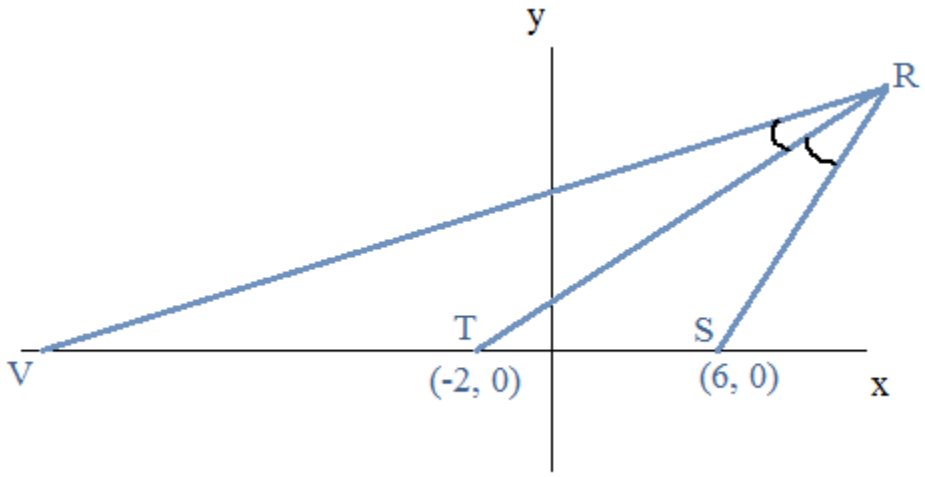
Geometry Similarity & Proportions Review Questions (and Answers)

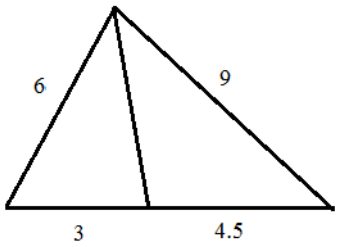


Topics include Angle Bisector Theorem, "Shadow Questions", Side-Splitter, Perimeter/Area/Volume Ratios, and more.

Mathplane.com

1) Angle Bisector Theorem

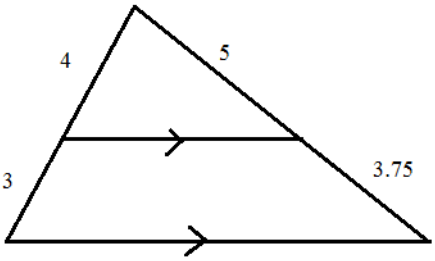
Similarity Concepts to Remember



$$\frac{6}{3} = \frac{9}{4.5}$$

$$\frac{6}{9} = \frac{3}{4.5}$$

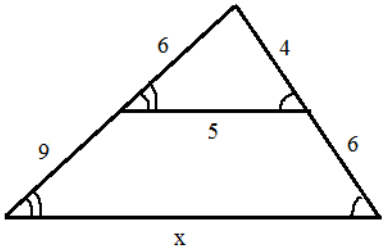
2) Side-Splitter Theorem



$$\frac{4}{3} = \frac{5}{3.75}$$

$$\frac{4}{5} = \frac{3}{3.75}$$

3) Corresponding triangles and angles



$$\frac{6}{5} \neq \frac{9}{x}$$

$$\frac{6}{5} = \frac{6+9}{x}$$

If 2 polygons are similar, then

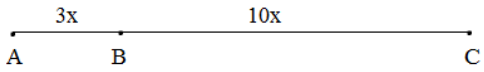
1) the ratio of their corresponding sides, perimeters, medians, diagonals, and angle bisectors are the same...

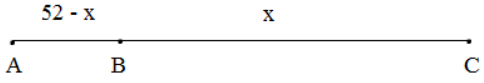
2) the ratio of their areas is equal to the square of the ratio of their corresponding sides...

3) the ratio of their volumes is equal to the cube of the ratio of their corresponding sides....

Example: AB : BC is 3 : 10

If AC = 52, what is BC ?





Approach 1:

$3x + 10x = 52$

$13x = 52$

$x = 4$

therefore, BC = 40

Approach 2:

$\frac{AB}{BC} = \frac{3}{10}$

$\frac{52 - x}{x} = \frac{3}{10}$

$520 - 10x = 3x$

$520 = 13x$

$x = 40$

Example: What is the ratio of x to y?

