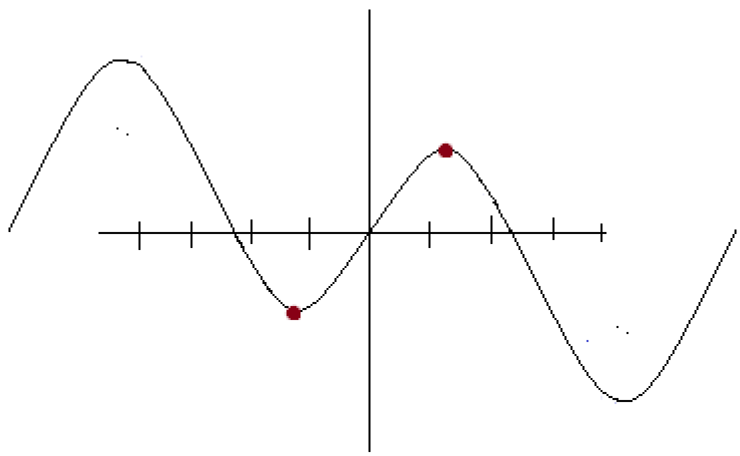
Graphing III

Identifying Functions

Notes, Examples, and Practice Questions (with answers)



Topics include EVEN/ODD functions, translation, transformations, and more.

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Notes on Symmetry, Even & Odd Functions

Even Functions

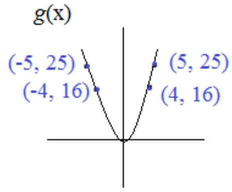
Reflect the y-axis
$f(-x) = f(x)$
If (x, y) is a point on the function, then (-x, y) is another point.

Examples: $g(x) = x^2$

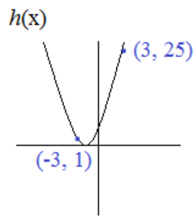
Does $(-x)^2 = (x)^2$?

YES $(-x)(-x) = (x)(x)$

If x = 5, $(-5)^2 = (5)^2$ 25 = 25
If x = -4, $(-(-4))^2 = (-4)^2$ 16 = 16



symmetry over the y-axis



It has symmetry, BUT it does not reflect over the y-axis

$h(x) = (x + 2)^2$

Does $(-x + 2)^2 = (x + 2)^2$?

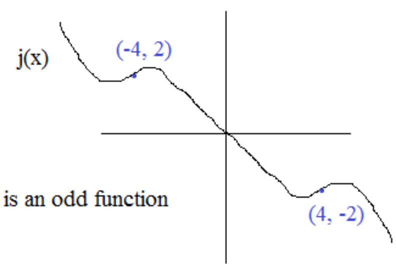
NO $(-x + 2)(-x + 2) \neq (x + 2)(x + 2)$
$x^2 - 4x + 4 \neq x^2 + 4x + 4$

$h(3) = 25$
$h(-3) = 1$

Odd Functions

Origin Symmetry
$f(-x) = -f(x)$
If (x, y) exists, then (-x, -y) exists!

Examples:



j(x) is an odd function

Is $y = \frac{1}{x}$ an odd function?

YES, because for every (x, y), there is a (-x, -y)

(Notice the function does not exist at x = 0.
Yet, it still has symmetry around the origin!)

Is $f(x) = x^3 + 5$ an odd function?

NO, because $f(0) = 5$

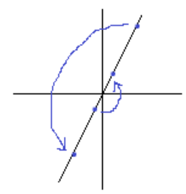
To have origin symmetry, (0, 0) must be a point on that function!

Is y = 2x an odd function?

YES, it has origin symmetry

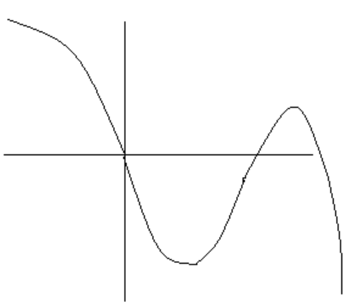
-y = -2x

(1, 2) (-1, -2)
(-2, -4) (2, 4)

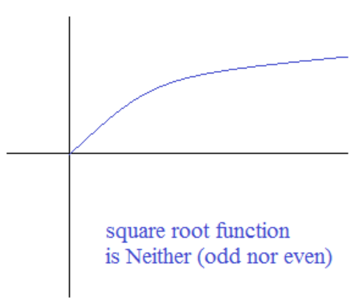


(A line through the origin is always odd)

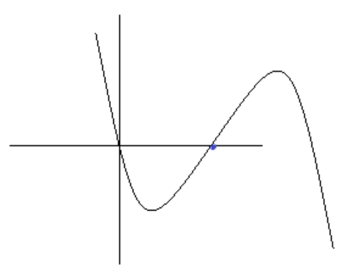
Neither Odd nor Even



No symmetry or correlations



square root function
is Neither (odd nor even)



The function has rotation symmetry around (4, 0)
(to be odd, it must have symmetry around (0, 0))

Determine if the functions are even, odd, or neither:

a) $x^3 + 5$

b) $x^2 + 5$

c) $(x + 5)^2$

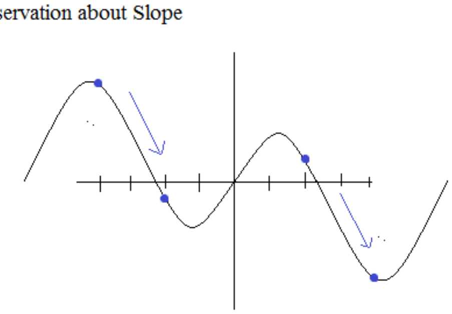
d) $|x + 5|$

e) $|x| + 5$

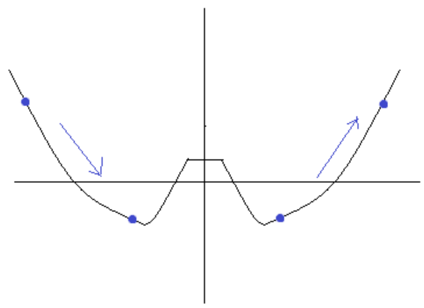
f) $-|x| - 5$

ODD, EVEN, or NEITHER
(Answers at the bottom half of the page)

Observation about Slope

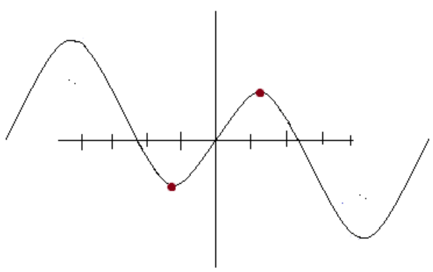


Odd function: decreasing on interval [-4, -2]
so, decreasin on interval [2, 4]

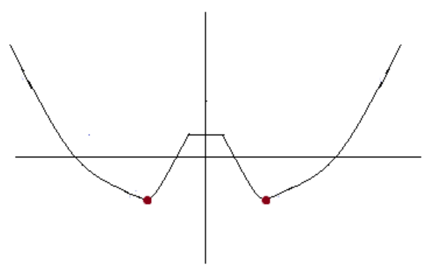


Even function: decreasing on interval [-5, -2]
so, increasing on interval [2, 5]

Observation about relative min/max



Odd function: relative min at (-1.2, -1.5)
so, relative max at (+1.2, +1.5)



Even function: relative min at (-1.8, -1.6)
so, relative min at (+1.8, -1.6)

Answers to ODD, EVEN, NEITHER

a) NEITHER. It looks like an odd function, but at x = 0, the output is 5. since it does not have origin symmetry, it is not odd.

b) EVEN. It has y-axis symmetry. And, any x has the same output as (-x). EX: (2, 9) (-2, 9) (1, 6) (-1, 6)

c) NEITHER. Obviously, it is not odd. (does not have origin symmetry) And, because it shifted 5 units to the left, it does not have y-axis symmetry.
Test points: at x = 3, y = 64. but, at x = -3, y = 4

d) NEITHER. (0, 5) -- instead of (0, 0) -- is a point (so, it can't be odd) And, because it shifts 5 units to the left, it does not have y-axis symmetry.

