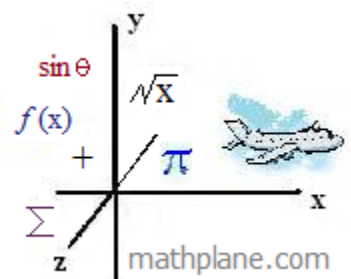
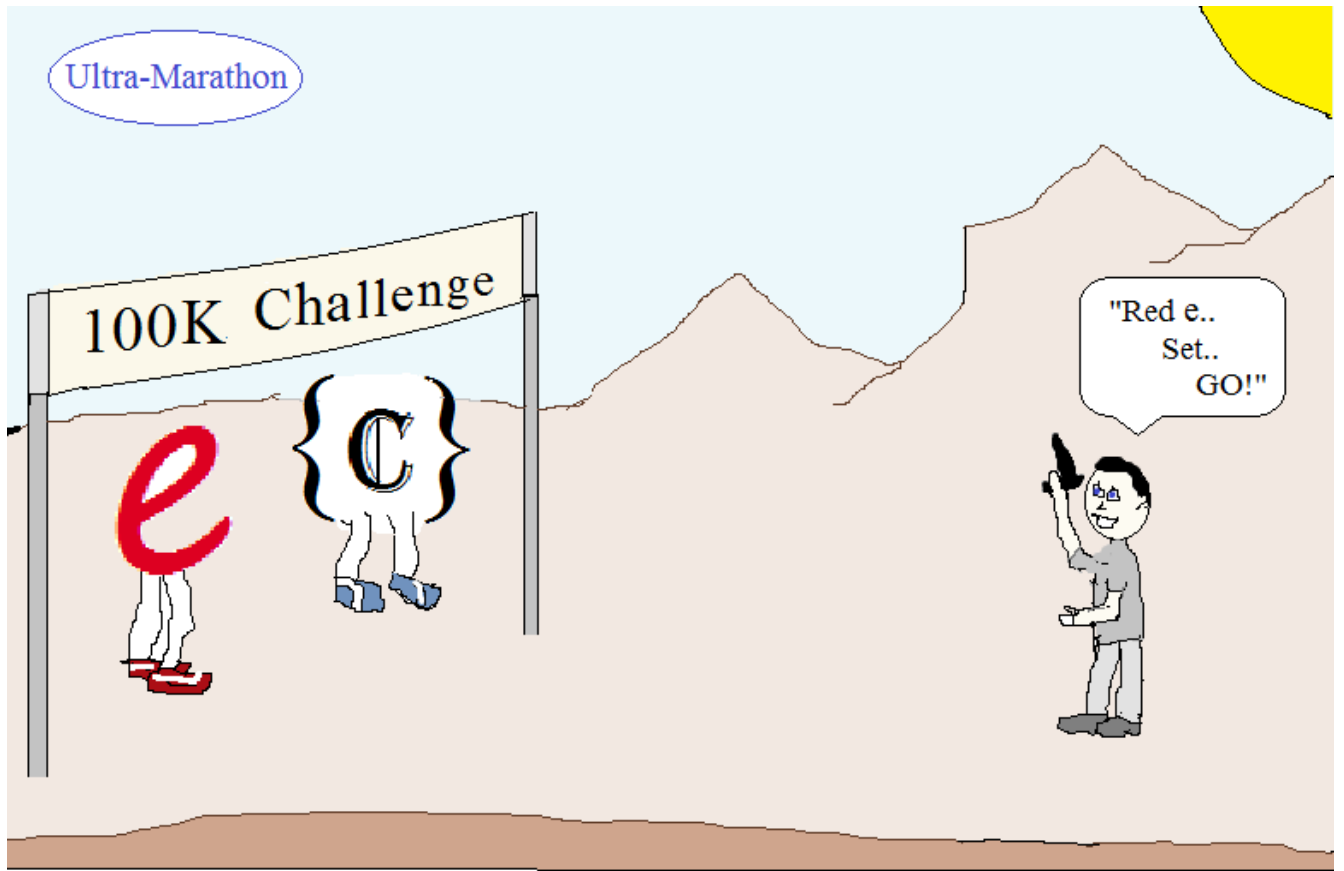


Algebra 2/Trig Final

Practice Exam (w/solutions)

Sections include trigonometry terms, conics, logarithms, linear systems, matrices, sequences & series, counting principles and more...





