# Variable Exponents & Higher Roots

## Notes, Examples, and Practice Questions

*Topics include rational exponents, exponential equations, using absolute value, negative exponents, and more.*

Simplifying Higher Roots: Notes and Examples

Write the following in $b^n$ form (if possible)

Look for common exponent.... *Example:* $7^3 \cdot 3^6$

$7^3 \cdot (3^2)^3 = 7^3 \cdot 9^3 = 63^3$

Look for common base.... *Example:* $3^6 \cdot 9^7$

$9^3 \cdot 9^7 = 9^{10}$

Note:

Simplify the following

Sometimes using exponent form is better *Example:* $\sqrt[3]{7} \cdot \sqrt{7} \Rightarrow 7^{1/3} \cdot 7^{1/2} = 7^{5/6}$

Sometimes using root form is better *Example:* $\frac{90^{1/2}}{10^{1/2}} \Rightarrow \frac{\sqrt{90}}{\sqrt{10}} = \sqrt{9} = 3$

*Example:* $\sqrt[4]{\frac{c}{32b^3}}$ $\frac{\sqrt[4]{c}}{2\sqrt[4]{2b^3}} \cdot \frac{\sqrt[4]{2^3 b}}{\sqrt[4]{2^3 b}} = \frac{\sqrt[4]{8bc}}{4b} \Rightarrow$

But, wait!! it should be $\frac{\sqrt[4]{8bc}}{4|b|}$

why the absolute value?

Applying an absolute value to simplified expression

suppose $b = -3$ and $c = -5$

| original | 1st simplified | correction |
|---|---|---|
| $\sqrt[4]{\frac{c}{32b^3}}$ | $\frac{\sqrt[4]{8bc}}{4b}$ | $\frac{\sqrt[4]{8bc}}{4\|b\|}$ |
| $\sqrt[4]{\frac{-5}{32(-27)}}$ | $\frac{\sqrt[4]{120}}{-12}$ | $\frac{\sqrt[4]{120}}{12}$ |
| (positive value) | this is not equivalent because it is negative! | (positive value) |

Simplify: (assume positive OR negative variables)



1) $\sqrt[3]{6} \cdot \sqrt{2}$

2) $\sqrt[3]{ab^2} \cdot \sqrt{ab}$

3) $\dfrac{\sqrt[3]{x+y} \cdot \sqrt[4]{(x+y)^2}}{\sqrt{(x+y)^3}}$

4) $\dfrac{\sqrt[3]{3}}{\sqrt[3]{25m}}$

5) $\dfrac{\sqrt[3]{9}}{2 + \sqrt[4]{9}}$

6) $\sqrt[4]{x^6 y^4 z^3}$

7) $\sqrt[5]{x} \cdot \sqrt[4]{x}$

8) $\dfrac{2 + \sqrt[5]{2}}{\sqrt[5]{9}}$

9) $\sqrt[3]{16x^5 y^{-2}}$

10) $\sqrt[3]{\dfrac{27^5}{9^2 \cdot 81^{-1}}}$

11) $\dfrac{\sqrt[3]{4}}{\sqrt[5]{8}}$

12) $\dfrac{\sqrt[5]{x^3}}{\sqrt[7]{x^4}}$

Simplify the radicals...

1) $\sqrt[5]{-64}$

2) $\sqrt[3]{375}$

3) $\sqrt[5]{128}$

4) $\sqrt[4]{162}$

5) $\sqrt[3]{192}$

6) $\sqrt{363}$

Write any and all solutions:

$x^{\frac{3}{5}} = -27$

$x^{\frac{2}{3}} = 49$

$x^{\frac{4}{3}} = -16$

$x^{\frac{3}{2}} = 8$

$x^{\frac{3}{4}} = -8$

Solve for x...

