Special Quadrilaterals 2

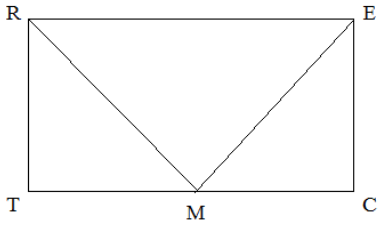
Notes, examples, practice questions (and, solutions)

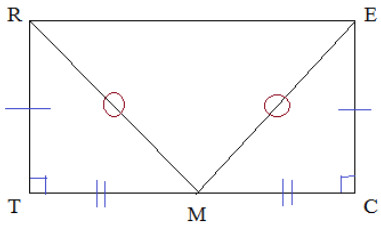
Topics include slope, distance, properties of quadrilaterals, proofs, and more…

Rectangle Property proofs

Given: Rectangle RECT
M is midpoint of $\overline{TC}$

Prove: $\triangle$REM is isosceles

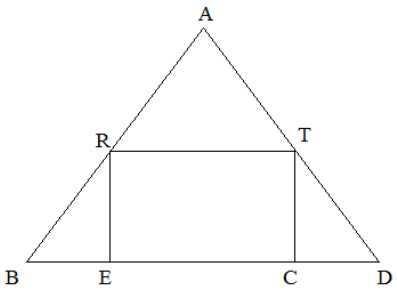


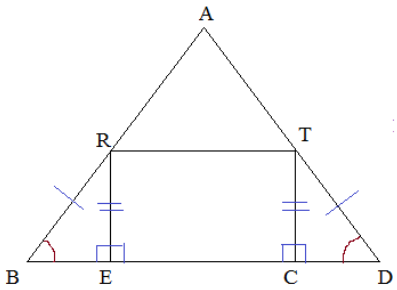


Statements	Reasons
1) Rectangle RECT	1) Given
2) $\overline{RT} \cong \overline{EC}$	2) Definition of Rectangle (opposite sides congruent)
3) M is midpoint of $\overline{TC}$	3) Given
4) $\overline{TM} \cong \overline{CM}$	4) Definiton of midpoint (midpoint divides segment into congruent halves)
5) $\angle$T and $\angle$C are right angles	5) Definiton of Rectangle (angles are 90 degrees)
6) $\angle$ T $\cong$ $\angle$C	6) All right angles are congruent
7) $\triangle$ RTM $\cong$ $\triangle$ECM	7) SAS (Side-Angle-Side) 2, 6, 4
8) $\overline{RM} \cong \overline{EM}$	8) CPCTC (corresponding parts of congruent triangles are congruent)
9) $\triangle$REM is isosceles	9) Definition of Isosceles Triangle (2 or more sides of triangle are congruent)

Given: RECT is a rectangle
$\overline{RB} \cong \overline{TD}$

Prove: $\triangle$ABD is isosceles





Statements	Reasons
1) Rectangle RECT	1) Given
2) $\overline{RE} \cong \overline{CT}$	2) Definition of Rectangle (opposite sides are congruent)
3) $\overline{RB} \cong \overline{TD}$	3) Given
4) $\angle$REC and $\angle$TCE are right angles	4) Definition of Rectangle (angles are right angles)
5) REB and REC are supplementary TCE and TCD are supplementary	5) Definition of Supplementary (angles that form a straight angle are supplementary)
6) REB and TCD are right angles	6) Subtraction property
7) $\triangle$REB = $\triangle$TCD	7) RHL (Right Angle- Hypotenuse - Leg) 6, 3, 2
8) $\angle$B = $\angle$D	8) CPCTC (corresponding parts of congruent triangles are congruent)
9) $\overline{AB} \cong \overline{AD}$	9) If congruent angles, then congruent sides (in triangle, if congruent angles, then opposite sides are congruent)
10) $\triangle$ABD is isosceles	10) Definition of Isosceles - 2 or more congruent sides (Also, base angles of triangle are congruent)

Coordinate Geometry: Verifying/Identifying Special Quadrilaterals

Quadrilaterals and Slope

Parallelogram: opposite sides parallel

Rectangle: opposite sides parallel; adjacent sides are perpendicular

Rhombus: opposite sides parallel; diagonals are perpendicular

square: diagonals are perpendicular; adjacent sides are perpendicular; opposite sides parallel

kite: diagonals are perpendicular; opposite sides are not parallel

trapezoid: one pair of opposite sides are parallel

Quadrilaterals and the Distance Formula

Parallelogram: opposite sides are congruent

Rhombus: all sides are congruent

Rectangle: opposite sides congruent AND diagonals congruent

Square: all sides congruent AND diagonals congruent

Kite: pair of consecutive sides are congruent

Isosoceles trapezoid: one pair of congruent (opposite) sides AND congruent diagonals

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Example: Verify using slope that the quadrilateral is a rhombus.

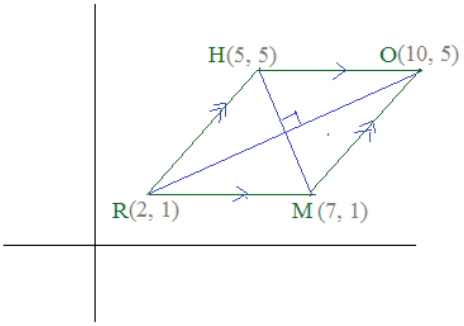
If the slopes of the opposites sides are equal, then it's a parallelogram...

RH: slope is 4/3
HO: slope is 0
OM: slope is 4/3
RM: slope is 0

Then, if the diagonals are perpendicular, then it's a rhombus...

HM: slope is -2
OR: slope is 1/2

(Note: since 4/3 and 0 are not opposite reciprocals, the sides are not perpendicular. Therefore, the figure is not a square.)



Example: Verify the quadrilateral is a rhombus using distance/length only

If the distances between the sides are the same, it's a rhombus or a square.

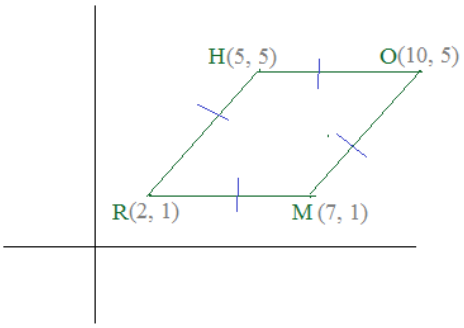
HO: distance is 5
MO: using distance formula --- $\sqrt{(7-10)^2+(1-5)^2} = 5$

