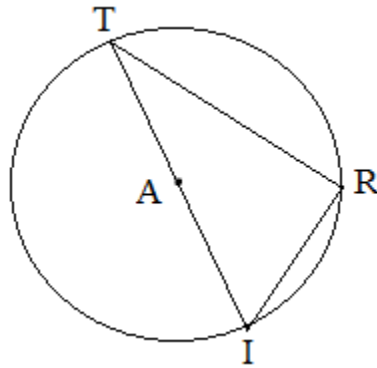# Circles and Inscribed Figures

Notes, Examples, and Exercises (with Solutions)
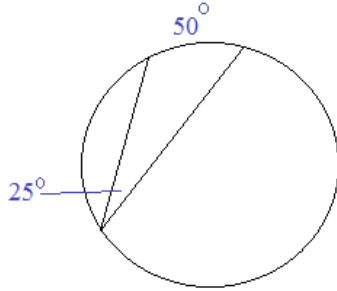

Topics include Arcs, Angles, Chords, Secants, Tangents, Quadrilaterals, and more.

Inscribed Angle

The vertex of an inscribed angle lies on the circle.
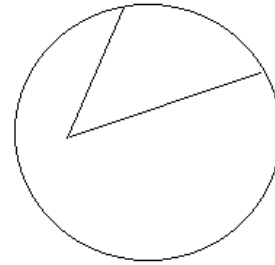
50°

25°

Not inscribed angle
(vertex is not on circumference
of the circle)

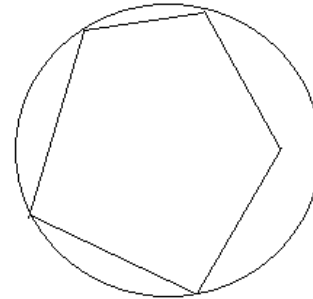measure of angle $= \dfrac{1}{2}$ (intercepted arc)

Inscribed Polygon

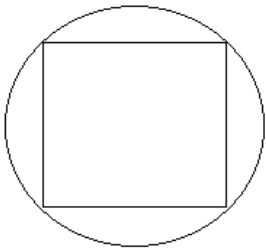A polygon is inscribed if every vertex lies on the circle.
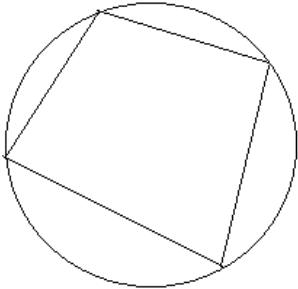
The pentagon is inside the circle, but
it is not inscribed in the circle.
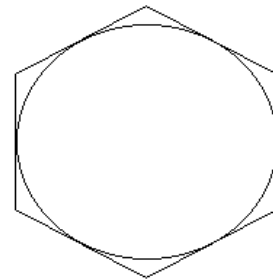

Inscribed vs. Circumscribed

Examples:

The rectangle is inscribed in the circle

OR

The circle is circumscribed about
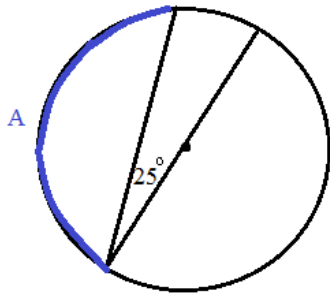the rectangle

The circle is inscribed in the regular hexagon.

OR,

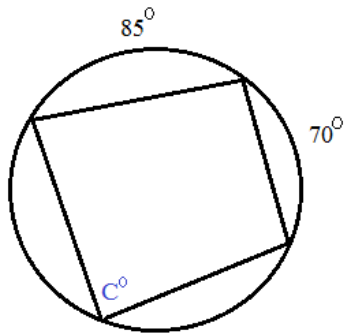A regular hexagon circumscribed about a
circle.

Example: What is the angle measure of arc A?

If an inscribed angle is 25 degrees, then the
corresponding arc on the circle is 50 degrees.

And, since it's a semi-circle, the remainder is 130^0

Example: What is the measure of angle C?

C is an inscribed angle.

The corresponding arc on the circle is $(85 + 70) = 155$ degrees.

Therefore, the inscribed angle is $\frac{1}{2}(155) =$ 77.5 degrees

Example: A regular hexagon is inscribed in a circle.
If the perimeter of the hexagon is 36, what
is the distance from the center of the circle
to each side?

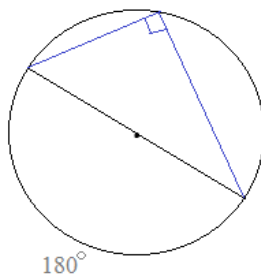Since perimeter is 36, each side is 6...

The measure of each exterior angle of regular
hexagon is 360/6 = 60 degrees..
So, measure of each interior angle is 120 degrees..

Therefore, the radius of the circle will be 6
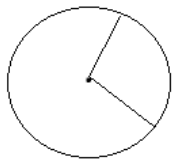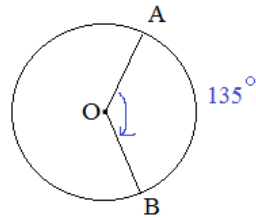and the distance to each side is $3\sqrt{3}$
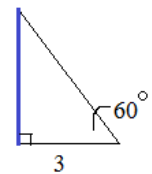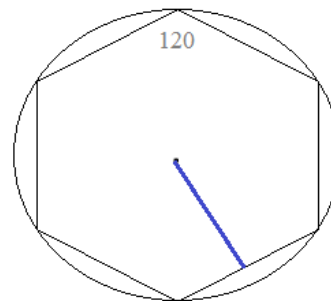
Application:

If one side of an inscribed triangle is the diameter of a circle,
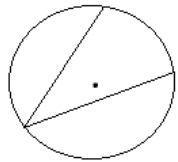then it must be a right triangle!