1) $3^{3x} = 9$

2) $4^{3x-1} = 8^{2x+7}$

3) $\dfrac{16^{x}}{4^{3x}} = 8^{6x}$

4) $\sqrt{125} = 5^{7x+2}$

5) $3 \cdot 9^{x+2} = \sqrt{27}$

6) $8^{x+3} = \dfrac{1}{32}$

Solve (and assume variables may be positive OR negative)

1) $(4x)^{-2} = 100$

2) $x - 3x^{\frac{1}{2}} = 4$

3) $x^{\frac{3}{4}} = 27$

4) $x^{\frac{3}{4}} = -27$

5) $4^{x} - 2^{x} - 2 = 0$

6) $(3^{x})^{x-5} = 1$

7) $\dfrac{A^{2}}{A^{-x}} = A^{3} \cdot A^{-x}$

8) $\dfrac{7^{-3x+2}}{343^{x}} = 49^{-2x-1}$

9) $x + 7x^{\frac{2}{3}} + 10x^{\frac{1}{3}} = 0$

10) $5^{x} + 125(5^{-x}) = 30$

11) $\left(\dfrac{7^{4x-3}}{7^{2x-3}}\right)^{x-7} = 1$

12) $x^{\frac{-1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}} = 0$

Miscellaneous Questions... 

A) Solve the system

$$3^{x} - 2^{y} = 23$$

$$3^{x+1} + 2^{y+1} = 89$$

B) Write as $b^{n}$ , where b and n are positive integers...

$$4^{3} \cdot 5^{2} =$$

C) Simplify. (Assume variables are negative or positive).

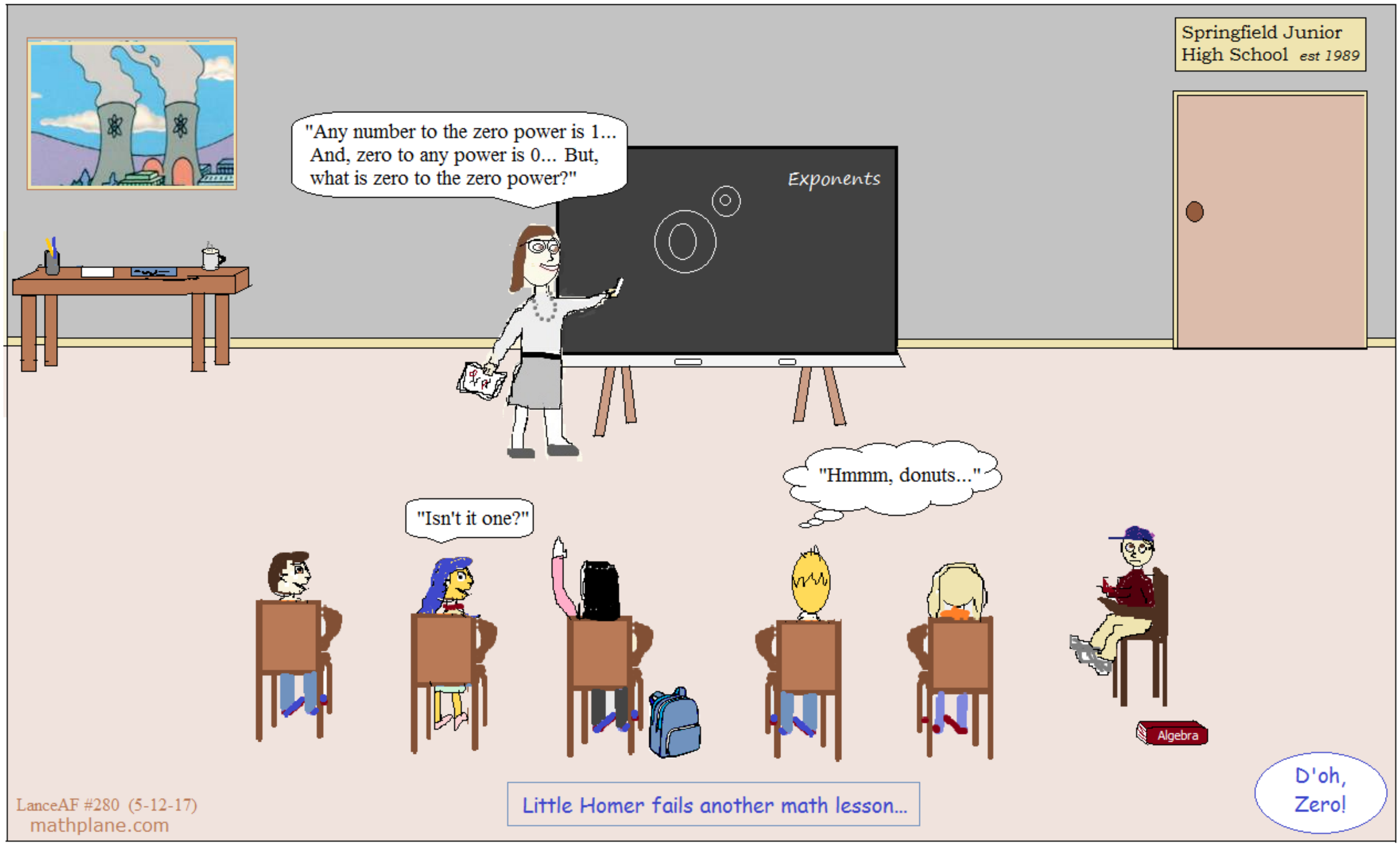
$$\sqrt[4]{\frac{c^{2}}{24a^{3}c^{-2}}}$$