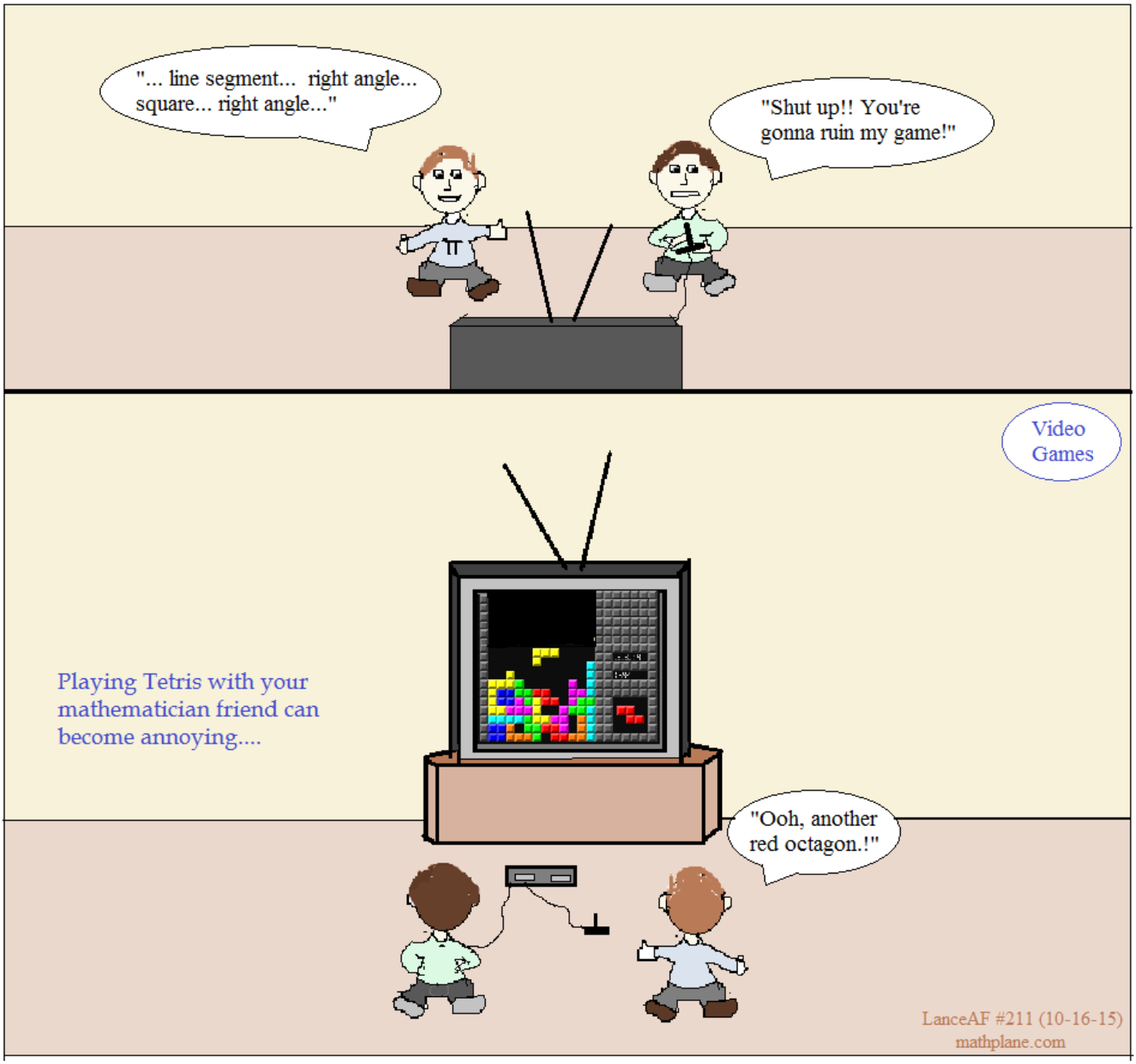
RM: length is 5 units
RH: using distance formula --- $\sqrt{(2-5)^2+(1-5)^2} = 5$

Then, if the diagonals are congruent, it's a square.. If not, it's only a rhombus..

RO: $\sqrt{(2-10)^2+(1-5)^2} = 4\sqrt{5}$

HM: $\sqrt{(7-5)^2+(1-5)^2} = 2\sqrt{5}$



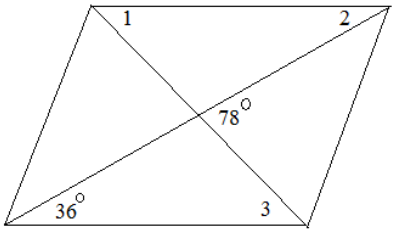

"... line segment... right angle... square... right angle..."
"Shut up!! You're gonna ruin my game!"
Video Games
Playing Tetris with your mathematician friend can become annoying....
"Ooh, another red octagon.!"
LanceAF #211 (10-16-15)
mathplane.com

Exercises -→

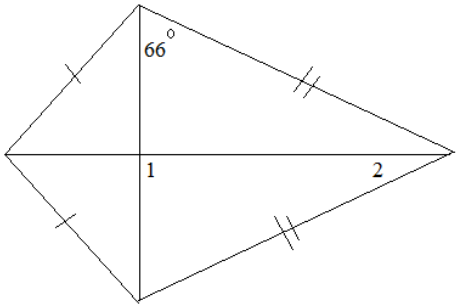
WARM-UP: Determine the angle measures

Special Quadrilateral Angle Properties

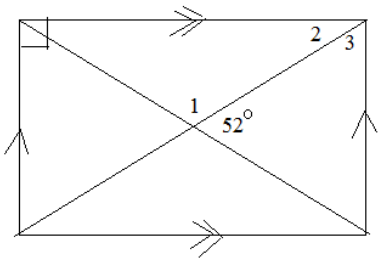
1) Parallelogram



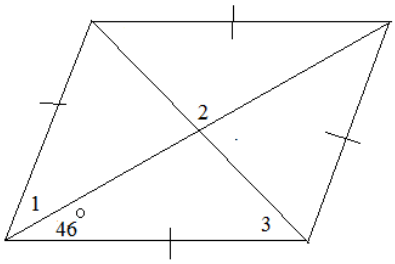
2) Kite



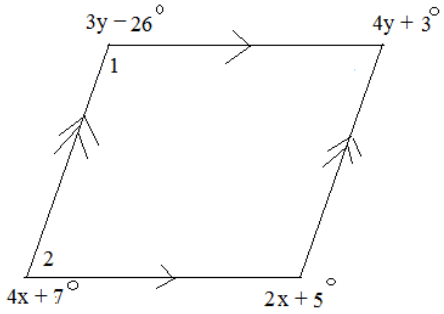
3) Rectangle



4) Rhombus

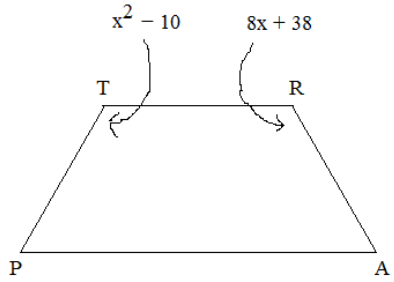


5) Parallelogram

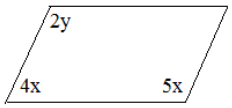


Applying Special Quadrilateral Properties

1) In the isosceles trapezoid TRAP,
what are the measures of $\angle P$ and $\angle T$?

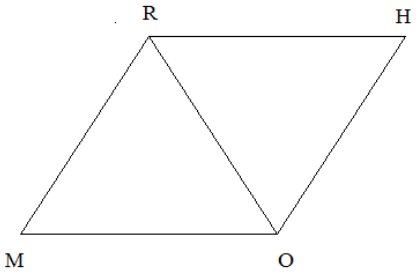


2) Find values for each variable for
the given parallelogram:



3) Triangles ROM and ROH are equilateral...

If the diagonal HM creates an angle HMO of measure 3x - 6 ,
and RO = 10x,
then what is the perimeter of rhombus RHOM?



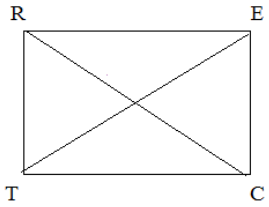
4) RECT is a rectangle...

RC = x^2

RE = 4 − x

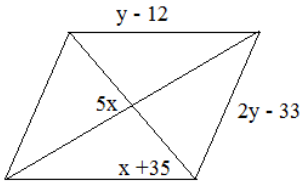
ET = 6x − 5

Find the possible values of x

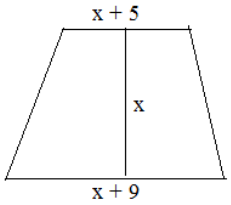


Applying Special Quadrilateral Properties

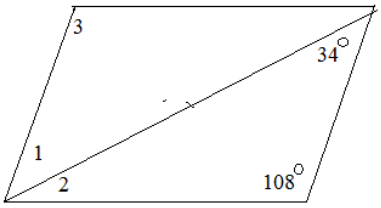
5) For the following rhombus , determine x and y...



6) If the area is 78 square units,
what is x?

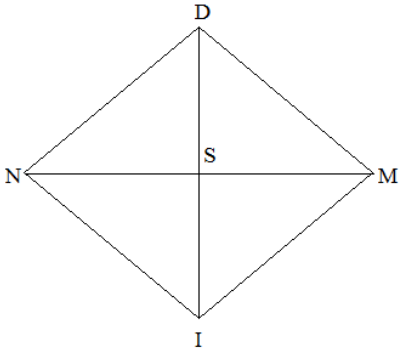


7) In the following parallelogram, find the angle measures of 1, 2, and 3...