180^0

central angle: angle equals arc

$\overset{\frown}{AB} = 135°$
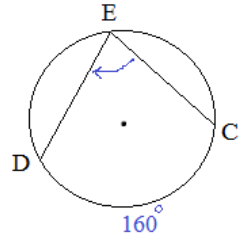
$\angle AOB = 135°$

Central Angle: The vertex
is on the center of the circle
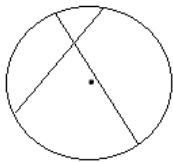
inscribed angle: angle = 1/2 arc

$\overset{\frown}{CD} = 160°$
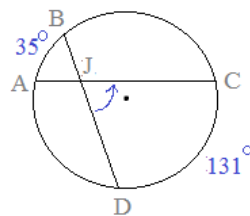
$\angle CED = 80°$

Inscribed Angle: The vertex
lies on the circle

chord-chord: 1/2 (sum of arcs)

$\angle CJD = \frac{1}{2}(35 + 131) = 83°$
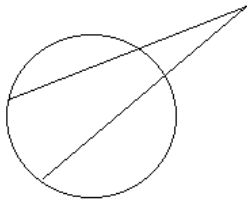
secant-secant: 1/2 (difference of arcs)
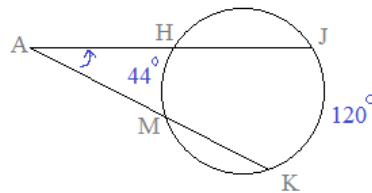
$\angle HAM = \frac{1}{2}(\overset{\frown}{JK} - \overset{\frown}{HM})$

big arc $-$ small arc

$= \frac{1}{2}(76°) = 38°$
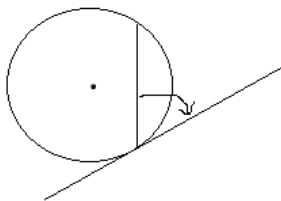
tangent-chord: angle = 1/2 arc
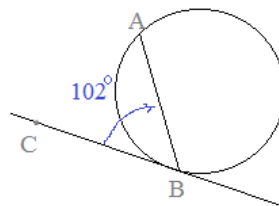
$\angle ABC = \frac{1}{2}(102) = 51°$

AB is the chord
CB is the tangent

$\overset{\frown}{AB}$ is the arc

similar to secant-secant...
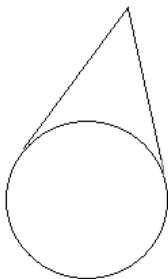
exterior point: angle = 1/2(difference of arcs)
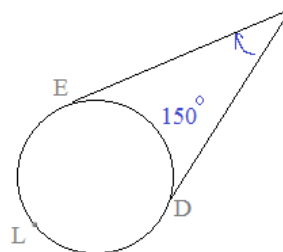
minor arc DE is 150

major arc DLE is 210 (360 - 150)

$\frac{1}{2}(210 - 150) = 30°$

observation/shortcut: exterior angle is
supplement to minor arc!

A and E are points of tangency

$$\overset{\frown}{AE} = 112°$$

$$\overset{\frown}{BC} = 70°$$

$$\overset{\frown}{AB} = 38°$$

Determine all of the numbered angles!

A and E are points of tangency

$$\overset{\frown}{AE} = 112^{\circ}$$

$$\overset{\frown}{BC} = 70^{\circ}$$

$$\overset{\frown}{AB} = 38^{\circ}$$

Answer Key

DE + AE = 180 (semicircle)

1) 68° (central angle)

AB + BC + CD = 180 (sum of angles is semicircle)

3) 90° (inscribed angle; also, triangle inscribed in a semicircle is a right triangle)

4) 35° (inscribed angle -- 1/2 of arc $\overset{\frown}{BC}$)

5) 55° (chord-chord theorem --- 1/2 ($\overset{\frown}{AB}$ + $\overset{\frown}{CD}$))

6) 90° (tangent and radius form right angle)

7) 19° (tangent-secant theorem)

8) 56° (inscribed angle --- 1/2 ($\overset{\frown}{AE}$))

9) 19° (inscribed angle --- 1/2 ($\overset{\frown}{AB}$))

10) 111° (chord-chord theorem --- 1/2 ($\overset{\frown}{CD}$ + $\overset{\frown}{EAB}$))

Note: To solve or to check answers, consider properties of angles and triangles.

sum of interior angles of triangle = 180°

vertical angles congruent

supplementary angles

11) 36^{0} (inscribed angle)