D) Evaluate...

$$-5^{-2} =$$

$$(0.6)^{-1} =$$

$$\left(\frac{4}{9}\right)^{\frac{2}{3}} =$$


Springfield Junior High School est 1989
"Any number to the zero power is 1... And, zero to any power is 0... But, what is zero to the zero power?"
Exponents
"Hmmm, donuts..."
"Isn't it one?"
Algebra
D'oh, Zero!
Little Homer fails another math lesson...
LanceAF #280 (5-12-17)
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SOLUTIONS-→

Simplify: (assume positive OR negative variables)

SOLUTIONS



1) $\sqrt[3]{6}\cdot\sqrt{2}$

$6^{\frac{1}{3}}\cdot 2^{\frac{1}{2}}$

$6^{\frac{2}{6}}\cdot 2^{\frac{3}{6}}$

$\sqrt[6]{6^2}\cdot\sqrt[6]{2^3} = \boxed{\sqrt[6]{288}}$

2) $\sqrt[3]{ab^2}\cdot\sqrt{ab}$

$(ab^2)^{\frac{1}{3}}\cdot(ab)^{\frac{1}{2}}$

$(ab^2)^{\frac{2}{6}}\cdot(ab)^{\frac{3}{6}}$

$\sqrt[6]{a^2b^4}\cdot\sqrt[6]{a^3b^3}$

$\sqrt[6]{a^5b^7}$

$\boxed{|b|\sqrt[6]{a^5b}}$

3) $\dfrac{\sqrt[3]{x+y}\cdot\sqrt[4]{(x+y)^2}}{\sqrt{(x+y)^3}}$

$\dfrac{(x+y)^{\frac{1}{3}}\cdot(x+y)^{\frac{2}{4}}}{(x+y)^{\frac{3}{2}}}$

$(x+y)^{\frac{1}{3}+\frac{2}{4}-\frac{3}{2}} = (x+y)^{\frac{-2}{3}} = \boxed{\dfrac{\sqrt[3]{(x+y)}}{(x+y)}}$

4) $\dfrac{\sqrt[3]{3}}{\sqrt[3]{25m}}$

$\dfrac{\sqrt[3]{3}}{\sqrt[3]{5\cdot5\cdot m}}\cdot\dfrac{\sqrt[3]{5m^2}}{\sqrt[3]{5m^2}}$

