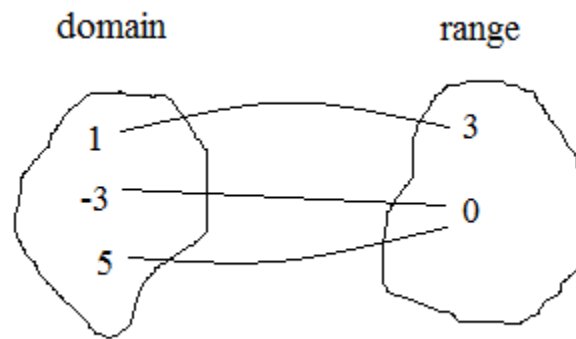


Domain/Range & Functions/Relations

Notes, examples, and quiz (with answers)



Topics include absolute value, graphs, radicals, vertical line test, piecewise functions, and more.

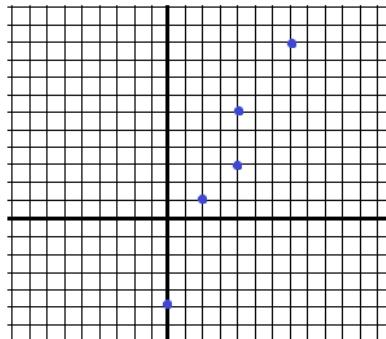
Domain & Range: Notes and Examples

What is the domain and range? Domain --- "all the X values"
 Range --- "all the Y values"

Example: For the relation, (4, 3) (2, 1) (0, -5) (4, 6) (7, 10)

Domain = {0, 2, 4, 7}
 Range = {-5, 1, 3, 6, 10}

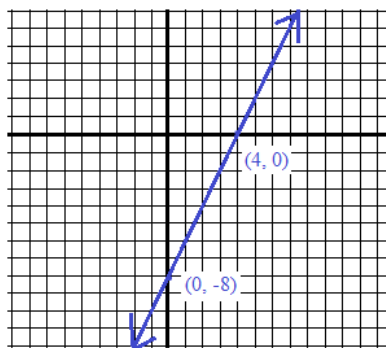
(numbers that are represented by
 points in the graph)



Example: For the function, $f(x) = 2x - 8$

Domain = {all real numbers}
 Range = {all real numbers}

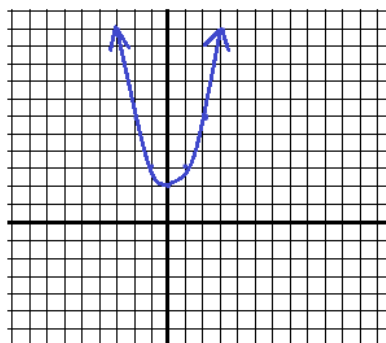
(any real number can go into x; and this
 will produce all possible real numbers)



Example: For the function, $y = x^2 + 2$

Domain = {all real numbers}
 Range = $\{y \geq 2\}$

(any number can go into x; but, it will only
 produce real numbers that are ≥ 2)

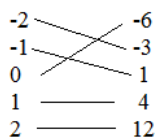


x	y
-3	11
-2	6
-1	3
0	2
1	3
2	6
3	11

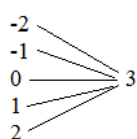
What is a relation? A function?

Relation --- a set containing pairs of related numbers (elements)
 Function --- a relation where *for each input value (X), there is only one output value (Y)*

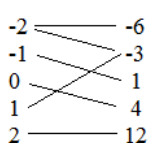
Note: All functions are relations... But, not all relations are functions.



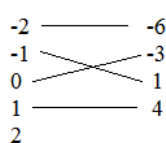
function



function



relation
(-2 has two range elements)



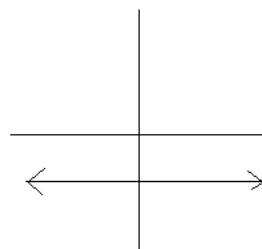
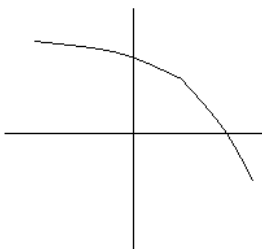
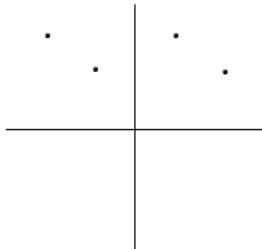
neither
(the 2 is in the domain; but, it has no range element!)

Vertical Line Test --- To determine if a set of pairs is a function, try the vertical line test.

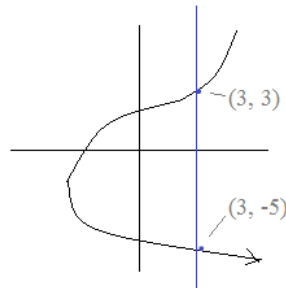
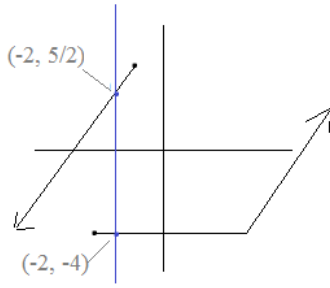
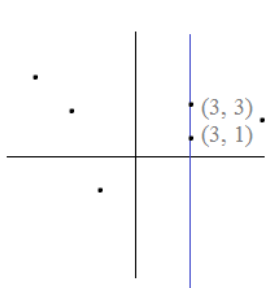
Graph the set.

If you can draw a vertical line through 2 or more points, then it is not a function.

(Note: "if a set of ordered pairs has 2 or more X values, then it is not a function")



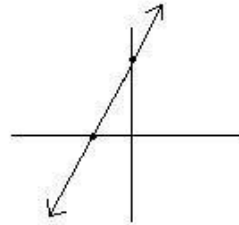
Functions: Every x has only one y value; passes vertical line test



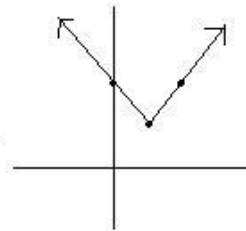
Relations that are not functions: sets of number pairs; but, each fails the vertical line test

Domain and Range examples and "exceptions"

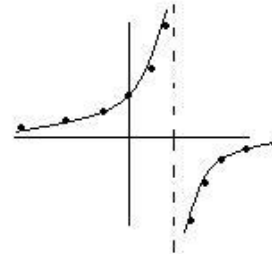
- 1) $y = 3x + 5$ linear equation; domain: all real numbers
 range: all real numbers
 It is a function...