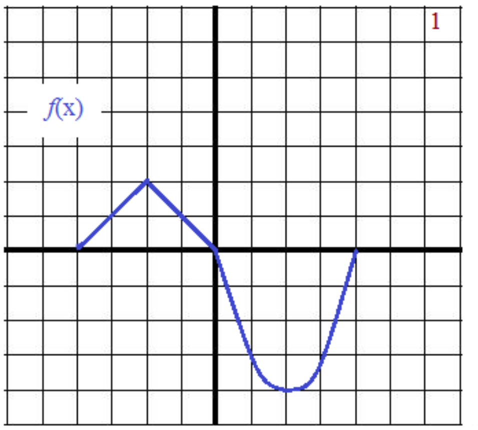
e) EVEN. Because the absolute value function shifts UP 5 units, the function maintains its y-axis symmetry.
Test points: at x = 3, y = 8 and at x = -3, y = 8.

f) EVEN. Graphically, it's an absolute value function facing down and shifted 5 units down. Algebraically, the output of -x would be the same as the output of x (because the absolute value immediately changes any input BEFORE the other operations)

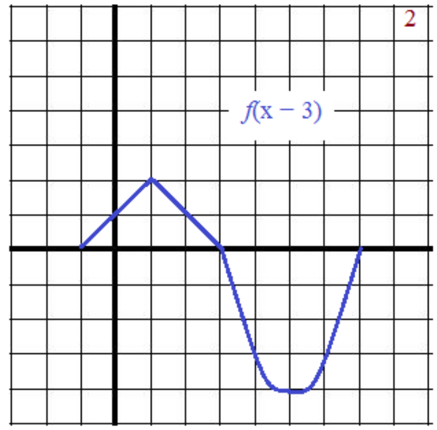
Transforming functions

Example: Given: $f(x)$

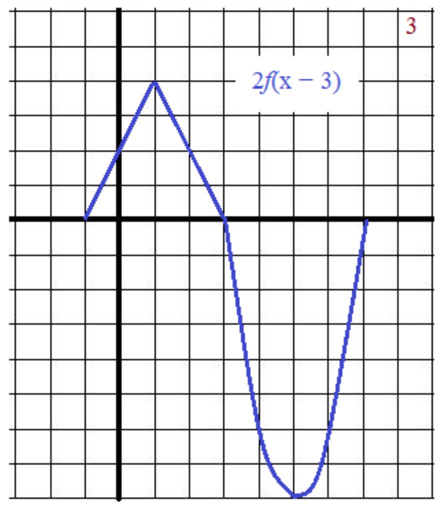
Find: $g(x) = -2f(x - 3) + 1$



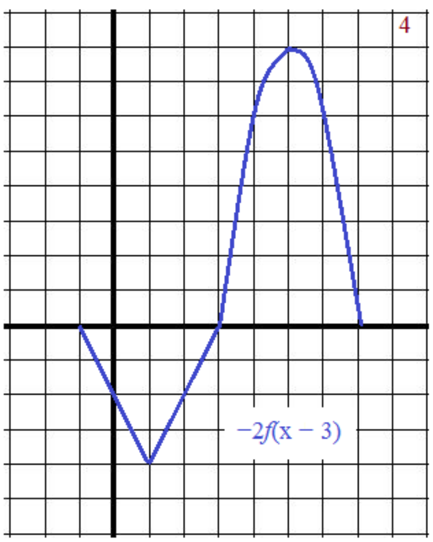
("horizontal shift 3 units to the right")



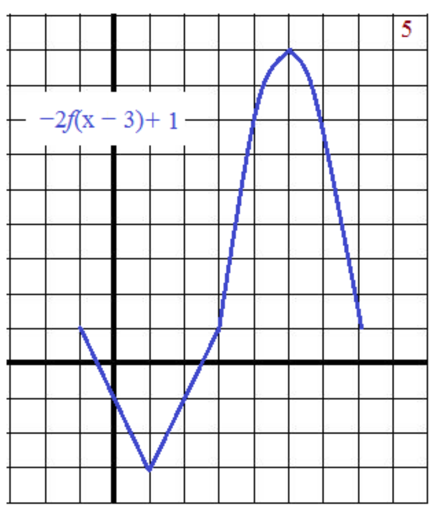
("stretch (or multiply) by a factor of 2")



("reflect over the x-axis")



("vertical shift up 1 unit")



Graphing and identifying transformations

Example: If the point (-3, 5) is on the ODD function $f(x)$, identify another point.

(3, -5)

If a function is odd, then for every point, there is *another point reflected over the origin.*
Definition of 'odd function' : $f(-x) = -f(x)$

Since $f(-3) = 5$, then $f(-(-3)) = -f(-3)$

$f(3) = -5$

Suppose $g(x) = -4f(\frac{1}{5}x + 2) - 1$

Determine 2 points in function $g(x)$

Approach 1: Finding the 2 inputs and solving

Since we know the outputs for $f(3)$ and $f(-3)$, we'll choose those points for $g(x)$

where does $f(\frac{1}{5}x + 2) = f(3)$?

$\frac{1}{5}x + 2 = 3$

$x = 5$

So, we'll use 5: $g(5) = -4f(\frac{1}{5}(5) + 2) - 1$

$= -4f(3) - 1$

$= -4(-5) - 1 = 19$

(5, 19)

Note: if we used another number, such as 10, what happens?

$g(10) = -4f(\frac{1}{5}(10) + 2) - 1$

$g(10) = -4f(2 + 2) - 1$

$= -4f(4) - 1$

Since we don't know what $f(4)$ equals, we can't determine that point!

Then, where does $f(\frac{1}{5}x + 2) = f(-3)$?

$\frac{1}{5}x + 2 = -3$

$x = -25$

So, we'll use -25: $g(-25) = -4f(\frac{1}{5}(-25) + 2) - 1$

$= -4f(-3) - 1$

$= -4(5) - 1 = -21$

(-25, -21)

Approach 2: Using transformations and translations

$g(x) = -4f(\frac{1}{5}x + 2) - 1$

$-4f\left(\frac{1}{5}(x + 10)\right) - 1$

$-4f\,\frac{1}{5}(x + 10) - 1$

(-) a b c d

Note: The order that each point is translated and transformed matters! Be careful..
(the shifts are last)

Taking a point in $f(x)$,	(-3, 5)	(3, -5)
(-) reflect over the x-axis	(-3, -5)	(3, 5)
a) vertical stretch of 4	(-3, -20)	(3, 20)
b) horizontal expansion by 5	(-15, -20)	(15, 20)
c) horizontal shift of 10 to the left	(-25, -20)	(5, 20)
d) vertical shift of 1 unit down	(-25, -21)	(5, 19)

			(-3, 5)	(3, -5)
change in x	(b)	expansion (inside function)	(-15, 5)	(15, -5)
	(c)	horizontal shift (inside)	(-25, 5)	(5, -5)
change in y	(a)	vertical stretch (outside)	(-25, 20)	(5, -20)
	(-)	reflection (outside)	(-25, -20)	(5, 20)
	(d)	vertical shift (outside)	(-25, -21)	(5, 19)

Identifying Properties and Transformations of Functions

Example: If the point (2, 7) is on the EVEN function $f(x)$, identify another point.

$(-2, 7)$ If a function is even, then for every point, there is another point reflected over the y-axis (the function's line of symmetry is the y-axis)

Definition of 'even function' : $f(-x) = f(x)$

Since $f(2) = 7$ and $f(-2) = f(2)$

then $f(-2) = 7$

Suppose $h(x) = \frac{1}{2} f(3 - x) + 5$

Determine 2 points in the function $h(x)$

Approach 1: Finding the 2 points and solving

Since we know $f(2)$ and $f(-2)$, we'll select these points for $h(x)$.

In other words, where does $f(3 - x) = f(2)$?