$\boxed{\dfrac{\sqrt[3]{15m^2}}{5m}}$

5) $\dfrac{\sqrt[3]{9}}{2+\sqrt[4]{9}}\cdot\dfrac{2-\sqrt[4]{9}}{2-\sqrt[4]{9}}$

$9^{\frac{1}{3}}\cdot 9^{\frac{1}{4}} = 9^{7/12}$

$= \sqrt[12]{9^7}$

$\dfrac{2\sqrt[3]{9}-\sqrt[12]{9^7}}{4-\sqrt[4]{81}}$ → 1

$\boxed{2\sqrt[3]{9}-\sqrt[12]{9^7}}$

6) $\sqrt[4]{x^6y^4z^3}$

$\sqrt[4]{x^4\cdot x^2\cdot y^4\cdot z^3}$

$\boxed{|x||y|\sqrt{x^2y^3}}$

7) $\sqrt[5]{x}\cdot\sqrt[4]{x}$

$x^{\frac{1}{5}}\cdot x^{\frac{1}{4}}$

$x^{\frac{1}{5}+\frac{1}{4}}$

$x^{\frac{9}{20}}$

$\boxed{\sqrt[20]{x^9}}$

8) $\dfrac{2+\sqrt[5]{2}}{\sqrt[5]{9}}$

$\dfrac{2+\sqrt[5]{2}}{\sqrt[5]{3\cdot3}}\cdot\dfrac{\sqrt[5]{3\cdot3\cdot3}}{\sqrt[5]{3\cdot3\cdot3}}$

$\boxed{\dfrac{2\sqrt[5]{27}+\sqrt[5]{54}}{3}}$

9) $\sqrt[3]{16x^5y^{-2}} = \dfrac{\sqrt[3]{8\cdot2\cdot x^3\cdot x^2}}{\sqrt[3]{y^2}}$

then, rationalize denominator...

$\dfrac{2x\sqrt[3]{2x^2}}{\sqrt[3]{y^2}}\cdot\dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$

$\boxed{\dfrac{2x\sqrt[3]{2x^2y}}{y}}$

10) $\sqrt[3]{\dfrac{27^5}{9^2\cdot81^{-1}}}$

$\sqrt[3]{\dfrac{(3^3)^5}{(3^2)^2\cdot(3^4)^{-1}}} = \sqrt[3]{\dfrac{3^{15}}{3^4\cdot3^{-4}}}$

$= \sqrt[3]{3^{15}} = 3^5 = \boxed{243}$

11) $\dfrac{\sqrt[3]{4}}{\sqrt[5]{8}} = \dfrac{4^{\frac{1}{3}}}{8^{\frac{1}{5}}} = \dfrac{(2^2)^{\frac{1}{3}}}{(2^3)^{\frac{1}{5}}}$

$\dfrac{2^{\frac{2}{3}}}{2^{\frac{3}{5}}} = 2^{\frac{2}{3}-\frac{3}{5}}$

$\boxed{2^{\frac{1}{15}}}$

12) $\dfrac{\sqrt[5]{x^3}}{\sqrt[7]{x^4}} = \dfrac{x^{\frac{3}{5}}}{x^{\frac{4}{7}}}$

$\boxed{x^{\frac{1}{35}}}$

Simplify the radicals...