- 2) $f(x) = |x - 2| + 3$ absolute value equation;
 domain: all real numbers
 (any number can go into x)
 range: all real numbers ≥ 3
 (there is no possible way to get an output less than 3)
 It is a function

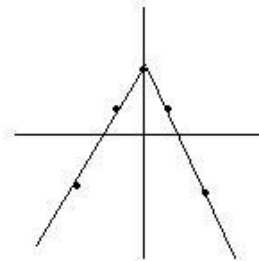


- 3) $f(x) = \frac{3}{2 - x}$
 domain: all real numbers $\neq 2$
 (can't have 0 in denominator)
 range: all real numbers $\neq 0$
 It is a function
 (passes vertical line test,
 It has an asymptote at $x=2$,
 so it is not continuous.
 Nevertheless, it is a
 function)
- | X | Y |
|----|-----------|
| -7 | 1/3 |
| -4 | 1/2 |
| -1 | 1 |
| 2 | undefined |
| 5 | -1 |
| 8 | -1/2 |
| 11 | -1/3 |



"Set Notation or Interval Notation"

- 4) $y = -2|x| + 3$ domain: all real numbers
 D: {all real numbers}
 domain: $(-\infty, \infty)$
 range: all real numbers ≤ 3
 R: {all real such that $y \leq 3$ }
 range: $(-\infty, 3]$
- set notation
 interval notation



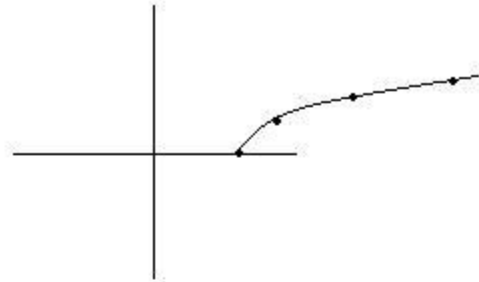
$$5) f(x) = \sqrt{x-3}$$

domain: all numbers ≥ 3

range: all numbers ≥ 0

x	f(x)
2	$\sqrt{1}$ not real
3	0
4	1
7	2
12	3

This is a square root function.



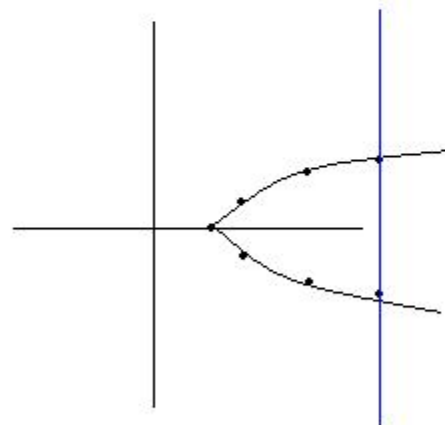
$$5a) y = \sqrt{x-3}$$

domain: $[3, \infty)$

range: $(-\infty, \infty)$

x	y
3	0
4	± 1
7	± 2
12	± 3
19	± 4

This is a square root relation.



fails "vertical line test"
Not a function

Mapping: Domain/Range, Functions/Relations

Try this exercise: for the following sets of ordered pairs, determine

- a) domain
- b) range
- c) whether the set is a function..

then,

- d) map the relationships

1) (1, 1) (1, 2) (1, 3) (1, 4)

2) (3, 3) (4, 3) (5, 3) (6, 3)

3) (-3, 7) (0, 0) (1, 6) (6, 1)

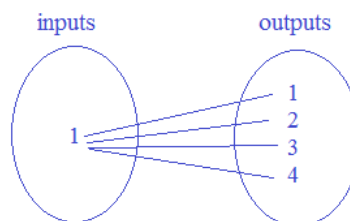
4) (1, 0) (0, 1) (2, 0) (0, 2)

SOLUTIONS....

1) (1, 1) (1, 2) (1, 3) (1, 4)

domain: { 1 }
range: { 1, 2, 3, 4 }

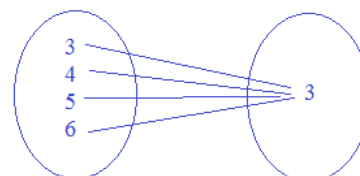
not a function



2) (3, 3) (4, 3) (5, 3) (6, 3)

domain: { 3, 4, 5, 6 }
range: { 3 }

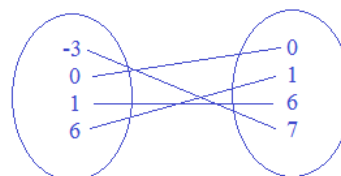
function
(each input has only one output..)



3) (-3, 7) (0, 0) (1, 6) (6, 1)

domain: { -3, 0, 1, 6 }
range: { 0, 1, 6, 7 }

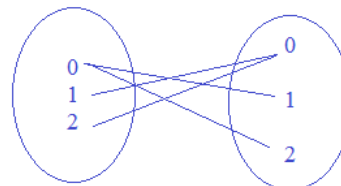
function



4) (1, 0) (0, 1) (2, 0) (0, 2)

