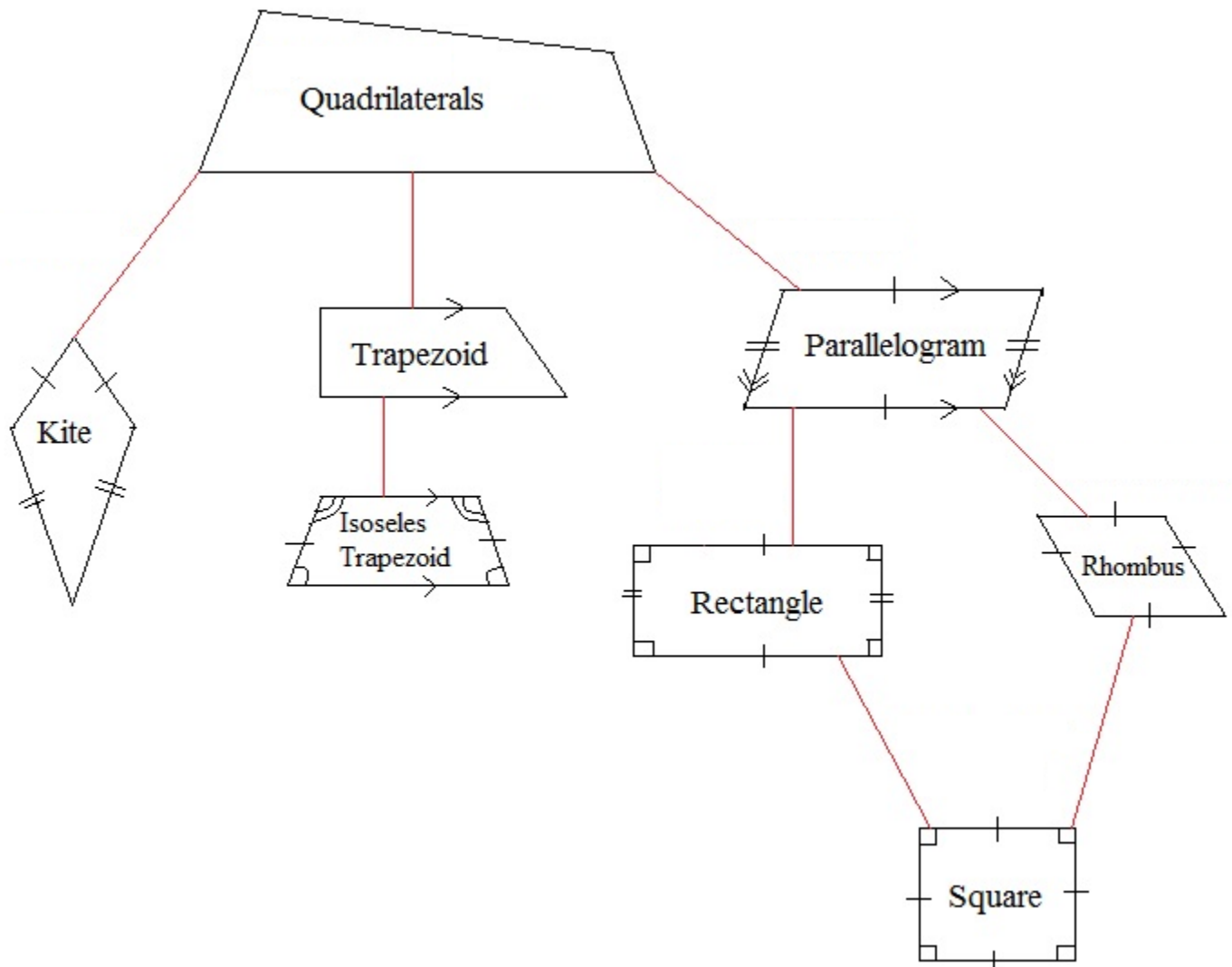
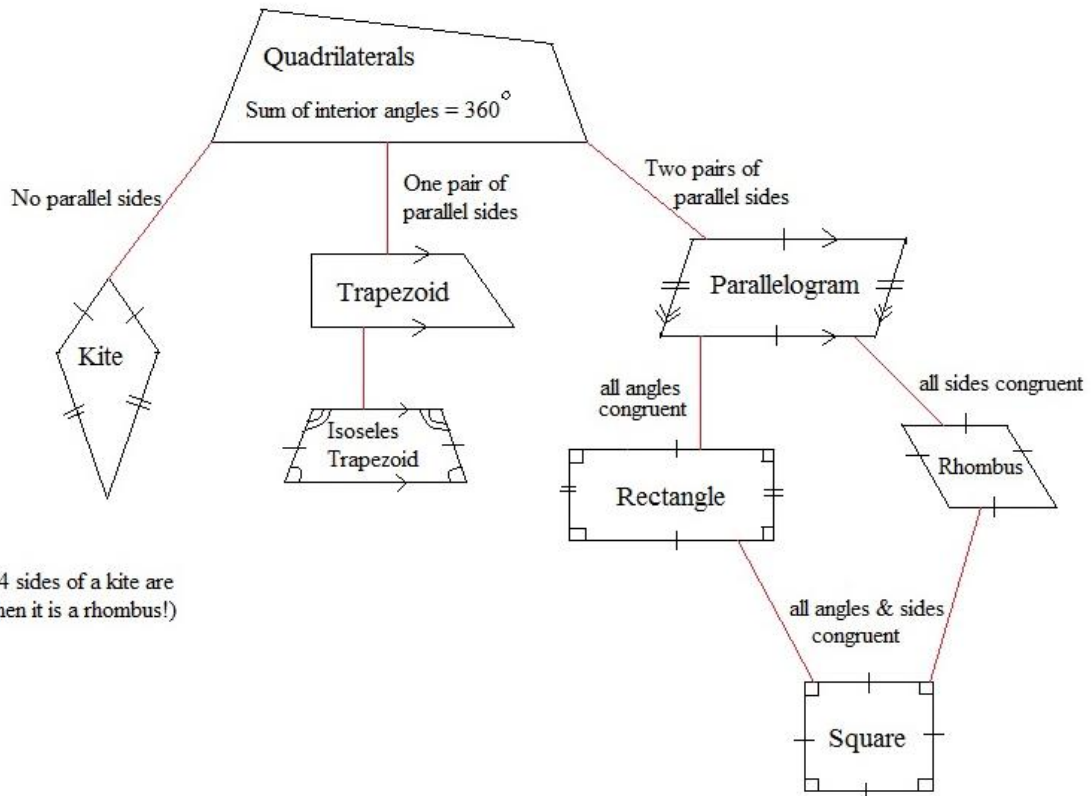


Geometry: Special Quadrilaterals

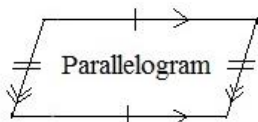


Definitions, notes, & practice test (w/solutions)



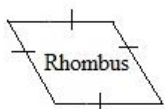
(Note: If all 4 sides of a kite are congruent, then it is a rhombus!)

