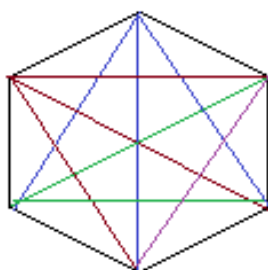
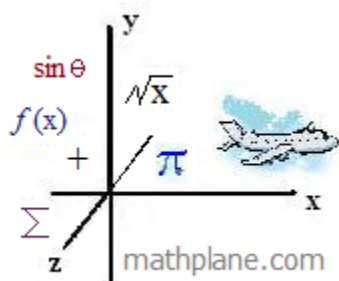


Polygons

Definitions, notes, examples, and practice test
(w/solutions)



Including concave/convex, exterior/interior angle sums,
diagonals, n-gon names, and more...

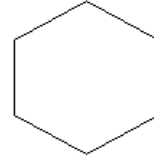
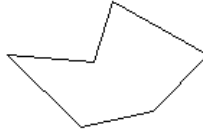


Polygons

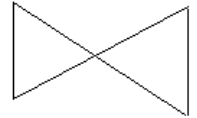
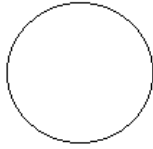
What is a *polygon*?

A closed figure in a plane where all the sides are line segments.

Polygons:



NOT polygons:



The number of sides determines the name of the polygon.

3 sides: triangle

4 sides: quadrilateral

5 sides: pentagon

6 sides: hexagon

7 sides: heptagon (or septagon)

8 sides: octagon

9 sides: nonagon

10 sides: decagon

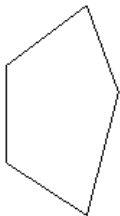
11 sides: undecagon

12 sides: dodecagon

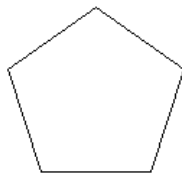
n -gon has n number of sides

example: a 25-gon polygon has 25 sides

If all the sides (and angles) are congruent, the polygon is a *regular polygon*.



Pentagon

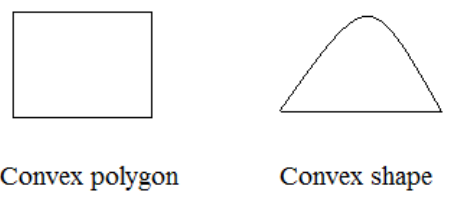


Regular Pentagon

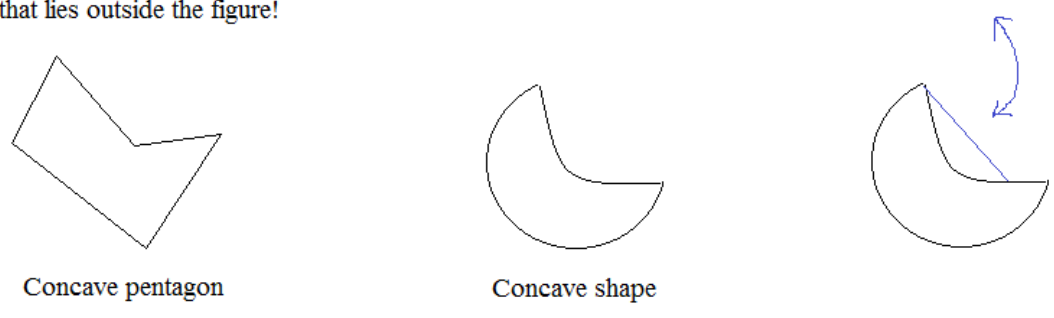
Polygons

Concave vs. Convex

A convex figure has no indentations, or if every line segment connecting every point is inside the figure.

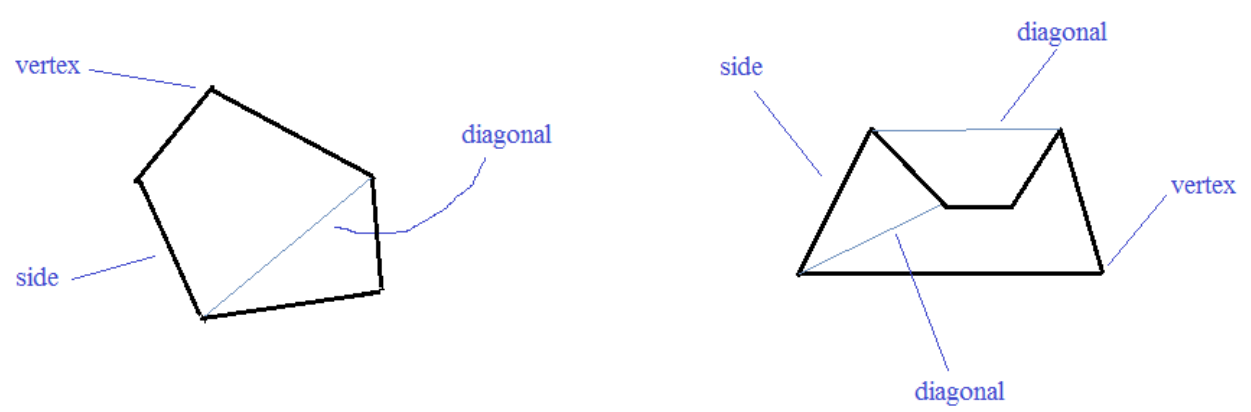


A concave figure has dents. One or more of the parts is "caved in". Therefore, there is at least one line segment connecting points that lies outside the figure!



Concave and Convex Polygons: working with vertices and diagonals

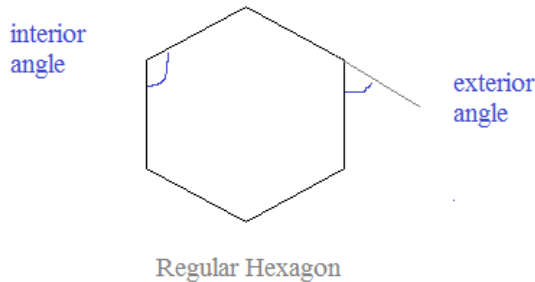
- What is a vertex? A corner point of a polygon
- What is a side? A line segment connecting 2 consecutive vertices.
- What is a diagonal? A line segment that connects 2 non-consecutive vertices



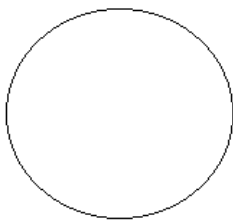
NOTE: all the diagonals in a convex polygon are inside the figure but, in a concave polygon, at least one of the diagonals goes outside the figure!