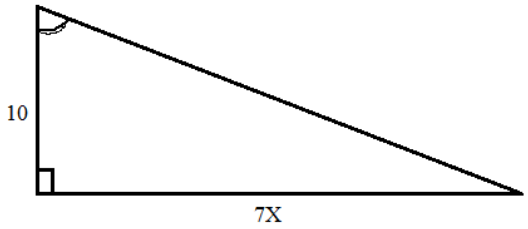
$\frac{6}{3x - 5y} = \frac{9}{6x - 8y}$

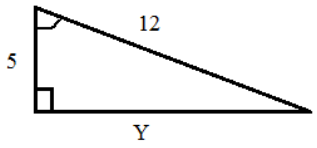
$36x - 48y = 27x - 45y$

$9x = 3y$

$\frac{9x}{y} = 3$ $\qquad \frac{x}{y} = \frac{1}{3}$ $\qquad$ 1:3

Example: Find X and Y





Step 1: Find Y (using Pythagorean Theorem) $\qquad 5^2 + Y^2 = 12^2$

$Y^2 = 119$

$Y = \sqrt{119}$

Step 2: Use proportions to find X

(Due to Angle-Angle, the triangles are similar)

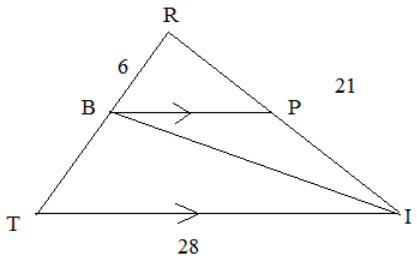
$\frac{10}{5} = \frac{7X}{\sqrt{119}}$ $\qquad 35X = 109.09$

X = 3.12

Example: $\overline{BP} \parallel \overline{TI}$

$\overline{BI}$ bisects angle I

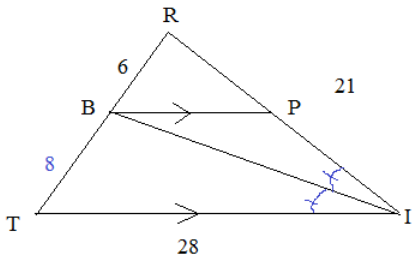
Find $\overline{BT}$, $\overline{RP}$, and $\overline{BP}$



Using Angle Bisector Theorem:

$$\frac{28}{21} = \frac{BT}{6}$$

$\boxed{\overline{BT} = 8}$



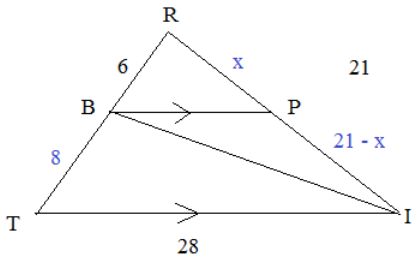
Using Side-Splitter Theorem:

$$\frac{6}{x} = \frac{8}{21 - x}$$

$8x = 126 - 6x$

$x = 9$

$\boxed{\overline{RP} = 9}$



Using Similar Triangles

$$\frac{9}{BP} = \frac{21}{28}$$ "right" / "bottom"

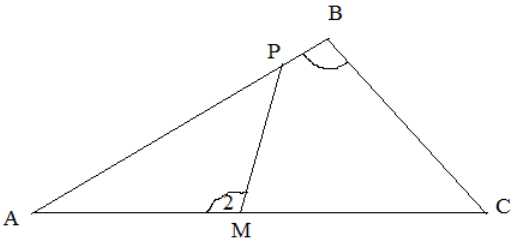
$\boxed{\overline{BP} = 12}$

Example: $\angle B = \angle 2$

$\overline{AP} = 8$

$\overline{PM} = 5$

What is $\frac{AC}{BC}$?

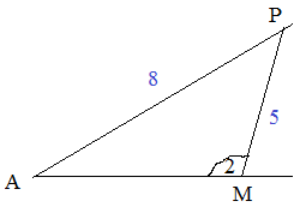


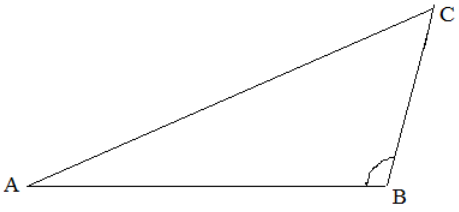
(Large Triangle "flipped over")