Characteristics of Special Quadrilaterals



- Opposite Sides are parallel
- Opposite Sides are congruent
- Opposite Angles are congruent

— Also, can be proven that diagonals bisect each other



- Opposite sides are parallel
- Opposite angles are congruent
- All Sides are congruent

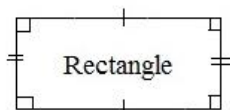
— Also, can be proven that

- 1) diagonals bisect each other
- 2) diagonals bisect the angles
- 3) diagonals are perpendicular to each other



- Two pairs of congruent sides
 - One diagonal bisects the angles (the other may or may not)
- This diagonal is "line of symmetry"

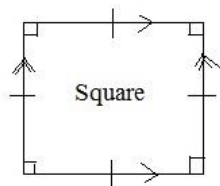
— Also, can be proven that diagonals are perpendicular



- All angles are congruent (90°)
- Opposite sides are congruent

— Also, can be proven that diagonals are congruent and bisect each other

Characteristics of Special Quadrilaterals (continued)

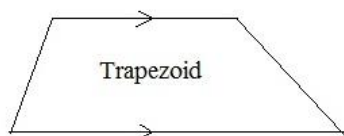


Square

- Opposite sides parallel
- All sides congruent
- All angles are right angles

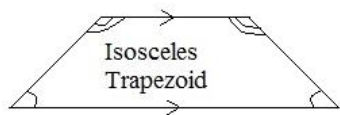
— Also, can be proven that

- 1) Diagonals bisect each other
- 2) Diagonals are perpendicular to each other
- 3) Diagonals are congruent
- 4) Diagonals form 4 congruent triangles



Trapezoid

- One pair of opposite sides parallel ('bases')
- (Other pair of opposite sides are 'legs')



Isosceles Trapezoid

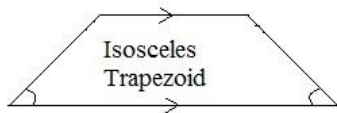
- One pair of opposite sides parallel
- Base angles are congruent
- Legs are congruent

— Also, can be proven that

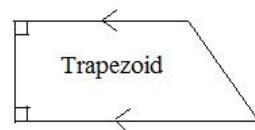
- 1) Lower base angles are supplementary to upper base angles
- 2) Diagonals are congruent

Question: If a trapezoid has 2 congruent angles, is it necessarily isosceles?

Not necessarily!!



Isosceles Trapezoid



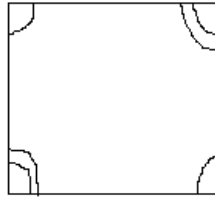
Trapezoid

Describing a quadrilateral from labels

What is this figure?

It *looks like* a square.

But, it is not...

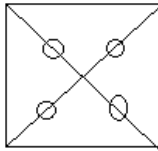


Since all sides may (or may not) be congruent, it is not necessarily a square... (or, a rhombus)

Since all angles are not (labeled) congruent, it is not necessarily a rectangle...

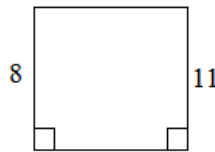
But, opposite angles are congruent... This occurs for ALL parallelograms...

Here are other examples:

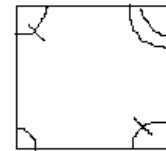


Rectangle

Diagonals congruent AND bisect each other.
(all squares, rectangles...
But, not all parallelograms)

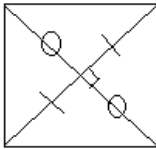


Trapezoid



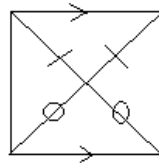
Kite

One pair of opposite angles are congruent.
The others are not..



Rhombus

Diagonals are perpendicular... So, could be a kite or square... However, kite may only have 1 perpendicular bisector. This quad. has 2.
A square has congruent diagonals. This quad. does not...



Isosceles Trapezoid

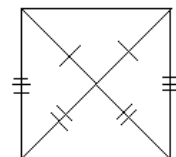


Quadrilateral

Identifying Special Quadrilaterals

Example: What 4-sided figure is best represented by the following diagram?

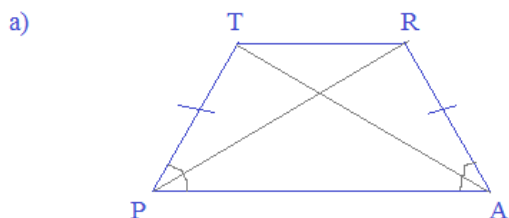
- Sketch the quadrilateral
- Write a 2-column proof to justify your answer
- Use a coordinate proof to verify your answer



The figure has opposite sides congruent and diagonals congruent.

It may be a square, rectangle, or isosceles trapezoid...

Since only 1 pair of opposite sides are congruent, it must be an *isosceles trapezoid*.



Non-parallel, opposite sides are congruent.

Base angles are congruent.

- b) Two-column proof: Prove the diagonals of an isosceles trapezoid are congruent.

Statements	Reasons
1. Isosceles trapezoid TRAP	1. Given
2. $\overline{TP} \cong \overline{RA}$	2. Definition of Isosceles Trapezoid
3. $\angle TPA = \angle RAP$	3. Def. of Isosceles Trapezoid
4. $\overline{PA} \cong \overline{PA}$	4. Reflexive Property
5. $\triangle TPA \cong \triangle RAP$	5. Side-Angle-Side (SAS) (2, 3, 4)
6. $\overline{RP} \cong \overline{TA}$	6. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

- c) Coordinate proof:

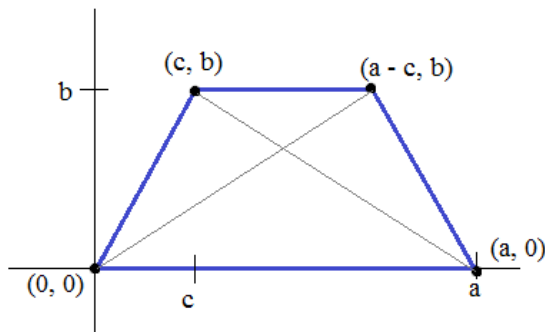
Using the distance formula, prove the lengths are the same!

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

distance from (c, b) to $(a, 0)$: $\sqrt{(c - a)^2 + b^2}$

distance from $(a - c, b)$ to $(0, 0)$: $\sqrt{(a - c)^2 + b^2}$

since $(a - c)^2 = (c - a)^2$, the lengths are equal!



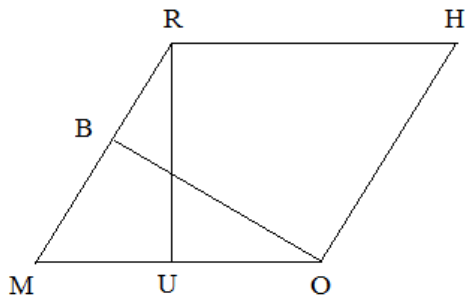
Here is a proof that utilizes the properties of a special quadrilateral!