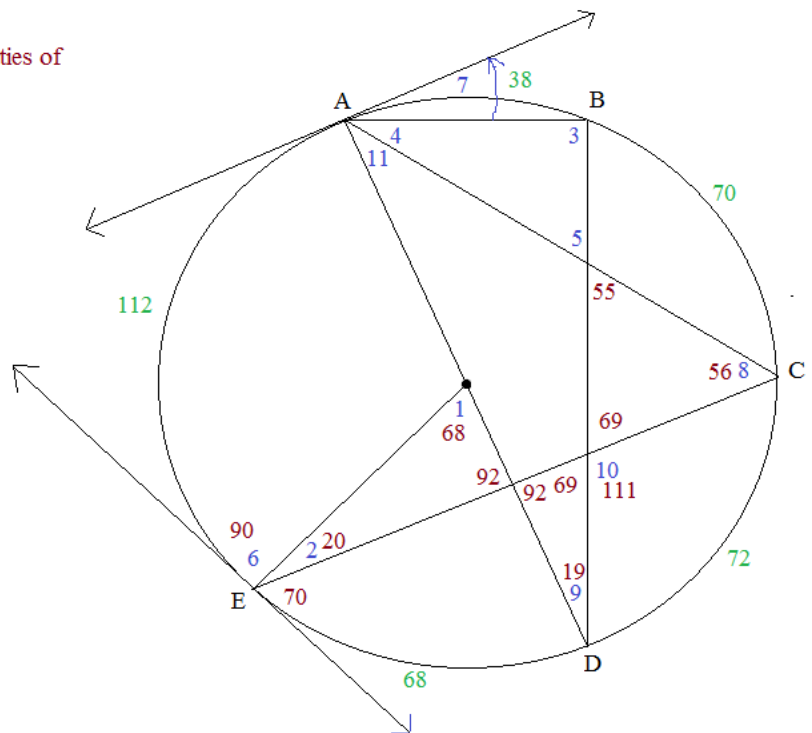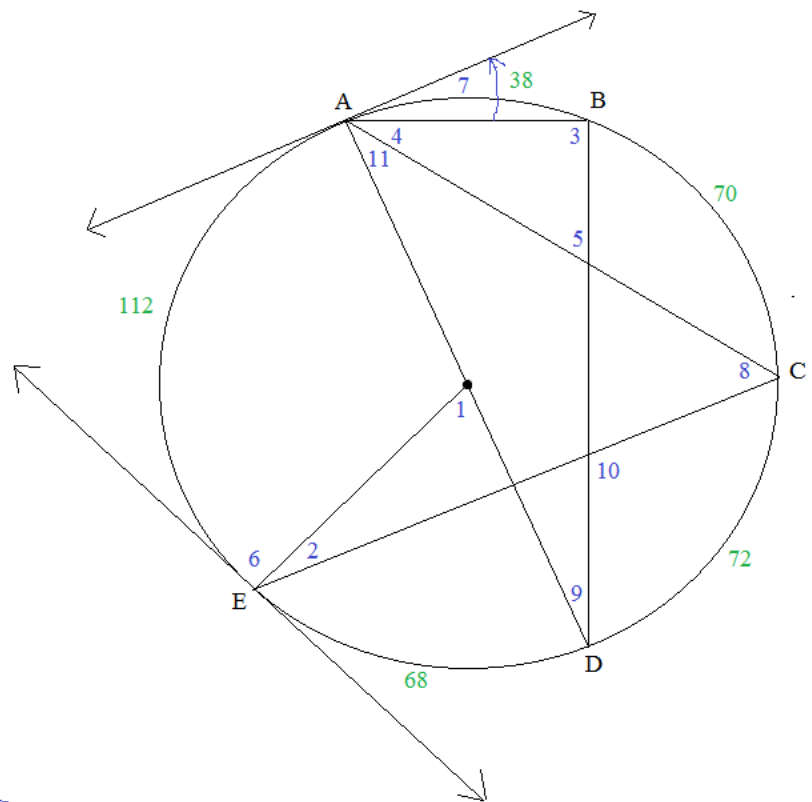.2) 20° (sum of interior angles of triangle)

also, we know angle 6 is 90 degrees

and the angle on the other side is

1/2 (68 + 72) = 70°...

therefore, angle 2 is 20^{0}

90 + 20 + 70 = 180°

Practice Test -→

Read carefully, and answer the following:

1) A square is inscribed in a circle.
   If the diameter of the circle is 20 feet,
   what is the area of the square?
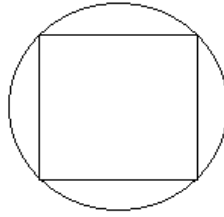
2) A parallelogram with sides 4 and 6 is inscribed in a circle.
   Find the *radius* of the circle.

   (Hint: The parallelogram must be a rectangle!)

3) QUAD is a quadrilateral inscribed in circle O.
   What is the measure(s) of angle Q ?

Read carefully, and answer the following:

4) Triangle TRI is inscribed in circle A
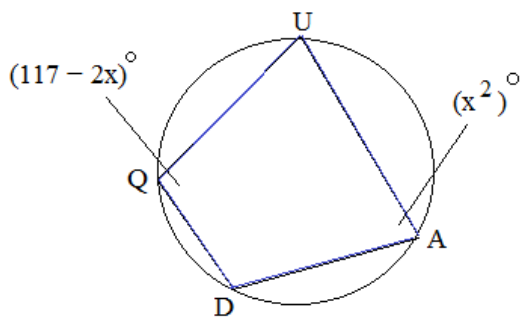
$\overset{\frown}{TR} = 120°$

$\overline{TI} = 10$

a) what is the measure of $\angle ATR$?

b) what is the perimeter of $\triangle TRI$?
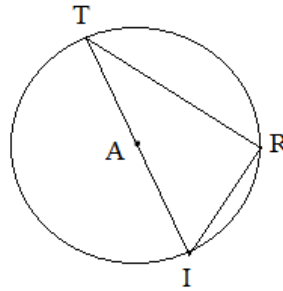
5) In the figure, $\angle A = 36°$

a) find $\angle B$

b) find $\overset{\frown}{DC}$

6) In the figure, central angle $\angle JOK = 40°$

$\overline{OK} = 14$

a) find the measure of $\angle LJO$

b) determine the arc length of $\overset{\frown}{JL}$

7) What is the measure of angle B?

40°

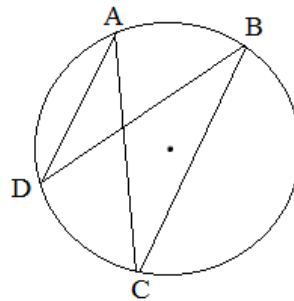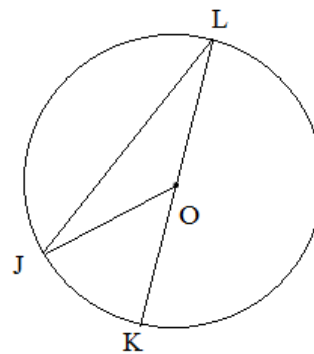120°

B

8) Given: Circle O

$\angle AOB = 74^{\circ}$
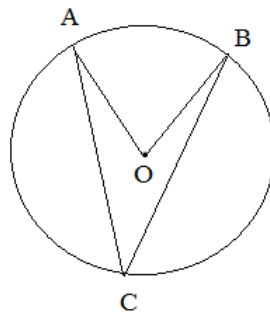
What is $\angle ACB$?