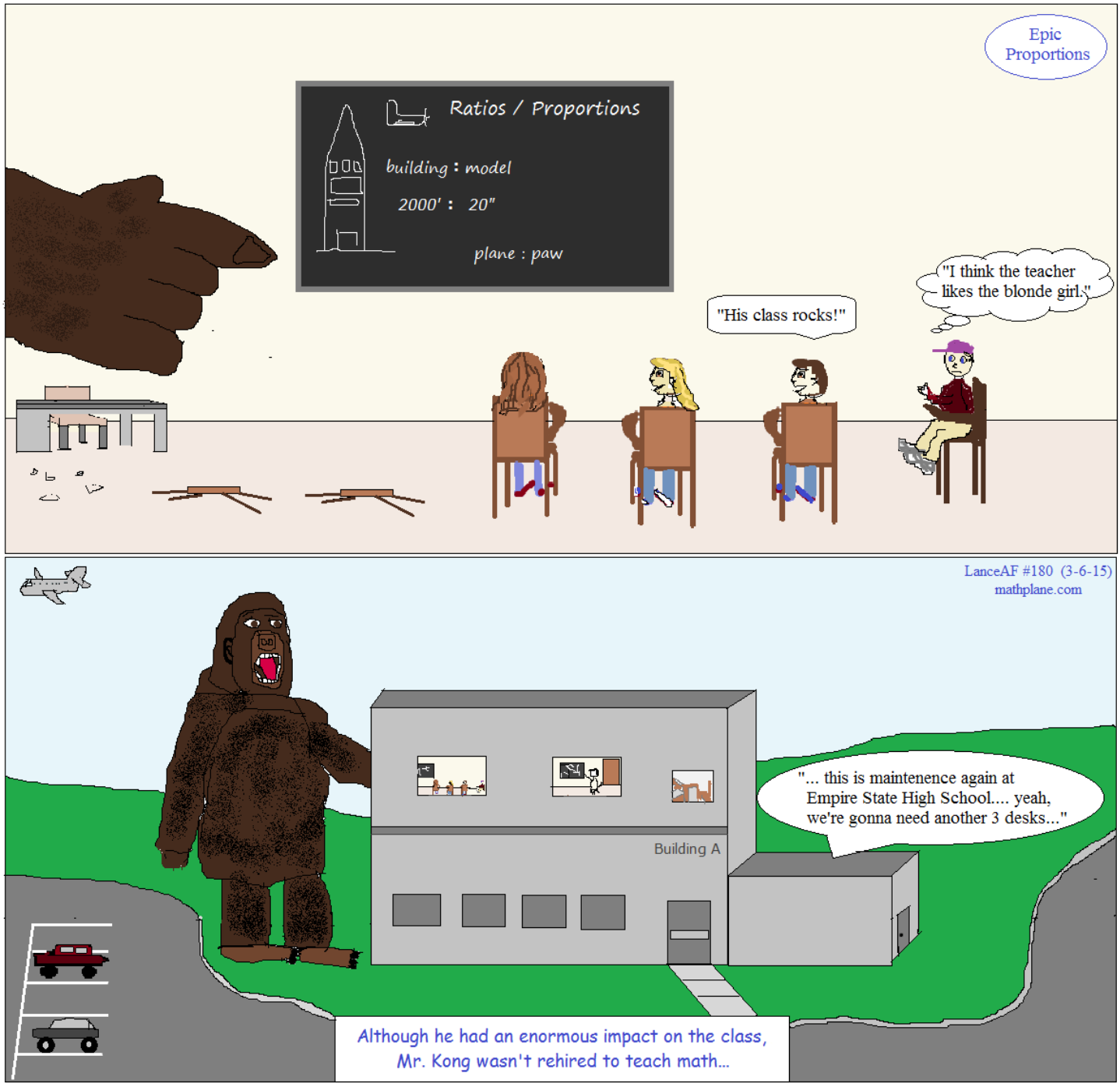
Triangles AMP and ABC are similar (because of Angle-Angle)

Since the ratio of AP to PM is 8/5,

the ratio of the corresponding segments AC to BC are also 8/5






Epic Proportions
Ratios / Proportions
building : model
2000' : 20"
plane : paw
"His class rocks!"
"I think the teacher likes the blonde girl."
LanceAF #180 (3-6-15)
mathplane.com
Building A
"... this is maintenence again at Empire State High School.... yeah, we're gonna need another 3 desks..."
Although he had an enormous impact on the class, Mr. Kong wasn't rehired to teach math...

Questions-→

I. Similarity Ratios: Surface Area and Volume

Each pair of solids are similar. Find the missing measurement.

A)

	Solid #1		Solid #2
Surface Area	2 square meters	Surface Area	98 square meters
Volume	5 cubic meters	Volume	?

B)

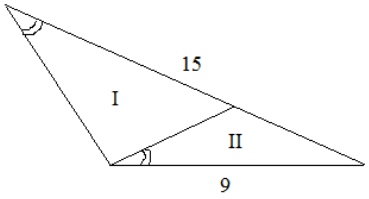
	Solid #3		Solid #4
Surface Area	1152 sq. feet	Surface Area	?
Volume	11,776 cubic feet	Volume	7,889 cubic feet

C)

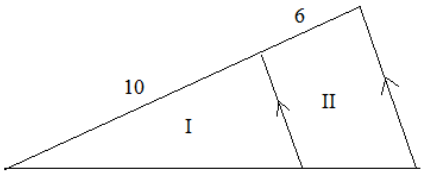
	Solid #5		Solid #6
SA	576 yds^2	SA	?
V	9216 yds^3	V	18 yds^3