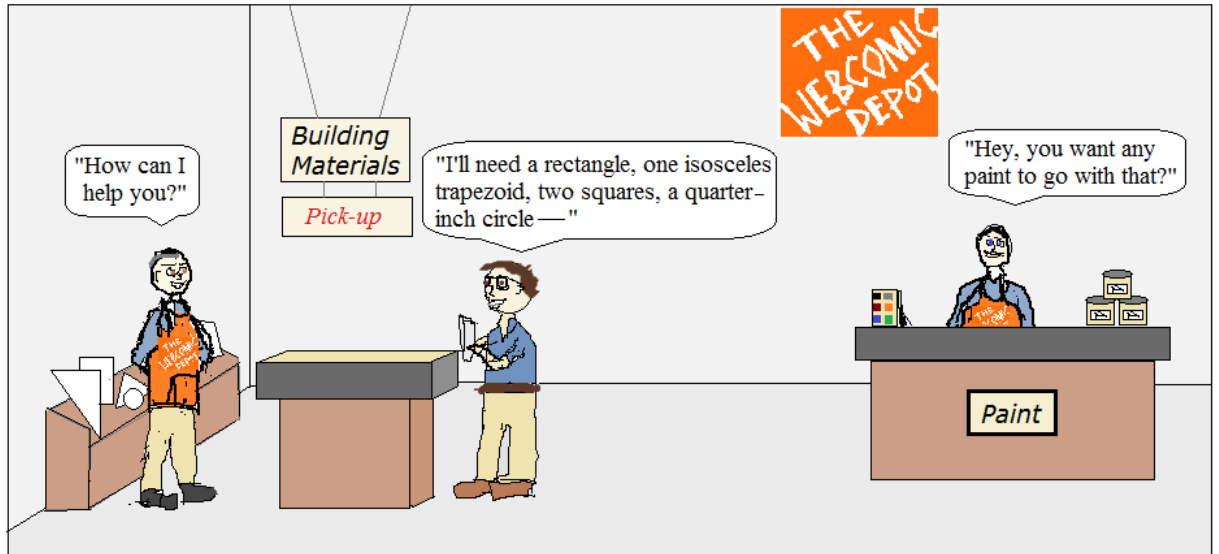
domain: { 0, 1, 2 }
range: { 0, 1, 2 }

not a function
(because input 0 has 2 different outputs!)



Math Homebuilders

Last Weekend...

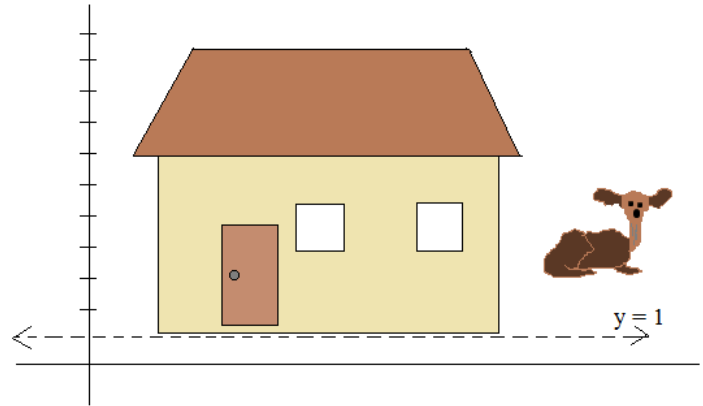


Today...

Home on the Range

Somewhere in The Great Planes of Kansas....

domain: $\{x \in \mathbb{R}\}$
 range: $\{y \in \mathbb{R} \mid 1 \leq y < 11\}$



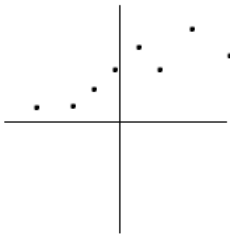
♪ "Home, home in a range ----where a deer and an asymptote lay.."

Practice Quiz-→

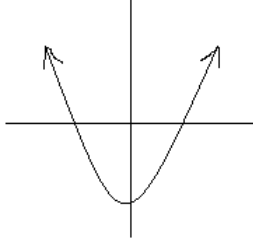
Relations and Functions

1) Which of the following relations are *functions*?

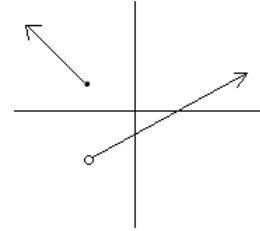
a)



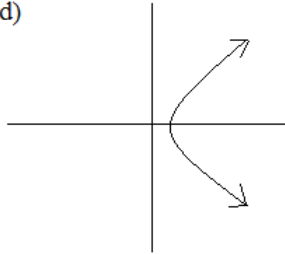
b)



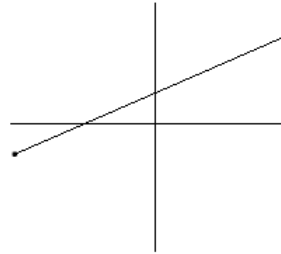
c)



d)



e)



2) The following points represent *relation* y : $(-1, 4)$ $(-1, 8)$ $(1, -4)$ $(6, -4)$ $(0, 0)$

Remove one point that would change the set to a *function*.

- a) $(-1, 4)$
- b) $(-1, 8)$
- c) $(1, -4)$
- d) $(6, -4)$
- e) $(0, 0)$

3) Determine the domain of $f(x) = \frac{5}{\sqrt{x-3}}$

- a) $x > 3$
- b) $x \geq 3$
- c) $x = 5$
- d) all real numbers
- e) all positive real numbers

What is the range of $y = -|x + 4| + 2$?

- a) $y \geq -4$
- b) $y \leq -4$
- c) $y \geq 2$
- d) $y \leq 2$
- e) all positive real numbers

4) Which of the following are functions?

a) $y = -6$

b) $x = 3$

c) $y = 3x^2$

d) $x = y^2 + 2$

e) $y = \frac{4}{x}$

5) Consider the 26 letters of the alphabet.

a) Identify 3 letters that when mapped on a coordinate plane, the result could be a function.