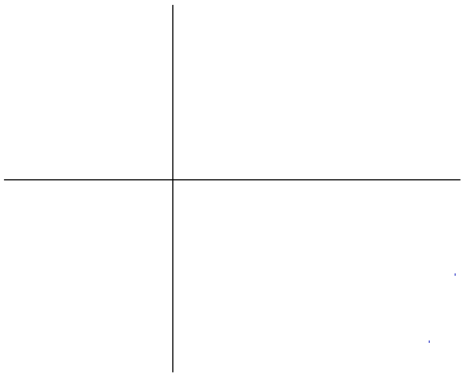
8) In rhombus MIND, $\angle DSN = 2x^2 + 5x + 15$ $DN = 2x + 3y$
$DM = 5y + 4$
$DS = xy$

Find x, y, and $\overline{ID}$

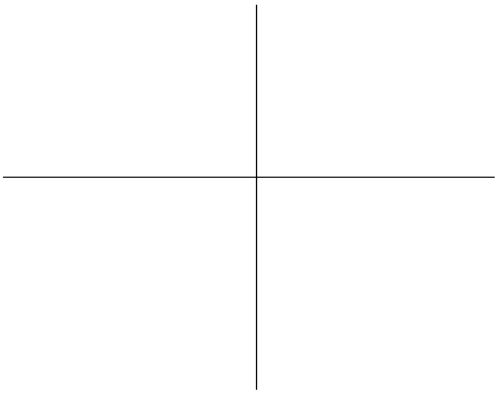


Describe the most exact quadrilateral *using distance formula only.*

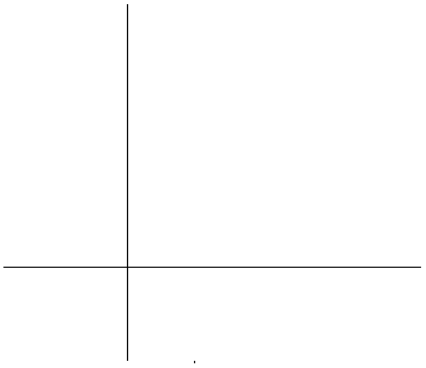
1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)



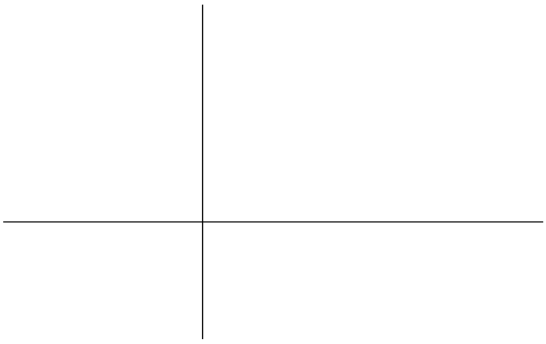
2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)



3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

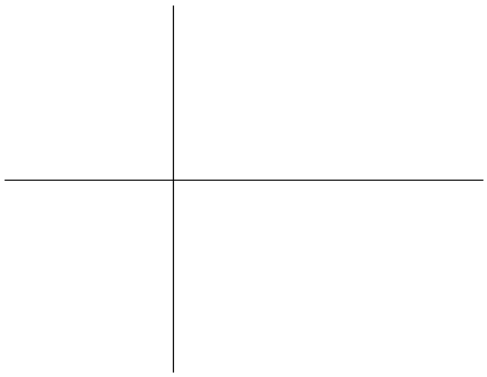


4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

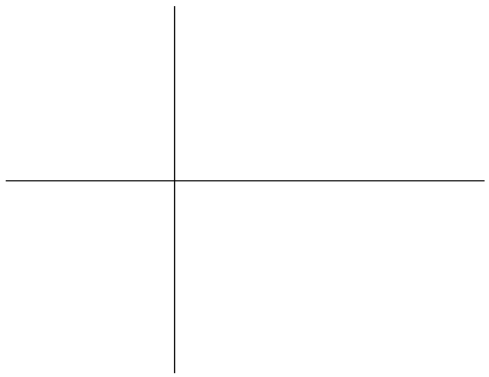


Describe the most exact quadrilateral *using distance formula only.*

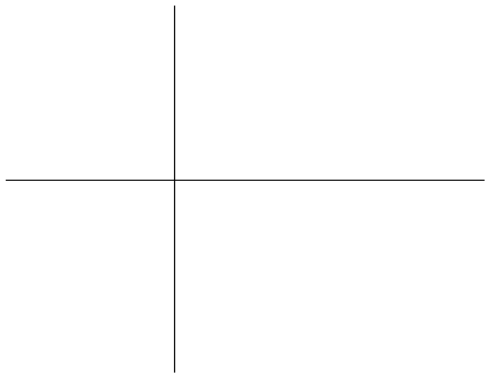
5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)



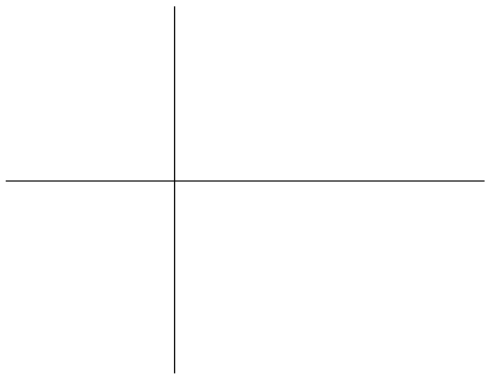
6) Consecutive Vertices: (-1, 8) (5, 2) (-3, 3) (7, -7)



7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

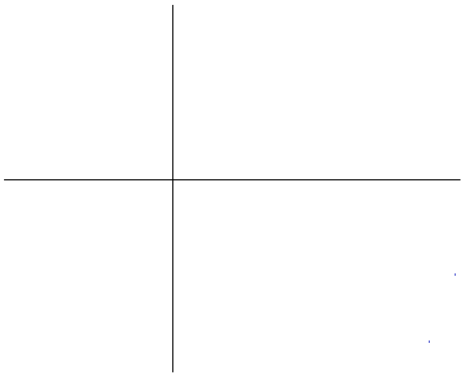


8) Consecutive Vertices: (0, 0) (-3, 4) (8, 4) (5, 0)

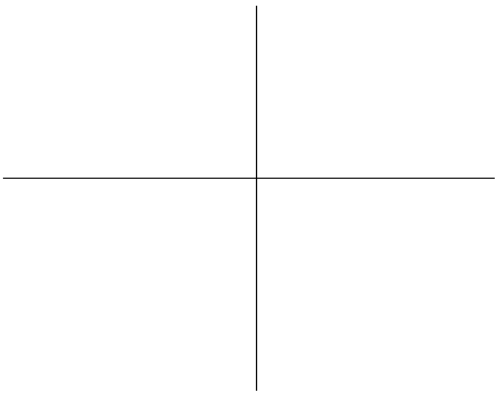


Describe the most exact quadrilateral *using slope only*.

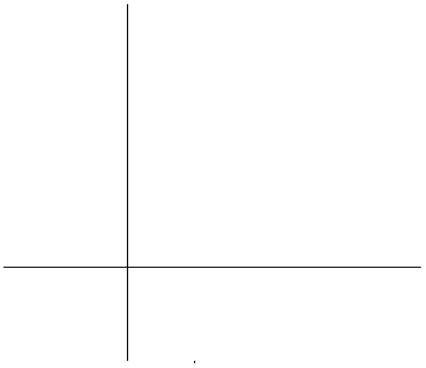
1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)



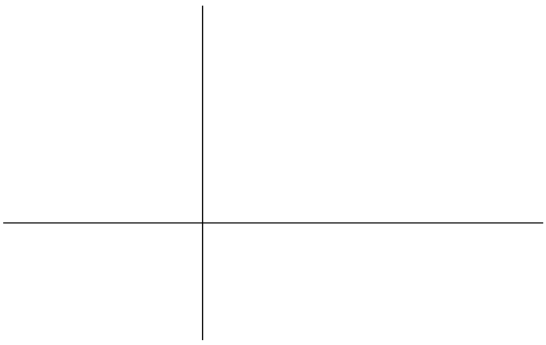
2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)