A   B

O

C

9) What is the radius of a circle inscribed in a 12-16-20 right triangle?

10) What is the degree measure of $\overset{\frown}{ST}$?

18°
40°
S
T
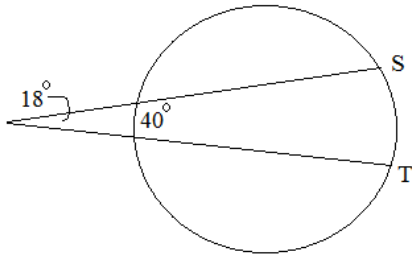
11) Find $\angle$ HOD

65°
C
H
O
R
43°
D
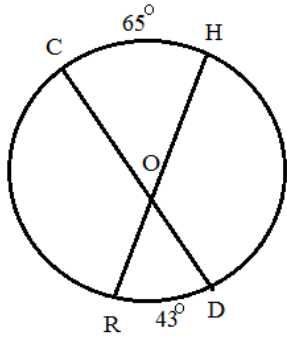
12) Determine a, b, and c

b
76
68
108
a
c
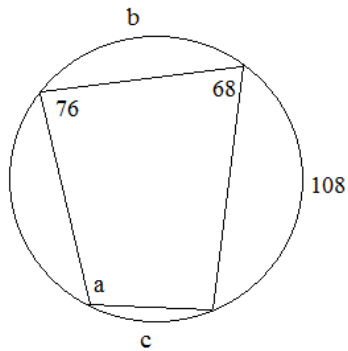
13) A circle with radius 10 is inscribed in a regular hexagon <u>and</u> circumscribed about another regular hexagon.

What is the ratio of the smaller hexagon's area to the larger hexagon's area?

14) Parallelogram GRAM is inscribed in the circle.

$$m\overset{\frown}{GR} = 4x + 25$$

$$m\overset{\frown}{RA} = 7x - 43$$

$$\overline{GM} = 8$$

$$\overline{MA} = 15$$

a) Find $m\overset{\frown}{GR}$

b) Find the arc length $\overset{\frown}{GR}$

15) Find x:

$x°$

16) The rhombus is 'inscribed' in the circle.

What is the measure of angle HRM ?

R H I
E D
W D
S E
N A N

Letter Key:

1	2	3	4	5	6	7	8	9	0
B	E	G	I	L	P	R	T	U	Y

1) π is approximately _____

3.[ ]4 _____

2) Area of shaded region (in units^2)

A

Radius = 5 units

O
B

$\angle AOB = 72°$

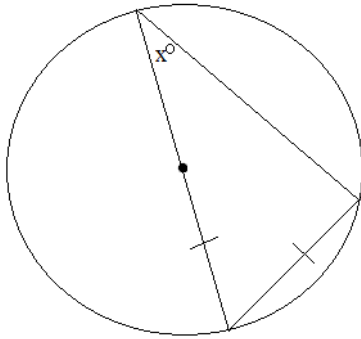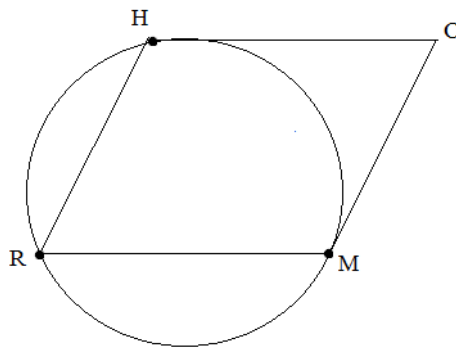[ ]π _____

3) Length of X:

4 3
12 X

[ ]$\rightarrow$ _____

4) $\dfrac{\text{Area of Circle O}}{\text{Perimeter of circle O}} =$

4 feet

o

[ ] feet _____

5) Find the arc length of $\overset{\frown}{BC}$

B C

$\overline{AC}$ = 44 feet

$\angle BAC = 45°$

A

1[ ]π feet _____

6)

A
B

$\dfrac{m\angle A}{m\angle B} =$

[ ]$\rightarrow$ _____

7) Find the length of Y

$2\sqrt{21}$

5

Y

[ ]$\rightarrow$ _____

8) Find the length of $\overline{MN}$

M N

Area of Circle M = 16π
Circumference of Circle N = 6π

[ ]$\rightarrow$ _____

9) Area of square = 400 sq. units
Find the area of the inscribed circle.

1[ ]0 π units^2 _____

10) Measure of the angle formed between the hour and minute hands at 2:00?

[ ]0 degrees _____

11) $\dfrac{\text{(Degrees in Circle)}}{\text{(Degrees in Right Angle)}} =$

[ ]$\rightarrow$ _____

Solutions -→

SOLUTIONS

1) A square is inscribed in a circle.
   If the diameter of the circle is 20 feet,
   what is the area of the square?

   The diameter of the circle is also the
   diagonal of the square.

   The diagonal of a square is also the
   hypotenuse of a 45-45-90 right triangle.

   The sides of the inscribed square are
   each $\frac{20}{\sqrt{2}}$ therefore, the area of the square is $\left(\frac{20}{\sqrt{2}}\right)^2 =$ 200 square feet

2) A parallelogram with sides 4 and 6 is inscribed in a circle.
   Find the *radius* of the circle.

   (Hint: The parallelogram must be a rectangle!)