1) $\sqrt[5]{-64}$ $\quad \sqrt[5]{-32} \cdot \sqrt[5]{2}$ $\quad \boxed{-2\sqrt[5]{2}}$

2) $\sqrt[3]{375}$ $\quad \sqrt[3]{125} \cdot \sqrt[3]{3}$ $\quad \boxed{5\sqrt[3]{3}}$

3) $\sqrt[5]{128}$ $\quad \sqrt[5]{2^7}$ $\quad 2\sqrt[5]{2^2} = \boxed{2\sqrt[5]{4}}$

4) $\sqrt[4]{162}$ $\quad \sqrt[4]{2 \cdot 81} = \boxed{3\sqrt[4]{2}}$

5) $\sqrt[3]{192}$ $\quad \sqrt[3]{3 \cdot 64} = \boxed{4\sqrt[3]{3}}$

6) $\sqrt{363}$ $\quad \sqrt{3 \cdot 121} = \boxed{11\sqrt{3}}$

SOLUTIONS

Write any and all solutions:

$x^{\frac{3}{5}} = -27$ $\quad \boxed{-243}$ odd radicals can have negatives

$x^{\frac{2}{3}} = 49$ $\quad \boxed{343, -343}$

$x^{\frac{4}{3}} = -16$ $\quad \boxed{\text{no solution}}$ any value to the 4th power will be positive

$x^{\frac{3}{2}} = 8$ $\quad \boxed{4}$ (-4 is NOT a solution)

$x^{\frac{3}{4}} = -8$ $\quad \boxed{\text{no solution}}$
(If you check, you'll see that neither 16 nor -16 are solutions)

Solve for x...

Find common roots (bases)..
Then, drop bases to solve..

1) $3^{3x} = 9$ $\quad 3^{3x} = 3^2$ $\quad 3x = 2$ $\quad \boxed{x = 2/3}$

2) $4^{3x-1} = 8^{2x+7}$ $\quad 2^{6x-2} = 2^{6x+21}$ $\quad 6x - 2 = 6x + 21$ $\quad \boxed{\text{no solution}}$

3) $\dfrac{16^x}{4^{3x}} = 8^{6x}$ $\quad \dfrac{2^{4x}}{2^{6x}} = 2^{18x}$ $\quad 2^{-2x} = 2^{18x}$ $\quad \boxed{x = 0}$

4) $\sqrt{125} = 5^{7x+2}$ $\quad 5^1 \cdot 5^{\frac{1}{2}} = 5^{7x+2}$ $\quad \frac{3}{2} = 7x + 2$ $\quad \boxed{x = \frac{-1}{14}}$

5) $3 \cdot 9^{x+2} = \sqrt{27}$ $\quad 3^1 \cdot 3^{2x+4} = 3^{\frac{3}{2}}$ $\quad 3^{2x+5} = 3^{\frac{3}{2}}$ $\quad 2x + 5 = \frac{3}{2}$ $\quad \boxed{x = \frac{-7}{4}}$

6) $8^{x+3} = \frac{1}{32}$ $\quad 2^{3x+9} = 2^{-5}$ $\quad 3x + 9 = -5$ $\quad \boxed{x = \frac{-14}{3}}$

Solve (and assume variables may be positive OR negative)

SOLUTIONS

1) $(4x)^{-2} = 100$

$\frac{1}{16x^2} = 100$

$1 = 1600x^2$

$x^2 = \frac{1}{1600}$

**$x = 1/40$ or $-1/40$**

2) $x - 3x^{\frac{1}{2}} = 4$

$x - 4 = 3x^{\frac{1}{2}}$ (square both sides)

$x^2 - 8x + 16 = 9x$

$(x - 1)(x - 16) = 0$

$x = 1, 16$

1 is extraneous...