Testing the limits of endurance,
these math figures will run on and on...

PRACTICE EXAM

Algebra II/Trig Practice Final

I. Trigonometry

A) Convert from Degrees to Radians (Express answer in terms of π)

$$135^\circ$$

$$15^\circ$$

Convert from Radians to Degrees

$$\frac{3\pi}{4} =$$

$$-\frac{5\pi}{2} =$$

B) For the angle $A = 890^\circ$

Find the reference angle. Then, find one positive coterminal angle and one negative coterminal angle..

C) What is the exact value of $\cos 330^\circ$? What is the exact value of $\cos \frac{3\pi}{4}$?

D) If the terminal side of angle B passes through $(-4, 5)$, find the 6 trigonometric functions of angle B.

E) Adam is looking at a tree 600 feet away. Looking up at a 35° angle of elevation, he spots a bird sitting at the top of the tree. How high is the bird?

F) Underdog is on top of a 1200 foot tall building. Looking at a 27° angle of depression, he sees SimonBarSinister harming SweetPollyPureBread at the bottom of a building 8 blocks away. How far must Underdog fly to directly get to Polly? How far apart are the 2 buildings (in feet)?

II. Logarithms

A) Find

$$\log_4 16 =$$

$$\log_b b =$$

$$\log 1000 =$$

B) Solve for x

$$3^{X+1} = 27^{X-1}$$

$$5^X = 9$$

C) Expand

$$\log_2 21x^2y$$

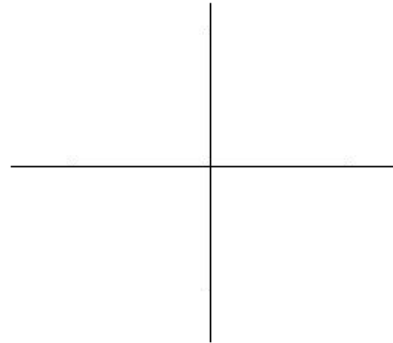
$$\log_a \left(\frac{4x^2}{y} \right)$$

D) Condense

$$\log_d 4 - 2\log_d X + 3\log_d Y$$

E) Convert to exponential form and graph

$$y = \log_2 x$$



F) Bonus Question: Earning 7% interest annually, how long will it take \$3000 to grow to \$6000?

III. Conics

1. $5x^2 - 80x + 5y^2 - 30y - 135 = 0$

2. $4x^2 - 24x + y^2 + 32 = 0$

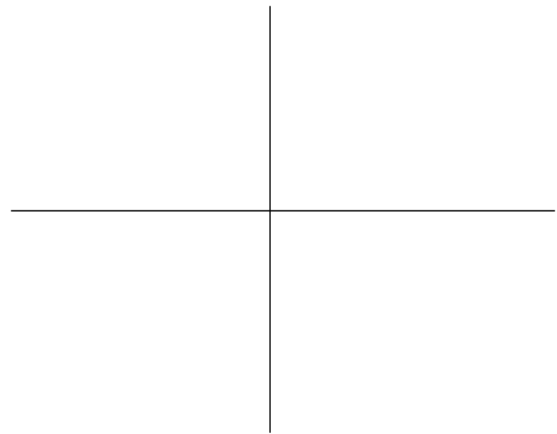
3. $6x^2 + 24x + 3y = 24$

4. $4x^2 - y^2 + 16 = 0$

A) Identify the 4 conics above.

B) What is the center of conic #1?