Given: RHOM is a rhombus

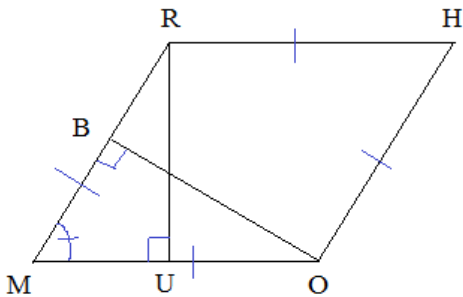
$$\overline{OB} \perp \overline{RM}$$

$$\overline{RU} \perp \overline{MO}$$

Prove: $\overline{OB} \cong \overline{RU}$



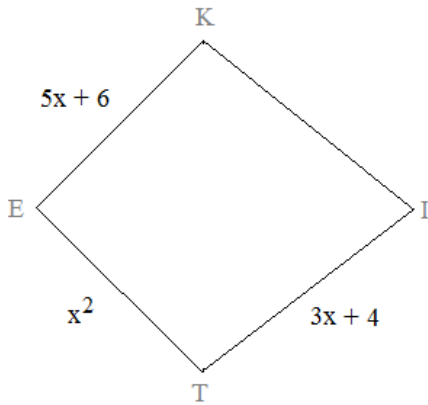
Statements	Reasons
1. RHOM is a rhombus	1. Given
2. $\overline{RM} \cong \overline{MO}$	2. Definition of Rhombus (all sides congruent)
3. $\overline{OB} \perp \overline{RM}$ $\overline{RU} \perp \overline{MO}$	3. Given
4. $\angle OBM$ and $\angle RUM$ are right angles	4. Definition of Perpendicular (perpendicular lines form right angles)
5. $\angle OBM \cong \angle RUM$	5. All right angles congruent
6. $\angle M = \angle M$	6. Reflexive property
7. $\triangle RUM = \triangle OBM$	7. AAS (Angle-Angle-Side) (5, 6, 2)
8. $\overline{OB} \cong \overline{RU}$	8. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)



Strategy: label all the given parts
Rhombus: all sides congruent

Look for congruent triangles (CPCTC)

Example: What is the perimeter of the kite?
 (NOTE: There is more than one answer!)



Definition of a kite: Quadrilateral with 2 pairs of adjacent congruent sides

Assume $\overline{KE} \cong \overline{ET}$

$$5x + 6 = x^2$$

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$x = -1 \text{ or } 6$$

then, $x = -1$

$$\begin{aligned} KE &= 1 \\ ET &= 1 \\ TI &= 1 \\ KI &= 1 \end{aligned}$$

4

and, $x = 6$

$$\begin{aligned} KE &= 36 \\ ET &= 36 \\ TI &= 22 \\ KI &= 22 \end{aligned}$$

116

Now, assume $\overline{ET} \cong \overline{TI}$

$$x^2 = 3x + 4$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = -1 \text{ or } 4$$

then, $x = 4$

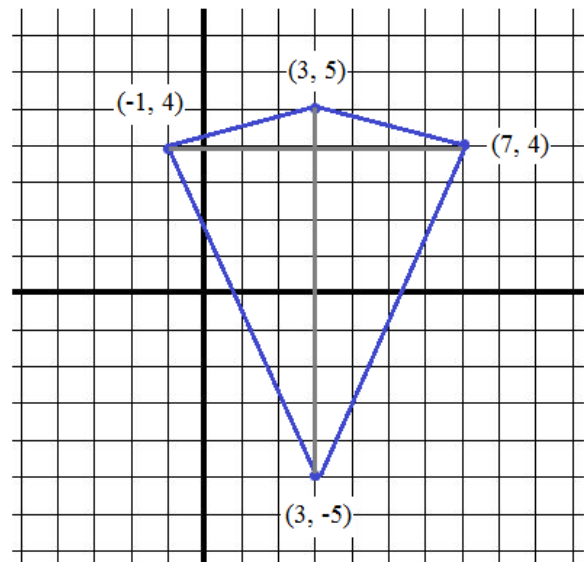
$$\begin{aligned} KE &= 26 \\ KI &= 26 \\ ET &= 16 \\ TI &= 16 \end{aligned}$$

84

Example: Describe the figure with vertices $(-1, 4)$ $(7, 4)$ $(3, 5)$ $(3, -5)$.

Diagonals are perpendicular
 (slopes are opposite reciprocals)

Two pairs of disjointed sides are congruent
 (distance formula verifies lengths)

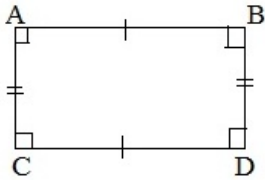


EXAMPLE: Prove diagonals of a rectangle are congruent and bisect each other.

Given: Rectangle $ABDC$

Prove: $\overline{BC} = \overline{AD}$ and $\overline{BC}, \overline{AD}$ bisect each other

PROOF

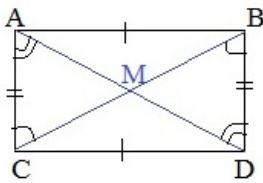


1) $AC = BD$
 $AB = CD$

Definition of Rectangle

(opposite sides are congruent)
 (all angles are congruent; right angles)

$AB \parallel CD$
 $AC \parallel BD$

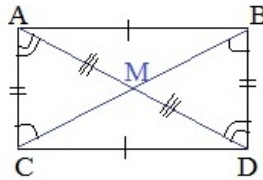


2) $\angle CBD \cong \angle BCA$
 $\angle CAD \cong \angle BDA$

Parallel lines cut by a transversal, then alternate interior angles are congruent

3) $\triangle ACM = \triangle DBM$

Congruent triangles
 Angle-Side-Angle



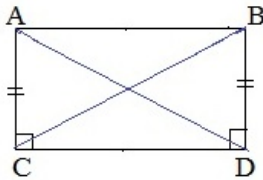
4) $\overline{AM} = \overline{DM}$
 $\overline{CM} = \overline{BM}$

CPCTC

5) \overline{BC} and \overline{AD} bisect each other

Definition of Bisector

(A line, ray, or segment that cuts a segment into 2 congruent parts)



6) $\overline{CD} = \overline{CD}$

Reflexive Property

7) $\angle C = \angle D = 90^\circ$
 $\overline{AC} = \overline{BD}$

Definition of Rectangle

8) $\triangle ACD = \triangle BDC$

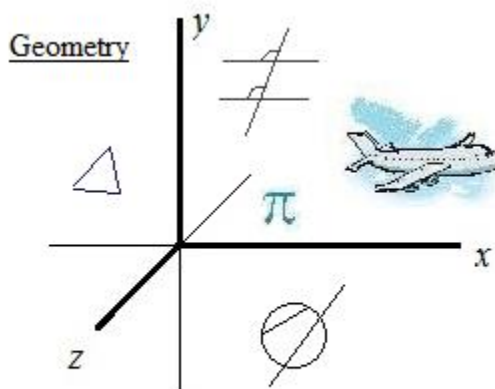
Congruent triangles
 Side-Angle-Side

9) $\overline{BC} = \overline{AD}$

CPCTC

Note: Pythagorean theorem can show that diagonals are equal

PRACTICE QUIZZES (w/ SOLUTIONS)