3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

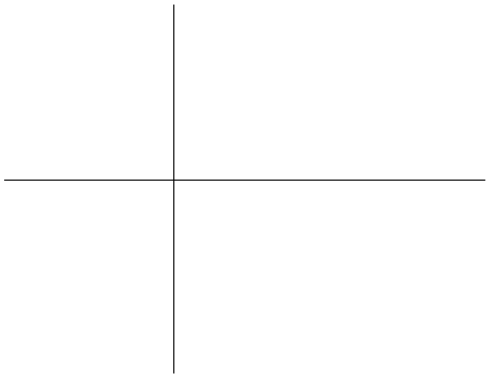


4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

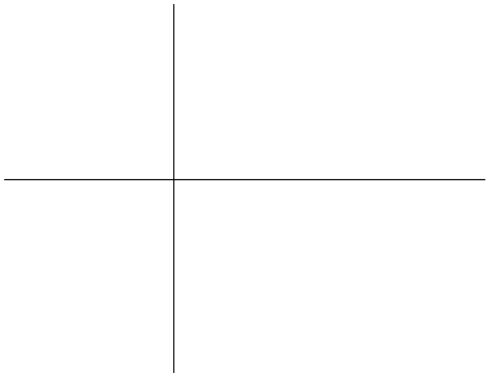


Describe the most exact quadrilateral *using slope only.*

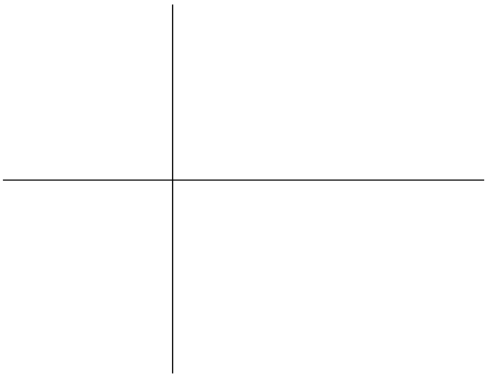
5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)



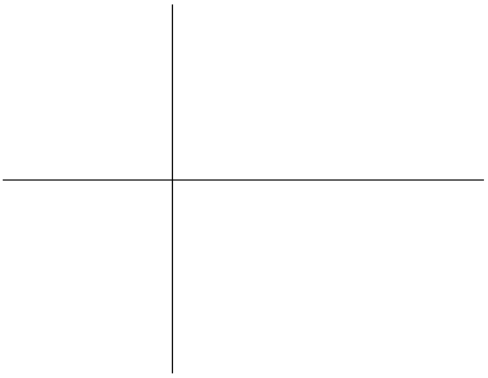
6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)



7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)



8) Consecutive Vertices: (0, 0) (-3, 4) (8, 4) (5, 0)

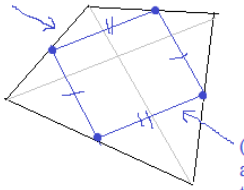


Identifying Special Quadrilaterals: Triangle Congruency and Midsegment Theorems

Determine the name of the figure formed by *connecting the midpoints of the sides* of each quadrilateral.

0) Quadrilateral

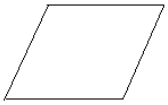
Connecting the midpoints...



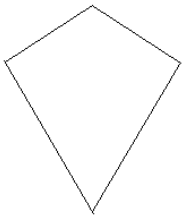
(These midsegments are 1/2 the length of the horizontal diagonal)

The inside is a parallelogram... (opposite sides are congruent)

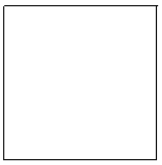
1) Rhombus:



2) Kite:



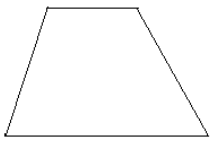
3) Square:



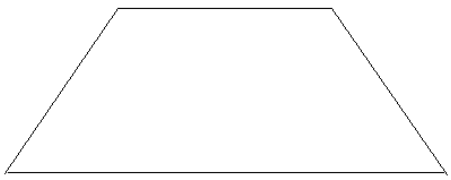
4) Rectangle:

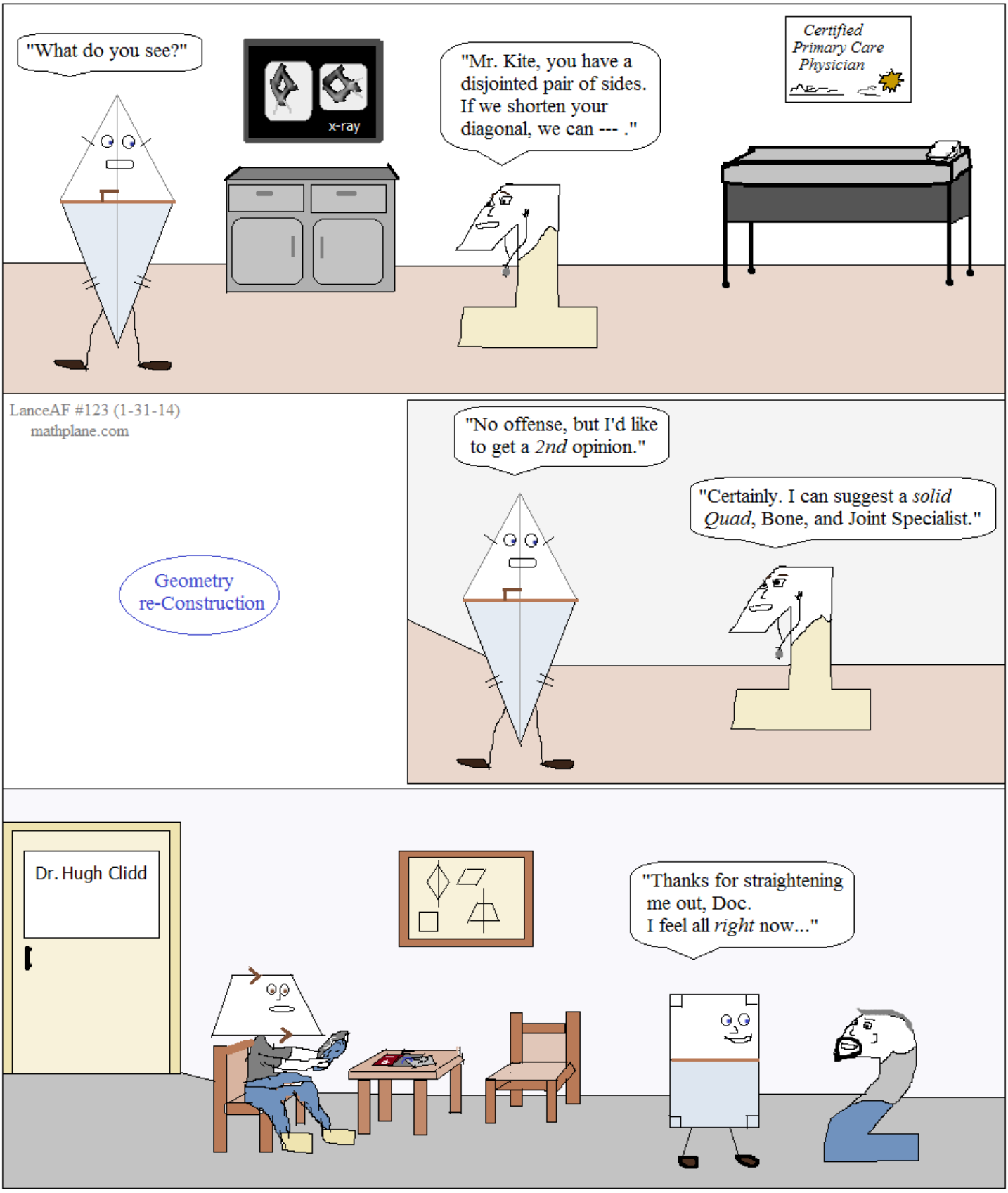
5) Parallelogram:

6) Trapezoid



7) Isosceles Trapezoid




"What do you see?"
x-ray
"Mr. Kite, you have a disjointed pair of sides. If we shorten your diagonal, we can --- ."
Certified Primary Care Physician
LanceAF #123 (1-31-14)
mathplane.com
Geometry re-Construction
"No offense, but I'd like to get a 2nd opinion."
"Certainly. I can suggest a solid Quad, Bone, and Joint Specialist."
Dr. Hugh Clidd
"Thanks for straightening me out, Doc. I feel all right now..."