$3 - x = 2$

$x = 1$

So, we'll use 1: $h(1) = \frac{1}{2} f(3 - 1) + 5$

$= \frac{1}{2} f(2) + 5$ and, we know $f(2) = 7$

$= \frac{1}{2} \cdot 7 + 5 = 17/2$

$(1, \frac{17}{2})$

And, where does $f(3 - x) = f(-2)$?

$3 - x = -2$

$x = 5$

Then, we'll use 5: $h(5) = \frac{1}{2} f(3 - 5) + 5$

$= \frac{1}{2} f(-2) + 5$ and, we know from above that $f(-2) = 7$

$= \frac{1}{2} \cdot 7 + 5 = 17/2$

$(5, \frac{17}{2})$

Approach 2: Recognizing translations/transformations

$h(x) = \frac{1}{2} f(3 - x) + 5$

If we rewrite the equation: $\frac{1}{2} f(-x + 3) + 5$

$\frac{1}{2} f(-x + 3) + 5$

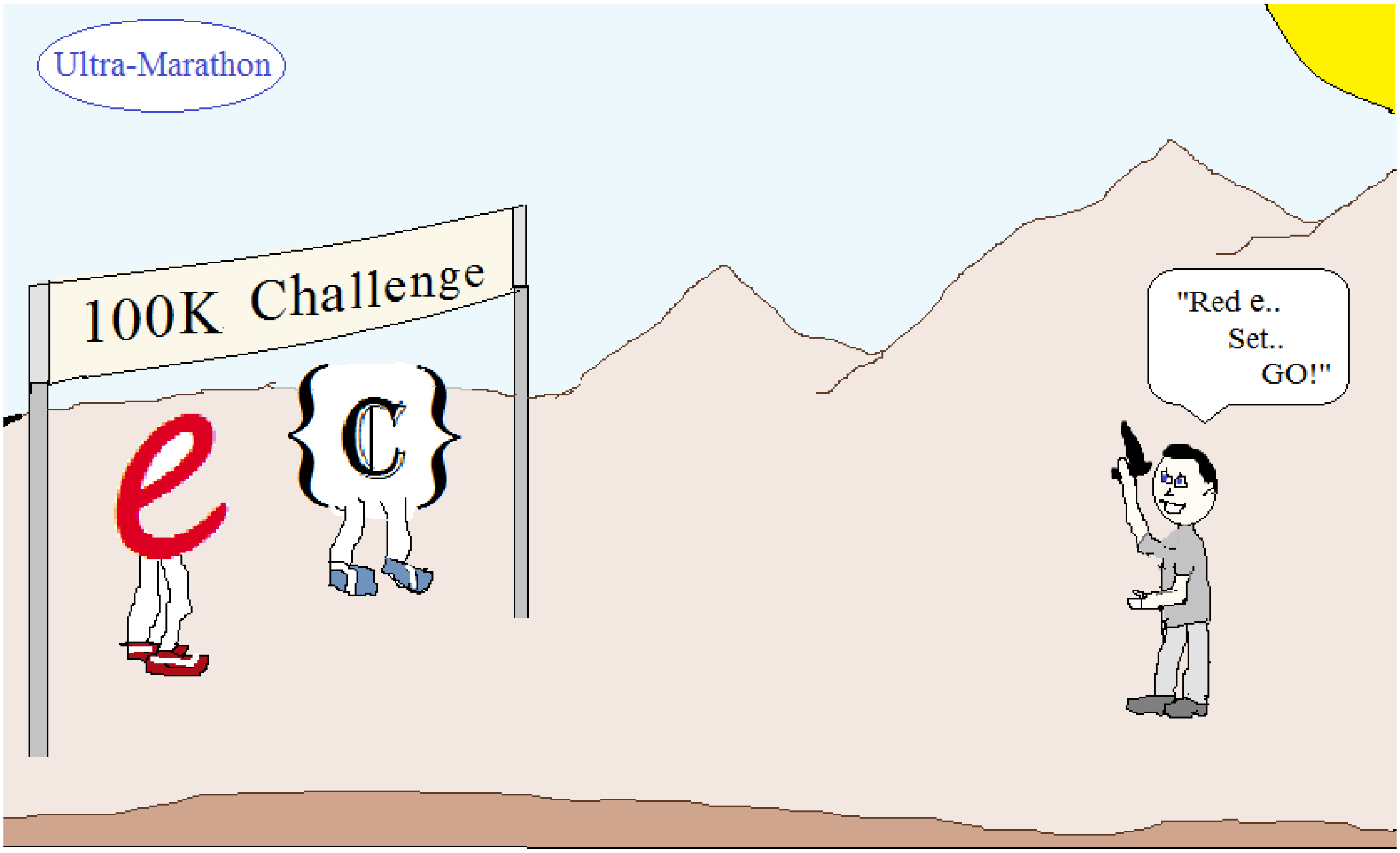
$\frac{1}{2} f\left(-(x - 3)\right) + 5$

$\frac{1}{2} f\left(-(x - 3)\right) + 5$

a (-) b c d

		(2, 7)	(-2, 7)
(b)	horizontal expansion is 1 (none)	(2, 7)	(-2, 7)
(-)	horizontal reflection over y-axis	(-2, 7)	(2, 7)
(c)	horizontal shift of 3 to the right	(1, 7)	(5, 7)
(a)	vertical shrink (x 1/2)	(1, 7/2)	(5, 7/2)
(d)	vertical shift of up 5	(1, 17/2)	(5, 17/2)

Observation: Because of the vertical shift, the function $h(x)$ is not an 'even' function any more



Testing the limits of endurance,
these math figures will run on and on...

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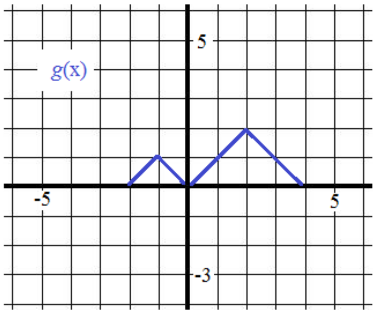
PRACTICE EXERCISES-→

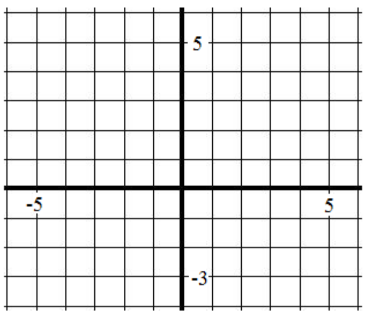
Algebra 2 / Graphing Transformations



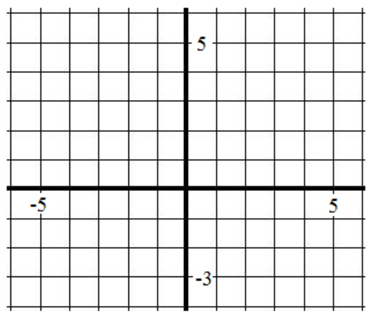
Given the parent function $g(x)$,

graph the following transformed functions...