II. What is the ratio of the areas of region I to region II?

A)



B)



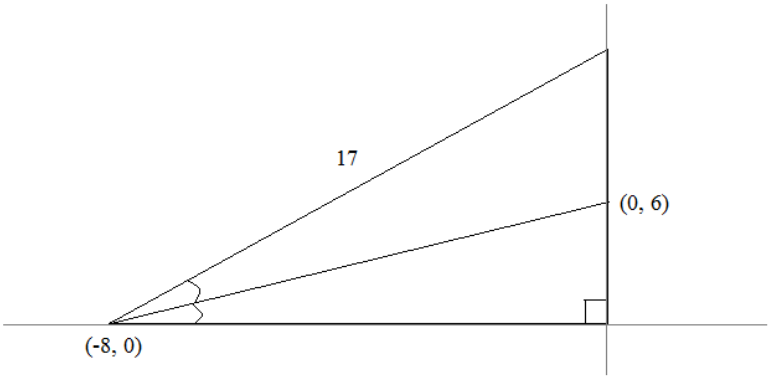
III. "Shadow Questions"

1) A 20 foot tree casts a 8 yard shadow.
How long is a shadow cast by a man 68 inches tall?

2) Jack is 6 feet tall. When standing near a 15 foot lamp post, his shadow is 4 feet.
If he walks 2 feet further from the lamp post, how much will his shadow increase?

IV. Concepts

1) What is wrong with this diagram?



2) If (quadrilateral) ABCD $\sim$ (quadrilateral) FGHI, which statement must be true?

a) $\angle A \cong \angle G$

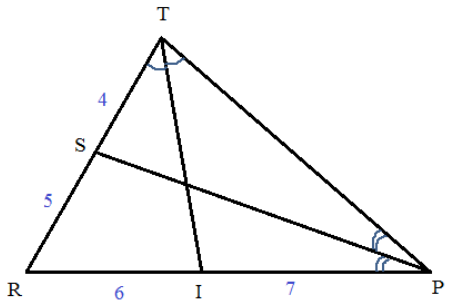
b) $\angle C \cong \angle H$

c) $\overline{BC} \cong \overline{GH}$

d) $\overline{AB} \cong \overline{HI}$

3) Is this diagram possible?

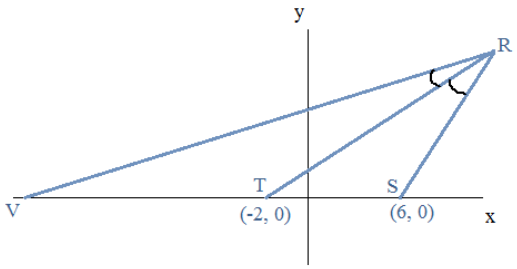
Justify your answer.



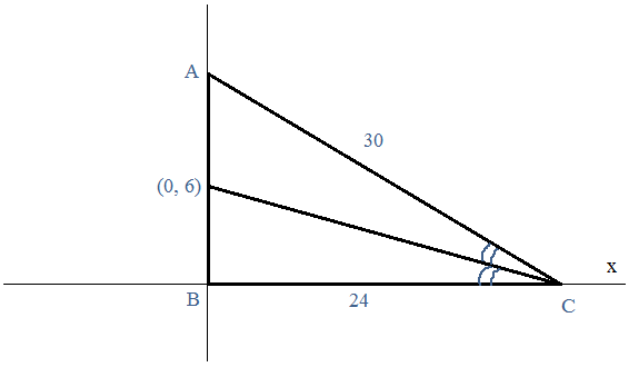
V. Angle Bisector

1) $\angle VRT \cong \angle TRS$ Coordinate R is (11, 12) $\overline{RV} = 28$

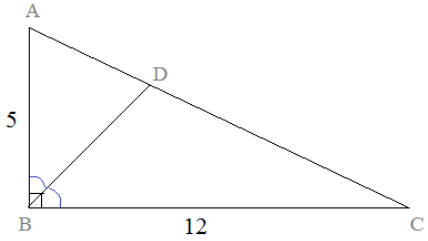
What is the coordinate of V?



2) What is the coordinate of A?



3) Find the length of $\overline{CD}$:



ABC is a right triangle, where $\angle ABD \cong \angle CBD$

VI. More Topics

A) Always/Sometimes/Never

1) If one base angle in an isosceles triangle is congruent to a base angle in another isosceles triangle, then the triangles are similar.

2) If one angle in an isosceles triangle is congruent to an angle in another isosceles triangle, then the triangles are similar.