C) Graph conic #2 (and, label the vertices)



IV. Solving Linear Systems

A) Solve using Cramer's Rule

$$\begin{aligned}7x + 5y &= 6 \\ -3x - 2y &= -2\end{aligned}$$

B) Solve

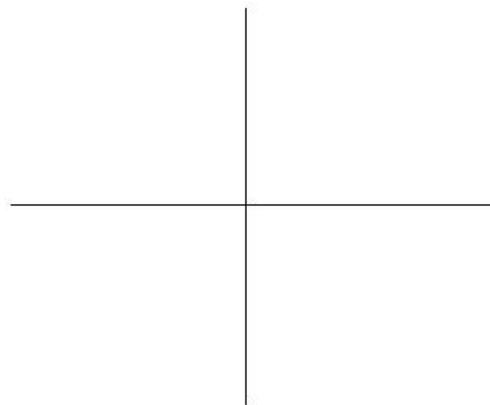
$$\begin{aligned}x^2 + y^2 &= 20 \\ x - y &= -2\end{aligned}$$

C) Solve

$$\begin{aligned}3x + 7y &= 37 \\ 10x + 14y &= 86\end{aligned}$$

D) Graph

$$\begin{aligned}x &\geq -4 \\ y &< -2x + 6 \\ -3x + y &\geq 1\end{aligned}$$



V. Matrix and Determinants

A) Evaluate

$$\begin{vmatrix} 6 & -3 \\ 1 & 2 \end{vmatrix} =$$

B)

$$A = \begin{bmatrix} -3 & 0 & -1 \\ -2 & 2 & 1 \\ -6 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 0 \\ -2 & -3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

1) Find $A + B$

2) Find BA

3) Find C^{-1}

4) Find $-4A$

VI. Sequences and Series

A) Find the 11th term of the sequence 4, 9, 14, 19, ...

B) Given the recursive sequence

$$a_n = 2a_{n-1} + 3 \quad \text{where } a_1 = 1$$

list the first 5 terms of a_n

C) What is the 9th term for the sequence $a_n = 3n^2$?

D) What is the 8th term of the following geometric series?

$$3 + 3\sqrt{2} + 6 + 6\sqrt{2} + \dots$$

E) In the following arithmetic series, what term has value 87?

$$3 + 7 + 11 + \dots$$

F)
$$\sum_{i=1}^5 \left(\frac{1}{4}\right)^{i-1}$$

G)
$$\sum_{i=1}^{20} 3i - 5$$

H)
$$\sum_{i=1}^{100} \frac{1}{(i+1)} - \frac{1}{(i+2)}$$
 (hint: find the pattern)

VII. Miscellaneous

A) $4^2 =$ $4^{-2} =$ $4^{1/2} =$ $4^{-1/2} =$

B) How many ways can you arrange the letters in the word VICTORY?

C) How many different arrangements can be made from the word SUCCESS?

D) What is the probability of drawing 2 Aces from a standard deck of cards?

E) Ice Cream Shoppe has 12 flavors. How many 3 flavor combinations could a customer make?

F) What are the ODDS of rolling a 2, 3, or 12 with two dice?

G) $(2e^2)^3 =$

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com



Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (t), or any beverage...

*Also, look for Honey Graham Squares
in the geometry section of your local store...*

SOLUTIONS

Algebra II/Trig practice final

I. Trigonometry

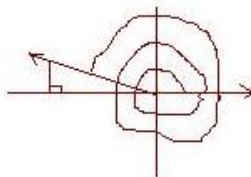
A) Convert from Degrees to Radians (Express answer in terms of π)

$$\frac{\pi}{180^\circ} \cdot \frac{135^\circ}{1} = \frac{3\pi}{4} \qquad \frac{\pi}{180^\circ} \cdot 15^\circ = \frac{\pi}{12}$$

Convert from Radians to Degrees

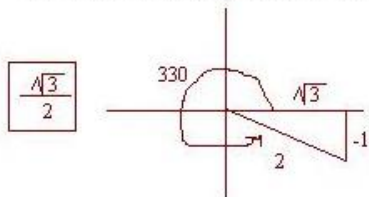
$$\frac{180^\circ}{\pi} \cdot \frac{3\pi}{4} = 135^\circ \qquad \frac{180^\circ}{\pi} \cdot -\frac{5\pi}{2} = -450^\circ$$

B) For the angle $A = 890^\circ$
 find the reference angle, then,
 find one positive coterminal angle and
 one negative coterminal angle.



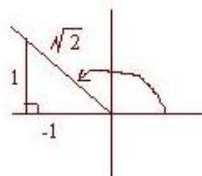
reference angle = 10°
 examples:
 positive coterminal
 angle $\rightarrow 170$
 negative coterminal
 angle $\rightarrow -190$

C) What is the exact value of $\cos 330^\circ$? What is the exact value of $\cos 3/4\pi$?