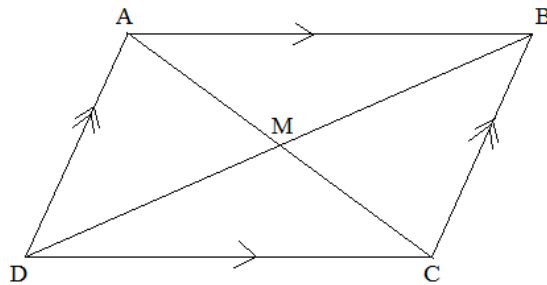
Parallelograms Quiz

I. List 5 properties of parallelograms.

- 1) Opposite sides are parallel
- 2)
- 3)
- 4)
- 5)

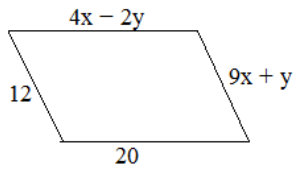
II. For parallelogram ABCD, answer and explain why:

- 1) $\overline{AB} \cong$ _____
- 2) $\overline{DM} \cong$ _____
- 3) $\angle AMD \cong$ _____
- 4) $\angle BCD \cong$ _____
- 5) $180^\circ - m\angle BAD = m\angle$ _____
- 6) $2\overline{AM} =$ _____

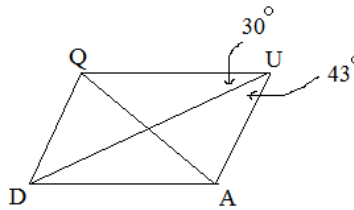


III. Solve:

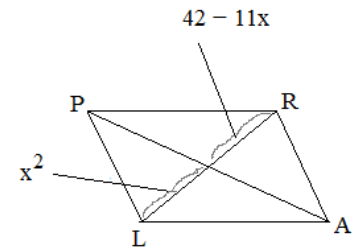
(Assume each quadrilateral is a parallelogram)



Find x and y:



Find: $\angle UDA$
 $\angle UQD$

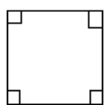


Find the length of \overline{LR} :

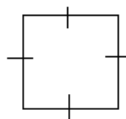
Special Quadrilateral Properties

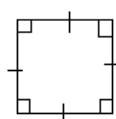
I. Give the most accurate description of each quadrilateral.

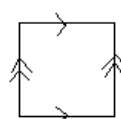
Example:

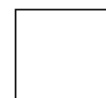


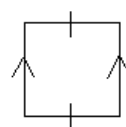
Answer: rectangle (4 congruent angles -- equiangular)

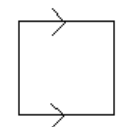


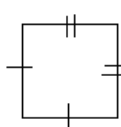


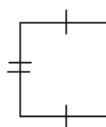










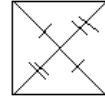
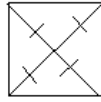
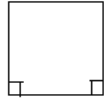
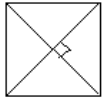


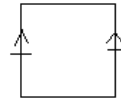
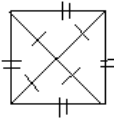
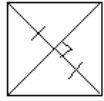
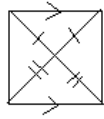
II. Always/Sometimes/Never

- 1) A square is a rhombus.
- 2) A rhombus is a square.
- 3) A quadrilateral is convex.
- 4) A kite is equiangular.
- 5) A parallelogram has 3 congruent sides.
- 6) Consecutive angles in a parallelogram are supplementary.
- 7) The diagonals of a kite are perpendicular.
- 8) A rhombus is a trapezoid.
- 9) The diagonals of a parallelogram are congruent.
- 10) If one angle of a parallelogram is 90 degrees, then *all angles* must be right angles.

Special Quadrilateral Properties

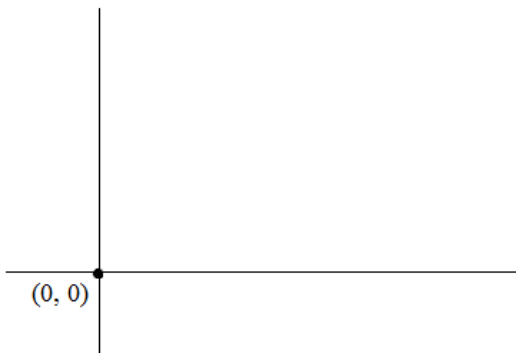
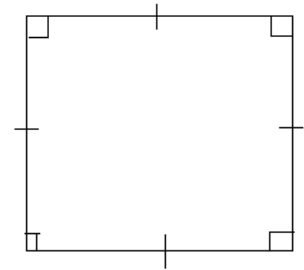
III. Give the most accurate description of the following quadrilaterals:





IV. Prove diagonals of a square are perpendicular bisectors. Use a 2-column proof. Then, verify with a coordinate proof.

Statements	Reasons



Coordinate Geometry and Quadrilaterals

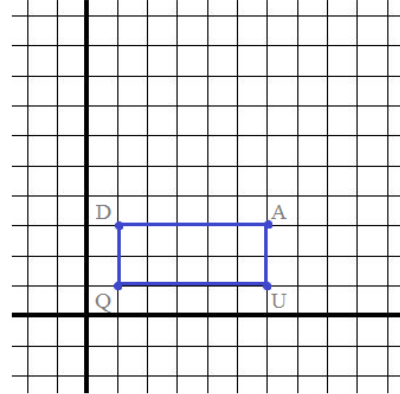
- 1) $Q(1, 1)$ $U(6, 1)$ $A(6, 3)$ $D(1, 3)$

What is the quadrilateral QUAD?

How can you move ('translate') \overline{DA} to make the figure a square?

How can you move ('translate') \overline{AU} to make the figure a square?

Describe a translation necessary to form a parallelogram.
(there are more than 1!)

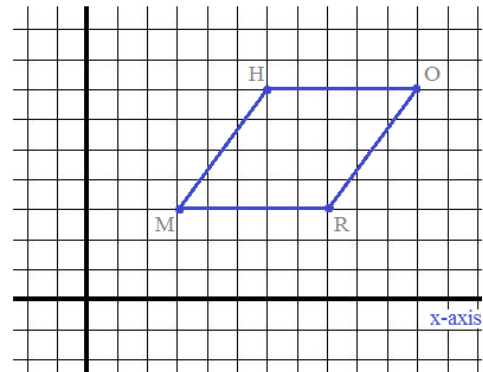


- 2) $H(6, 7)$ $O(11, 7)$ $R(8, 3)$ $M(3, 3)$

Find the length of \overline{MR} .