3) If ratio of 2 sides of polygon is 3:4, then ratio of perimeters is 5:6

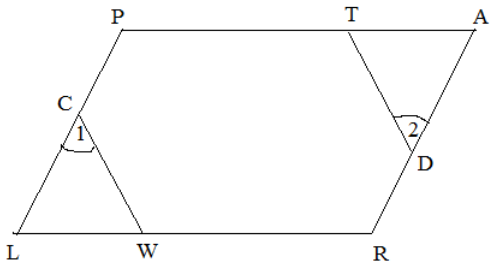
4) If ratio of all sides of polygons is 3:4, then ratio of perimeters is 5:6

B) Proof

Given: Parallelogram PARL

$\angle 1 \cong \angle 2$

Prove: (CL)(AT) = (DA)(LW)



Statements	Reasons

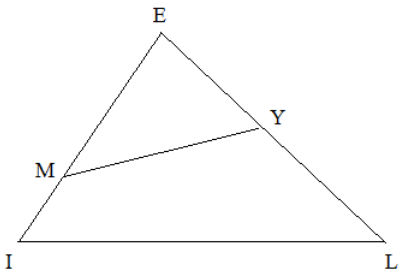
C) Solve $\angle EYM = \angle I$

$\overline{EM} = 14$

$\overline{MI} = 6$

$\overline{EY} = 12$

What is the length of $\overline{YL}$?



I. Similarity Ratios: Surface Area and Volume

SOLUTIONS

Each pair of solids are similar. Find the missing measurement.

A)

	Solid #1		Solid #2
Surface Area	2 square meters	Surface Area	98 square meters
Volume	5 cubic meters	Volume	1715 cubic meters

Step 1: Find the similarity ratio

$\frac{SA1}{SA2} = \frac{2}{98}$

similarity ratio is $\frac{\sqrt{2}}{7\sqrt{2}}$ ---> $\frac{1}{7}$

Step 2: Find ratio of volumes

$\frac{1^3}{7^3} = \frac{1}{343}$

Step 3: Apply ratio to solids

$\frac{1}{343} = \frac{5}{?}$ $V = 1715 \text{ meters}^3$

B)

	Solid #3		Solid #4
Surface Area	1152 sq. feet	Surface Area	882 sq feet
Volume	11,776 cubic feet	Volume	7,889 cubic feet

$\frac{V3}{V4} = \frac{11776}{7889}$ $\frac{\sqrt[3]{11776}}{\sqrt[3]{7889}} = \frac{8}{7}$ similarity ratio of #3 to #4 is 8:7

$\frac{SA3}{SA4} = \frac{8^2}{7^2} = \frac{64}{49}$ ratio of the areas

$\frac{64}{49} = \frac{1152 \text{ sq ft}}{SA}$ SA of 3 is 882 sq feet

C)

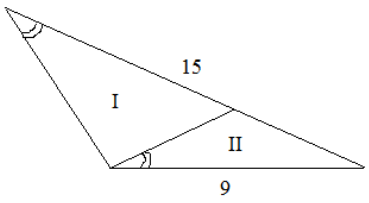
	Solid #5		Solid #6
SA	576 yds^2	SA	9 sq. yards
V	9216 yds^3	V	18 yds^3

$\frac{\sqrt[3]{9216}}{\sqrt[3]{18}} = 8$ similarity ratio of S5:S6 is 8:1, so ratio of areas is 64:1

$\frac{64}{1} = \frac{576}{S6}$ surface area of S6 is 9

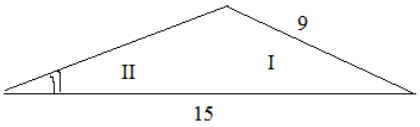
II. What is the ratio of the areas of region I to region II?

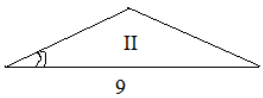
A)



Triangle II and Big triangle (I and II) are similar triangles... (Angle - Angle)

Ratio is 15 : 9 or 5 : 3 therefore, area ratio is $5^2 : 3^2$





25 : 9

entire : II

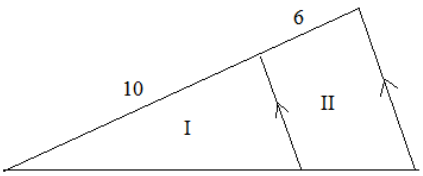
If 25 : 9 is whole : II

then, ratio of whole : I

is 25 : 16

ratio of I to II is 16 : 9

B)



10 : 16 or 5 : 8 (triangle I to whole)

25 : 64 (area of triangle I to area of whole triangle)

39 : 64 (area of II to area of whole)

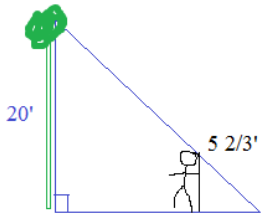
25 : 39 is ratio of area I to area II

III. "Shadow Questions"

SOLUTIONS

1) A 20 foot tree casts a 8 yard shadow.
How long is a shadow cast by a man 68 inches tall?