Polygons

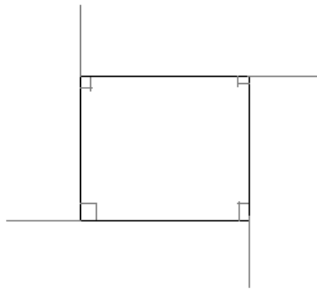
Interior and Exterior Angle measures



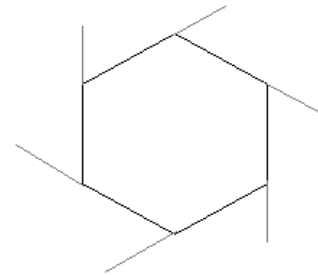
Finding the sum of the exterior angles:



360 degrees in a circle



Each exterior angle of a square is 90 degrees.
The sum: 360 degrees



In fact, the sum of the exterior angles of any polygon is 360 degrees!

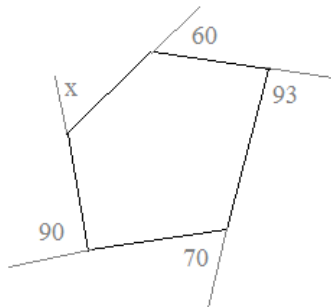
Question: what is the measure of an exterior angle of a regular hexagon?

Answer: since a regular hexagon has 6 equal sides, it has 6 congruent exterior angles.
Since the sum of the angles is 360, each angle is 60 degrees.

Question: what is the measure of an exterior angle of a non-regular pentagon?

Answer: it depends on the measure of the other 4 angles! Nevertheless, all 5 of the exterior angles must add up to 360 degrees.

Example:



$$x + 60 + 93 + 70 + 90 = 360$$

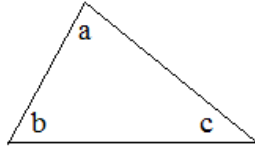
$$x + 313 = 360$$

$$x = 47 \text{ degrees}$$

polygons

Finding the sum of the interior angles of a polygon:

The sum of the interior angles of a triangle is 180 degrees



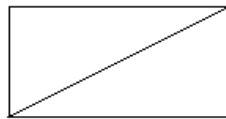
$$a^\circ + b^\circ + c^\circ = 180^\circ$$

For any polygon, the sum of the interior angles is $(n - 2) \times 180$

where n is the number of sides

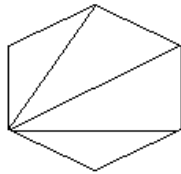
Why? Because, polygons can be cut into triangles.

Examples:



$$\begin{aligned} n &= 4 \text{ (sides)} \\ (n - 2) &\longrightarrow 2 \text{ triangles} \\ (n - 2) \times 180^\circ &= 360^\circ \end{aligned}$$

Interior angles of a quadrilateral
add up to 360 degrees...



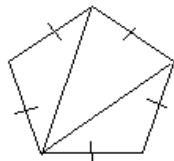
$$\begin{aligned} n &= 6 \text{ (sides)} \\ (n - 2) &\longrightarrow 4 \text{ triangles} \\ (n - 2) \times 180^\circ &= 720^\circ \end{aligned}$$

The sum of the interior angles of a
hexagon is 720 degrees.

Therefore, for any *regular polygon*, each interior angle is

$$\frac{(n - 2) \times 180^\circ}{n}$$

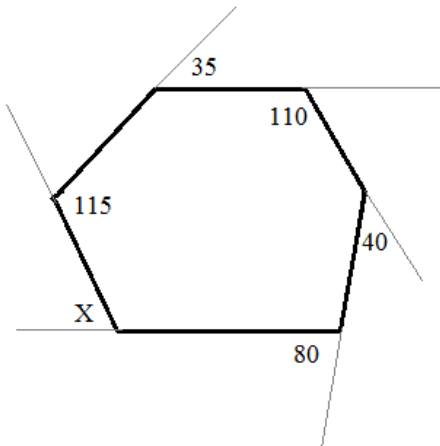
Example:



$$\begin{aligned} n &= 5 \\ (n - 2) \times 180^\circ &= 540^\circ \\ \text{then, } \frac{180(n - 2)}{n} &= 108^\circ \end{aligned}$$

Each interior angle of a *regular*
pentagon is 108 degrees.

Example:



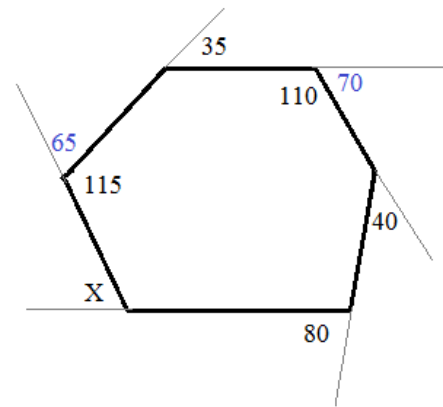
What is the measure of X?

Using supplementary angles, we can add all the exterior angles....

$$35 + 70 + 40 + 80 + x + 65 = 360$$

$$x + 290 = 360$$

$$x = 70$$



Example: What is the measure of each interior angle of a regular 18-gon?

method 1: Using the formula

$$\text{measure of interior angle of an } n\text{-gon} = \frac{180^\circ(n-2)}{n}$$

$$\text{measure in 18-gon} = \frac{180^\circ(18-2)}{18}$$

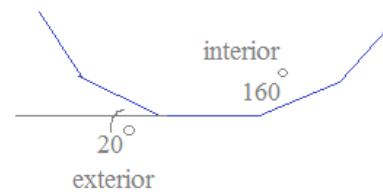
$$= \frac{2880^\circ}{18} = 160^\circ$$

method 2: Find "supplement of the exterior angle"

$$\text{measure of exterior angle of an } n\text{-gon} = \frac{360^\circ}{n}$$

$$\text{exterior angle measure in 18-gon} = \frac{360^\circ}{18} = 20^\circ$$

since each exterior angle is 20 degrees, each interior angle is 160 degrees!



Assume all the figures are *regular* polygons.

Find the angle measures.