So, **$x = 16$**

3) $x^{\frac{3}{4}} = 27$

$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = 27^{\frac{4}{3}}$

**$x = 81$**

4) $x^{\frac{3}{4}} = -27$

$\left(x^{\frac{1}{4}}\right)^3 = -27$

$x^{\frac{1}{4}} = -3$

**No solution (because the 1/4 root of a number is positive)**

5) $4^x - 2^x - 2 = 0$

$(2^2)^x - 2^x - 2 = 0$

equal

$(2^x)^2 - 2^x - 2 = 0$

$A^2 - A - 2 = 0$

$(A - 2)(A + 1) = 0$

$A = 2$ or $-1$

$2^x = 2$ or ~~$-1$~~

**$x = 1$**

6) $(3^x)^{x-5} = 1$

$3^{x^2 - 5x} = 3^0$

$x^2 - 5x = 0$

**$x = 0$ or $5$**

7) $\frac{A^2}{A^{-x}} = A^3 \cdot A^{-x}$

$A^{2+x} = A^{3-x}$

$2 + x = 3 - x$

**$x = 1/2$**

8) $\frac{7^{-3x+2}}{343^x} = 49^{-2x-1}$

$\frac{7^{-3x+2}}{7^{3x}} = 7^{-4x-2}$

$7^{-6x+2} = 7^{-4x-2}$

$-6x + 2 = -4x - 2$

**$x = 2$**

9) $x + 7x^{\frac{2}{3}} + 10x^{\frac{1}{3}} = 0$

Let $A = x^{\frac{1}{3}}$

$A^3 + 7A^2 + 10A = 0$

$A(A^2 + 7A + 10) = 0$

$A(A + 2)(A + 5) = 0$

$A = 0, -2, -5$

**$x = 0, -8, -125$**

10) $5^x + 125(5^{-x}) = 30$

$5^x + 125(5^{-x}) - 30 = 0$

multiply by $5^x$

$5^{2x} + 125 - 30(5^x) = 0$

let $A = 5^x$

$A^2 - 30A + 125 = 0$

$(A - 5)(A - 25) = 0$

$A = 5, 25$

$5^x = 5$ **$x = 1$**

$5^x = 25$ **$x = 2$**

11) $\left(\frac{7^{4x-3}}{7^{2x-3}}\right)^{x-7} = 1$

$\left(7^{2x}\right)^{x-7} = 1$

$7^{2x^2 - 14x} = 7^0$

$2x^2 - 14x = 0$

**$x = 0$ or $7$**

12) $x^{\frac{-1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}} = 0$

$x^{\frac{-1}{2}} \cdot \left(1 + 2x + x^2\right) = 0$

$x^{\frac{-1}{2}} \cdot (x + 1)(x + 1) = 0$

$x^{\frac{-1}{2}} = 0 \Rightarrow \frac{1}{\sqrt{x}} = 0$ **no solutions.** ✗

$(x + 1)(x + 1) = 0 \Rightarrow x = -1$

but, if $x = -1$, then $2x^{\frac{1}{2}} = 2(-1)^{\frac{1}{2}}$

however, $2\sqrt{-1}$ is not real ✗

Miscellaneous Questions...



A) Solve the system

$$3^x - 2^y = 23$$
$$3^{x+1} + 2^{y+1} = 89$$

$$2\cdot 3^x - 2\cdot 2^y = 46$$

$$2\cdot 3^x - 2^{y+1} = 46$$
$$3^{x+1} + 2^{y+1} = 89$$

$$2\cdot 3^x + 3\cdot 3^x = 135$$

$$5\cdot 3^x = 135$$

$$x = 3$$

If $x = 3$, then $y = 2$

SOLUTIONS

B) Write as $b^n$, where b and n are positive integers...

$$4^3 \cdot 5^2 =$$

$$64\cdot 5^2 = 8^2\cdot 5^2 = 40^2$$

C) Simplify. (Assume variables are negative or positive).

$$\sqrt[4]{\frac{c^2}{24a^3c^{-2}}}$$

$$\sqrt[4]{\frac{c^4}{2\cdot 2\cdot 2\cdot 3\cdot a^3}} \cdot \frac{\sqrt[4]{2\cdot 3\cdot 3\cdot 3\cdot a}}{\sqrt[4]{2\cdot 3\cdot 3\cdot 3\cdot a}} = \frac{|c|\sqrt[4]{54a}}{6a}$$

since original has $a^3$, absolute value isn't necessary

since original had $c^2$ and $c^{-2}$, absolute value of c must be maintained to keep expressions equal!