Draw a diagram and convert the units!



Set up the ratios:

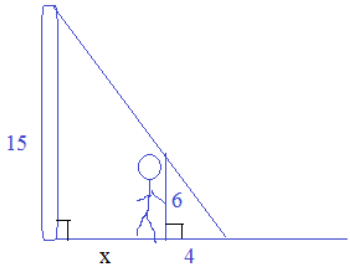
tree shadow $\frac{20'}{24'} = \frac{5\ 2/3'}{X}$ man shadow

$20X = 136'$

$X = 6.8$ feet

2) Jack is 6 feet tall. When standing near a 15 foot lamp post, his shadow is 4 feet.
If he walks 2 feet further from the lamp post, how much will his shadow increase?

Step 1: Determine proportion (to find distance from lamp post)



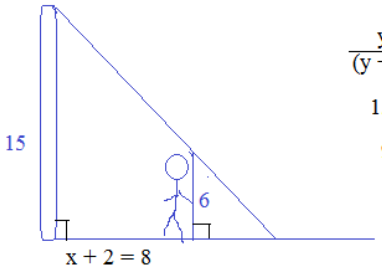
$\frac{6}{15} = \frac{4}{(x+4)}$

$60 = 6x + 24$

$36 = 6x$

$x = 6$

Step 2: Redraw diagram with Jack 2 feet further...



$\frac{y}{(y+8)} = \frac{6}{15}$

$15y = 6y + 48$

$9y = 48$

$y = 5\ 1/3$

The shadow goes from 4 feet to 5 1/3 feet....

So, it increases by 1' 4"

IV. Concepts

1) What is wrong with this diagram?

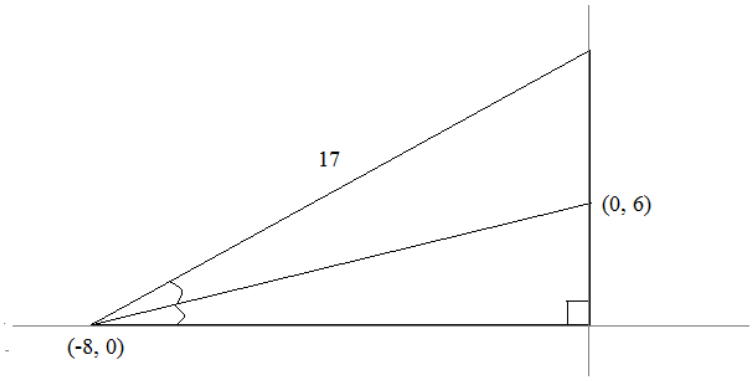
According angle bisector theorem, $\frac{17}{8} = \frac{x}{6}$

$x = 12.75$

But, according to the pythagorean theorem,

$8^2 + (\text{right side})^2 = 17^2$

the right side must equal 15..
but, it is 18.75.... (which is greater than the hypotenuse)



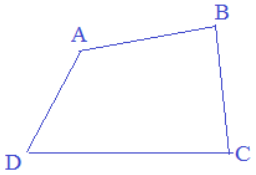
2) If (quadrilateral) ABCD $\sim$ (quadrilateral) FGHI, which statement must be true?

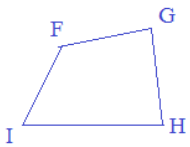
a) $\angle A \cong \angle G$

b) $\angle C \cong \angle H$

c) $\overline{BC} \cong \overline{GH}$

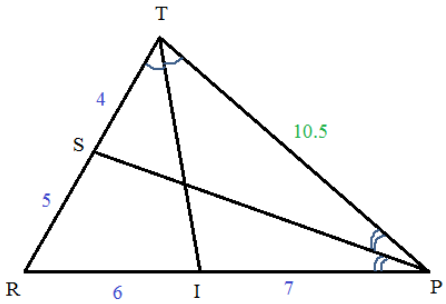
d) $\overline{AB} \cong \overline{HI}$





3) Is this diagram possible? NO it is not...

Justify your answer.



SOLUTIONS

Utilizing the angle bisector theorem:

$$\frac{TR}{RI} = \frac{TP}{PI}$$

$$\frac{9}{6} = \frac{TP}{7} \qquad TP = 10.5$$

Then,

$$\frac{TP}{TS} = \frac{PR}{PS}$$

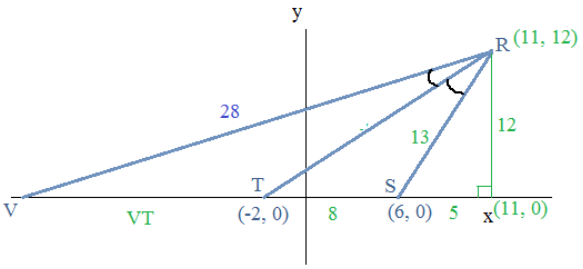
$$\frac{10.5}{4} = \frac{13}{5}$$

BUT, this proportion is not equal!!