a) _____

b) _____

c) _____

d) _____

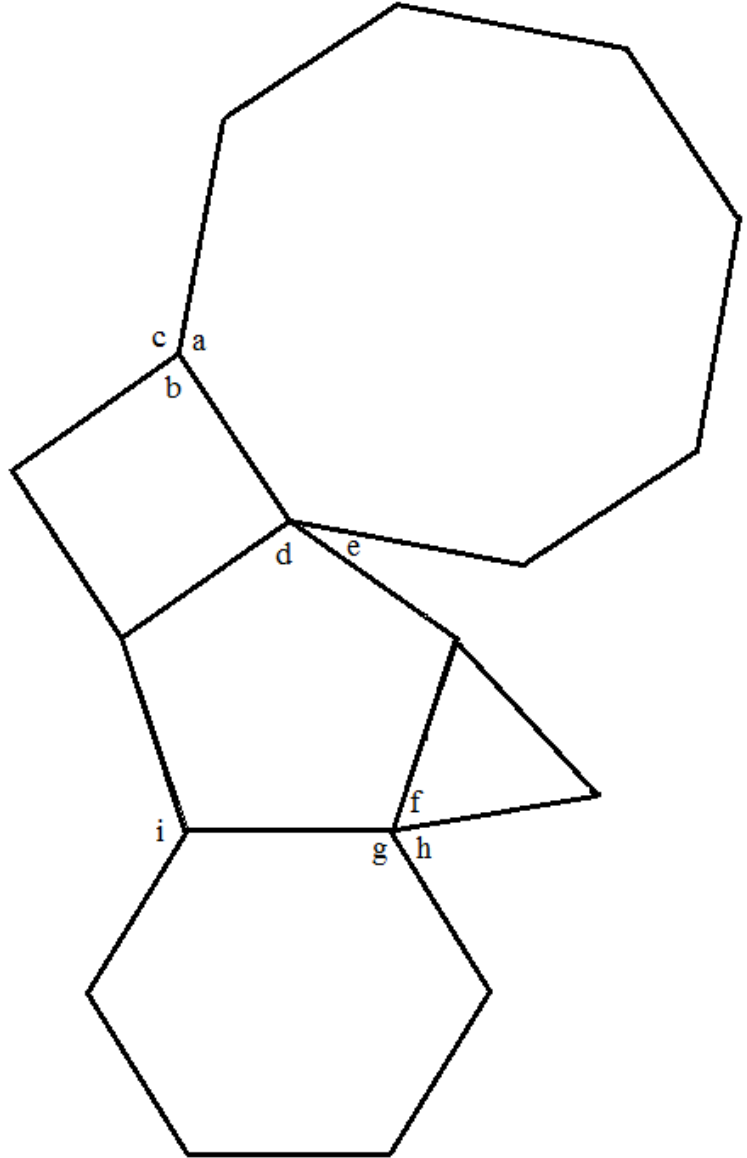
e) _____

f) _____

g) _____

h) _____

i) _____



Answers on next page →

Assume all the figures are *regular* polygons.

Find the angle measures.

SOLUTIONS

a) 135

b) 90

c) 135

d) 108

e) 27

f) 60

g) 120

h) 72

i) 132

To find the 'letter' angles, we first determine the measure of each interior angle...

triangle: 60

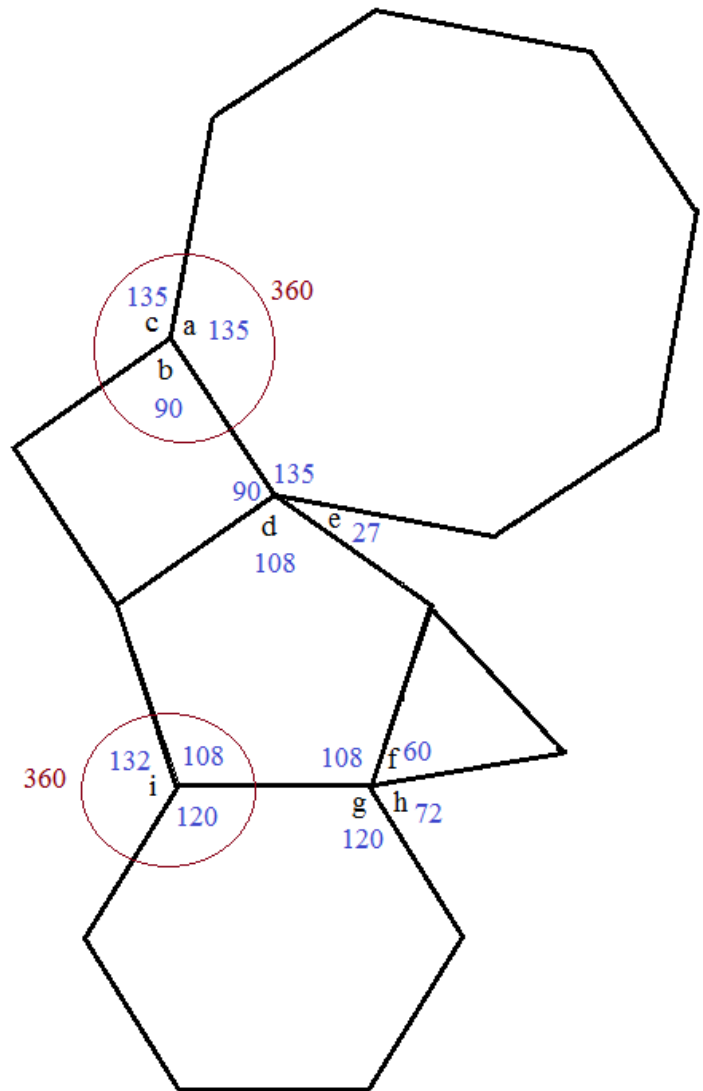
square: 90

pentagon: 108

hexagon: 120

octagon: 135

Then, we recognize that each "cluster" must have angle measures add up to 360 degrees!



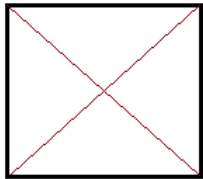
For regular polygons with n sides:	exterior angle = $\frac{360}{n}$	interior angle = $\frac{180(n-2)}{n}$
		interior angle = $180 - \left(\frac{360}{n}\right)$

What is a diagonal?

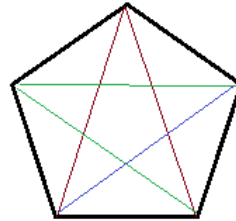
A line segment that joins 2 non-adjacent vertices of a polygon.

How many diagonals are in a polygon?

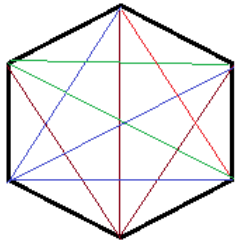
$$\frac{n(n-3)}{2} = \text{number of diagonals in n-gon}$$



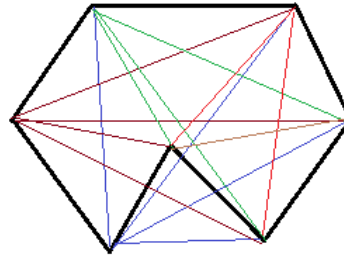
Square: 4 sides
2 diagonals



Pentagon: 5 sides
5 diagonals



Hexagon: 6 sides
9 diagonals



(convex)
Heptagon: 7 sides
14 diagonals
(some go outside the figure)



Octagon: 8 sides
How many diagonals?

answer: $n = 8$ sides

$$\# \text{ of diagonals} = \frac{8(8-3)}{2}$$

= 20 diagonals

Formula Explanation:

You can draw a line segment to every other vertex except 3: the 2 adjacent vertices and itself...

$$(n - 3)$$

And, there are n vertices in an n -sided polygon..

$$n(n - 3)$$

Then, to eliminate the overlap (i.e. double counting: a segment from A to B is identical to a segment from vertex B to vertex A), divide the total in half!

$$\frac{n(n-3)}{2} = \text{number of diagonals in n-gon}$$

Product Placement

"Suppose you have three Ford Mustangs. Inside each -- on the plush, leather seats -- is a six-pack of Coca-Cola...."