   Step 1: Draw a picture, labeling the given parts
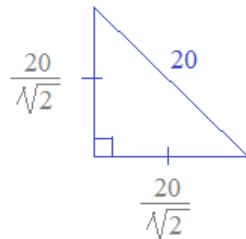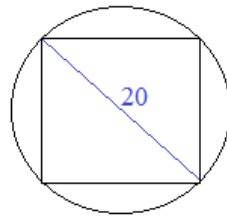
   Step 2: Recognizing that it's a rectangle, we can
           see the diagonal is also the diameter of
           the circle.
           Take out the right triangle and solve.

   Step 3: Answer the question.

   Since the diameter (d) = $2\sqrt{13}$, the radius = $\sqrt{13}$
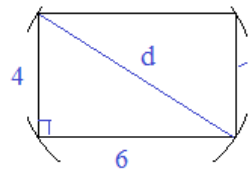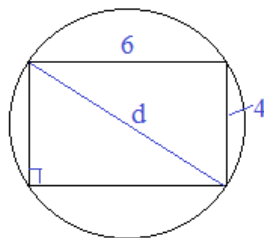
   pythagorean theorem:

   $(4)^2 + (6)^2 = d^2$

   $d = \sqrt{52} = 2\sqrt{13}$

3) QUAD is a quadrilateral inscribed in circle O.
   What is the measure(s) of angle Q ?

   $m\angle Q + m\angle A = 180°$

   $(117 - 2x) + (x^2) = 180$

   $x^2 - 2x - 63 = 0$

   $(x - 9)(x + 7) = 0$

   $x = -7, 9$

   If x = -7, Q = 117 − 2(-7) = 131°

   and A = $(-7)^2 = 49°$

   If x = 9, Q = 117 − 2(9) = 99°
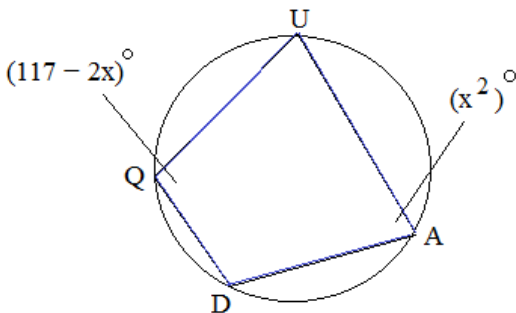
   and A = $(9)^2 = 81°$

   Note: opposite angles of inscribed quadrilateral
         are *supplementary*!

Read carefully, and answer the following:

4) Triangle TRI is inscribed in circle A

$\overset{\frown}{TR} = 120°$

$\overline{TI} = 10$

a) what is the measure of $\angle ATR$?

$\boxed{30 \text{ degrees}}$

b) what is the perimeter of $\triangle TRI$?

$\boxed{15 + 5\sqrt{3}}$

**Note: Any triangle inscribed in a *semi-circle* must be a right triangle!

a) since $\overset{\frown}{TR} = 120°$, $\overset{\frown}{RI} = 60°$..
   therefore, $\angle ATR = (1/2)60 = 30°$

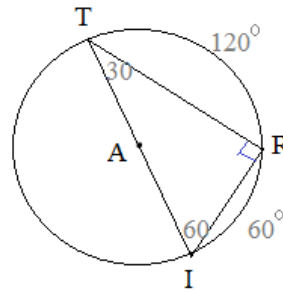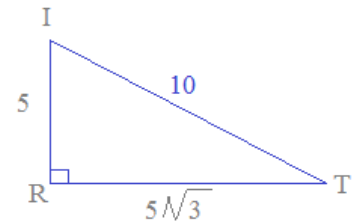b) we can recognize that this triangle is 30-60-90 special right triangle:

therefore, the perimeter is

$5 + 5\sqrt{3} + 10 = 15 + 5\sqrt{3}$

5) In the figure, $\angle A = 36°$

a) find $\angle B$

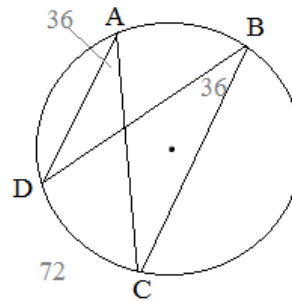Since angle A = 36, arc DC = 72...
Then, since DC = 72, angle B = $\boxed{36 \text{ degrees}}$

b) find $\overset{\frown}{DC}$

since $\angle DAC$ is an inscribed angle,
$\overset{\frown}{DC}$ is 2(DAC) = 2(36) = $\boxed{72 \text{ degrees...}}$

**Note: $\angle DAC$ and $\angle DBC$ are overlapping angles..

JOK = 40 (given)
JOL = 140 (supplementary angles)
JLO = 20
(inscribed angle = 1/2(arc length)

6) In the figure, central angle $\angle JOK = 40°$

$\overline{OK} = 14$

a) find the measure of $\angle LJO$

JOL + JLO + LJO = 180 degrees $\boxed{20 \text{ degrees}}$
140   20   20   = 180

b) determine the arc length of $\overset{\frown}{JL}$

circumference of circle O = $2\pi(r) = 28\pi$

$\dfrac{\overset{\frown}{JL}}{circle} = \dfrac{140}{360} = \dfrac{7}{18}$

therefore, arc length of $\overset{\frown}{JL} = \dfrac{7}{18} \cdot 28\pi = \boxed{\dfrac{98}{9}\pi}$

www.mathplane.com

SOLUTIONS

7) What is the measure of angle B?