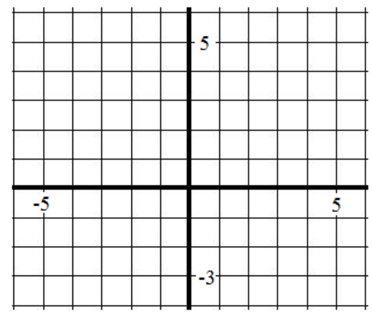
a) $g(x + 1)$



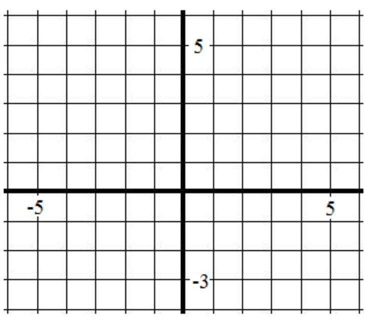
b) $g(-x)$



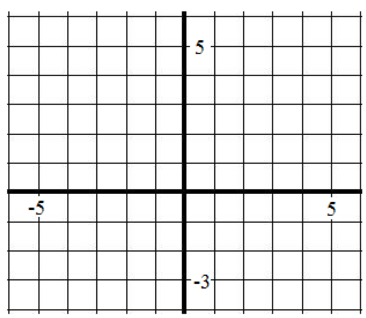
c) $-g(x)$



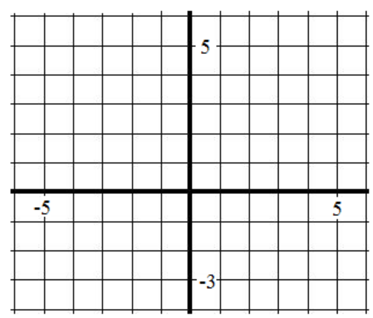
d) $2g(x)$



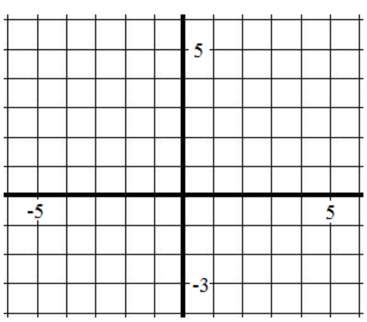
e) $g(2x)$



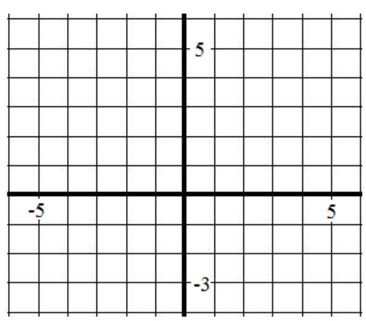
f) $g(2x + 4)$



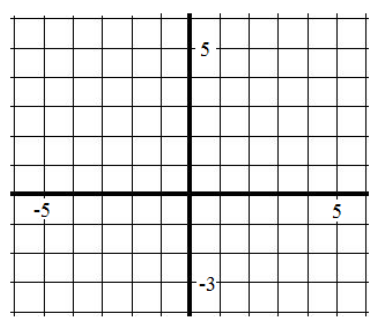
g) $g(1 - x)$



h) $-2g(x) + 3$



i) $3g(\frac{1}{2}x) - 1$



Understanding Transformations of Functions

A) Given $y = f(x)$ Write equations to describe the following:

1) $f(x)$ is translated left 8 units and down 2 units

2) $f(x)$ is vertically stretched by a factor of 5 and shifted to the right 4 units

3) $f(x)$ is horizontally dilated by 1/4 and reflected over the x-axis

4) $f(x)$ is reflected over the y-axis, vertically shrunk by 1/3, and shifted up 10 units

B) Answer the following:

1) If $f(x) = -2(x+3)^2 + 4$

$g(x) = f(x+1) - 5$

What is the vertex of $g(x)$?

2) $h(x)$ has a domain $(3, 8)$ and a range $[4, 14]$.

What is the domain and range of $3h(x+2)+1$?

3) $g(x) = 3f(x) + 7$

$f(2) = 5$

What point MUST exist in $g(x)$?

4) $h(x) = 4f(x+6) - 10$

$f(2) = 5$

What point MUST exist in $h(x)$?

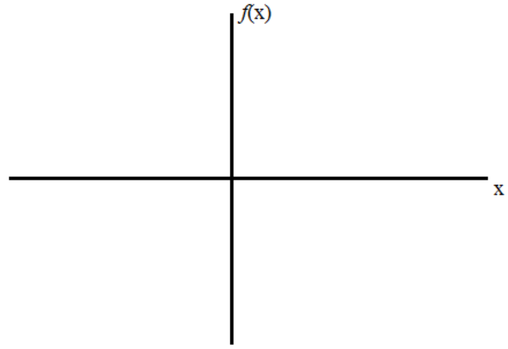
Practice Exercises: Identifying graphs and functions

Given $f(x)$ is an odd function; decreasing on the interval [-2, 0]; a relative maximum exists at the point (-4, 7)

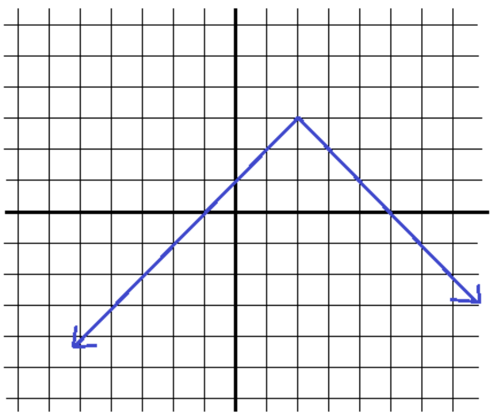
a) Is the function increasing or decreasing at x = 1?

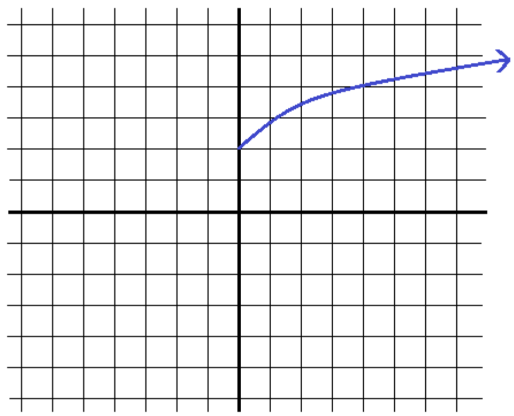
b) Identify a relative minimum point?

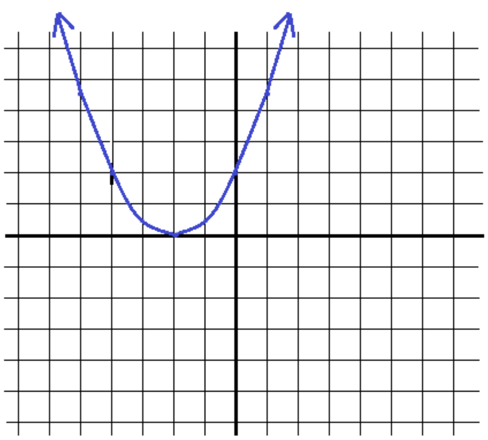
c) Sketch a graph that satisfies the characteristics of $f(x)$

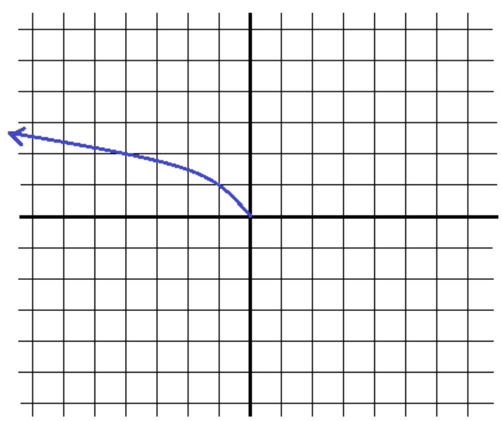


Write equations that describe each of the following graphs

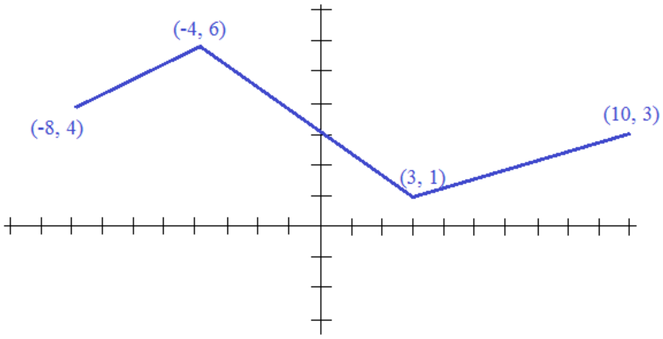








The following is a graph of $f(x)$ on the interval [-8, 10]



What is the relative minimum of $f(x)$?

What is the range of $f(x)$?

If $g(x) = f(x) + 3$, what is the relative maximum of $g(x)$?