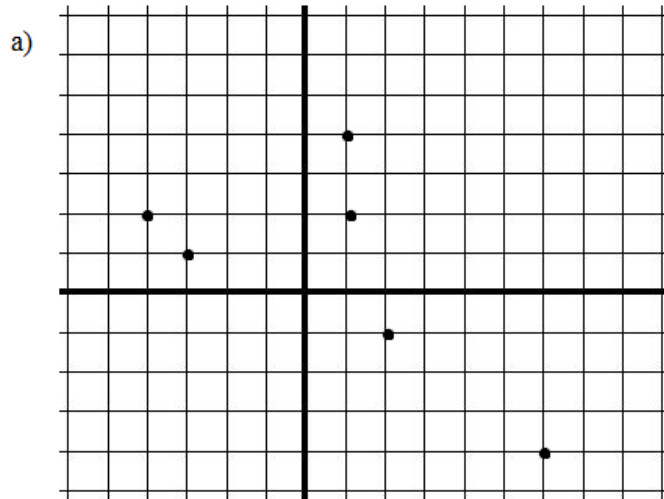
b) Identify 3 letters that when mapped on a coordinate plane, the result would not be a function.

6) What is the domain and range of $g(x) = x^2 - 6x + 5$?

7) What is the domain and range of the following *piecewise* function?

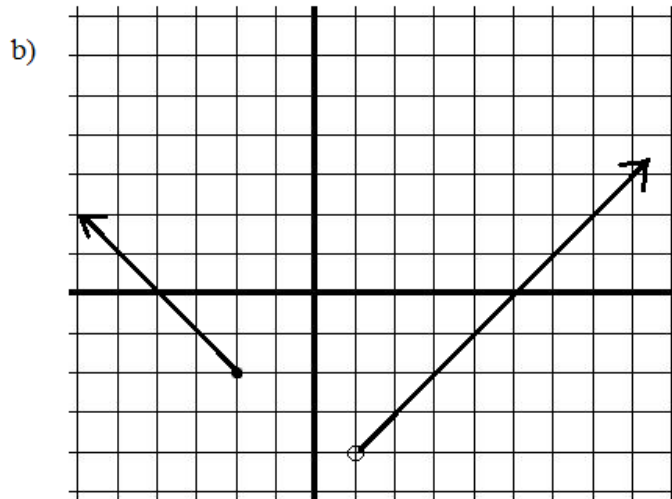
$$f(x) = \begin{cases} 5 - x & \text{if } x < 0 \\ 2 & \text{if } 0 \leq x < 5 \\ x - 9 & \text{if } x \geq 5 \end{cases}$$

8) Write the domain and range of each relation/function (in set notation or interval notation).



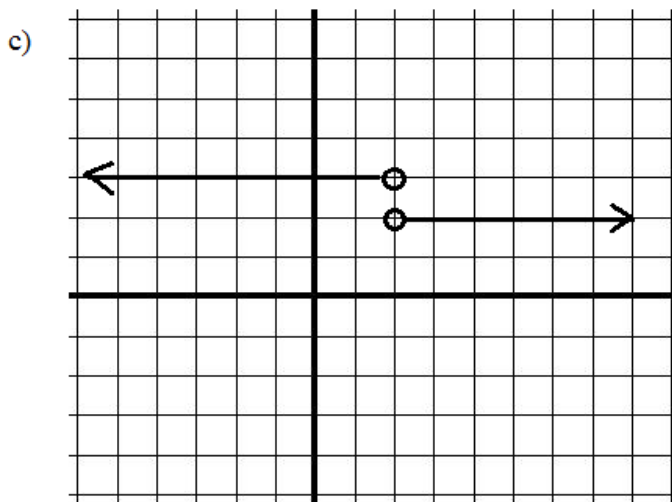
Domain:

Range:



Domain:

Range:

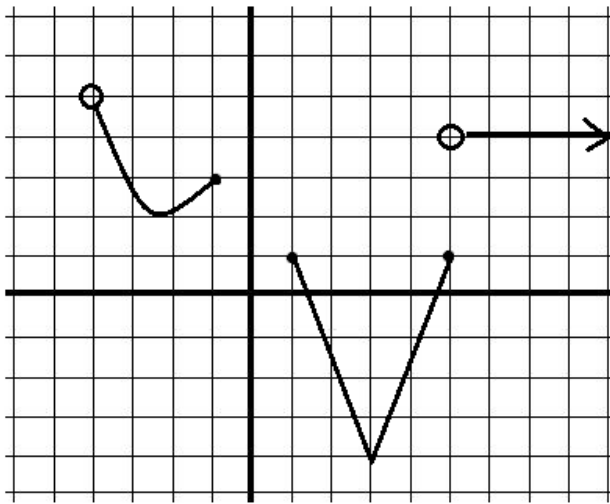


Domain:

Range:

Write the domain and range of each relation/function (in set notation or interval notation).

d)



Domain:

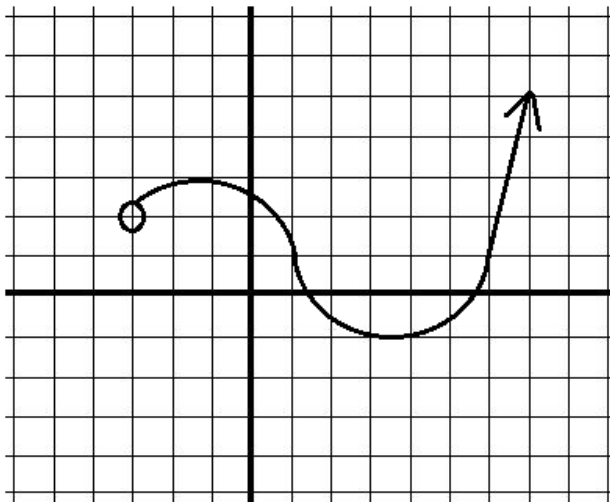
Range:

Increasing Intervals:

Decreasing Intervals:

Constant Intervals:

e)



Domain:

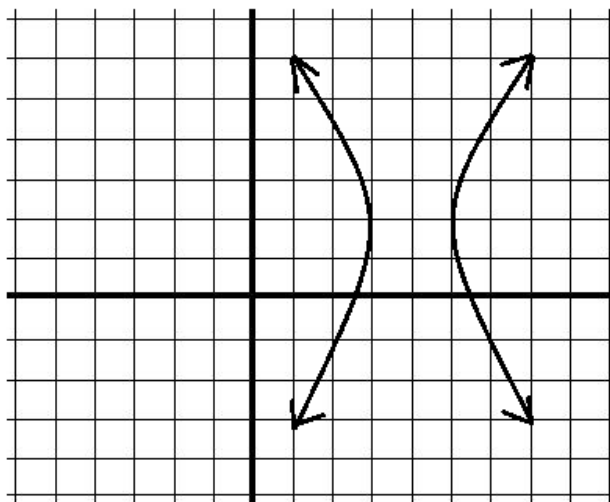
Range:

Increasing Intervals:

Decreasing Intervals:

Constant Intervals:

f)



Domain:

Range:

Increasing Intervals:

Decreasing Intervals:

Constant Intervals:

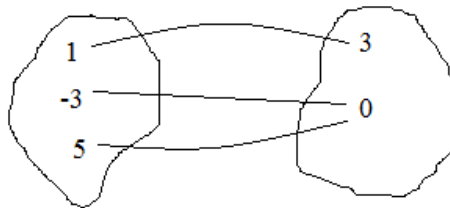
- 9) Determine if the relation given by the set of ordered pairs shown is a function. Justify your answer.

a) $\{(2, 4) (3, 4) (5, -4) (0, 0) (1, 1)\}$

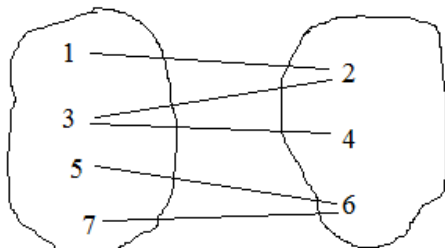
b) $\{(1, 2) (2, 3) (3, 4) (4, 5) (5, 6)\}$

c) $\{(10, 5) (10, -5) (5, 0) (-5, 0) (0, 0)\}$

- d) domain range



- e) domain range



- f) domain range



Domain Worksheet

Determine the domain of the following functions.

1) $y = \frac{x+2}{3x-2}$

2) $y = x + \sqrt[3]{6}$

3) $f(x) = \frac{\sqrt{x+4}}{x}$

4) $g(x) = \frac{3}{x^2+5}$

5) $y = \sqrt{3-x} + 5$

6) $h(x) = \sqrt[3]{x+1}$

7) $y = \frac{3}{x^2+12x}$

8) $f(x) = \frac{1}{\sqrt{x^2-10x+21}}$

9) $y = \frac{x+7}{|x-3|-5}$

10) $y = \ln(x+5)$

11) $y = \frac{1}{\log_2(x)}$

12) $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$

Numbers of you get into problems...

.... For help, call 1-800-EQUATES ...

Expert advice from
Dr. Maxwell Nathan Teger
Math Psychologist

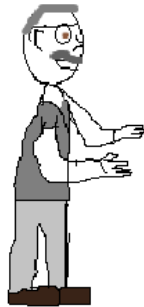


"We order pairs, so that
relations can function."

Math
Mediator

LanceAF #55 (10-20-12)
www.mathplane.com

"It's helpful to compromise
and be more rational."



"I admit, I'm difficult to
figure out.... But, you're
the only one for me."



$$\left[\sqrt[3]{3} \right] = 1$$

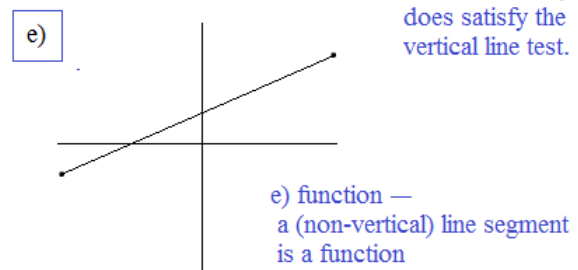
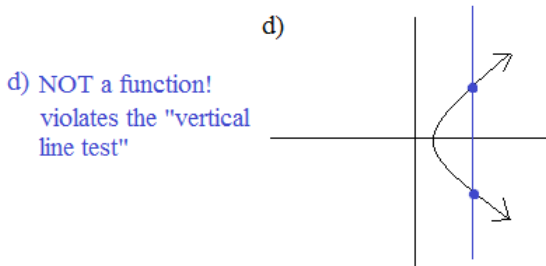
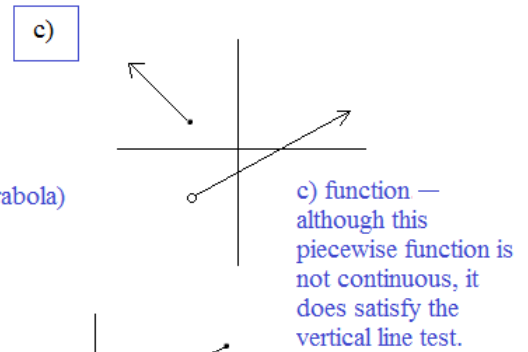
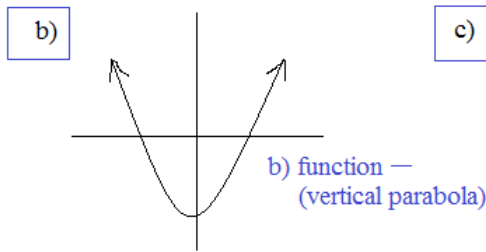
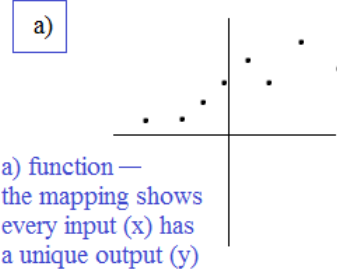
Thanks to Max N. Teger, countless pairs
have ended up together.

Solutions-→

Relations and Functions

SOLUTIONS

1) Which of the following relations are *functions*?



2) The following points represent *relation* y : $(-1, 4)$ $(-1, 8)$ $(1, -4)$ $(6, -4)$ $(0, 0)$
 Remove a point that would change the set to a *function*.