40°

120°
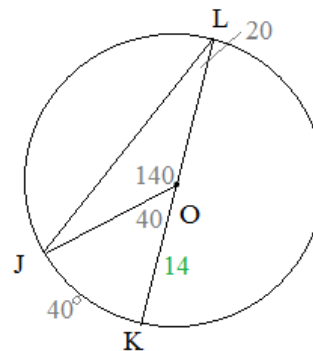
B
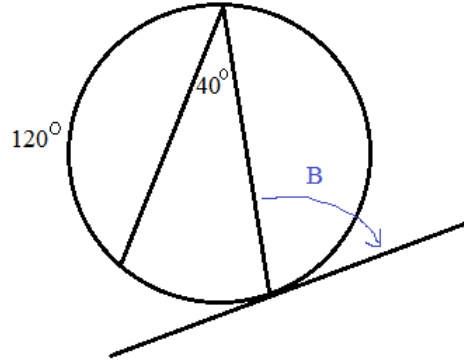
Inscribed angle 40, so
opposite arc is 80 degrees.
80 + 120 = 200 degrees..

Remainder of circle
is 160 degrees..
Angle B = 1/2 (160)

80

8) Given: Circle O

$\angle AOB = 74°$

What is $\angle ACB$?

A   74   B

74

O

37

C
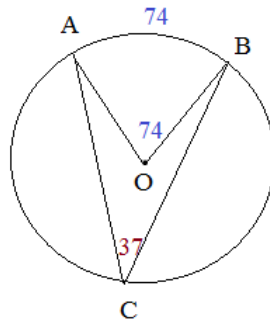
Since *central angle* AOB is 74
degrees, the angle measure of
arc AB is 74 degrees..

Therefore, inscribed angle
ACB is 1/2 of 74...

37 degrees...

9)   What is the radius of a circle inscribed in a 12-16-20 right triangle?

$12 - x$

$12 - x$

$20 - (12 - x) = x + 8$

x

x   x + 8

$x + (x + 8) = 16$

$x = 4$

Radius = 4

8

8

4

4

4

12

4

4   12

SOLUTIONS

10) What is the degree measure of $\overset{\frown}{ST}$?
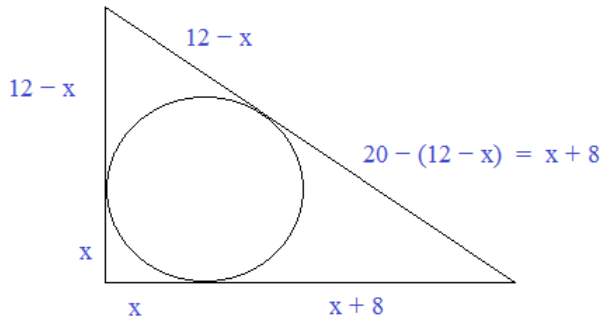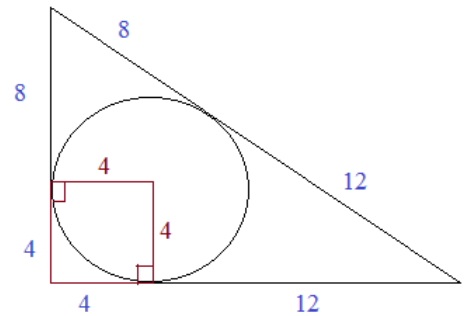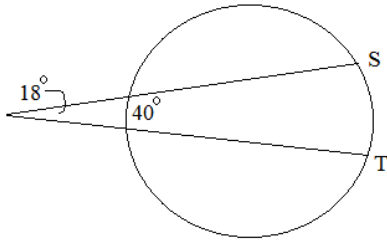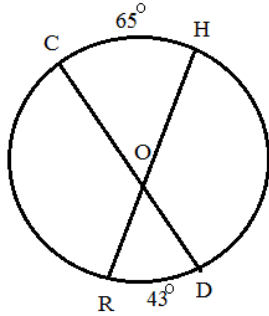
(secant-secant arc theorem)

$$\frac{1}{2}(ST - 40) = 18$$

$$ST - 40 = 36$$

$$\boxed{\overset{\frown}{ST} = 76^\circ}$$
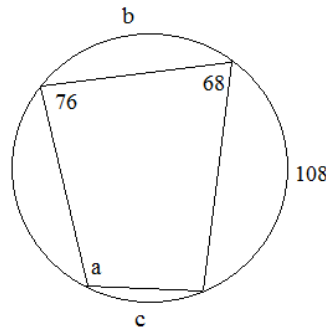
11) Find $\angle$ HOD

(chord-chord theorem)

$$\angle COH = \frac{1}{2}(\overset{\frown}{CH} + \overset{\frown}{RD})$$

$$= \frac{1}{2}(108) = 54$$

COH is supplementary to HOD

$$\boxed{\angle HOD = 126^\circ}$$

12) Determine a, b, and c

Since the quadrilateral is inscribed in the circle,

a is supplementary to 68 .

$$\boxed{a = 112}$$

$$76 = \frac{1}{2}(108 + c)$$

$$\boxed{c = 44}$$

$$112 = \frac{1}{2}(108 + b)$$

$$\boxed{b = 116}$$

13) A circle with radius 10 is inscribed in a regular hexagon __and__ circumscribed about another regular hexagon.

What is the ratio of the smaller hexagon's area to the larger hexagon's area?