"How many cans of refreshing, ice cold Coke are there?"

Multiplication: Word Problems



LanceAF #41 (7-14-12)
www.mathplane.com

"... and, the product of 6 and 3 is 18 cans of Coke."

Multiplication: Word Problems

$$\frac{6 \text{ cans}}{\text{mustang}} \times 3 \text{ mustangs} = 18 \text{ cans}$$

JUST DO IT.



SLOPE

$$\text{McDonald's logo} = \frac{\text{rise}}{\text{run}}$$

Volume of Sphere

$$\text{AT&T globe logo} = \frac{4}{3} \pi r^3$$

presented by AT&T

A new way of funding education

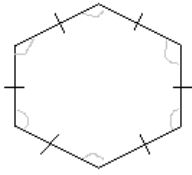
PRACTICE QUIZ (with SOLUTIONS)

Polygons Quiz

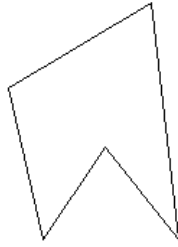
I. Classifying Polygons

Match the figure with its description:

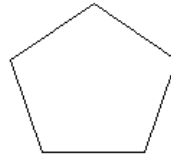
1)



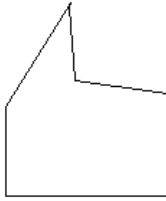
2)



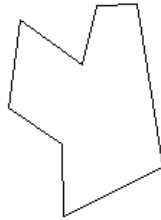
3)



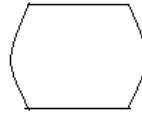
4)



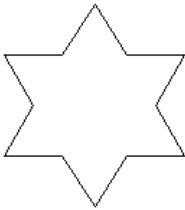
5)



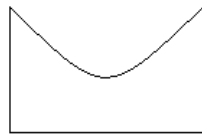
6)



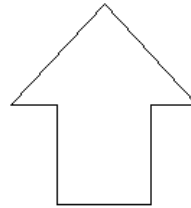
7)



8)



9)

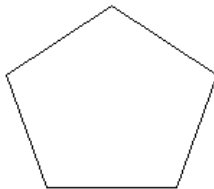


- a) regular pentagon
- b) concave pentagon
- c) regular hexagon
- d) convex pentagon
- e) non-polygon
- f) concave octagon
- g) convex octagon
- h) hexagon
- i) quadrilateral
- j) dodecagon
- k) heptagon

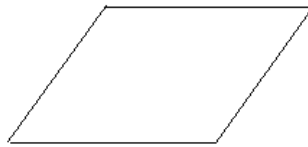
II. Polygon Parts

1) Draw the diagonals in the polygons

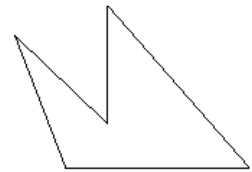
a)



b)



c)



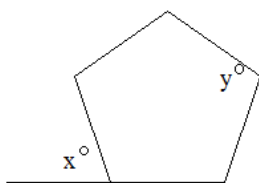
2) How many vertices are in a nonagon?

3) How many diagonals are in a triangle?

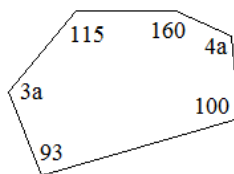
III. Interior and Exterior Angles

Determine the variables:

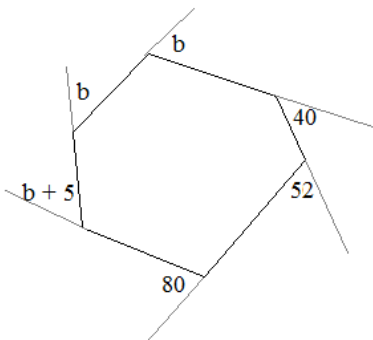
1) Given: A regular pentagon



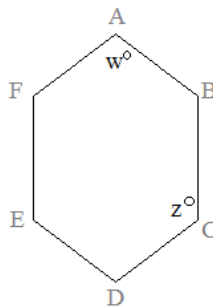
2) Given: Degree measures of interior angles



3) Given: Degree measures of exterior angles



4) Given: The polygram as shown



$$\angle B \cong \angle C \cong \angle E \cong \angle F$$

$$\angle A \cong \angle D$$

Angle A is 15 degrees less than angle B

IV. Miscellaneous

1) In a regular octagon, what is the measure of an interior angle? Exterior angle?

2) What is the sum of the interior angles of a 18-gon (polygon)?

3) An interior angle of an n -sided regular polygon is 144 degrees. How many sides are there? (i.e. what is n ?)

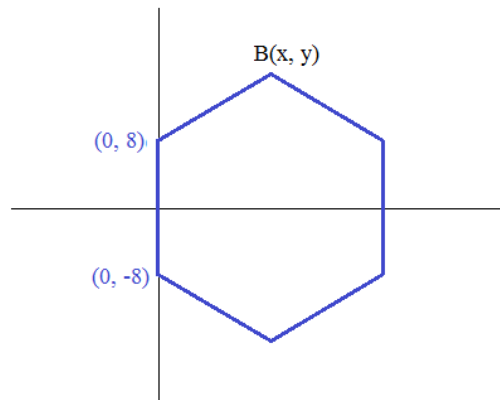
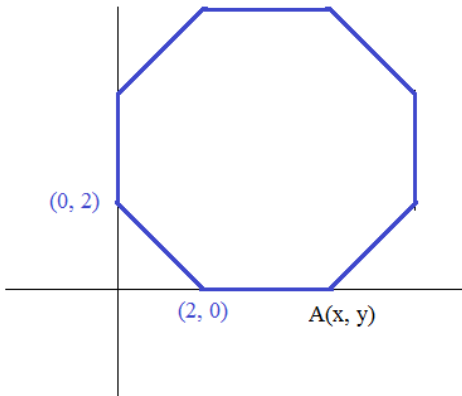
Challenge: How many diagonals are in a convex octagon?
 How many diagonals are in a regular 20-gon?
 How many diagonals exist for a given polygon?

V. Extra math topics

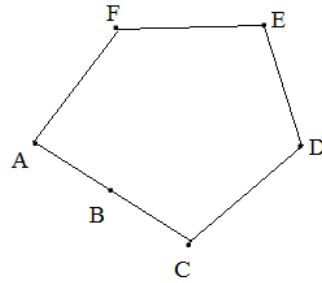
- 1) A decagon contains 7 angles that total 1220.
Of the 3 remaining angles, exactly 2 are supplementary and exactly 2 are complementary.
What are the 3 angles?

- 2) In an equiangular polygon, each *exterior* angle is 25% of the measure of each *interior* angle.
What is the name of the polygon?

- 3) Identify the coordinates of each vertex in the following regular polygons:
(contains topics beyond basic geometry)



4) Is the figure ABCDEF a polygon? Explain.