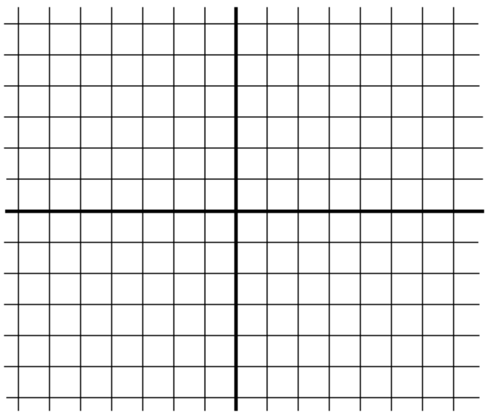
What interval(s) is $g(x)$ increasing?

If $h(x) = -f(x)$, what is the relative maximum of $h(x)$?

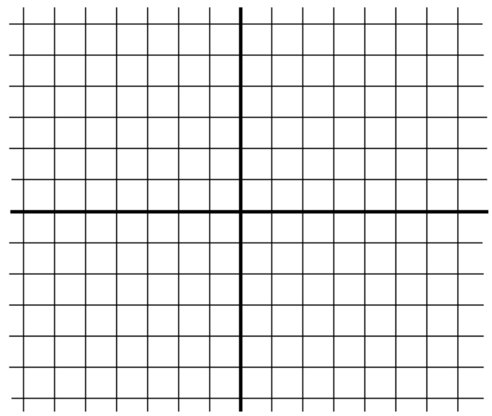
What is the range of $h(x)$?

$h(x) = |x|$ Graph each of the following; determine if the function is even, odd, or neither...

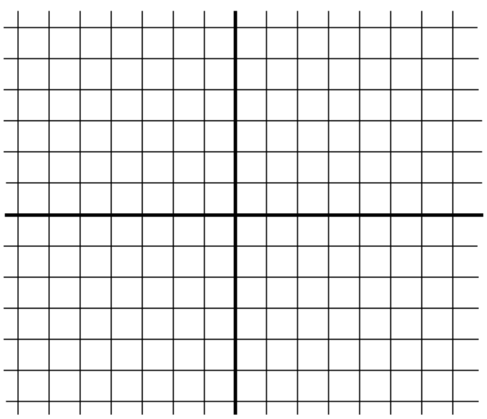
$h(x) - 4$



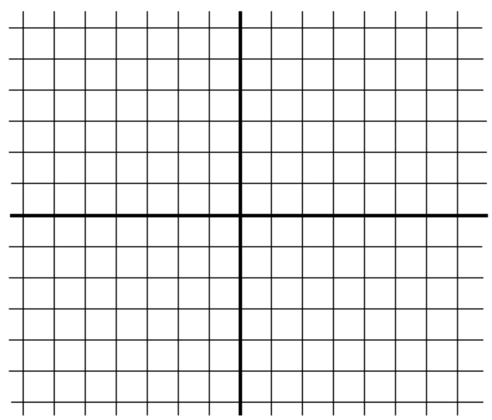
$h(x + 3)$



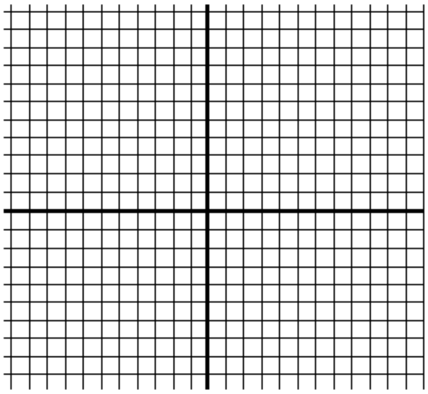
$-h(x)$



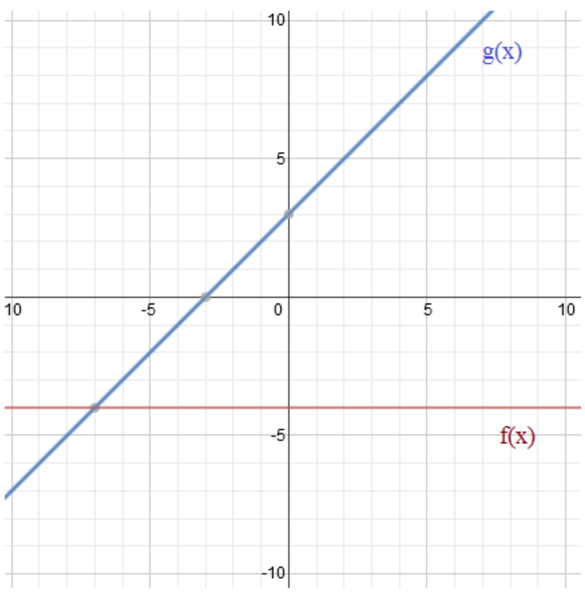
$h(2x)$



1) Sketch a graph of a quadratic with a negative discriminant and no minimum.



2) Answer the questions for the following graph:



a) $(f+g)(3) =$

b) $(f \circ g)(3) =$

c) $(g \circ f)(3) =$

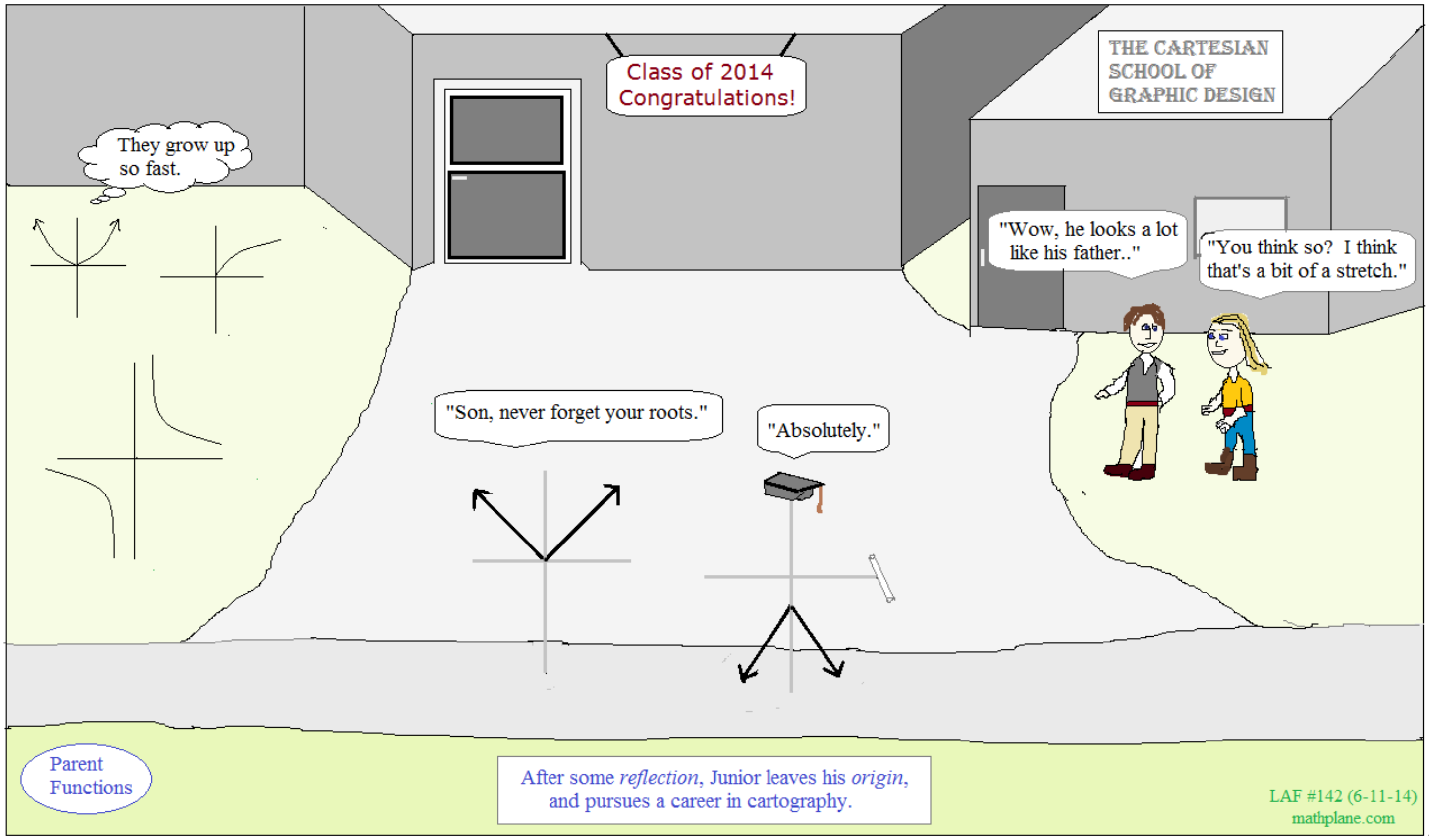
d) $(f \circ f)(1) =$

e) $g(g(4)) =$

f) $g^{-1}(3) =$

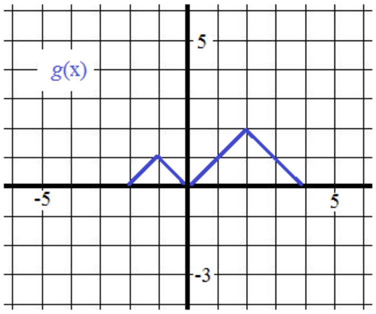
g) $f^{-1}(3) =$

h) $(f-g)(0) =$


They grow up so fast.
Class of 2014 Congratulations!
THE CARTESIAN SCHOOL OF GRAPHIC DESIGN
"Wow, he looks a lot like his father.."
"You think so? I think that's a bit of a stretch."
"Son, never forget your roots."
"Absolutely."
Parent Functions
After some *reflection*, Junior leaves his *origin*, and pursues a career in cartography.
LAF #142 (6-11-14)
mathplane.com

SOLUTIONS-→

Algebra 2 / Graphing Transformations

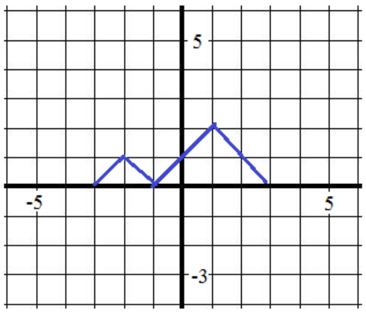


Given the parent function $g(x)$,

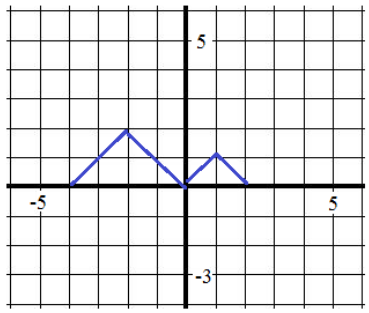
graph the following transformed functions...

SOLUTIONS

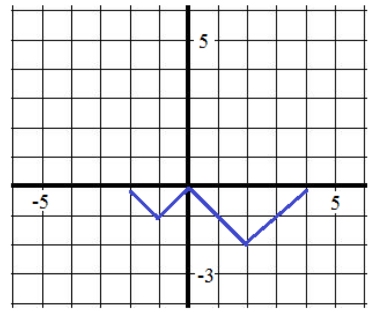
a) $g(x + 1)$ shift 1 unit to the left...



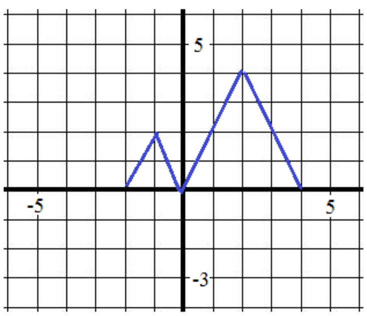
b) $g(-x)$ reflect (horizontally) over the y-axis



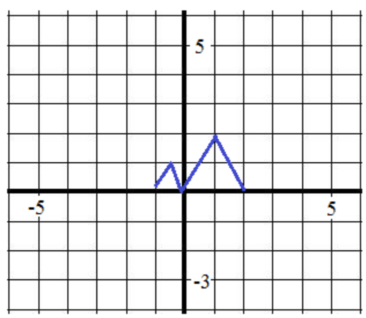
c) $-g(x)$ reflect (vertically) over the x-axis



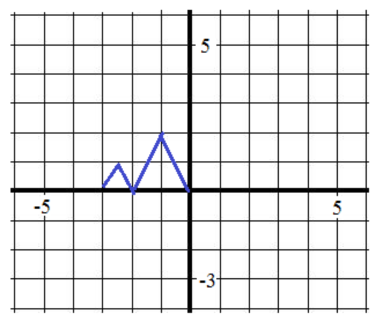
d) $2g(x)$ vertical dilation (stretch) by 2



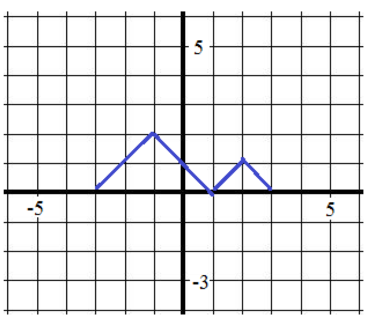
e) $g(2x)$ horizontal compression by 1/2



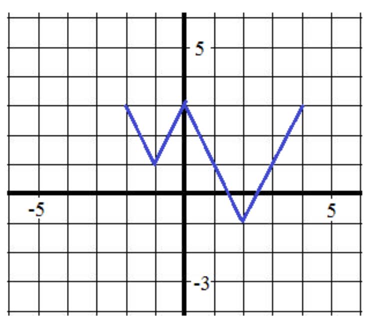
f) $g(2x + 4)$ $g(2(x + 2))$ horizontal compression by 1/2; shift left 2 units



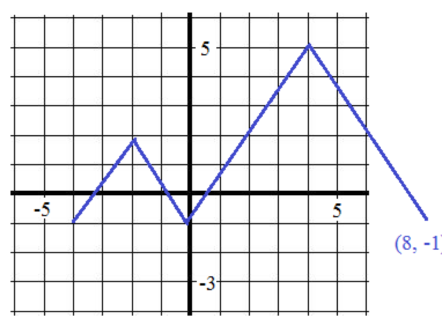
g) $g(1 - x)$ $g(-(x - 1))$ horizontal reflection and shift right 1 unit



h) $-2g(x) + 3$ vertical reflection over x-axis vertical dilation by factor of 2 shift up 3 units



i) $3g(\frac{1}{2}x) - 1$ horizontal expansion by 2 vertical stretch by 3 vertical shift down 1



Understanding Transformations of Functions

SOLUTIONS

A) Given $y = f(x)$ Write equations to describe the following:

1) $f(x)$ is translated left 8 units and down 2 units

$f(x + 8) - 2$

2) $f(x)$ is vertically stretched by a factor of 5 and shifted to the right 4 units

$5f(x - 4)$

3) $f(x)$ is horizontally dilated by 1/4 and reflected over the x-axis

$-f(4x)$

4) $f(x)$ is reflected over the y-axis, vertically shrunk by 1/3, and shifted up 10 units

$\frac{1}{3} f(-x) + 10$

B) Answer the following:

1) If $f(x) = -2(x + 3)^2 + 4$

$g(x) = f(x + 1) - 5$

What is the vertex of $g(x)$?

Answer: The vertex of $f(x)$ is (-3, 4)

Since $g(x)$ is a transformation of $f(x)$ --- shift 1 to the left and 5 down -- (translation)

the vertex is (-4, -1)

2) $h(x)$ has a domain (3, 8) and a range [4, 14].

What is the domain and range of $3h(x + 2) + 1$?

Changes "inside the function" affect left/right (i.e. domain)..

$(x + 2)$ -----> shift to the left 2 units... new domain: (1, 6)

Changes "outside the function" affect up/down (i.e. range)..