- a) $(-1, 4)$
 - b) $(-1, 8)$
 - c) $(1, -4)$
 - d) $(6, -4)$
 - e) $(0, 0)$
- a) or b)

In the relation, -1 has TWO outputs. Therefore, removing either point would change the set of a function.

3) Determine the domain of $f(x) = \frac{5}{\sqrt{x-3}}$

- a) $x > 3$
- b) $x \geq 3$
- c) $x = 5$
- d) all real numbers
- e) all positive real numbers

- a) correct: any real number > 3 can go into the function
- b) wrong: at $x = 3$, the function is undefined! $f(0) = 5/0$
- c) wrong: $x = 5$ is only one element in the infinite domain
- d) wrong: $x = 3$, $f(x)$ is undefined; $x < 3$, there is a negative number under the radical
- e) wrong: counter-example: let $x = 2$

What is the range of $y = -|x + 4| + 2$?

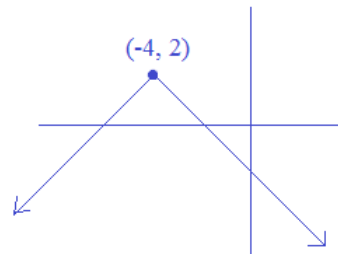
- a) $y \geq -4$
- b) $y \leq -4$
- c) $y \geq 2$
- d) $y \leq 2$
- e) all positive real numbers

The range represents all the possible "y values/outputs"

2 is the maximum value; and, there is no minimum.

$(-\infty, 2]$

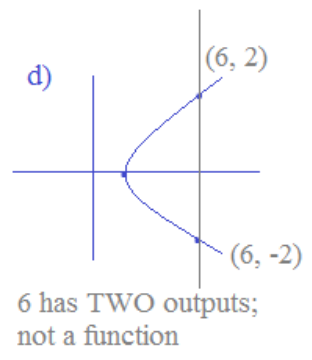
Note: $|x + 4| \geq 0$; therefore, $-|x + 4| \leq 0$



Solutions

4) Which of the following are functions?

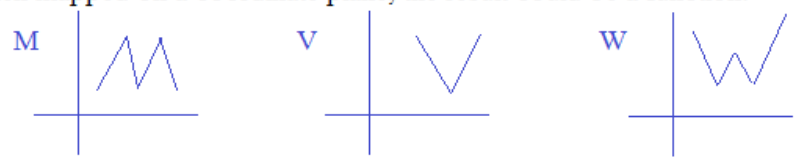
- a) $y = -6$ function (horizontal line)
- b) $x = 3$ NOT a function (vertical line)
- c) $y = 3x^2$ function (parabola -- satisfies the "vertical line test")
- d) $x = y^2 + 2$ NOT a function ('horizontal parabola' -- fails "vertical line test")
- e) $y = \frac{4}{x}$ function (inverse function with asymptotes) every x input has a unique y input!



5) Consider the 26 letters of the alphabet.

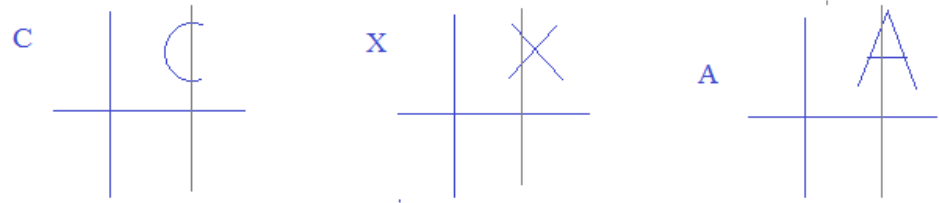
a) Identify 3 letters that when mapped on a coordinate plane, the result could be a function.

Examples may include M, V, and W



b) Identify 3 letters that when mapped on a coordinate plane, the result would not be a function.

Examples include C, A, and X... All fail the vertical line test...



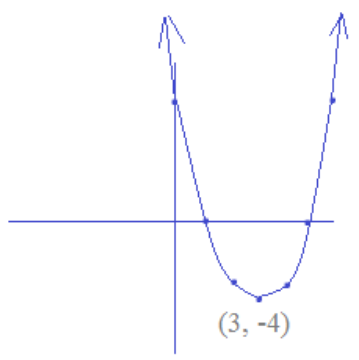
6) What is the domain and range of $g(x) = x^2 - 6x + 5$?

Domain is all real numbers... (Any real number can be put into the function)

Range is $[-4, \infty)$

$y \geq -4$

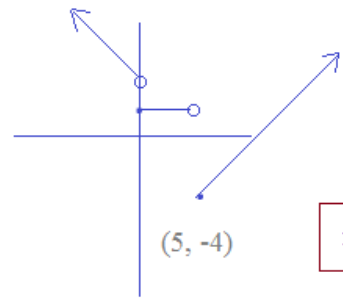
3 is the minimum value (the vertex of the parabola/quadratic)



7) What is the domain and range of the following piecewise function?

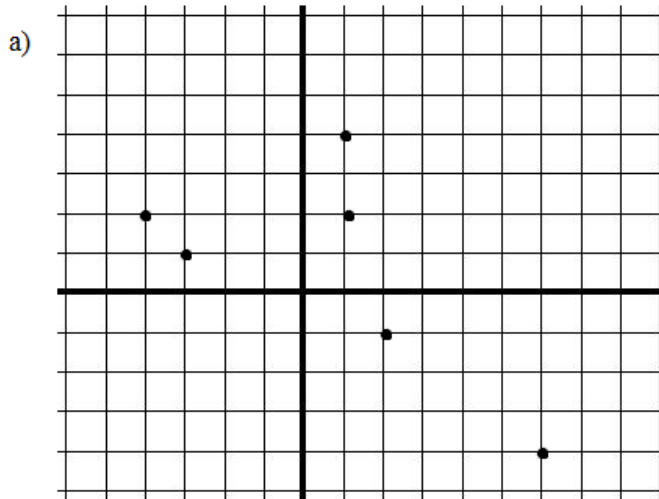
$$f(x) = \begin{cases} 5 - x & \text{if } x < 0 \\ 2 & \text{if } 0 \leq x < 5 \\ x - 9 & \text{if } x \geq 5 \end{cases}$$

domain: all real numbers



range: $[-4, \infty)$

8) Write the domain and range of each relation/function (in set notation or interval notation).