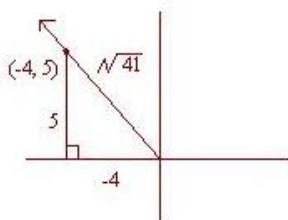


$$3/4\pi = 135^\circ$$

$$\frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$



D) If the terminal side of angle B passes through $(-4, 5)$,
 find the 6 trigonometric functions of angle B.

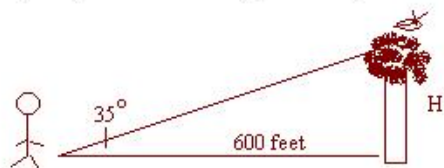


$$\cos B = \frac{-4}{\sqrt{41}} \qquad \sec B = \frac{-\sqrt{41}}{4}$$

$$\sin B = \frac{5}{\sqrt{41}} \qquad \csc B = \frac{\sqrt{41}}{5}$$

$$\tan B = \frac{-5}{4} \qquad \cot B = \frac{-4}{5}$$

E) Adam is looking at a tree 600 feet away. Looking up at a 35° angle of elevation, he spots a bird sitting at the top of the tree. How high is the bird?



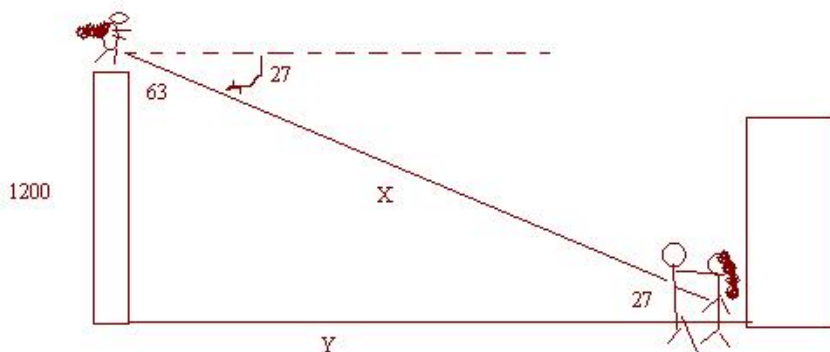
$$\tan 35 = H/600 \text{ feet}$$

$$.70 \times 600 = 420.12 \text{ feet}$$

F) Underdog is on top of a 1200 foot tall building. Looking at a 27° angle of depression, he sees SimonBarSinister harming SweetPollyPureBread at the bottom of a building 8 blocks away. How far must Underdog fly to directly get to Polly? How far apart are the 2 buildings (in feet)?

a) $\cos 63 = 1200/X$
 $X = 2643.22 \text{ feet}$

b) $\tan 63 = Y/1200$
 $Y = 2355.13 \text{ feet}$



(use pythagorean theorem to check)

II Logarithms

A) Find

$$\log_4 16 = 2$$

$$\log_b b = 1$$

$$\log_{10} 1000 = 3$$

$$b^1 = b$$

$$10^3 = 1000$$

B) Solve for X

$$3^{X+1} = 27^{X-1}$$

convert to "common bases"

$$3^{(X+1)} = (3^3)^{(X-1)}$$

$$3^{(X+1)} = 3^{3X-3}$$

then, solve for X

$$X+1 = 3X-3 \rightarrow X=2$$

check answer:

$$3^{2+1} = 27^{2-1} \quad 27 = 27 \checkmark$$

$$5^X = 9$$

take log of both sides

$$\log 5^X = \log 9$$

solve for X

$$X \log 5 = \log 9$$

$$X (699) = .954$$

$$X = 1.365$$

check answer:

$$5^{1.365} = 8.997 \checkmark$$

C) Expand

$$\log_2 21X^2Y$$

$$\log_2 21 + 2 \log_2 X + \log_2 Y$$

$$\log_a \left(\frac{4X^2}{Y} \right)$$

$$\log_a 4 + 2 \log_a X - \log_a Y$$

note: we separate the constant, the X, and the Y when we expand

D) Condense

$$\log_d 4 - 2 \log_d X + 3 \log_d Y$$

condense the coefficients first!