$3h$ -----> vertical stretch by factor of 3... new range: [12, 42]

THEN, $+ 1$ -----> vertical shift up 1 unit... final range: [13, 43]

3) $g(x) = 3f(x) + 7$

$f(2) = 5$

What point MUST exist in $g(x)$?

since $f(2) = 5$, we can use $x = 2$

$g(2) = 3f(2) + 7$

$= 3 \cdot 5 + 7$

(2, 22)

4) $h(x) = 4f(x + 6) - 10$ What point MUST exist in $h(x)$?

$f(2) = 5$

Since we know the output for $f(2)$, we must "adjust" $f(x + 6)$ with a horizontal shift to the left..

So, we'll let $x = -4$

Then, we have a vertical stretch/multiplier of 4... So, the output of 5 becomes 20..

And, we have a vertical shift of 10 units down... So, the output of 20 becomes 10...

Therefore, the point $(-4, 10)$ MUST exist...

A quick check: $h(-4) = 4f(-4 + 6) - 10$

$= 4 \cdot f(2) - 10$

$= 4 \cdot 5 - 10 = 10$

Practice Exercises: Identifying graphs and functions

SOLUTIONS

Given $f(x)$ is an odd function; decreasing on the interval [-2, 0]; a relative maximum exists at the point (-4, 7)

a) Is the function increasing or decreasing at x = 1?

Decreasing

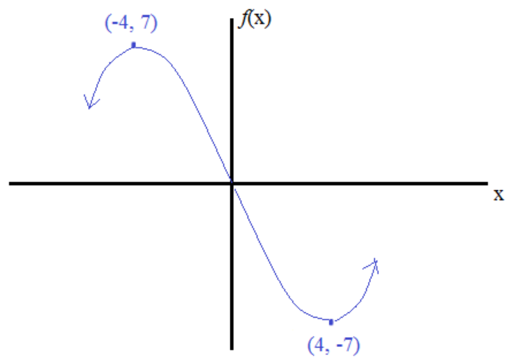
b) Identify a relative minimum point?

Since it's an odd function, it has origin symmetry.

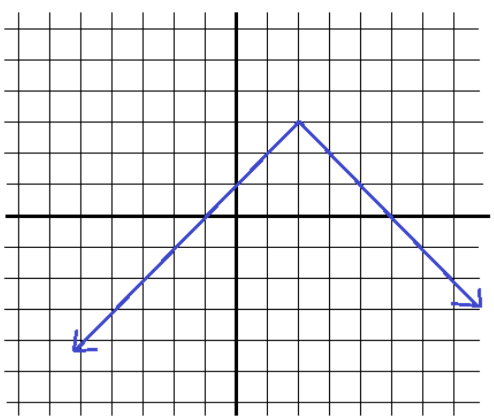
If (-4, 7) is a relative max, then, (4, -7) is a relative min.

c) Sketch a graph that satisfies the characteristics of $f(x)$

This sketch is decreasing [-2, 0], has relative maximum at (-4, 7), and passes through (0, 0)

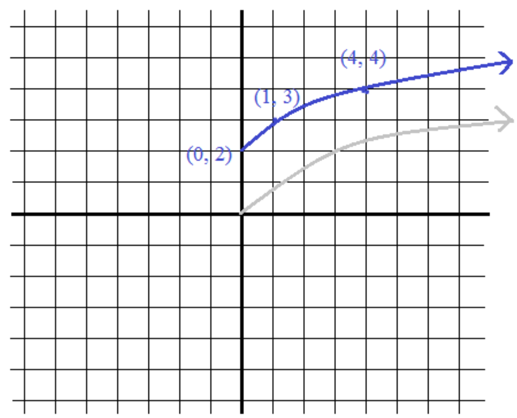


Write equations that describe each of the following graphs



Absolute value function facing down, shifted 2 units to the right and 3 units up.... $y = -|x - 2| + 3$

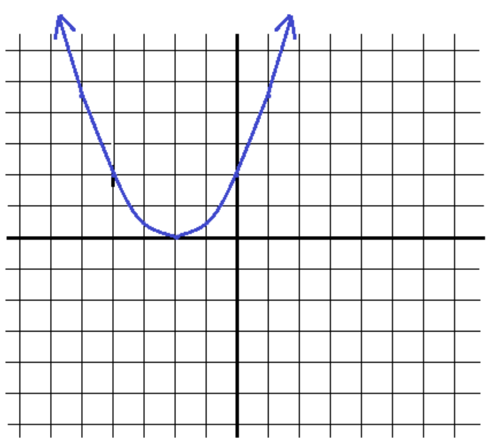
abs. value: $y = a|x - h| + k$



Square root function shifted up 2 units... $y = \sqrt{x} + 2$

square rt: $y = a\sqrt{x - h} + k$

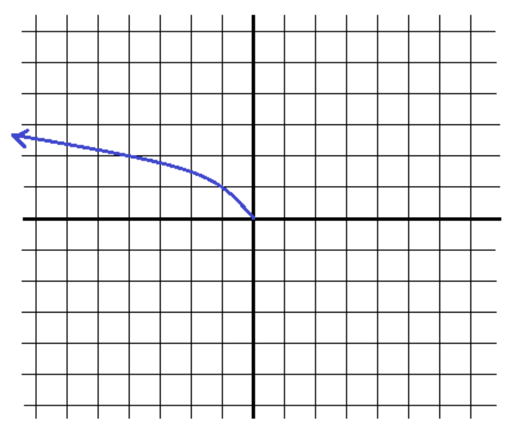
test points to verify:
(0, 2)
(1, 3)
(4, 4)



quadratic function: x^2

shifted left 2 units: $(x + 2)^2$

graph represents 1/2 of output: $\frac{1}{2}(x + 2)^2$

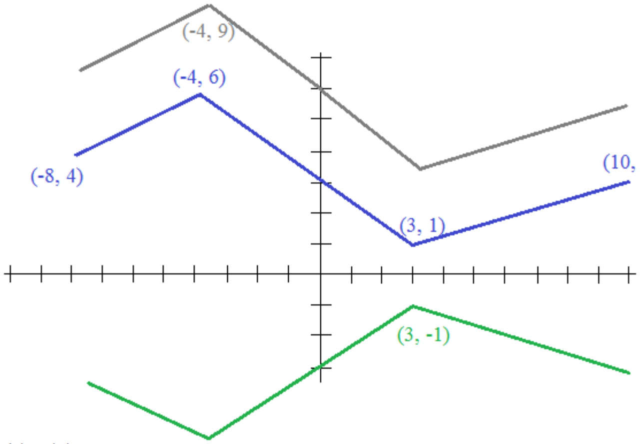


absolute value reflected over y-axis (negative)

$y = \sqrt{(-x)}$

SOLUTIONS

The following is a graph of $f(x)$ on the interval [-8, 10]



What is the relative minimum of $f(x)$? (3, 1)

What is the range of $f(x)$? [1, 6] all the y values

If $g(x) = f(x) + 3$, what is the relative maximum of $g(x)$? (-4, 9)

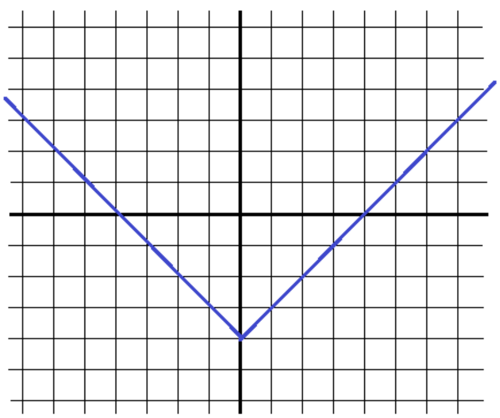
What interval(s) is $g(x)$ increasing? [-8, -4) and (3, 10]

If $h(x) = -f(x)$, what is the relative maximum of $h(x)$? (3, -1)

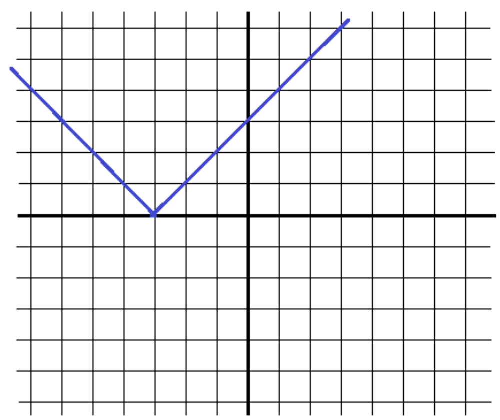
What is the range of $h(x)$? [-6, -1]

$h(x) = |x|$ Graph each of the following; determine if the function is even, odd, or neither...