D) Evaluate...

$$-5^{-2} = (-1)(5)^{-2} = (-1)\left(\frac{1}{5^2}\right) = \frac{-1}{25}$$

$$(0.6)^{-1} = (3/5)^{-1} = 5/3$$

$$\left(\frac{4}{9}\right)^{\frac{2}{3}}$$

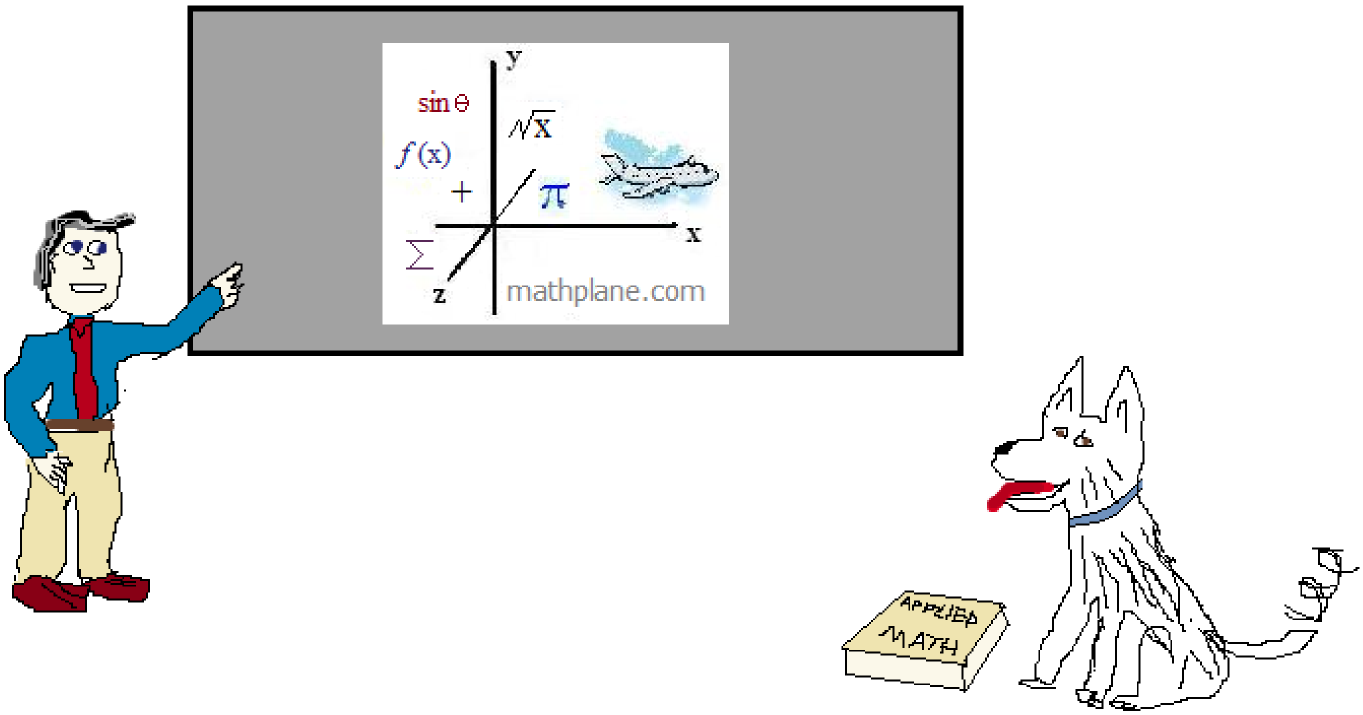
It's not $\frac{8}{27}$!!

$$\left(\left(\frac{4}{9}\right)^2\right)^{\frac{1}{3}} = \sqrt[3]{\frac{16}{81}} = \frac{\sqrt[3]{16}}{\sqrt[3]{9\cdot 9}}\cdot\frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{\sqrt[3]{16\cdot 9}}{9}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

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