Domain: (all the x values)

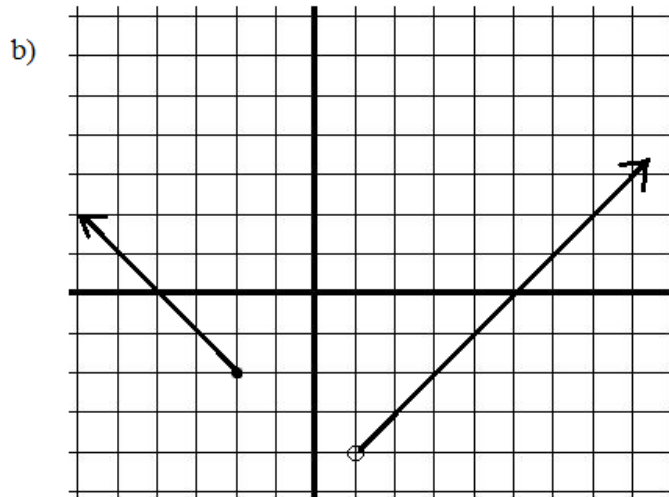
$$\{-4, -3, 1, 2, 6\}$$

Range: (all the y values)

$$\{-4, -1, 1, 2, 4\}$$

(this is a relation because $x = 1$ has TWO possible outputs!)

Solutions



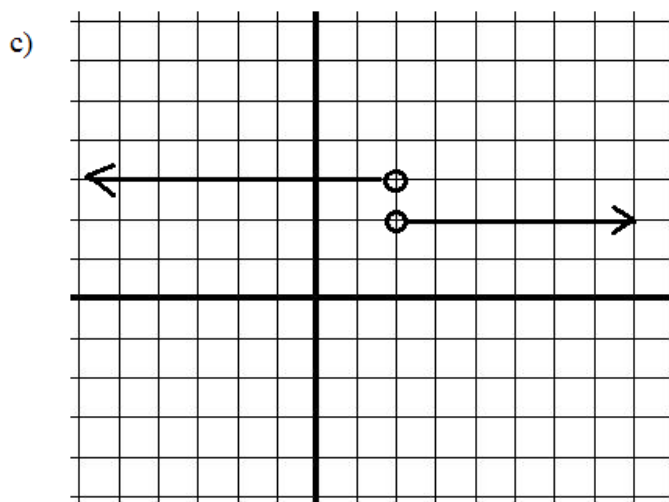
Domain: $(-\infty, -2] \cup (1, \infty)$

$$D = \{x \leq -2 \text{ or } x > 1\}$$

Range: $(-4, \infty)$

$$R = \{y > -4\}$$

(outputs/y values will be every number 'above' -4)



Domain: $(-\infty, 2) \cup (2, \infty)$

$$D = \{\text{all real numbers, where } x \neq 2\}$$

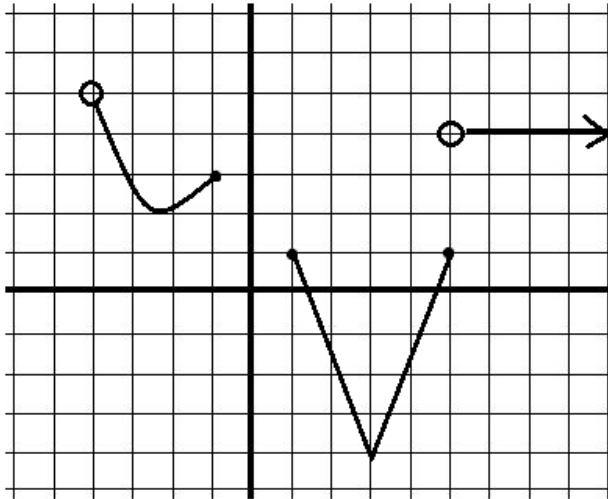
Range: $R = \{2, 3\}$

(this is a function with an infinite domain and a range of just 2 values)

Write the domain and range of each relation/function (in set notation or interval notation).

Solutions

d)



Domain: $(-4, -1] \cup [1, 5] \cup (5, \infty)$

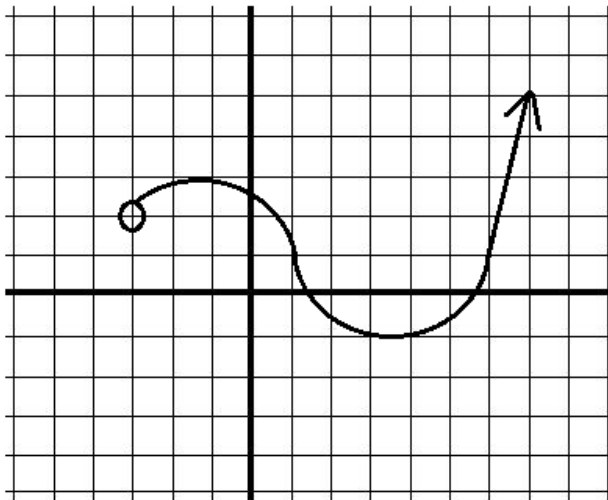
Range: $[-4, 1] \cup [2, 5)$

Increasing Intervals: $(-2.5, 1) \cup (3, 5)$

Decreasing Intervals: $(-4, -2.5) \cup (1, 3)$

Constant Intervals: $(5, \infty)$

e)