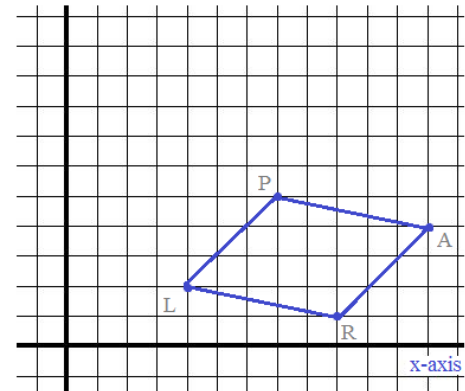
Find the length of \overline{HM} .

What is the quadrilateral ROHM?



- 3) $P(7, 5)$ $A(12, 4)$ $R(9, 1)$ $L(4, 2)$

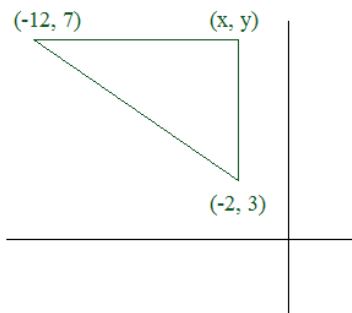
Verify that PARL is a parallelogram.
(Hint: What is the definition of a parallelogram?)



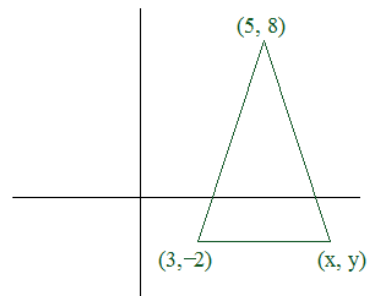
Identify the missing coordinates. Then, find the area of each figure.

Quadrilaterals, Triangles, and Coordinates

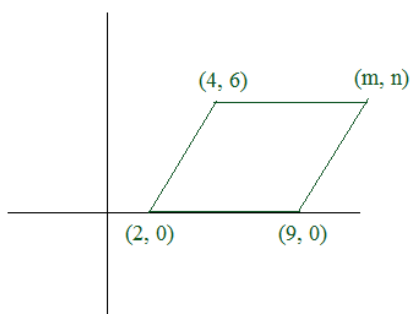
1) Right Triangle



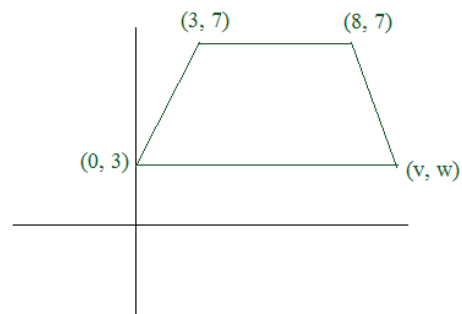
2) Isosceles Triangle



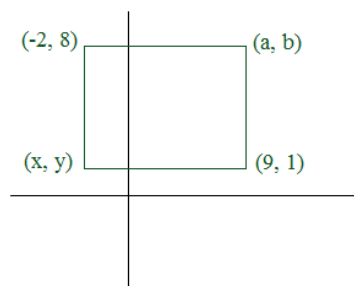
3) Parallelogram



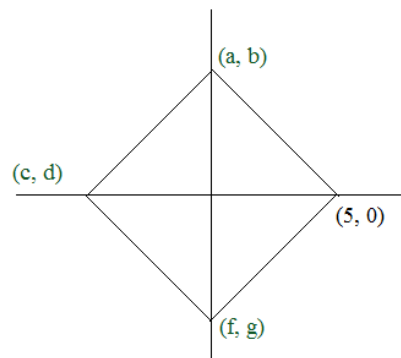
4) Isosceles Trapezoid



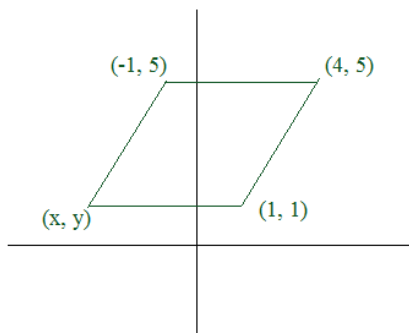
5) Rectangle



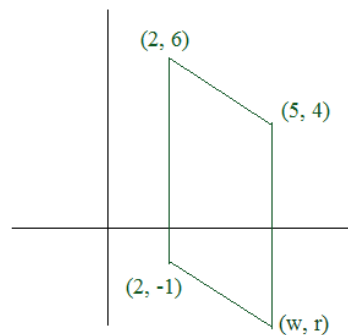
6) Square

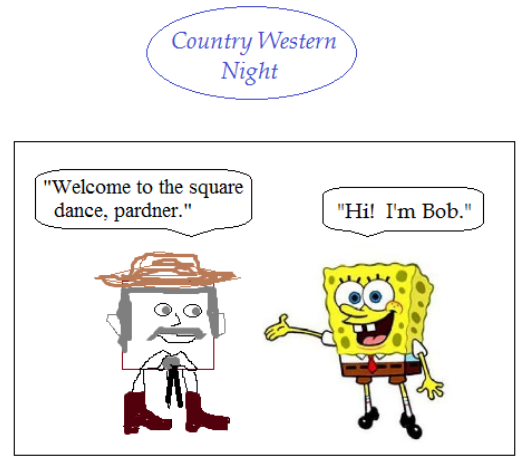
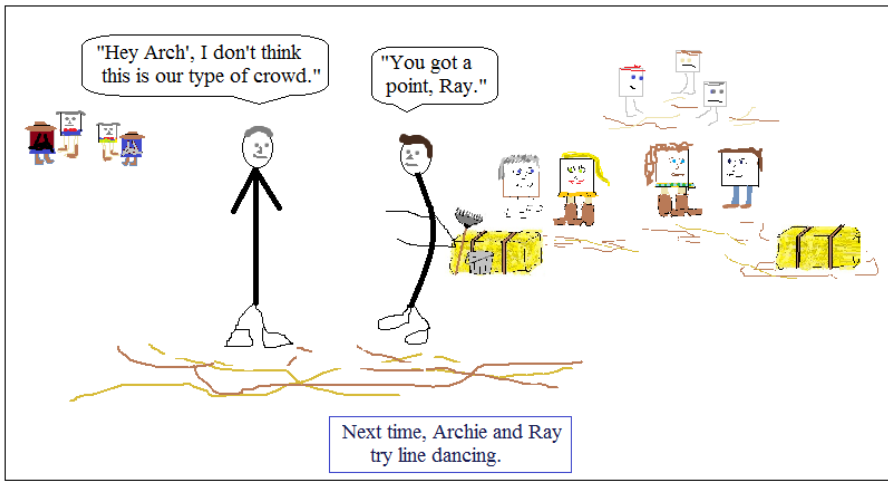
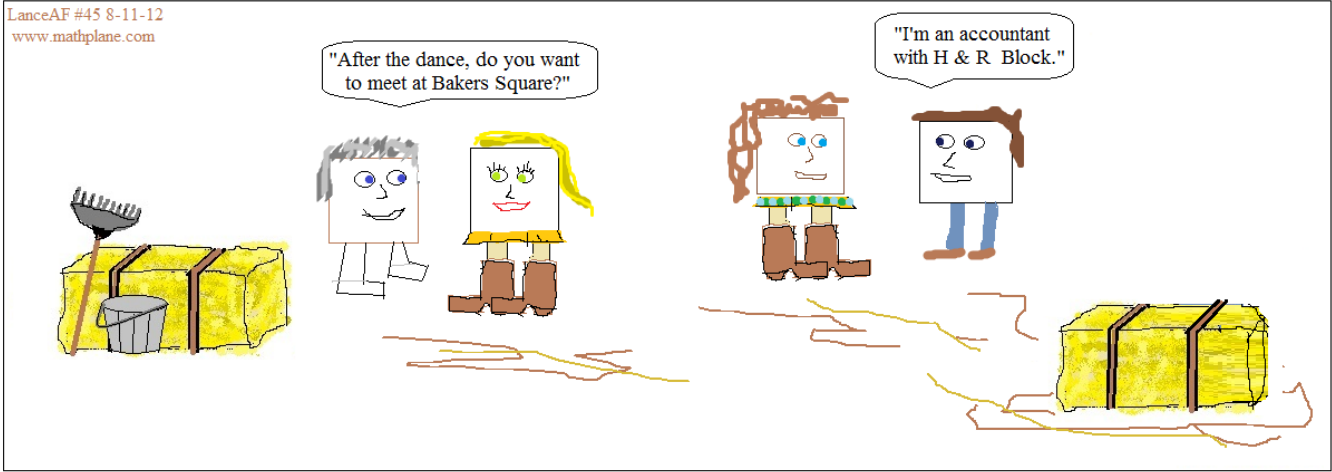