Step 1: Sketch the figure(s)

(suggestion: separate the hexagons)

Step 2: Find side lengths

exterior angles of regular hexagon are 60 degrees, so interior angles are 120 degrees...
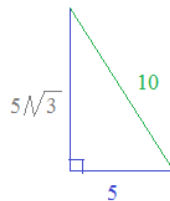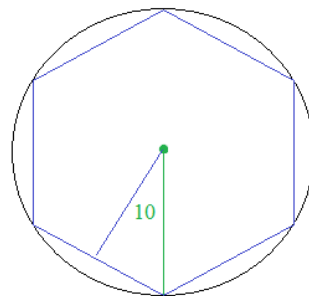
Extract right triangles to find lengths

small hexagon side: 10

large hexagon side: $\dfrac{20\sqrt{3}}{3}$
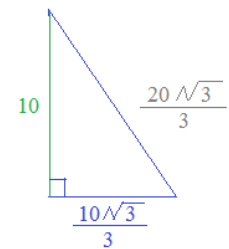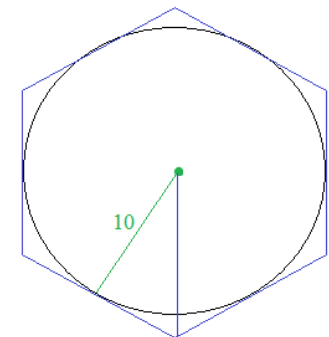
Step 3: Determine ratios

$$\frac{small}{large} = \frac{10}{\dfrac{20\sqrt{3}}{3}} = \frac{30}{20\sqrt{3}} = \frac{\sqrt{3}}{2}$$

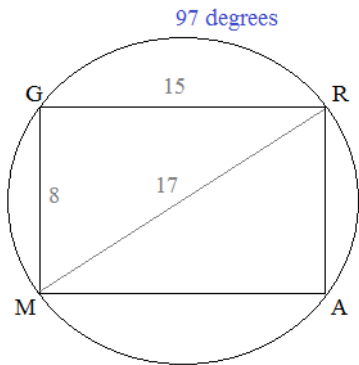$$\boxed{\text{Then, the ratio of the } areas \text{ is } \frac{3}{4}}$$

30-60-90 right triangles

14) Parallelogram GRAM is inscribed in the circle.

97 degrees

$$m\widehat{GR} = 4x + 25$$

$$m\widehat{RA} = 7x - 43$$

$$\overline{GM} = 8$$

$$\overline{MA} = 15$$

a) Find $m\widehat{GR}$

b) Find the arc length $\widehat{GR}$

A parallelogram inscribed in a circle must be a rectangle!

$$m\widehat{GR} + m\widehat{RA} = 180$$
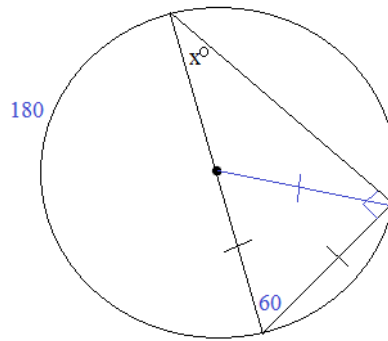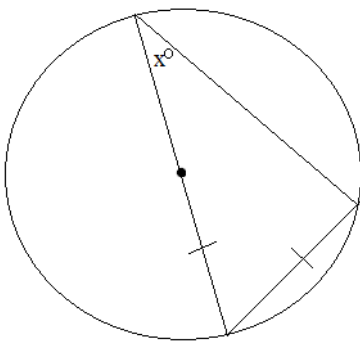
$$4x + 25 + 7x - 43 = 180$$

$$11x = 198$$

$$x = 18$$

$$m\widehat{GR} = 4(18) + 25 = 97^\circ$$

Diameter RM is the hypotenuse of the legs 8 and 15
Therefore, diameter of circle is 17.

Arc length GR = $17 \pi \dfrac{97}{360}$

15) Find x:

180

60

triangle inscribed in a semicircle must be a right triangle...

Since all radii are congruent, the lower triangle is an equilateral triangle!

Therefore, x is 30...

16) The rhombus is 'inscribed' in the circle.

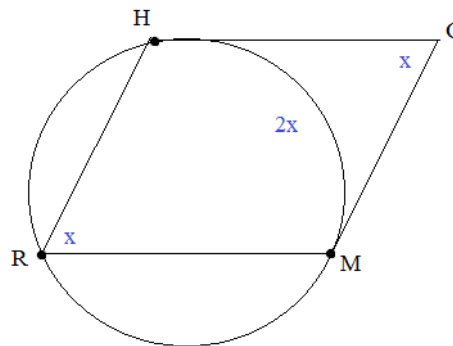What is the measure of angle HRM ?
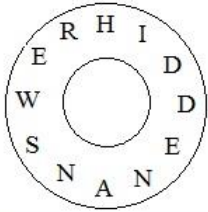
Angle O and measure of arc HM are supplementary...

(external angle formed by 2 tangents is supplementary to arc)

Angle R is 1/2 of HM (inscribed angle)

$$x + 2x = 180$$

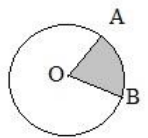$$x = 60$$

Riddle Clue: "A Math Dessert: Partially Eaten"

Letter Key:

1	2	3	4	5	6	7	8	9	0
B	E	G	I	L	P	R	T	U	Y

mathplane.com

1) π is approximately __3.14__

$$3.\boxed{1}4 \quad \underline{B}$$

2) Area of shaded region (in units^2)

A
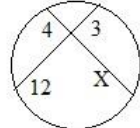Radius = 5 units
O
$\angle AOB = 72°$
B

$\text{Area}_{circle} = \pi r^2 = 25\pi$

% of circle = 72/360 = 1/5

Area of wedge = $1/5(25\pi)$

$= 5\pi$

$$\boxed{5}\pi \quad \underline{L}$$

3) Length of X:

4   3
12   X

$(4)(x) = (3)(12)$

$x = 9$

$$\boxed{9} \rightarrow \underline{U}$$
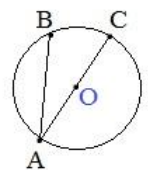
"BLUEBERRY PI"

4) $\dfrac{\text{Area of Circle O}}{\text{Perimeter of circle O}} =$
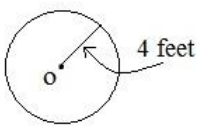
4 feet
o