5) The sum of 5 angles inside an "octagon" is 400 degrees.
What can you conclude about this figure?

6) Find the angles formed by

a) 2 consecutive radii

b) the radius and adjoining side

in a regular 1) pentagon

a)

b)

2) hexagon

a)

b)

3) octagon

a)

b)

4) decagon

a)

b)

7) The sum of the measures of the interior angles of a regular polygon is 5040.
How many sides in this polygon?

8) True or False

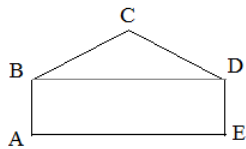
- a) A regular polygon is equilateral
- b) An equilateral polygon is regular.
- c) When the midpoints of each side of a rhombus are consecutively joined, the figure is a rhombus.
- d) A scalene quadrilateral can have 2 congruent angles.

9) ABCDE is a regular semi-hexagon

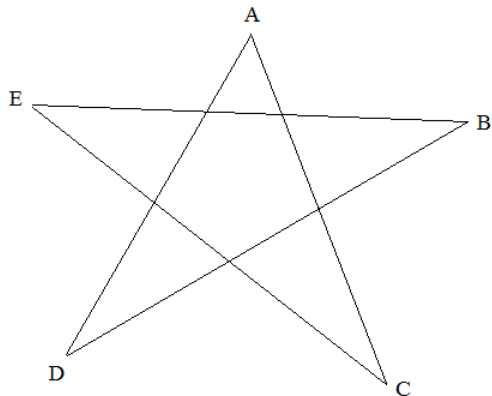
If $\angle C = 2x + 3y + 10$

Find x and y.

$\angle CDB = x - 3y + 20$



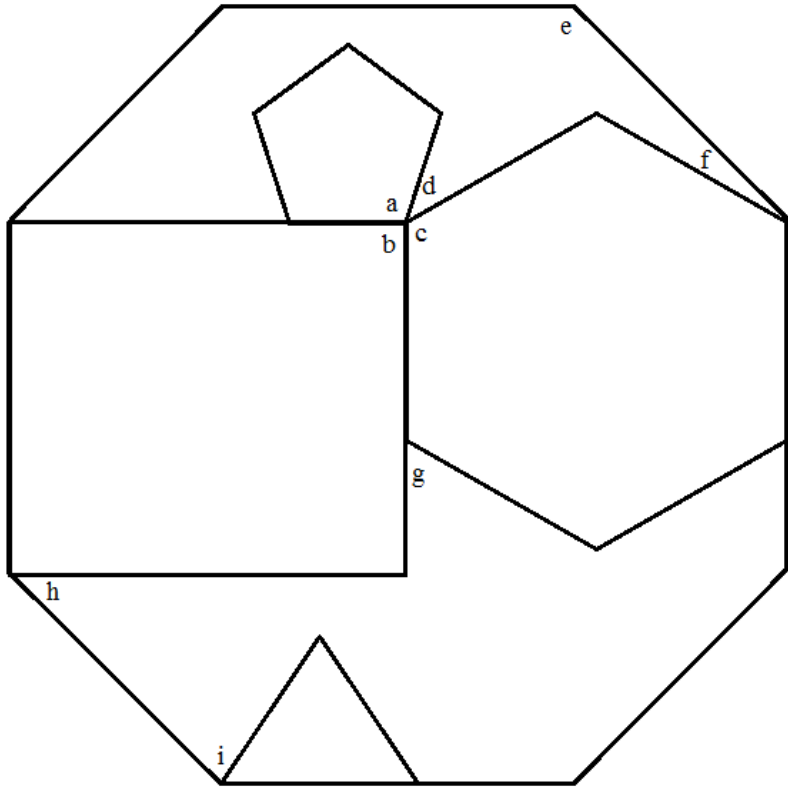
10) ***Challenge: What is the sum of the angle measures A, B, C, D, and E?

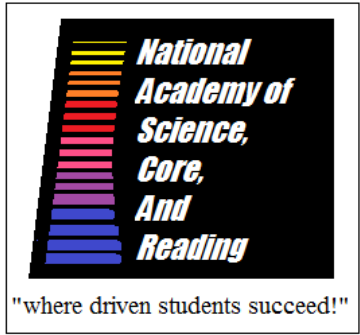
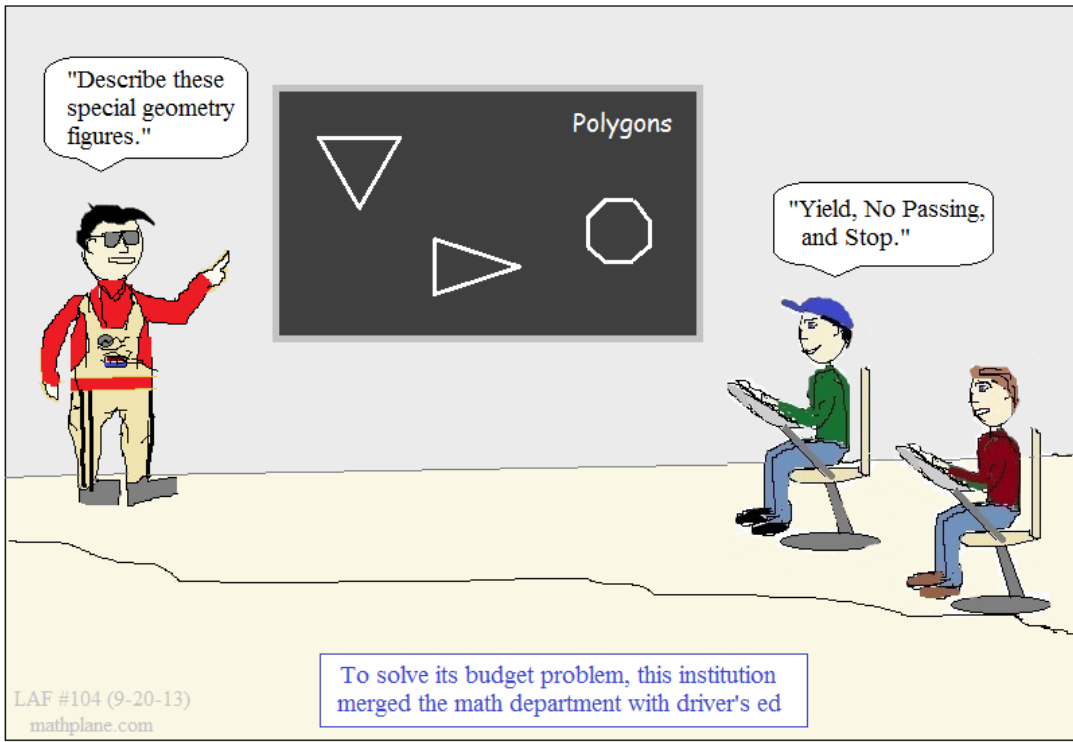


Assume all the figures are *regular* polygons.

Find the angle measures.