7) Rhombus



8) Parallelogram

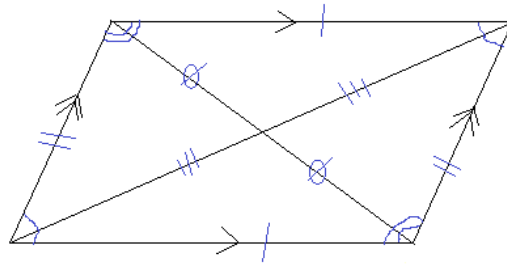




SOLUTIONS

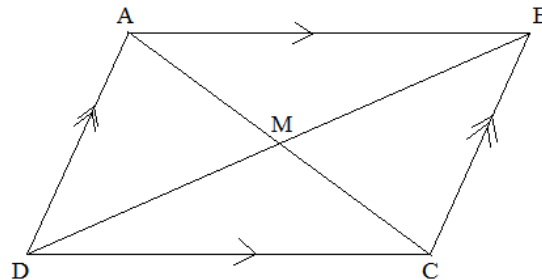
I. List 5 properties of parallelograms.

- 1) Opposite sides are parallel
- 2) Opposite sides are congruent
- 3) Opposite angles are congruent
- 4) Consecutive angles are supplementary
- 5) Diagonals bisect each other



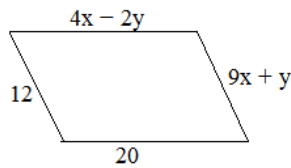
II. For parallelogram ABCD, answer and explain why:

- 1) $\overline{AB} \cong \overline{DC}$ opposite sides are congruent
- 2) $\overline{DM} \cong \overline{BM}$ diagonals bisect each other
- 3) $\angle AMD \cong \angle BMC$ vertical angles congruent
- 4) $\angle BCD \cong \angle BAD$ opposite angles congruent
- 5) $180^\circ - m\angle BAD = m\angle ABC$ OR $\angle ADC$
consecutive angles are supplementary
- 6) $2\overline{AM} = \overline{AC}$ diagonals bisect each other...



III. Solve:

(Assume each quadrilateral is a parallelogram)



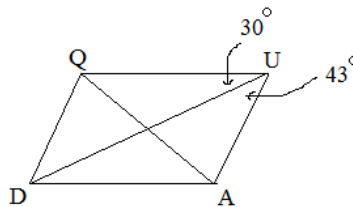
Find x and y:

$$\begin{aligned} 9x + y &= 12 \\ 4x - 2y &= 20 \end{aligned}$$

use combination method

$$\begin{aligned} 18x + 2y &= 24 \\ 4x - 2y &= 20 \\ \hline 22x &= 44 \\ x &= 2 \end{aligned}$$

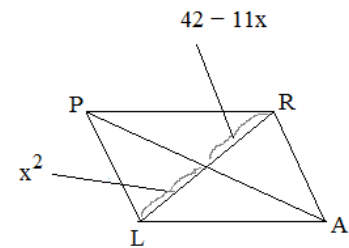
If $x = 2$, then $y = -6$



Find: $\angle UDA$ 30 degrees
(because alternate interior angles are congruent)

$\angle UQD$ 107 degrees
(because QUA is 73 degrees, and consecutive angles are supplementary)

since $x = 3$, the lengths are 9 and 9... $\overline{LR} = 18$



Find the length of \overline{LR} :

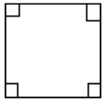
since diagonal is bisected, the segments are congruent:

$$\begin{aligned} x^2 &= 42 - 11x \\ x^2 + 11x - 42 &= 0 \\ (x + 14)(x - 3) &= 0 \\ x &= -14 \text{ or } 3 \end{aligned}$$

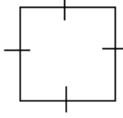
or, if $x = -14$, lengths are 196 and 196... $\overline{LR} = 392$

I. Give the most accurate description of each quadrilateral.

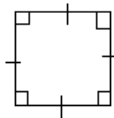
Example:



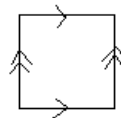
Answer: rectangle (4 congruent angles -- equiangular)



Rhombus



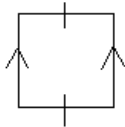
Square



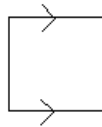
Parallelogram



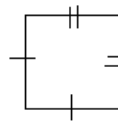
Quadrilateral



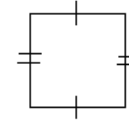
Isosceles Trapezoid



Trapezoid





Kite

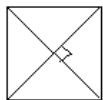


Parallelogram

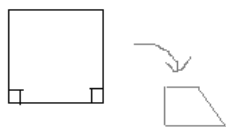
II. Always/Sometimes/Never

- 1) A square is a rhombus. Always... (rhombus is any quad. with 4 congruent sides)
- 2) A rhombus is a square. Sometimes.... (if rhombus has right angles)
- 3) A quadrilateral is convex. Sometimes....  convex  concave
- 4) A kite is equiangular. Sometimes... (if the kite is a square)
- 5) A parallelogram has 3 congruent sides. Never... It can have 2 pairs or all 4 congruent sides.
- 6) Consecutive angles in a parallelogram are supplementary. Always...
- 7) The diagonals of a kite are perpendicular. Always...
- 8) A rhombus is a trapezoid. It depends: If a trapezoid (geometry) is quad. with *exactly* one pair of parallel sides, then NEVER.... But, if a trapezoid (calculus) is quad. with *at least one* pair of parallel sides, then ALWAYS!
- 9) The diagonals of a parallelogram are congruent. Sometimes.. (EX: If the parallelogram is a rectangle.)
- 10) If one angle of a parallelogram is 90 degrees, then *all angles* must be right angles. Always....