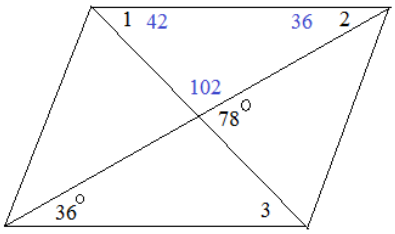
SOLUTIONS-→

WARM-UP: Determine the angle measures

SOLUTIONS

Special Quadrilateral Angle Properties

1) Parallelogram

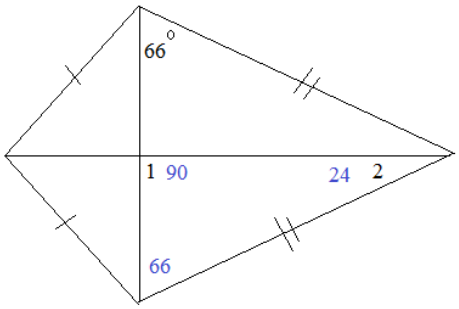


angle 2 = 36 (because alternate interior angles are congruent)

and, since 1 + 2 + 102 = 180, angle 1 = 42

and, therefore, angle 3 = 42

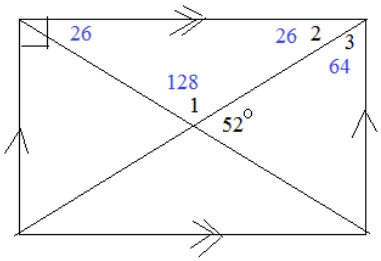
2) Kite



angle 1 = 90 (because diagonals of kite are perpendicular)

therefore, angle 2 = 24 (because sum of angles in triangle is 180)

3) Rectangle

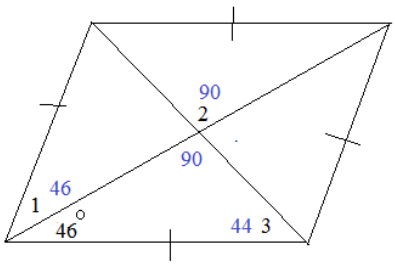


angle 1 is supp to 52 ⟹ 128 degrees

angle 2 + angle 1 + (angle 2) = 180
x + 128 + x = 180
2x = 52
x = 26

angle 3 + 52 + (angle 3) = 180
angle 3 = 64

4) Rhombus



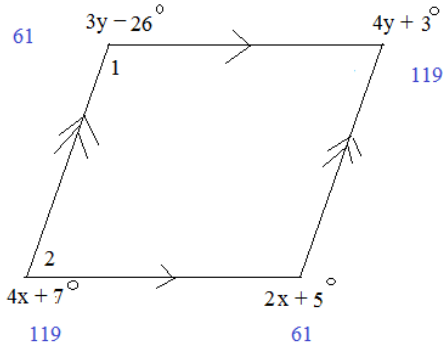
Rhombus:

angle 1: since diagonals are angle bisectors, 46

angle 2: since diagonals are perpendicular, 90

angle 3: since triangles interior equal 180, 44

5) Parallelogram



method 1: consecutive angles are supplementary

7y − 23 = 180
7y = 203
y = 29

6x + 12 = 180
6x = 168
x = 28

method 2: opposite angles congruent

4x + 7 = 4y + 3
3y - 26 = 2x + 5

4x - 4y = -4
2x - 3y = -31
4x - 6y = -62

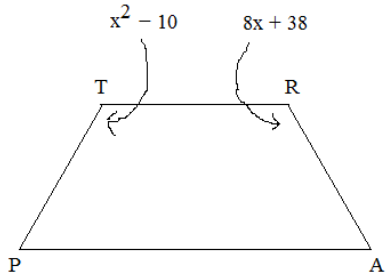
2y = 58
y = 29

61 and 119

Applying Special Quadrilateral Properties

SOLUTIONS

1) In the isosceles trapezoid TRAP,
what are the measures of $\angle P$ and $\angle T$?



$x^2 - 10 = 8x + 38$

base and upper angles are congruent (isosceles trapezoid)

$x^2 - 8x - 48 = 0$

$(x - 12)(x + 4) = 0$

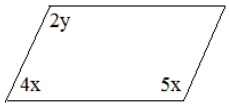
$x = 12$ or -4

If $x = 12$, then angle T = 134, angle R = 134, and so, angle P and angle A = 46

If $x = -4$, then angle T = 6, angle R = 6, and so, angle P and angle A = 174...

It doesn't look like the diagram, but it is a possiblity!!

2) Find values for each variable for the given parallelogram:

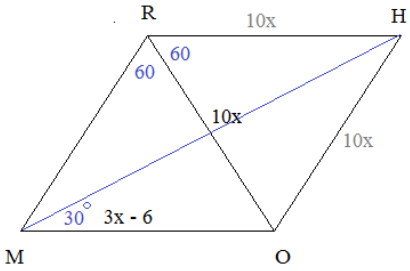


Consecutive angles are supp.
$4x + 5x = 180$
$x = 20$

Opposite angles are congruent
$5x = 100$..
So, $2y = 100$
$y = 50$

3) Triangles ROM and ROH are equilateral...

If the diagonal HM creates an angle HMO of measure $3x - 6$, and RO = $10x$,
then what is the perimeter of rhombus RHOM?



since angle M is 60 degrees, the diagonal bisects it ----> 30 degrees

$3x - 6 = 30$ $x = 12$

Then, $RO = 12(10) = 120$

therefore, perimeter is 480 units

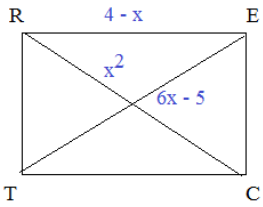
4) RECT is a rectangle...

$RC = x^2$

$RE = 4 - x$

$ET = 6x - 5$

Find the possible values of x



$x^2 = 6x - 5$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$

$x = 1, 5$

no solution

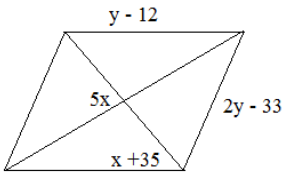
x cannot be 5, because RE would have a negative length!

x cannot be 1, because ET and RC would have length 1...
And, RE would have length 3...
That's not possible! because the hypotenuse cannot be smaller than the side!

Applying Special Quadrilateral Properties

mathplane.com

5) For the following rhombus , determine x and y...



$y - 12 = 2y - 33$ (all sides congruent)

$5x = 90$ (diagonals are perpendicular)

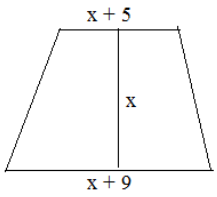
$(x + 35$ is irrelevant to solving)

$x = 18$

$y = 21$

SOLUTIONS

6) If the area is 78 square units, what is x?



$78 = 1/2(x)(2x + 14)$

$78 = (x)(x + 7)$

$78 = x^2 + 7x$

$x^2 + 7x - 78 = 0$

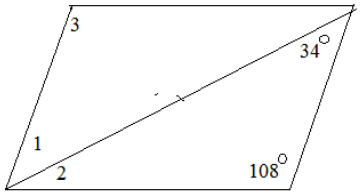
$(x + 13)(x - 6) = 0$

$x = -13$ or 6

since side length must be positive, we eliminate -13..

Therefore, $x = 6$

7) In the following parallelogram, find the angle measures of 1, 2, and 3...



$3 = 108$ opposite angles are congruent

since consecutive angles are supplementary, $1 + 2 = 72$....

$1 = 34$ and $2 = 38$

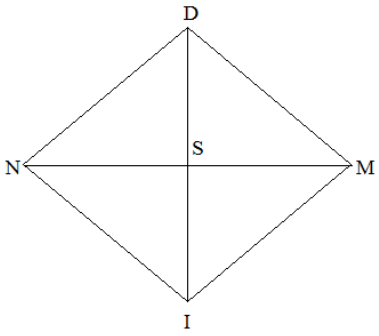
8) In rhombus MIND, $\angle DSN = 2x^2 + 5x + 15$

$DN = 2x + 3y$

$DM = 5y + 4$

$DS = xy$

Find x, y, and $\overline{ID}$



Diagonals are perpendicular

$DSN = 2x^2 + 5x + 15 = 90$

$2x^2 + 5x - 75 = 0$

$(2x + 15)(x - 5) = 0$

$x = -15/2$ or 5

All sides are congruent

$2x + 3y = 5y + 4$

$2x - 2y = 4$

$x - y = 2$

so, if $x = 5$, then $y = 3$

OR

if $x = -15/2$, then $y = -19/2$

(**However, this is impossible because the sides would be negative!)

Note: $DS = xy$.. so, if $y > 0$, then x must be > 0...

therefore, $x = 5$, $y = 3$

and, $ID = 2(xy) = 30$

Diagonals bisect each other...

quick check: sides = 19
angle = 90
and xy = 15

Describe the most exact quadrilateral *using distance formula only.*

1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

(1, 1) to (4, 5) $\sqrt{(1-4)^2+(1-5)^2} = 5$

(4, 5) to (9, -7) $\sqrt{(9-4)^2+(-7-5)^2} = 13$

(9, -7) to (6, -11) $\sqrt{(9-6)^2+(-7--11)^2} = 5$

(6, -11) to (1, 1) $\sqrt{(1-6)^2+(1--11)^2} = 13$

Since opposite sides are congruent, it must be a parallelogram...