- a) _____
- b) _____
- c) _____
- d) _____
- e) _____
- f) _____
- g) _____
- h) _____
- i) _____

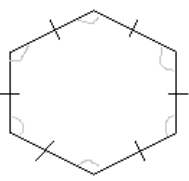
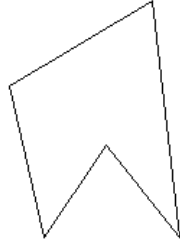
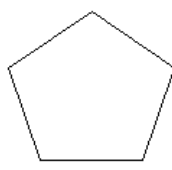
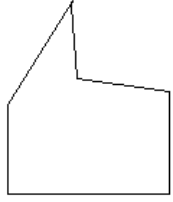
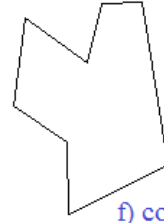
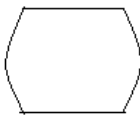
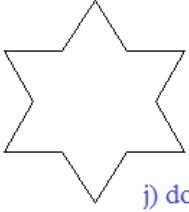
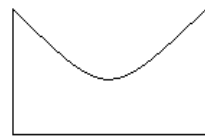
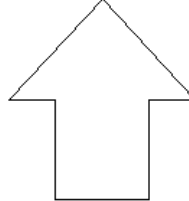




Solutions-→

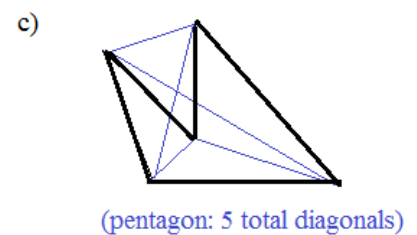
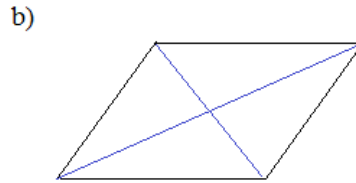
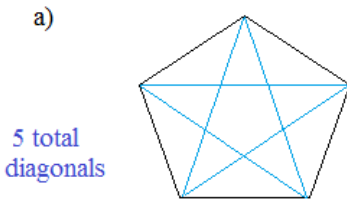
I. Classifying Polygons

Match the figure with its description:

1) 	2) 	3) 	(if the sides/angles were congruent, then it is regular as well...)	a) regular pentagon
c) regular hexagon	b) concave pentagon	d) convex pentagon		b) concave pentagon
4) 	5) 	6) 	e) non-polygon (not all sides are line segments)	c) regular hexagon
h) hexagon (concave)	f) concave octagon (8-sides)			d) convex pentagon
7) 	8) 	9) 	k) heptagon (7-sided)	e) non-polygon
j) dodecagon (12-sided polygon)	e) non-polygon			f) concave octagon
				g) convex octagon
				h) hexagon
				i) quadrilateral
				j) dodecagon
				k) heptagon

II. Polygon Parts

1) Draw the diagonals in the polygons



- 2) How many vertices are in a nonagon? A nonagon has 9 sides, so it has 9 vertices
- 3) How many diagonals are in a triangle? A triangle has ZERO diagonals (because there are no 'non-consecutive' sides)

III. Interior and Exterior Angles

SOLUTIONS

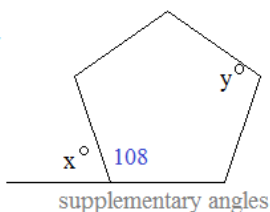
Polygons Quiz

Determine the variables:

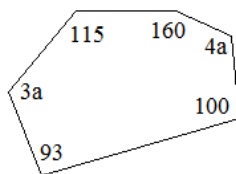
1) Given: A regular pentagon

$$\frac{360}{5} = 72$$

$$\begin{aligned} x &= 72 \\ y &= 108 \end{aligned}$$



2) Given: Degree measures of interior angles



6 sides, so the sum of interior angles is

$$(6 - 2) \times 180 = 720 \text{ degrees}$$

$$3a + 4a + 115 + 160 + 100 + 93 = 720$$

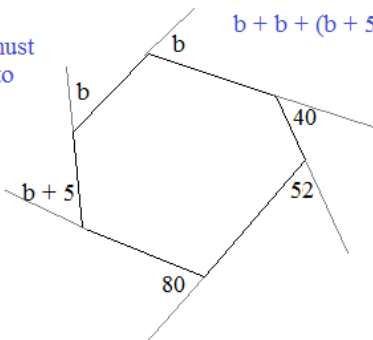
$$7a = 252$$

$$a = 36$$

3) Given: Degree measures of exterior angles

4) Given: The polygon as shown

exterior angles must add up to 360

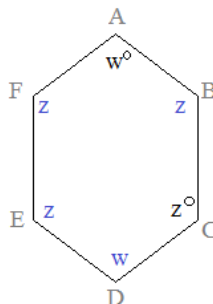


$$b + b + (b + 5) + 40 + 52 + 80 = 360$$

$$3b + 177 = 360$$

$$3b = 183$$

$$b = 61$$



$$\angle B \cong \angle C \cong \angle E \cong \angle F$$

$$\angle A \cong \angle D$$

Angle A is 15 degrees less than angle B

Sum of interior angles is 720

$$2w + 4z = 720$$

$$w + 15 = z$$

using substitution:

$$2w + 4(w + 15) = 720$$

$$6w + 60 = 720$$

$$6w = 660$$

$$w = 110 \quad z = 125$$

IV. Miscellaneous

1) In a regular octagon, what is the measure of an interior angle? Exterior angle?

the sum of exterior angles is 360...

therefore, each exterior angle is $360/8 = 45$ degrees

then, each interior angle is supplementary:

$$180 - 45 = 135 \text{ degrees}$$

2) What is the sum of the interior angles of a 18-gon (polygon)?

$$n = 18 \text{ sides}$$

sum of interior angles

$$= (n - 2) \times 180 = (18 - 2) \times 180 = 2880 \text{ degrees}$$

3) An interior angle of an n -sided regular polygon is 144 degrees. How many sides are there? (i.e. what is n ?)

If the interior angle of a regular polygon is 144 degrees, then each exterior angle is 36 degrees..

$360/36$ degrees ----> 10 sided figure....

$$\text{OR, } \frac{(n - 2) \times 180}{n} = 144$$

$$144n = 180(n - 2)$$

$$144n = 180n - 360$$

$$-36n = -360$$

$$n = 10$$

NOTE: each interior and exterior angle are supplementary

Challenge: How many diagonals are in a convex octagon?

How many diagonals are in a regular 20-gon?

How many diagonals exist for a given polygon?

Find a pattern:

3 sides: 0 diagonals

4 sides: 2 diagonals

5 sides: 5 diagonals

6 sides: 9 diagonals

7 sides: 14 diagonals

From each vertex, there are $n - 3$ non-consecutive vertices...

So, for octagon there are 5 available vertices for each vertex... $5 \times 8 = 40$ total diagonals..