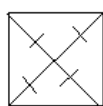
III. Give the most accurate description of the following quadrilaterals:



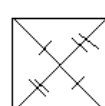
rhombus
(or a kite)



trapezoid

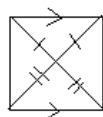


rectangle

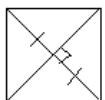


parallelogram

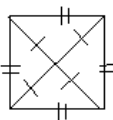
SOLUTIONS



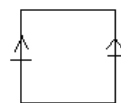
isosceles trapezoid



kite



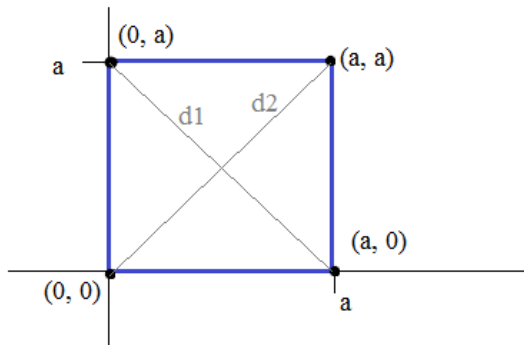
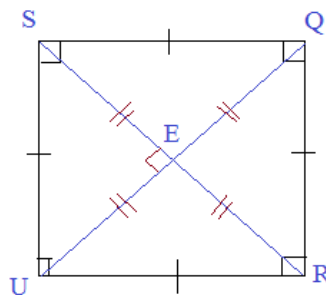
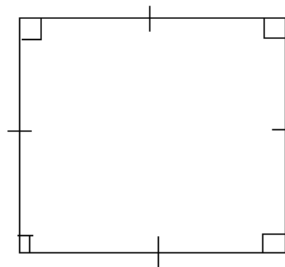
square (diagonals
and sides congruent)



parallelogram

IV. Prove diagonals of a square are perpendicular bisectors. Use a 2-column proof. Then, verify with a coordinate proof.

Statements	Reasons
1. SQRU is a square	1. Given
2. $\overline{US} \cong \overline{UR}$	2. Definition of a square (all sides congruent)
3. $\overline{QS} \cong \overline{QR}$	3. Def. of a square
4. \overline{UQ} is perpendicular bisector of \overline{SR}	4. Equidistance Theorem (if 2 points are each equidistant from the endpoints of a segment, then the 2 points determine the perpendicular bisector of that segment)
5. $\overline{SU} \cong \overline{SQ}$	5. Def. of square
6. $\overline{RU} \cong \overline{RQ}$	6. Def. of square
7. \overline{SR} is \perp bis. of \overline{UQ}	7. Equidistance Theorem



a) If the diagonals are perpendicular, then there slopes will be opposite reciprocals.

slope of d1: $\frac{(a - 0)}{(0 - a)} = -1$ slope of d2: $\frac{(a - 0)}{(a - 0)} = 1$ ✓

b) If the diagonals bisect each other, the midpoints will be the same:

midpoint of d1: $\left(\frac{a}{2}, \frac{a}{2}\right)$ midpoint of d2: $\left(\frac{a}{2}, \frac{a}{2}\right)$ ✓

- 1) Q(1, 1) U(6, 1) A(6, 3) D(1, 3)

What is the quadrilateral QUAD?

Opposite sides congruent; Equiangular; all right angles **RECTANGLE**

How can you move ('translate') \overline{DA} to make the figure a square?

shift 3 units up; D'(1, 6) A'(6, 6) (also, consider 7 units down)

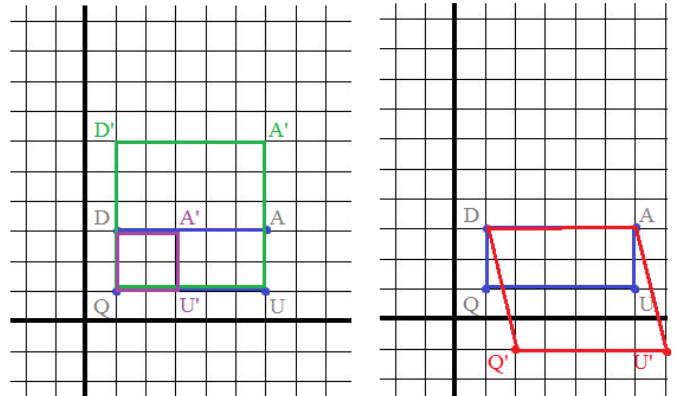
How can you move ('translate') \overline{AU} to make the figure a square?

shift 3 units left; A'(3, 3) U'(3, 1) (also, consider 7 units to the left)

Describe a translation necessary to form a parallelogram.
(there are more than 1!)

Move Q and U together: horizontally 1 unit (right) and vertically 2 units (down)
< 1, -2 >

(other possibilities: move D and A together; D and Q together;
or move A and U together)



- 2) H(6, 7) O(11, 7) R(8, 3) M(3, 3)

Find the length of \overline{MR} . M(3, 3) and R(8, 3) are 5 units apart

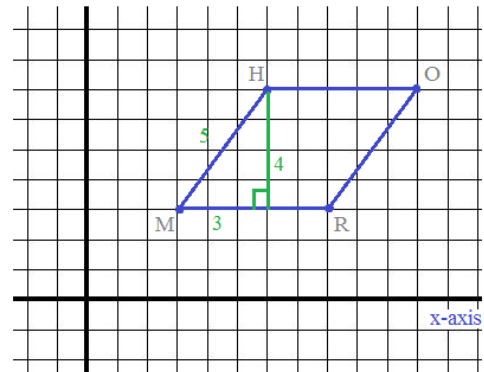
Find the length of \overline{HM} . To find \overline{HM} , use distance formula or pythagorean theorem

$$HM = \sqrt{3^2 + 4^2} = 5 \text{ units}$$

What is the quadrilateral ROHM?

Lengths of each side are 5 units; opposite sides parallel; not equiangular

RHOMBUS



- 3) P(7, 5) A(12, 4) R(9, 1) L(4, 2)

Verify that PARL is a parallelogram.

(Hint: What is the definition of a parallelogram?)