$$\log_d 4 - \log_d X^2 + \log_d Y^3$$

then, condense the other elements

$$\log_d 4Y^3 - \log_d X^2$$

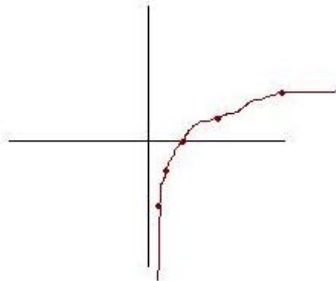
$$\log_d \frac{4Y^3}{X^2}$$

E) Convert to exponential form and graph

$$Y = \log_2 X$$

$$2^Y = X$$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2



F) Bonus question: Earning 7% interest annually, how long will it take \$3000 to grow to \$6000?

$$6000 = 3000 (1 + .07)^t$$

divide both sides by 3000

$$2 = (1 + .07)^t$$

take the log of both sides

$$\log 2 = \log (1 + .07)^t$$

solve for t

$$\log 2 = t \log (1.07)$$

$$t = \frac{\log 2}{\log (1.07)}$$

$$t = \frac{.301}{.029} = 10.24$$

note: check answer using

"rule of 72"...

approx: $\frac{72}{7}$ just over 10 years

III Conics

1. $5X^2 - 80X + 5Y^2 - 30Y - 135 = 0$

circle (coefficients of X^2 and Y^2 same)

2. $4X^2 - 24X + Y^2 + 32 = 0$

ellipse (coefficients of X^2 and Y^2 different and positive)

3. $6X^2 + 24X + 3Y = 24$

parabola (coefficient of Y is 0)

4. $4X^2 - Y^2 + 16 = 0$

hyperbola (coefficients of X^2 and Y^2 are different and not both positive)

A) Identify the 4 conics above.

B) What is the center of conic #1?

change to "vertex form"

divide by 5

$$5X^2 - 80X + 5Y^2 - 30Y = 135$$

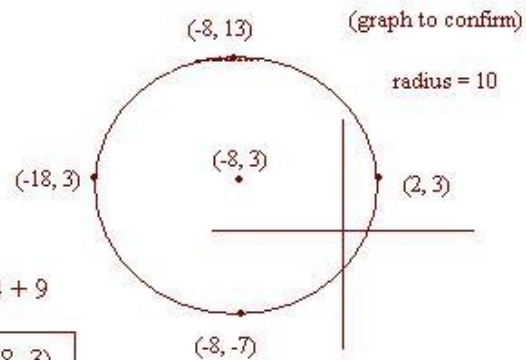
$$X^2 - 16X + Y^2 - 6Y = 27$$

complete the square

$$X^2 - 16X + 64 + Y^2 - 6Y + 9 = 27 + 64 + 9$$

$$(X + 8)^2 + (Y - 3)^2 = 100$$

center is (-8, 3)



C) Graph conic #2. (and, label the vertices)

change to vertex form

$$4X^2 - 24X + Y^2 = -32$$

$$4(X^2 - 6X) + Y^2 = -32$$

$$4(X^2 - 6X + 9) + Y^2 = -32 + 36$$

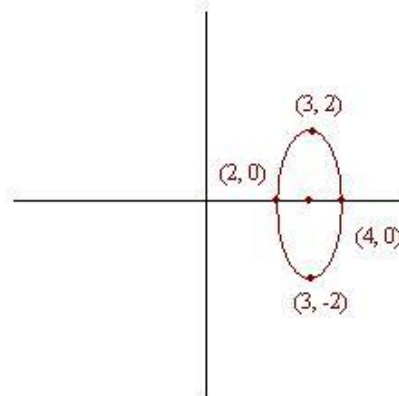
$$4(X - 3)^2 + Y^2 = 4$$

$$\frac{(X - 3)^2}{1} + \frac{Y^2}{4} = 1$$

center: (3, 0)

major axis y length = 2

minor axis x length = 1



*** to check answers, plug the 4 vertex values into the equation!!

IV. Solving Linear Systems

A) Solve using Cramer's Rule

$$\begin{array}{l} 7X + 5Y = 6 \\ -3X - 2Y = -2 \end{array} \quad D = \begin{vmatrix} 7 & 5 \\ -3 & -2 \end{vmatrix} = 1 \quad D_x = \begin{vmatrix} 6 & 5 \\ -2 & -2 \end{vmatrix} = -2 \quad D_y = \begin{vmatrix} 7 & 6 \\ -3 & -2 \end{vmatrix} = 4$$

$$X = \frac{D_x}{D} = -2 \quad Y = \frac{D_y}{D} = 4$$

check: $7(-2) + 5(4) = 6$ ✓
 $-3(-2) - 2(4) = -2$ ✓

B) Solve.