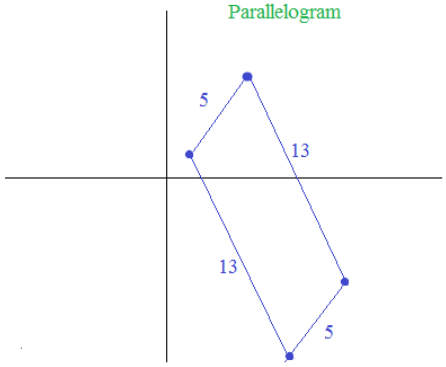
(Since *not all* sides are same, we can eliminate square and rhombus)

diagonals: (1, 1) to (9, -7) $8\sqrt{2}$

(4, 5) to (6, -11) $2\sqrt{65}$

Since the diagonals are NOT congruent, it cannot be a rectangle..

Parallelogram



2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

(0, 3) to (3, 0) $3\sqrt{2}$

(3, 0) to (-6, -9) $9\sqrt{2}$

(-6, -9) to (-9, -6) $3\sqrt{2}$

(-9, -6) to (0, 3) $9\sqrt{2}$

Since opposite sides are congruent, the quadrilateral is a parallelogram...

(Since the 4 sides are not all the same, it eliminates rhombus and square)

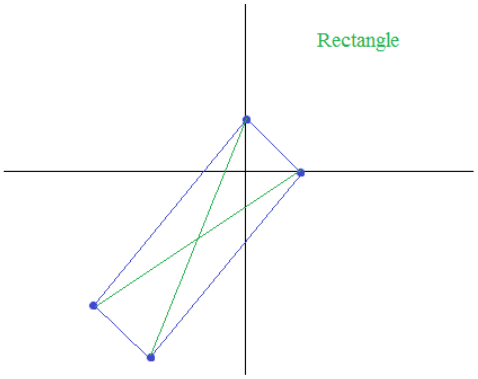
diagonals:

(0, 3) to (-6, -9) $6\sqrt{5}$

(3, 0) to (-9, -6) $6\sqrt{5}$

Then, since the diagonals are congruent, it is a rectangle...

Rectangle



3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

(0, 8) to (3, 4) 5

(3, 4) to (3, 9) 5

(3, 9) to (0, 13) 5

(0, 13) to (0, 8) 5

Since all the sides are the same, it's a rhombus or a square...

Then, since the diagonals are NOT congruent, then it cannot be a square..

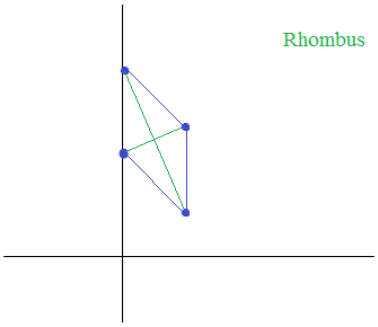
The quadrilateral is a rhombus...

diagonals:

(0, 8) to (3, 9) $\sqrt{10}$

(3, 4) to (0, 13) $3\sqrt{10}$

Rhombus



4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

(2, 5) to (3, -2) $5\sqrt{2}$

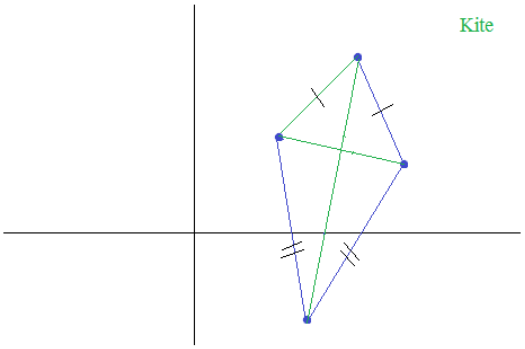
(3, -2) to (8, 3) $5\sqrt{2}$

(8, 3) to (6, 7) $2\sqrt{5}$

(6, 7) to (2, 5) $2\sqrt{5}$

Since there are 2 pairs of *consecutive* congruent sides, the figure is a kite!

Kite



Describe the most exact quadrilateral *using distance formula only.*

5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)

(3, 5) to (7, 4) $\sqrt{(3-7)^2+(5-4)^2} = \sqrt{17}$

(7, 4) to (6, 0) $\sqrt{(7-6)^2+(4-0)^2} = \sqrt{17}$

(6, 0) to (2, 1) $\sqrt{(6-2)^2+(0-1)^2} = \sqrt{17}$

(2, 1) to (3, 5) $\sqrt{(2-3)^2+(1-5)^2} = \sqrt{17}$

since the distances between consecutive points are all the same, sides are congruent...

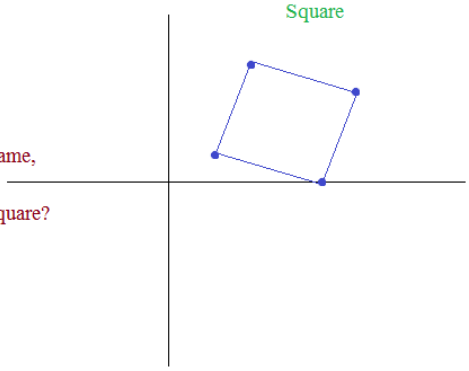
It's a rhombus... But, is it a square?

diagonals: (3, 5) to (6, 0) distance = $\sqrt{34}$

(7, 4) to (2, 1) distance = $\sqrt{34}$

the lengths of the diagonals are congruent...
Therefore, it is a square!

Square



6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)

(-3, 3) to (7, -7) $10\sqrt{2}$

(7, -7) to (5, 2) $\sqrt{85}$

(5, 2) to (1, 6) $4\sqrt{2}$

(1, 6) to (-3, 3) 5

the lengths of all 4 sides are different, so it's either a quadrilateral or trapezoid..

Midpoint distances: (2, -2) to (3, 4) $\sqrt{37}$

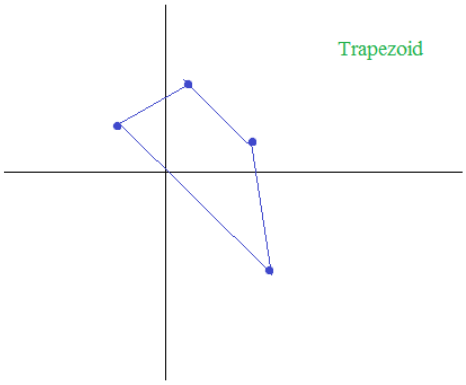
(6, -5/2) to (-1, 9/2) $7\sqrt{2}$

this length is middle, so it's a midsegment..

diagonals: (-3, 3) to (5, 2) $\sqrt{65}$

(7, -7) to (1, 6) $\sqrt{205}$

Trapezoid



7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

(-10, 5) to (0, 3) $2\sqrt{26}$

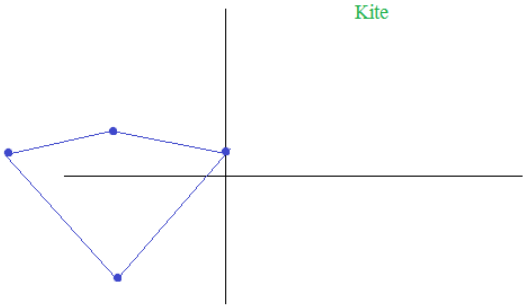
(0, 3) to (-10, -10) $\sqrt{269}$

(-10, -10) to (-20, 3) $\sqrt{269}$

(-20, 3) to (-10, 5) $2\sqrt{26}$

2 pairs of congruent sides...
It's a kite..