$\text{Area}_{circle} = \pi r^2 = 16\pi \text{ feet}^2$

$\text{Perimater} = 2\pi r = 8\pi \text{ feet}$

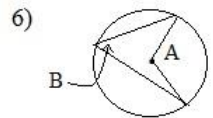$\dfrac{16\pi \text{ feet}^2}{8\pi \text{ feet}} = 2 \text{ ft}$

$$\boxed{2} \text{ feet} \quad \underline{E}$$

5) Find the arc length of $\overset{\frown}{BC}$

B   C
O
A

$\overline{AC}$ = 44 feet $\angle BOC = 90°$

$\angle BAC = 45°$

$\overset{\frown}{BC} = \dfrac{90}{360}(44\pi)$

$= 11\pi$

$$1\boxed{1}\pi \text{ feet} \quad \underline{B}$$

6)

B
A

$\dfrac{m\angle A}{m\angle B} = \dfrac{X}{(1/2)X} = 2$

$$\boxed{2} \rightarrow \underline{E}$$

$Y(Y + 5) = (2\sqrt{21})^2$

$Y^2 + 5Y = 84$

$Y^2 + 5Y - 84 = 0$

$(Y + 12)(Y - 7) = 0$

$Y = \cancel{-12} \text{ or } 7$

7) Find the length of Y

$2\sqrt{21}$
5
Y
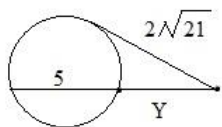
$$\boxed{7} \rightarrow \underline{R}$$

8) Find the length of $\overline{MN}$

M   N
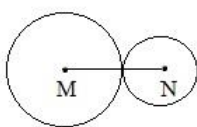
Area of Circle M = 16π radius M = 4

Circumference of Circle N = 6π radius N = 3

$\overline{MN}$ = 7

$$\boxed{7} \rightarrow \underline{R}$$

9) Area of square = 400 sq. units
Find the area of the inscribed circle.
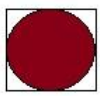
diameter = square side = 20
radius = 10
area = 100π

$$1\boxed{0}0\pi \text{ units}^2 \quad \underline{Y}$$

10) Measure of the angle formed between the hour and minute hands at 2:00?

$\dfrac{2}{12} = \dfrac{X}{360}$

x = 60

(angle is 2/12 of clock)

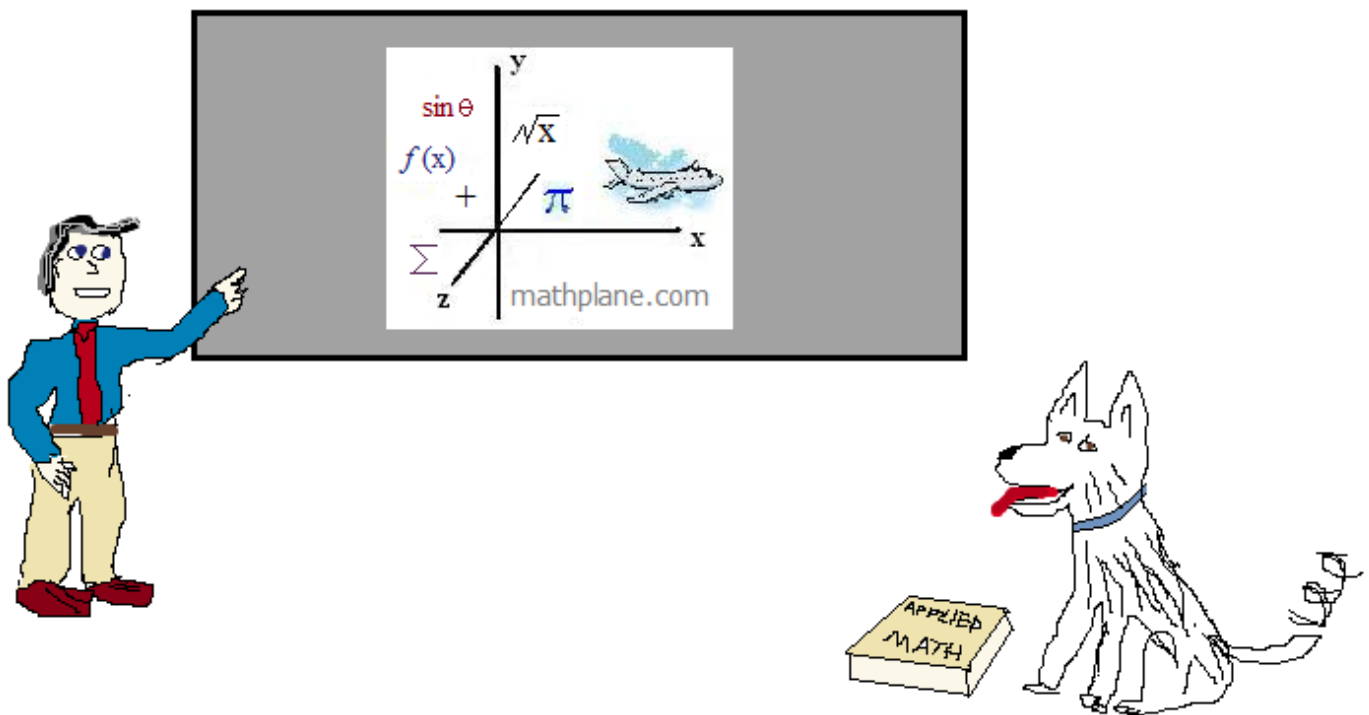$$\boxed{6}0 \text{ degrees} \quad \underline{P}$$

11) $\dfrac{\text{(Degrees in Circle)}}{\text{(Degrees in Right Angle)}} = \dfrac{360}{90} = 4$

$$\boxed{4} \rightarrow \underline{I}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy

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And, *Mathplane Express* for mobile and tablets at mathplane.ORG