$$\begin{array}{l} X^2 + Y^2 = 20 \\ X - Y = -2 \\ X = Y - 2 \end{array}$$

(use substitution method)

$$\begin{array}{l} (Y - 2)^2 + Y^2 = 20 \\ Y^2 - 4Y + 4 + Y^2 = 20 \\ 2Y^2 - 4Y - 16 = 0 \\ Y^2 - 2Y - 8 = 0 \end{array}$$

$$\begin{array}{l} (Y - 4)(Y + 2) = 0 \\ Y = 4, -2 \end{array}$$

$$\begin{array}{l} X - 4 = -2 \quad X = 2 \\ X - (-2) = -2 \quad X = -4 \end{array}$$

$$\boxed{(2, 4) \quad (-4, -2)}$$

C) Solve.

$$\begin{array}{l} 3X + 7Y = 37 \\ 10X + 14Y = 86 \\ 5X + 7Y = 43 \end{array}$$

(use combination or elimination method)

$$\begin{array}{r} -3X - 7Y = -37 \\ \underline{5X + 7Y = 43} \\ 2X = 6 \\ X = 3 \end{array}$$

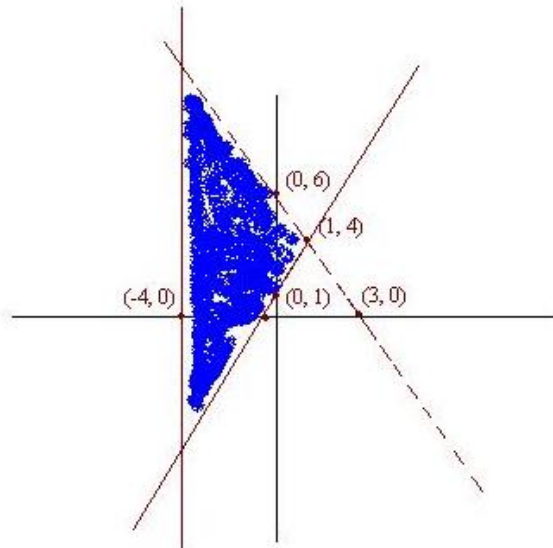
$$\begin{array}{r} 3(3) + 7Y = 37 \\ 9 + 7Y = 37 \\ 7Y = 28 \\ Y = 4 \end{array}$$

check: $10(3) + 14(4) = 86$ ✓

$$\boxed{(3, 4)}$$

D) Graph

$$\begin{array}{l} X \geq -4 \\ Y < -2X + 6 \\ -3X + Y \geq 1 \\ Y \geq 3X + 1 \end{array}$$



V. Matrix and Determinants

A) Evaluate

$$\begin{vmatrix} 6 & -3 \\ 1 & 2 \end{vmatrix} = (6 \times 2) - (-3 \times 1) = 15$$

B)

$$A = \begin{bmatrix} -3 & 0 & -1 \\ -2 & 2 & 1 \\ -6 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 0 \\ -2 & -3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

1) Find A + B

$$\begin{bmatrix} -3+1 & 0+2 & -1+(-1) \\ -2+3 & 2+0 & 1+0 \\ -6+(-2) & 1+(-3) & 4+4 \end{bmatrix} = \begin{bmatrix} -2 & 2 & -2 \\ 1 & 2 & 1 \\ -8 & -2 & 8 \end{bmatrix}$$

2) Find BA

(note: BA = AB)