Kite



8) Consecutive Vertices (0, 0) (-3, 4) (8, 4) (5, 0)

(0, 0) to (-3, 4) 5

(-3, 4) to (8, 4) 11

(8, 4) to (5, 0) 5

(5, 0) to (0, 0) 5

distances between midpoints:

(-3/2, 2) to (6.5, 2) 8

(5/2, 4) to (5/2, 0) 4

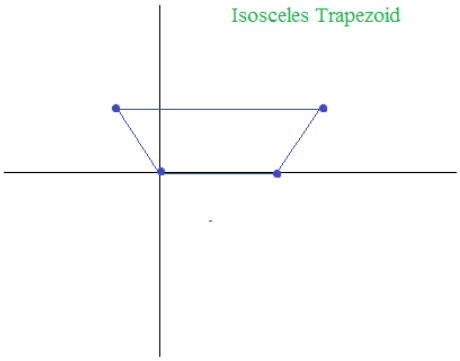
diagonals: (0, 0) to (8, 4) $4\sqrt{5}$

(-3, 4) to (5, 0) $4\sqrt{5}$

the length of the segment connecting midpoints is the average of sides..
therefore, it is a midsegment...

diagonals are congruent: either a rectangle or isosceles trapezoid

Isosceles Trapezoid



Describe the most exact quadrilateral *using slope only.*

1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

(1, 1) to (4, 5) $\frac{5-1}{4-1} = \frac{4}{3}$

(4, 5) to (9, -7) $\frac{-7-5}{9-4} = \frac{-12}{5}$

(9, -7) to (6, -11) $\frac{-11--7}{6-9} = \frac{-4}{-3} = \frac{4}{3}$

(6, -11) to (1, 1) $\frac{1--11}{1-6} = \frac{12}{-5}$

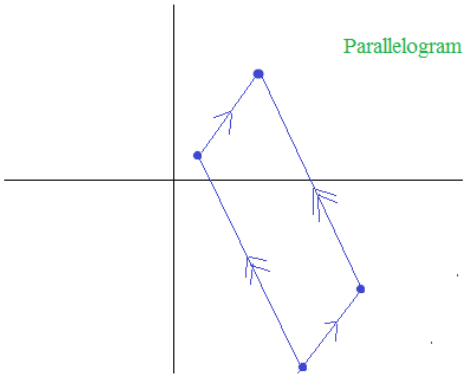
diagonals: (1, 1) (9, -7) slope: -1

(4, 5) (6, -11) slope: -8

since opposite sides have the same slopes, it must be a parallelogram...

Then, since the slopes are not opposite reciprocals, it cannot be square or rectangle...

And, since diagonals are not perpendicular, it cannot be a rhombus



Parallelogram

2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

Slopes of sides:

(0, 3) to (3, 0) -1

(3, 0) to (-6, -9) 1

(-6, -9) to (-9, -6) -1

(-9, -6) to (0, 3) 1

Slopes of diagonals:

(0, 3) to (-6, -9) 2

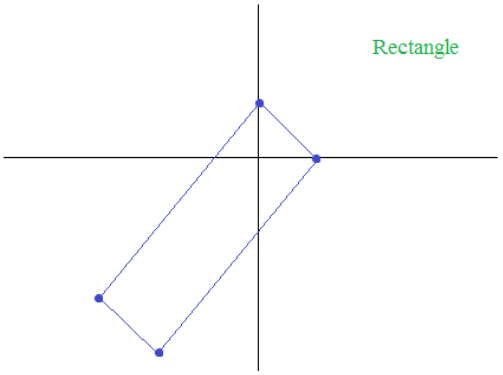
(3, 0) to (-9, -6) 1/2

Since opposite sides are parallel, the quadrilateral is a parallelogram...

Then, since the consecutive sides have slopes that are opposite reciprocals, the sides are perpendicular; right angles...
So, it's a rectangle or square...

Then, since the diagonals are NOT perpendicular (slopes are not opposite reciprocals), the figure can't be a square...

So, it's a rectangle...



Rectangle

3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

Slopes of sides:

(0, 8) to (3, 4) -4/3

(3, 4) to (3, 9) undefined

(3, 9) to (0, 13) -4/3

(0, 13) to (0, 8) undefined

Slopes of the diagonals:

(0, 8) to (3, 9) 1/3

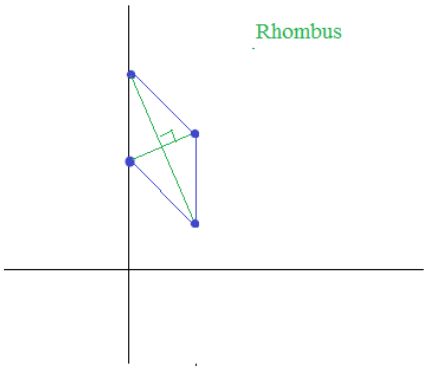
(3, 4) to (0, 13) -3

Since slopes of opposite sides are the same, the quadrilateral is a parallelogram...

(and, since the slopes are not opposite reciprocals, the corners are NOT right angles..)

Then, since the slopes of the diagonals are opposite reciprocals, the diagonals are perpendicular...

Therefore, it is a rhombus...



Rhombus

4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

(2, 5) to (3, -2) -7

(3, -2) to (8, 3) 1

(8, 3) to (6, 7) -2

(6, 7) to (2, 5) 1/2

Slopes of diagonals:

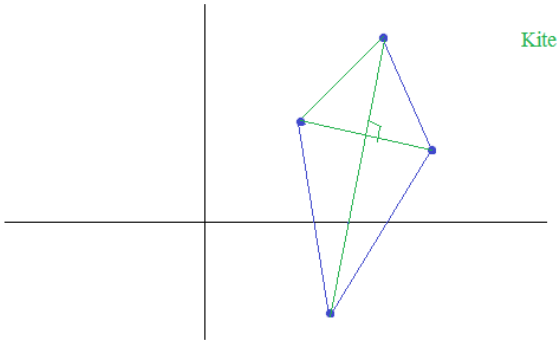
(2, 5) to (8, 3) -1/3

(3, -2) to (6, 7) 3

Since none of the side slopes are congruent, the figure is not a trapezoid or parallelogram...

Then, since the slopes of the diagonals are opposite reciprocals, the diagonals are perpendicular...

Therefore, the quadrilateral is a kite



Kite

Describe the most exact quadrilateral *using slope only.*

5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)

(3, 5) to (7, 4) -1/4
(7, 4) to (6, 0) 4
(6, 0) to (2, 1) -1/4
(2, 1) to (3, 5) 4

since opposite sides have same slope, they are parallel.. (parallelogram)

And, since consecutive sides have opposite reciprocals, the sides are perpendicular..

(rectangle)

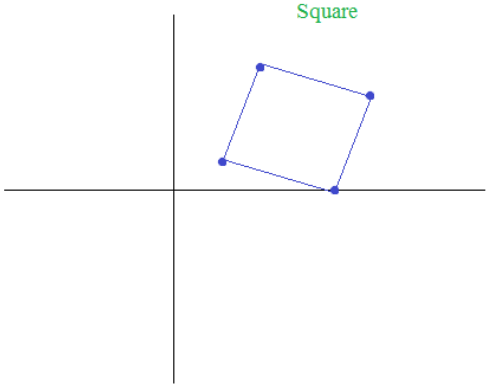
slope of diagonals:

(3, 5) to (6, 0) -5/3
(7, 4) to (2, 1) 3/5

since diagonals are opposite reciprocals, they are perpendicular...

Therefore, figure is a square

Square



6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)

(-3, 3) to (7, -7) -1
(7, -7) to (5, 2) -9/2
(5, 2) to (1, 6) -1
(1, 6) to (-3, 3) 3/4

since one pair of opposite sides have the same slope, then there are only 2 parallel sides...

(trapezoid)

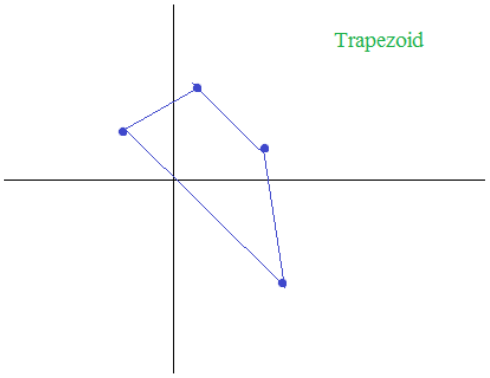
since slopes of other opposite sides are NOT reciprocals, it is not isosceles...

slope of diagonals

(-3, 3) to (5, 2) -1/8
(7, -7) to (1, 6) -13/6

since slopes of diagonals are not opposites, this is NOT an isosceles trapezoid...