V. Angle Bisector

1) $\angle VRT \cong \angle TRS$ Coordinate R is (11, 12) $\overline{RV} = 28$

What is the coordinate of V?



$$\frac{RS}{TS} = \frac{RV}{VT}$$

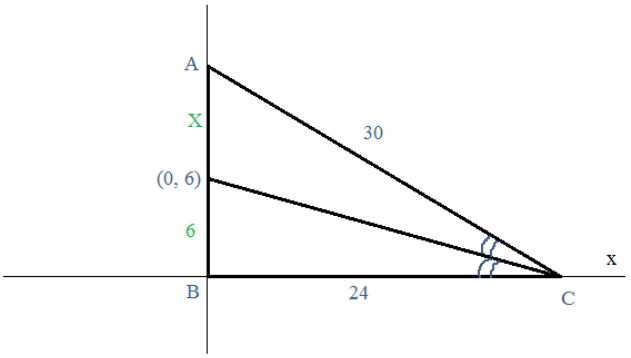
$$\frac{13}{8} = \frac{28}{VT}$$

$$13(VT) = 8(28)$$

$$VT = 17.23$$

Coordinate V is (-19.23, 0)

2) What is the coordinate of A?



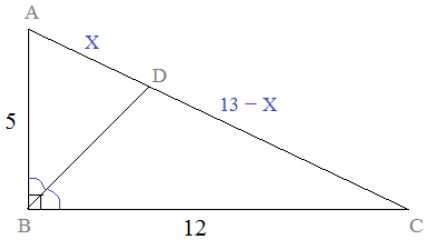
$$\frac{30}{24} = \frac{X}{6}$$

OR $X = 7.5$

$$\frac{30}{X} = \frac{24}{6}$$

Coordinate A is (0, 13.5)

3) Find the length of $\overline{CD}$:



ABC is a right triangle, where $\angle ABD \cong \angle CBD$

Pythagorean Theorem: $AB^2 + BC^2 = AC^2$

$$5^2 + 12^2 = AC^2$$

$$AC = 13$$

$$AD + DC = AC$$

Angle Bisector Theorem: $\frac{AB}{AD} = \frac{BC}{CD}$

$$\frac{5}{X} = \frac{12}{13 - X}$$

$$65 - 5X = 12X$$

$$65 = 17X$$

$X = 3.824$

CD = 9.176

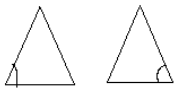
VI. More Topics

SOLUTIONS

A) Always/Sometimes/Never

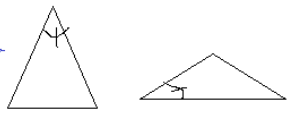
1) If one base angle in an isosceles triangle is congruent to a base angle in another isosceles triangle, then the triangles are similar.

ALWAYS



2) If one angle in an isosceles triangle is congruent to an angle in another isosceles triangle, then the triangles are similar.

SOMETIMES



3) If ratio of 2 sides of polygon is 3:4, then ratio of perimeters is 5:6

SOMETIMES (all the corresponding sides must be proportional!)

4) If ratio of all sides of polygons is 3:4, then ratio of perimeters is 5:6

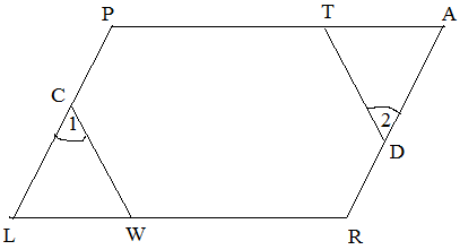
NEVER

B) Proof

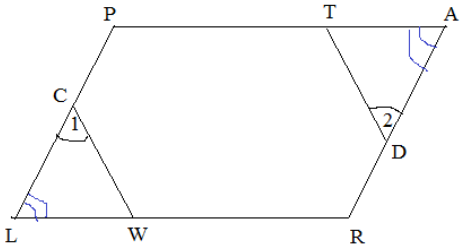
Given: Parallelogram PARL

$\angle 1 \cong \angle 2$

Prove: (CL)(AT) = (DA)(LW)



Statements	Reasons
1) ▱PARL	1) Given
2) $\angle 1 \cong \angle 2$	2) Given
3) $\angle A \cong \angle L$	3) Definition of Parallelogram (opposite angles congruent)
4) $\triangle TAD \sim \triangle WLC$	4) Angle-Angle triangle similarity
5) $\frac{CL}{DA} = \frac{LW}{AT}$	5) CSSTP (Corresponding Sides of Similar Triangles are Proportional)
6) (CL)(AT) = (DA)(LW)	6) MEPT Means-Extremes Product Theorem