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 0 \\ -2 & -3 & 4 \end{bmatrix} \begin{bmatrix} -3 & 0 & -1 \\ -2 & 2 & 1 \\ -6 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1(-3)+2(0)+(-1)(-6) & 1(0)+2(2)+(-1)(1) & 1(-1)+2(1)+(-1)(4) \\ 3(-3)+0(0)+0(-6) & 3(0)+0(2)+0(1) & 3(-1)+0(1)+0(4) \\ -2(-3)+(-3)(-2)+4(-6) & -2(0)+(-3)(2)+4(1) & -2(-1)+(-3)(1)+4(4) \end{bmatrix} = \begin{bmatrix} -1 & 3 & -3 \\ -9 & 0 & -3 \\ -12 & -2 & 15 \end{bmatrix}$$

3) Find C^{-1}

$$\begin{array}{l} 4R_1 + R_2 \quad \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 11 & 0 & 4 & 1 \end{array} \right] \quad -2R_2 + R_1 \quad \left[\begin{array}{cc|cc} 0 & -1 & 3/11 & -2/11 \\ 1 & 0 & 4/11 & 1/11 \end{array} \right] \\ \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right] \quad 1/11 R_2 \quad \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 1 & 0 & 4/11 & 1/11 \end{array} \right] \quad -1R_1 \quad \left[\begin{array}{cc|cc} 0 & 1 & -3/11 & 2/11 \\ 1 & 0 & 4/11 & 1/11 \end{array} \right] \end{array}$$

(check answer)

$$C^{-1}C = \begin{bmatrix} -3/11 & 2/11 \\ 4/11 & 1/11 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 11/11 \\ 11/11 & 0 \end{bmatrix}$$

(identity matrix)

4) Find $-4A$

$$-4A = \begin{bmatrix} -4(-3) & -4(0) & -4(-1) \\ -4(-2) & -4(2) & -4(1) \\ -4(-6) & -4(1) & -4(4) \end{bmatrix} = \begin{bmatrix} 12 & 0 & 4 \\ 8 & -8 & -4 \\ 24 & -4 & -16 \end{bmatrix}$$

VI Sequences and Series

A) Find the 11th term of the sequence 4, 9, 14, 19, ...

4, 9, 14, 19, 24, 29, 34, 39, 44, 49, 54, 59

B) Given the recursive sequence

$$a_n = 2a_{n-1} - 3 \quad \text{where } a_1 = 1$$

list the first 5 terms of a_n

$$a_1 = 1$$

$$a_2 = 2(1) - 3 = -1$$

$$a_3 = 2(-1) - 3 = -5$$

$$a_4 = 2(-5) - 3 = -13$$

$$a_5 = 2(-13) - 3 = -29$$

C) What is the 9th term for the sequence $a_n = 3n^2$?

$$a_9 = 3(9)^2 = 243$$

D) What is the 8th term of the following geometric series?

$$3 + 3\sqrt{2} + 6 + 6\sqrt{2} + 12 + 12\sqrt{2} + 24 + \boxed{24\sqrt{2}}$$

(multiplying each term by $\sqrt{2}$)

E) In the following arithmetic series, what term has value 87?

$$3 + 7 + 11 + \dots + 83 + 87$$

(arithmetic --> adding 4)

$$\frac{(87 - 3)}{4} = 21$$

22nd term

(spaces between 3 & 87)

F)

$$\sum_{i=1}^5 \left(\frac{1}{4}\right)^{i-1} = 1 + 1/4 + 1/16 + 1/64 + 1/256$$

$$= 1 + 5/16 + 5/256 = 21/16 + 5/256 = 378/256 + 5/256 = 1 \frac{127}{256}$$

G)

$$\sum_{i=1}^{20} 3i - 5 \quad \sum_{i=1}^{20} -5 = -100 \quad 3 \sum_{i=1}^{20} i = 3 \cdot \frac{20 \cdot 21}{2} = 630 \quad 630 - 100 = \boxed{530}$$

10 'pairs' value of each pair - (1 + 20)
(2 + 19) (3 + 18)
etc...

H)

$$\sum_{i=1}^{100} \frac{1}{(i+1)} - \frac{1}{(i+2)} = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{101} - \frac{1}{102}$$

$$\frac{1}{2} - \frac{1}{102} = \boxed{\frac{25}{51}}$$

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers,

Lance@mathplane.com