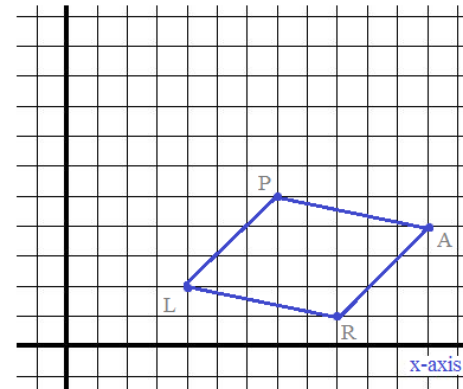
Definition of parallelogram: opposite sides parallel;
opposite side congruent

$$\text{slope of PA} = \frac{5-4}{7-12} = \frac{-1}{5} \quad \text{slope of LR} = \frac{2-1}{4-9} = \frac{-1}{5} \quad \checkmark$$

$$\text{slope of PL} = 1 \quad \text{slope of RA} = 1 \quad \checkmark$$

$$\text{also, length of PA} = \sqrt{26} \quad \text{length of LR} = \sqrt{26} \quad \checkmark$$

$$\text{length of PL} = 3\sqrt{3} \quad \text{length of RA} = 3\sqrt{3} \quad \checkmark$$

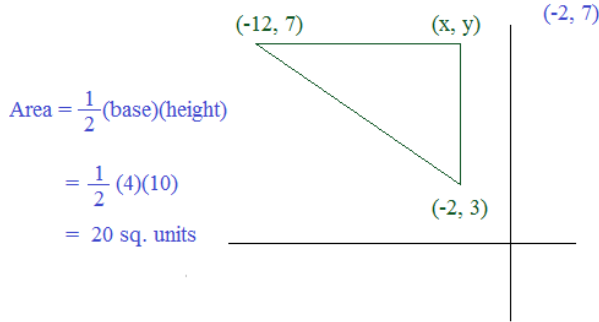


Identify the missing coordinates. Then, find the area of each figure.

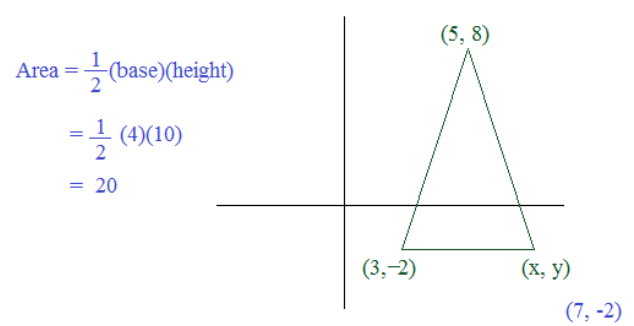
SOLUTIONS

Quadrilaterals, Triangles, and Coordinates

1) Right Triangle

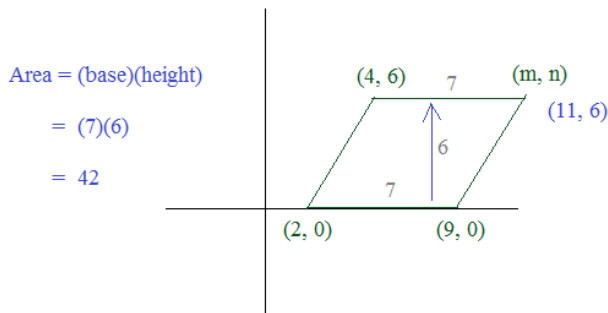


2) Isosceles Triangle

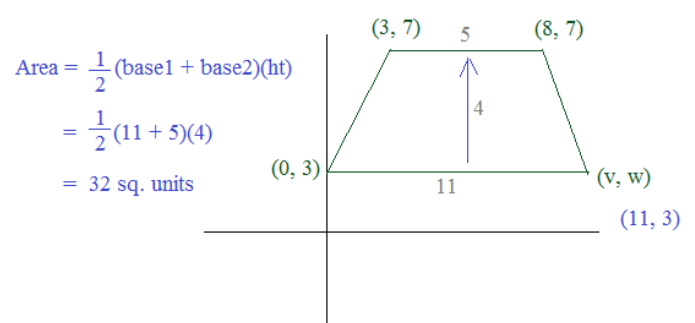


3) Parallelogram

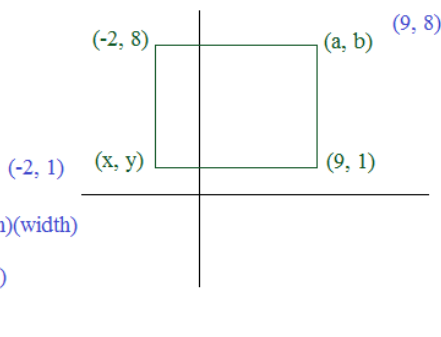
(opposite sides congruent)



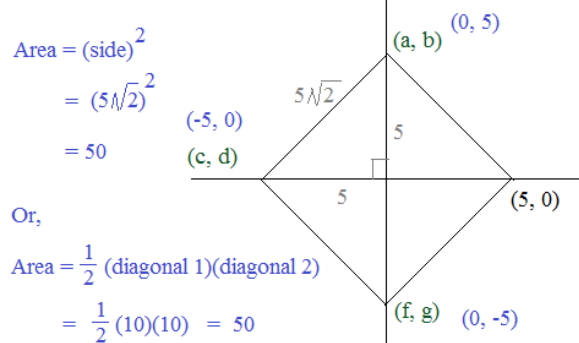
4) Isosceles Trapezoid



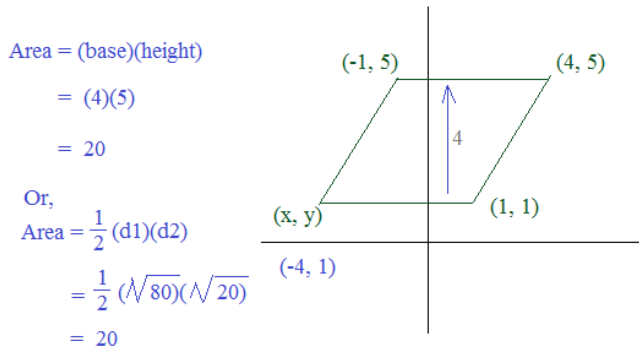
5) Rectangle



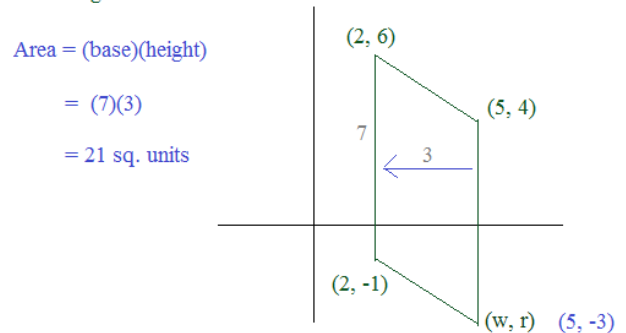
6) Square



7) Rhombus



8) Parallelogram



Thanks for visiting. (Hope it helped!)

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