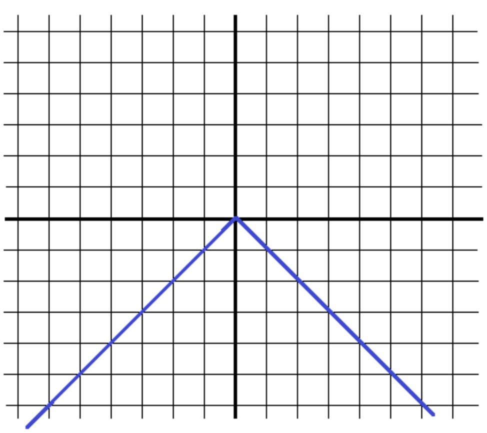
$h(x) - 4$ EVEN



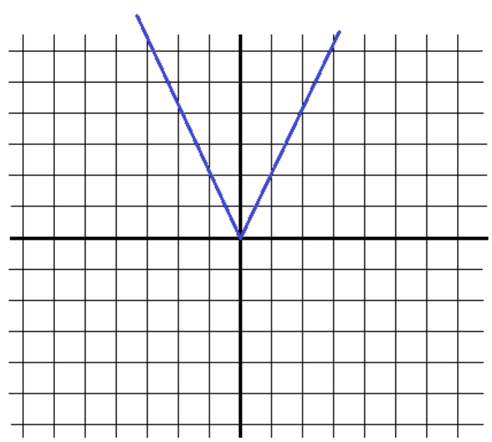
$h(x+3)$ NEITHER



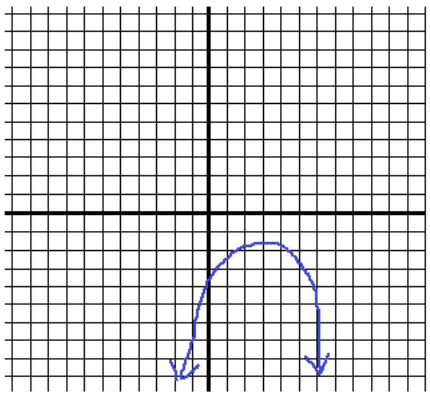
$-h(x)$ EVEN



$h(2x)$ EVEN

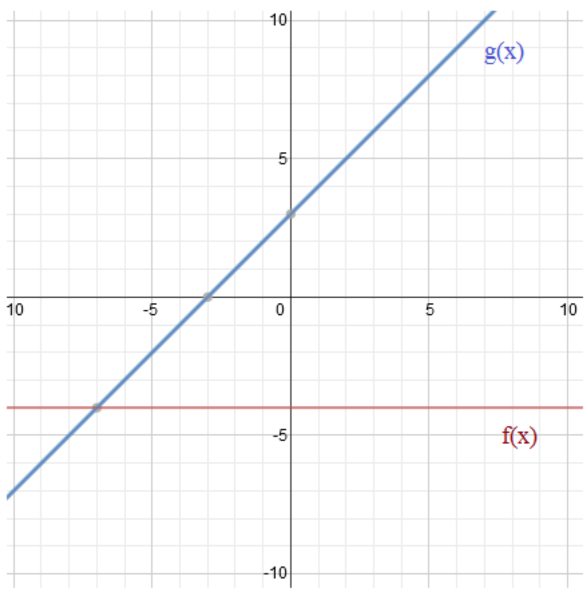


1) Sketch a graph of a quadratic with a negative discriminant and no minimum.



While there is a maximum (at the vertex), there is no minimum.
And, since the quadratic/parabola does not intersect the x-axis, it has no zeros (and must have a negative discriminant)

2) Answer the questions for the following graph:



a) $(f + g)(3) =$ $f(3) + g(3) = -4 + 6 = 2$

b) $(f \circ g)(3) =$ $g(3) = 6$ and $f(6)$ -4

c) $(g \circ f)(3) =$ $f(3) = -4$ and then $g(-4) = -1$

d) $(f \circ f)(1) =$ $f(1) = -4$ and then $f(-4) = -4$

e) $g(g(4)) =$ $g(4) = 7$ and then $g(7) = 10$

f) $g^{-1}(3) =$ "g of what number equals 3" ?
0 (because $g(0) = 3$)

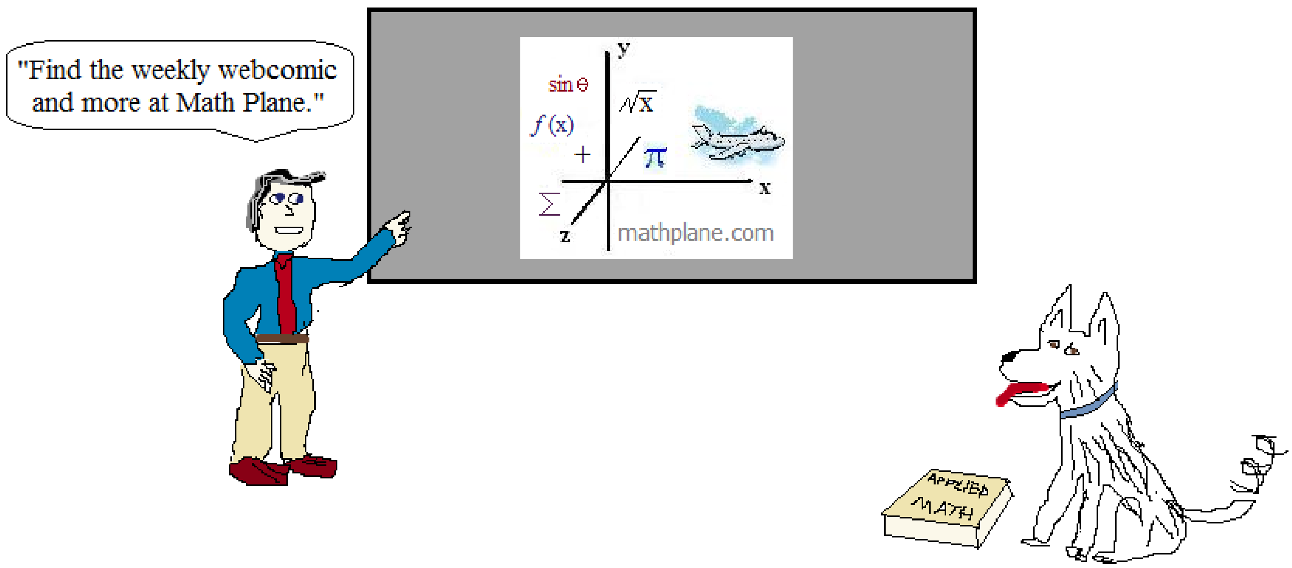
g) $f^{-1}(3) =$ since no input into f(x) would produce 3, there is no solution ϕ

h) $(f - g)(0) =$ $f(0) - g(0) = -4 - 3 = -7$

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know..

Cheers.



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

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One more question: Even, Odd, or Neither?

$$\sqrt{x^2 + 6}$$

Answer on Next Page-→

Even, Odd, or Neither?

$$\sqrt{x^2 + 6}$$

Even!

Ordinarily, square root functions are neither (because the negative side isn't part of the domain).
But, in this case, all real numbers are in the domain...
Then, there is symmetry over the y-axis.

$$f(-x) = \sqrt{(-x)^2 + 6} = \sqrt{x^2 + 6} = f(x)$$