THEN, divide by two to avoid 'double counting'..

20 diagonals...

Again, 20 sides implies 17 diagonals for each vertex...

340 total diagonals..

then, divide by 2 to discount 'double counting'.

$$340/2 = 170 \text{ diagonals..}$$

$$\# \text{ of diagonals} = \frac{(n - 3)n}{2}$$

- 1) A decagon contains 7 angles that total 1220.
 Of the 3 remaining angles, exactly 2 are supplementary and exactly 2 are complementary.
 What are the 3 angles?

Let x , y , and z be the 3 angles:

sum of interior angles of decagon

$$1440$$

$$1440 - 1220 = 220$$

2 angles are complementary $\rightarrow x + y + z = 220$

2 angles are supplementary $\rightarrow x + y = 90$

$\rightarrow y + z = 180$

(solve using substitution)

$$x = 90 - y$$

$$z = 180 - y$$

$$(90 - y) + y + (180 - y) = 220$$

$$270 - y = 220$$

$$y = 50$$

since $y = 50$,

$x = 40$,

and $z = 130$

- 2) In an equiangular polygon, each exterior angle is 25% of the measure of each interior angle.
 What is the name of the polygon?

Method A: Use the formulas

$$\text{Exterior Angle} = \frac{360}{n}$$

$$\text{Interior Angle (regular polygon)} = \frac{(n-2)180}{n}$$

$$\frac{360}{n} = .25 \left(\frac{(n-2)180}{n} \right)$$

$$\frac{360}{n} = \frac{.25(180n - 360)}{n}$$

$$360 = 45n - 90$$

$$n = 10$$

Decagon

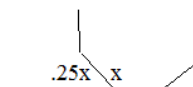
Method B: Recognize exterior/interior angles are supplementary

$$x + .25x = 180$$

$$1.25x = 180$$

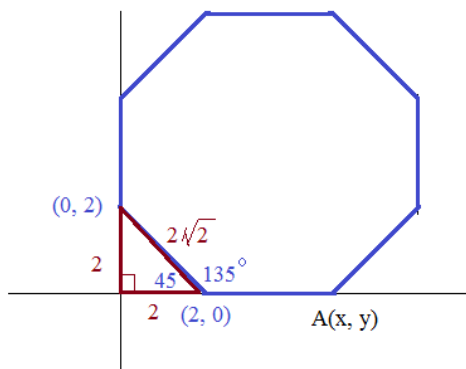
$$x = 144$$

and, $.25x = 36$



Since the exterior angle is 36, the polygon is 10-sided

- 3) Identify the coordinates of each vertex in the following regular polygons:
 (contains topics beyond basic geometry)

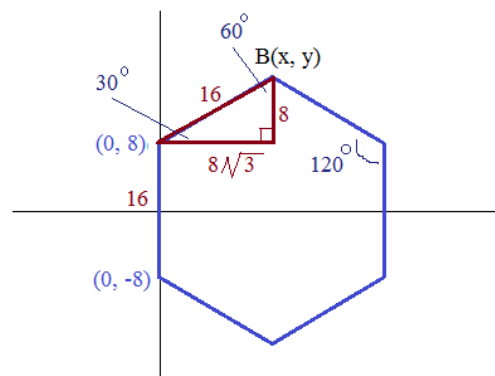


The exterior angles of a regular octagon: $360/8 = 45$ degrees
 The interior angles are 135 degrees...

Note: a 45-45-90 right triangle has side ratios $1:1:\sqrt{2}$

Since the length of each side is $2\sqrt{2}$,

the coordinate of A is $(2 + 2\sqrt{2}, 0)$



The exterior angles of a regular hexagon are $360/6 = 60$

The interior angles are $\frac{(6-2)180}{6} = 120$

Distance from $(0, 8)$ to $(0, -8)$ is 16 units..
 so, each side is 16 units...

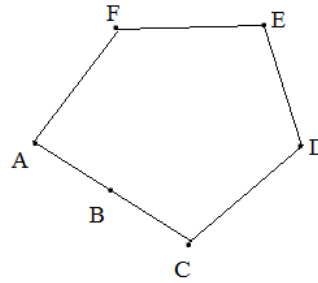
Note: a 30-60-90 right triangle has side ratios $1:\sqrt{3}:2$

the coordinate of B is $(8\sqrt{3}, 16)$

4) Is the figure ABCDEF a polygon? Explain.

This is NOT a polygon..

The polygon ABCDEF describes a figure with 6 vertices, and therefore 6 sides...
However, this figure only has 5 sides...



SOLUTIONS

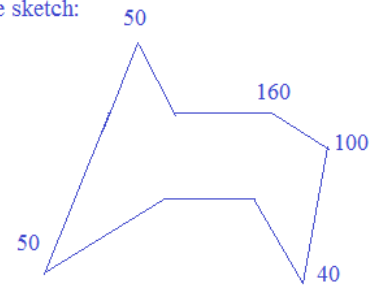
5) The sum of 5 angles inside an "octagon" is 400 degrees.
What can you conclude about this figure?

The sum of the interior angles of an octagon would be

$(8 - 2)(180) = 1440$... So, if 5 of the angles are 400 degrees, then the other 3 must add up to 890?!?!?

The figure must be CONCAVE..

possible sketch:



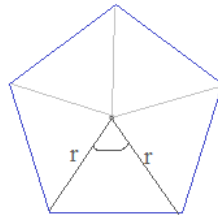
6) Find the angles formed by

a) 2 consecutive radii

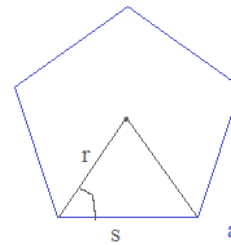
b) the radius and adjoining side

in a regular 1) pentagon

- a) 72°
- b) 54°



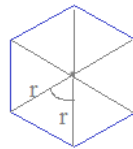
$$360 \div 5 = 72^\circ$$



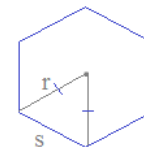
Since the 5 triangles are isosceles....
angle + angle + $72 = 180$
angle = 54°

2) hexagon

- a) 60°
- b) 60°



$$360 \div 6 = 60^\circ$$



Since the radii angle is 60, the other two angles add up to 180..

3) octagon

- a) 45°
- b) 67.5°

An octagon has 8 congruent radii angles...

$$360 \div 8 = 45^\circ$$

Since the radii angle is 45 degrees.. the 2 base angles must add up to 135 degrees...

$$67.5 \text{ and } 67.5$$

4) decagon

- a) 36°
- b) 72°

Decagon has 10 sides.. 10 congruent radii angles.. 36 degrees

The radius/side angles are 72 degrees...

7) The sum of the measures of the interior angles of a regular polygon is 5040.
How many sides in this polygon?