Trapezoid



7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

(-10, 5) to (0, 3) -1/5
(0, 3) to (-10, -10) 13/10
(-10, -10) to (-20, 3) -13/10
(-20, 3) to (-10, 5) 1/5

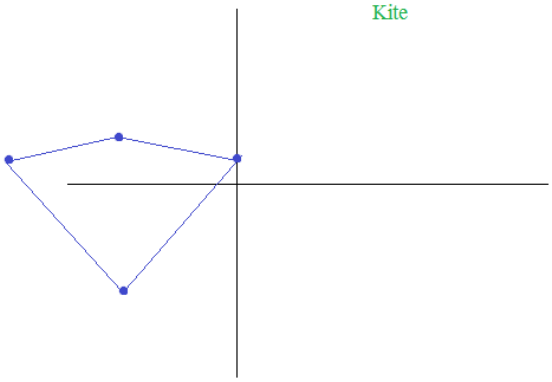
The slopes are opposites... Suspect it's a kite....

slope of diagonals:

(-10, 5) to (-10, -10) undefined
(0, 3) to (-20, 3) 0

Diagonals are perpendicular... KITE

Kite



8) Consecutive Vertices (0, 0) (-3, 4) (8, 4) (5, 0)

(0, 0) to (-3, 4) -4/3
(-3, 4) to (8, 4) 0
(8, 4) to (5, 0) 4/3
(5, 0) to (0, 0) 0

One pair of parallel sides so, trapezoid...

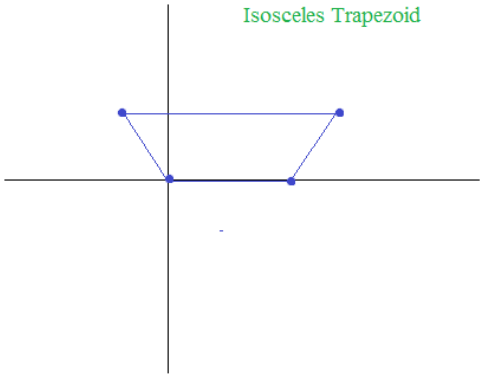
and, non-parallel sides are opposites... isosceles trapezoid

diagonal slopes:

(0, 0) to (8, 4) 1/2
(-3, 4) to (5, 0) -1/2

diagonal slopes are opposites...

Isosceles Trapezoid



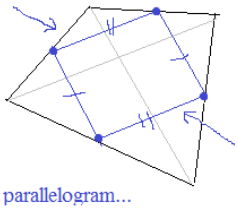
Identifying Special Quadrilaterals: Triangle Congruency and Midsegment Theorems

SOLUTIONS

Determine the name of the figure formed by *connecting the midpoints of the sides* of each quadrilateral.

0) Quadrilateral

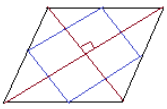
Connecting the midpoints...



(These midsegments are 1/2 the length of the horizontal diagonal)

The inside is a parallelogram... (opposite sides are congruent)

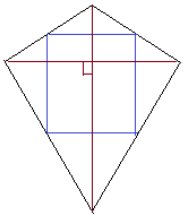
1) Rhombus:



Rectangle

Diagonals of rhombus are perpendicular. And, each segment is parallel to a diagonal. Therefore, consecutive sides are perpendicular.

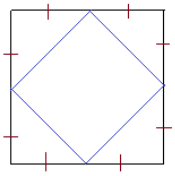
2) Kite:



Rectangle

(Triangle) Midsegment Theorem: If a segment joins the *midpoints of angle sides* of a triangle, then the segment is parallel to the base and 1/2 the length of the base.

3) Square:

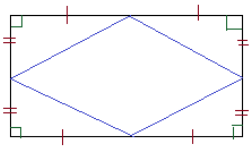


Square

Four congruent triangles
(side-angle-side) - interior quadrilateral sides congruent

Since triangles are isosceles w/ vertex 90 degrees. Then, base angles are 45 degrees... So, interior quadrilateral angles are all 90 degrees

4) Rectangle:

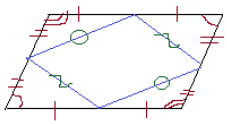


Rhombus

For congruent triangles
(side-angle-side) - interior quadrilateral sides congruent

However, only the opposite angles are congruent, so the interior quadrilateral is not equiangular.

5) Parallelogram:

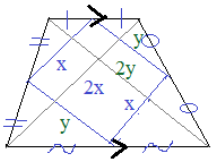


Parallelogram

Opposite triangles are congruent, so *opposite sides* of interior quadrilateral are congruent

6) Trapezoid

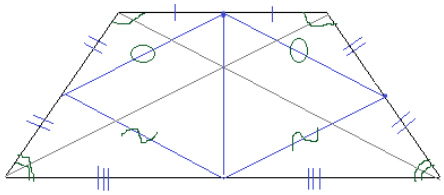
Parallelogram



Using midsegment theorem, we know that opposite sides are congruent..
Therefore, this is a parallelogram

(Note: Since the diagonals are not necessarily congruent or perpendicular, the interior figure is not necessarily a rectangle or rhombus)

7) Isosceles Trapezoid



Rhombus

Since the base angles are congruent.. And the sides are congruent, the bisectors form 2 pairs of congruent triangles.

(Using CPCTC, we have 2 pairs of consecutive congruent sides)

Kite?

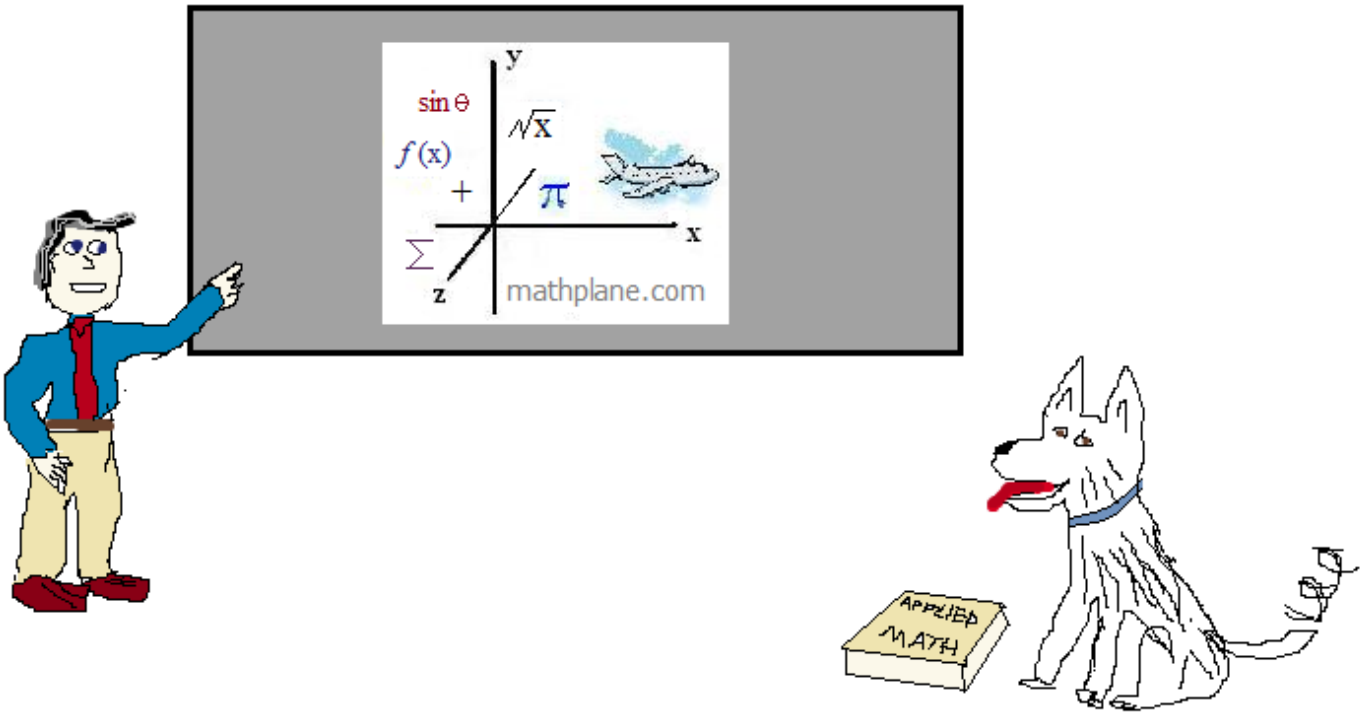
But, remember, the diagonals are congruent... Therefore, the 4 midsegments (which are 1/2 the length of the diagonals) are all congruent!

It's a rhombus..

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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