C) Solve

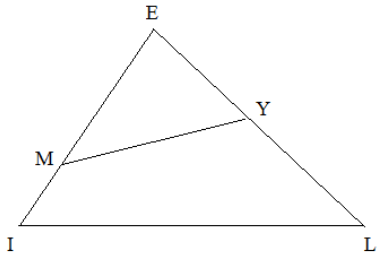
$\angle EYM = \angle I$

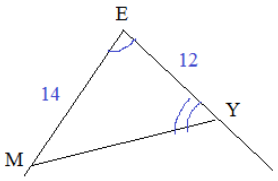
$\overline{EM} = 14$

$\overline{MI} = 6$

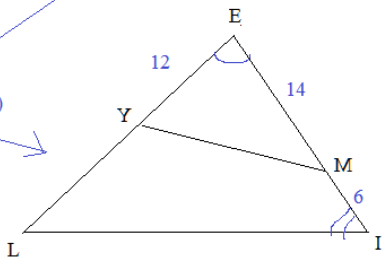
$\overline{EY} = 12$

What is the length of $\overline{YL}$?





Triangles are similar (angle-angle)



$\frac{20}{12} = \frac{EL}{14}$

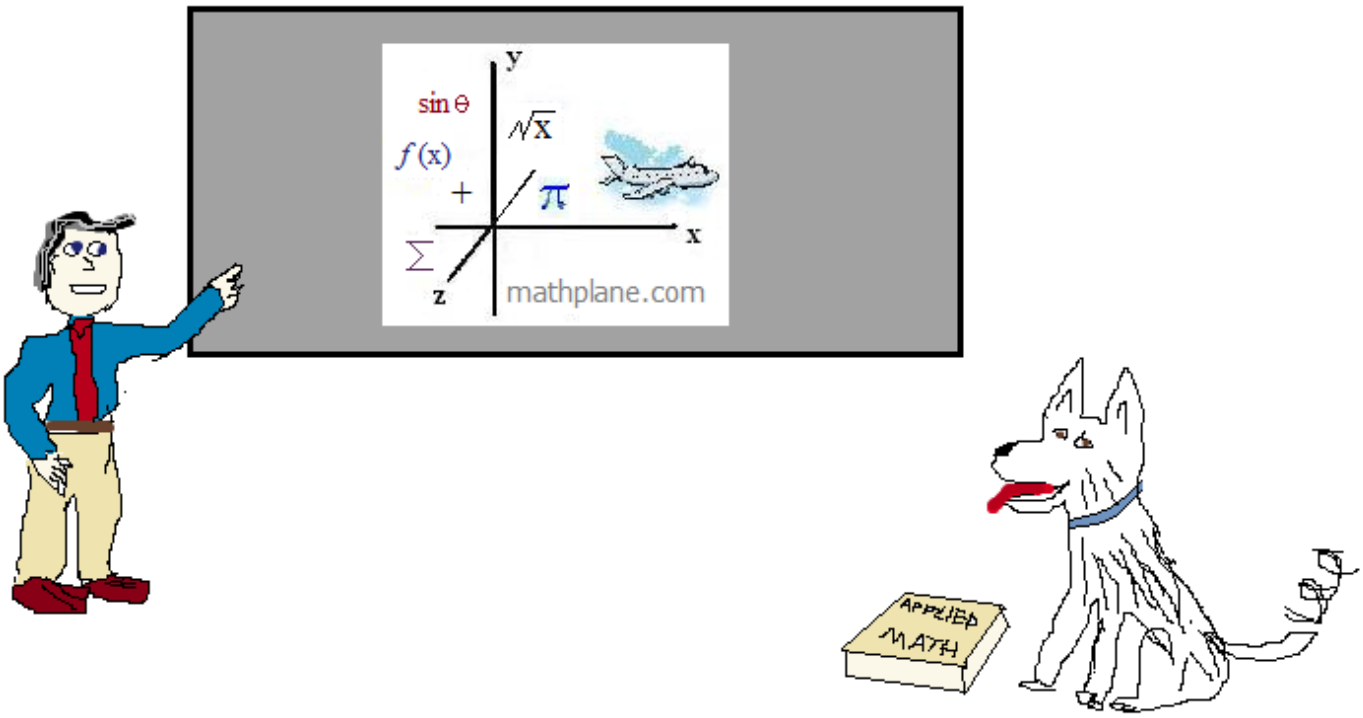
$EL = 70/3$

therefore, YL is 70/3 - 36/3 = 34/3

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



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