SOLUTIONS

$$\text{Sum of measures of interior angles} = (n - 2) \cdot 180$$

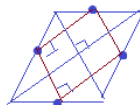
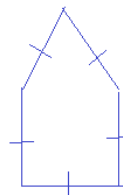
$$5040 = (n - 2) \cdot 180$$

$$28 = n - 2$$

$$n = 30 \text{ sides}$$

8) True or False

- a) A regular polygon is equilateral True: a regular polygon is equilateral and equiangular
- b) An equilateral polygon is regular. False: a polygon can be equilateral, but not equiangular..
- c) When the midpoints of each side of a rhombus are consecutively joined, the figure is a rhombus. False: the result is a rectangle
- d) A scalene quadrilateral can have 2 congruent angles.



True : here is an example

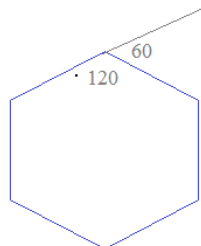
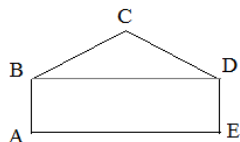


A scalene quadrilateral has 4 different side lengths...

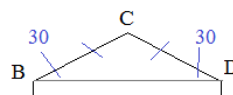
9) ABCDE is a regular semi-hexagon

$$\text{If } \angle C = 2x + 3y + 10$$

$$\angle CDB = x - 3y + 20$$



since $\triangle BCD$ has 2 congruent sides, it is isosceles.. and, the opposite angles are congruent... (sum of 3 angles = 180)



$$120 + 30 + 30 = 180 \text{ degrees..}$$

$$120 = 2x + 3y + 10$$

$$30 = x - 3y + 20$$

$$110 = 2x + 3y$$

$$10 = x - 3y$$

$$120 = 3x$$

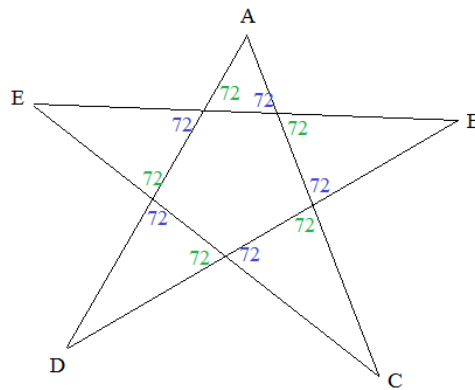
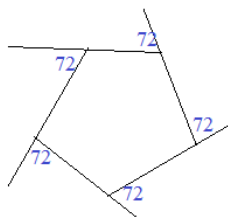
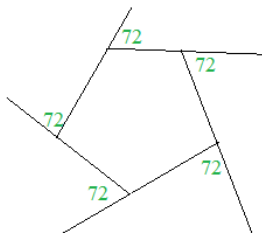
$$x = 40 \text{ and } y = 10$$

10) ***Challenge: What is the sum of the angle measures A, B, C, D, and E?

Looking at the inside pentagon, we know the exterior angle measures add up to 360 degrees..

(Let's assume it's a regular pentagon...)

Then, looking at each isosceles triangle, we know the vertex angle must be 36 (because $36 + 72 + 72$ equals 180)



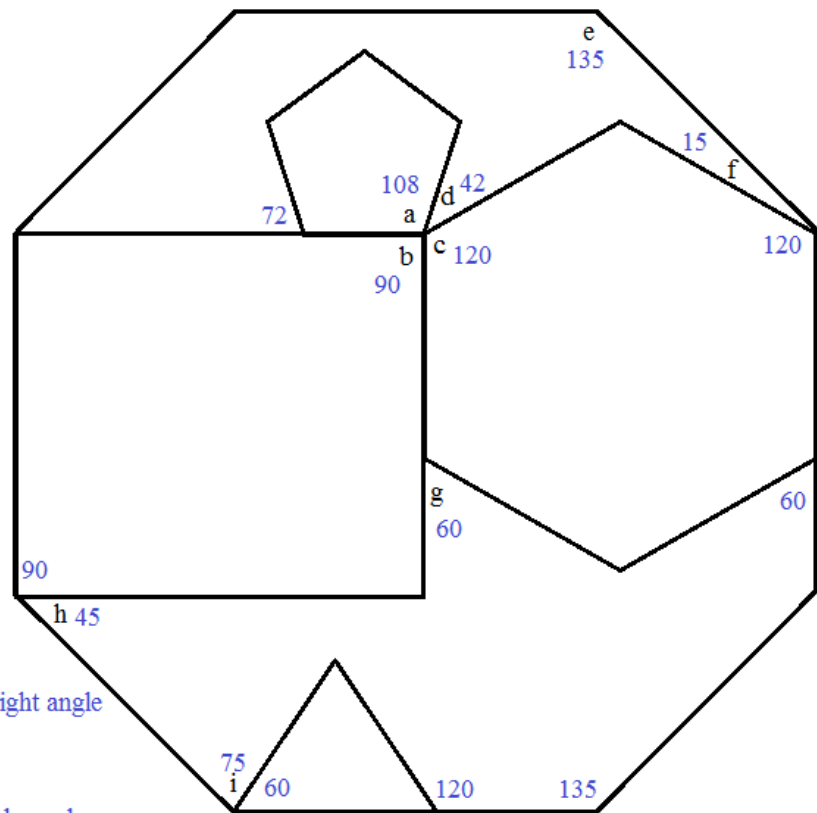
$$A + B + C + D + E = 180 \text{ degrees...}$$

SOLUTIONS

Assume all the figures are *regular* polygons.

Find the angle measures.

- a) 108 interior angle of pentagon
- b) 90 each angle in a square is a right angle
- c) 120 hexagon -- 6 sides: each exterior angle is 60.. each interior is 120
- d) 42 sum of a, b, c, and d must be 360 degrees
- e) 135 regular octagon has exterior angles 45.. interior angles 135
- f) 15 octagon angle – hexagon angle
 $135 - 120 = 15$
- g) 60 exterior angle of regular hexagon
- h) 45 octagon interior angle – square right angle
 $135 - 90 = 45$
- i) 75 octagon angle – equilateral triangle angle
 $135 - 60 = 75$



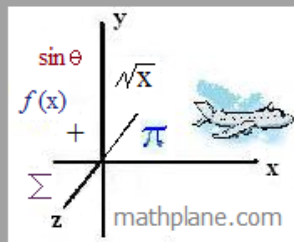
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Good luck,

Lance

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One more Polygon Question:

What is the name of the regular polygon whose ratio of interior angle measure to exterior angle measure is 4:1?

Answer on next page....

What is the name of the regular polygon
whose ratio of each interior angle measure to each exterior angle measure is 4:1?

In a polygon, each interior angle is supplementary to its exterior angle!

If the measures are 4:1, then $4x + 1x = 180$

$$5x = 180$$

$$x = 36 \quad \text{and} \quad 4x = 144$$

If a regular polygon has an exterior angle measure of 36, then it has $\frac{360}{36} = 10$ sides

A polygon with 10 sides is a *decagon*.

