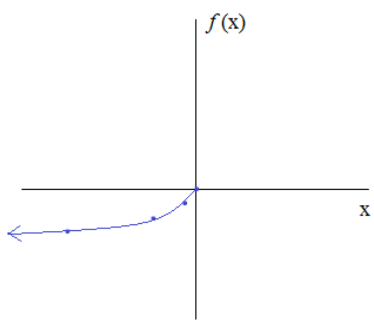
4) $f(x) = -\sqrt{-x}$

Parent Function: $\sqrt{x}$ (square root)

Domain: $(-\infty, 0]$ or $x \leq 0$

Range: $(-\infty, 0]$ or $f(x) \leq 0$

Symmetry: NONE



$f(3) = -\sqrt{-(3)} =$ not real; does not exist

$f(-4) = -\sqrt{-(-4)} = -2$ (-4, -2)

$f(-1) = -\sqrt{-(-1)} = -1$ (-1, -1)

$f(-9) = -\sqrt{-(-9)} = -3$ (-9, -3)

(square root function reflected over the y-axis, then reflected over the x-axis.)

Domain, Range, Symmetry, and graphing functions

Graph the following functions; Label three points. Then, identify the parent function, domain, range, and any symmetry (if it exists).

SOLUTION

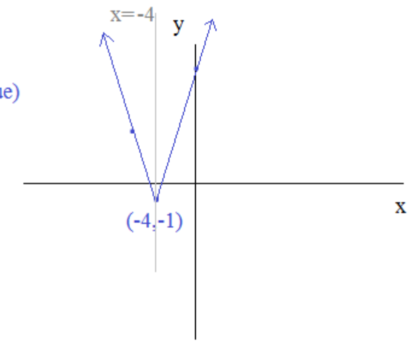
5) $y = 3|x + 4| - 1$

Parent Function: $y = |x|$ (absolute value)

Domain: all real numbers

Range: $[-1, \infty)$

Symmetry: $x = -4$



$x = 0$; $y = 3|(0) + 4| - 1 = 11$ (0, 11)

$x = -4$: $y = 3|(-4) + 4| - 1 = -1$ (-4, -1)

$x = -6$; $y = 3|(-6) + 4| - 1 = 5$ (-6, 5)

(absolute value function shifted to the left 4 units and down 1 unit; Then, it is "stretched" by a factor of 3)

Vertex is (-4, -1)

6) $f(x) = \dfrac{1}{x - 2}$

Parent Function: $\dfrac{1}{x}$ (inverse function)

Domain: all real numbers $\neq 2$

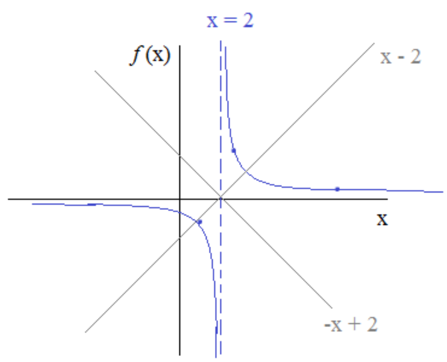
$(-\infty, 2) \cup (2, \infty)$

Range: all non-zero numbers

$(-\infty, 0) \cup (0, \infty)$

Symmetry: $f(x) = x - 2$

$f(x) = -x + 2$



$f(2) = \dfrac{1}{0}$ undefined; asymptote

$f(1) = \dfrac{1}{(1) - 2} = -1$ (1, -1)

$f(2\ 1/3) = \dfrac{1}{(1/3)} = 3$ (2 1/3, 3)

$f(10) = 1/8$ (10, 1/8)

(inverse function shifted to the right 2 units..
vertical asymptote: $x = 2$
horizontal asymptote $y = 0$
(i.e. the x-axis))

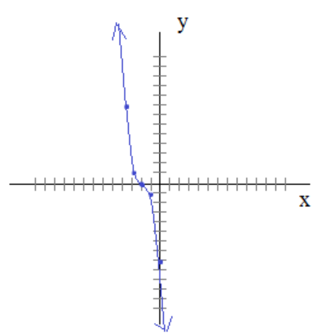
7) $y = -(x + 2)^3$

Parent Function: $y = x^3$ (cubic)

Domain: all real numbers

Range: all real numbers

Symmetry: rotation (180°) around the point (-2, 0)



$x = -2$; $y = -(-2 + 2)^3 = 0$ (-2, 0)

$x = 0$; $y = -(0 + 2)^3 = -8$ (0, -8)

$x = -4$; $y = -(-4 + 2)^3 = 8$ (-4, 8)

(cubic function shifted 2 units to the left, and 'flipped over' because it's negative)

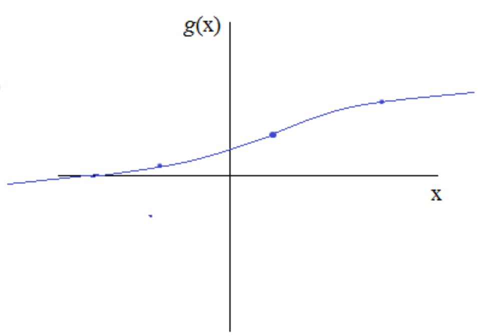
8) $g(x) = \sqrt[3]{x - 3} + 3$

Parent Function: $\sqrt[3]{x}$ (cube root)

Domain: all real numbers

Range: all real numbers

Symmetry: rotation (180°) around the point (3, 3)



$x = 3$; $g(3) = 0 + 3 = 3$ (3, 3)

$x = 11$; $g(11) = \sqrt[3]{8} + 3 = 5$ (11, 5)

$x = -5$; $g(-5) = \sqrt[3]{-8} + 3 = 1$ (-5, 1)

(cube root function shifted to the right 3 units and shifted up 3 units)