Domain: $(-3, \infty)$

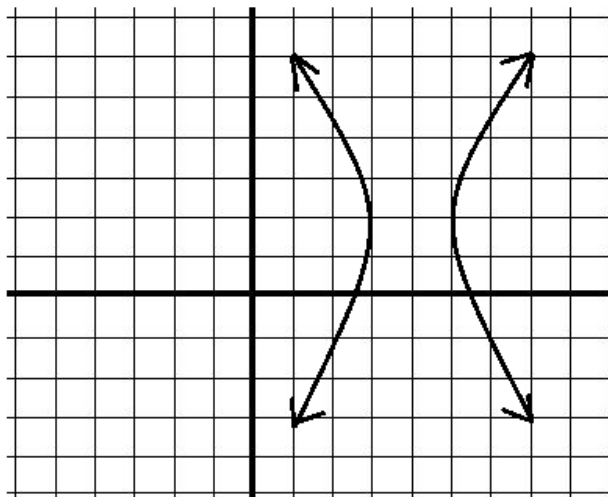
Range: $(-1, \infty)$

Increasing Intervals: $(-3, -1.5) \cup (3.5, \infty)$

Decreasing Intervals: $(-1.5, 3.5)$

Constant Intervals: None

f)



Domain: $(-\infty, 3) \cup (5, \infty)$

Range: $(-\infty, \infty)$

Increasing Intervals: Ambiguous
Decreasing Intervals: (It's not a function!)

Constant Intervals: None

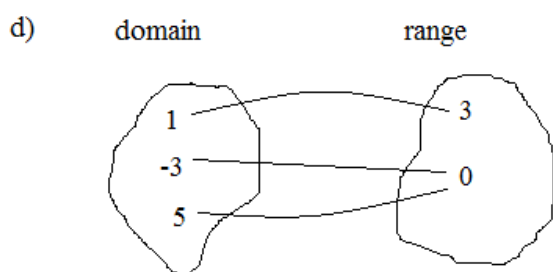
9) Determine if the relation given by the set of ordered pairs shown is a function. Justify your answer.

Solutions

a) $\{(2, 4) (3, 4) (5, -4) (0, 0) (1, 1)\}$ **Function:** every input -- 2, 3, 5, 0, and 1 -- has a unique output

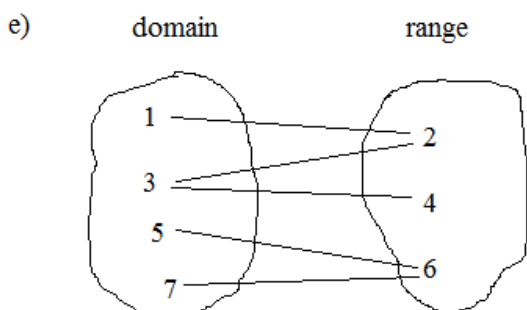
b) $\{(1, 2) (2, 3) (3, 4) (4, 5) (5, 6)\}$ **Function:** every input -- x value in each ordered pair -- has a unique output y

c) $\{(10, 5) (10, -5) (5, 0) (-5, 0) (0, 0)\}$ **Relation:** The input 10 has more than one possible output --- 5 or -5 **(Not a function)**



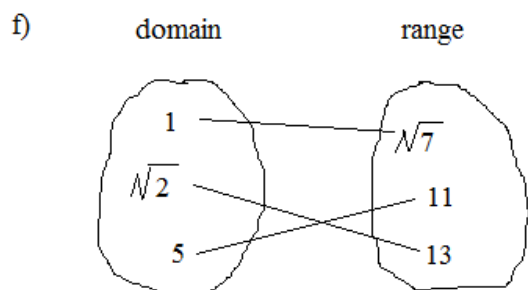
mapping: $(1, 3)$
 $(-3, 0)$
 $(5, 0)$

Function: each input has a specific output...



NOT a function:

If $x = 3$, then y can be 2 OR 4



one-to-one **Function**

Domain Worksheet

Determine the domain of the following functions.

1) $y = \frac{x+2}{3x-2}$

$3x - 2 \neq 0$

all reals where $x \neq 2/3$
 $(-\infty, 2/3) \cup (2/3, \infty)$

4) $g(x) = \frac{3}{x^2 + 5}$

since x^2 is positive, the denominator will never equal 0

all real numbers
 $(-\infty, \infty)$

7) $y = \frac{3}{x^2 + 12x}$

$x^2 + 12x \neq 0$

$x(x+12) \neq 0$

$x \neq 0, -12$

all reals, except -12 or 0

10) $y = \ln(x+5)$

$x + 5 > 0$

$x > -5$

$(-5, \infty)$

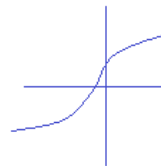
2) $y = x + \sqrt{6}$

it's a line...
 all real numbers
 $(-\infty, \infty)$

5) $y = \sqrt{3-x} + 5$

find where $3 - x \geq 0$

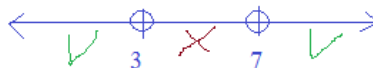
$x \leq 3$
 $(-\infty, 3]$



8) $f(x) = \frac{1}{\sqrt{x^2 - 10x + 21}}$

$x^2 - 10x + 21 > 0$

$(x-3)(x-7) > 0$ (test points on number line)



$(-\infty, 3) \cup (7, \infty)$

11) $y = \frac{1}{\log_2(x)}$

$x > 0$ (log must be > 0)

then, $\log_2(x)$ cannot equal zero in the denominator

$\log_2(1) = 0$

$x > 0$ and $x \neq 1$

$(0, 1) \cup (1, \infty)$

3) $f(x) = \frac{\sqrt{x+4}}{x}$

$x \neq 0$

$x+4 \geq 0$ $x \geq -4$

$[-4, 0) \cup (0, \infty)$

6) $h(x) = \sqrt[3]{x+1}$

cube root of negative is acceptable..
 so, domain is

all real numbers

9) $y = \frac{x+7}{|x-3|-5}$

$|x-3|-5 \neq 0$

$|x-3| \neq 5$

$x \neq 8$ or -2

all reals, except -2 or 8

12) $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$

$-1 \leq \frac{x}{3} \leq 1$

$-3 \leq x \leq 3$

$[-3, 3]$

SOLUTIONS

What to look for:

- a) denominator cannot equal 0
- b) no negatives under square root
- c) logarithms > 0
- d) inverse cosine/sine: $-1 \leq x \leq 1$

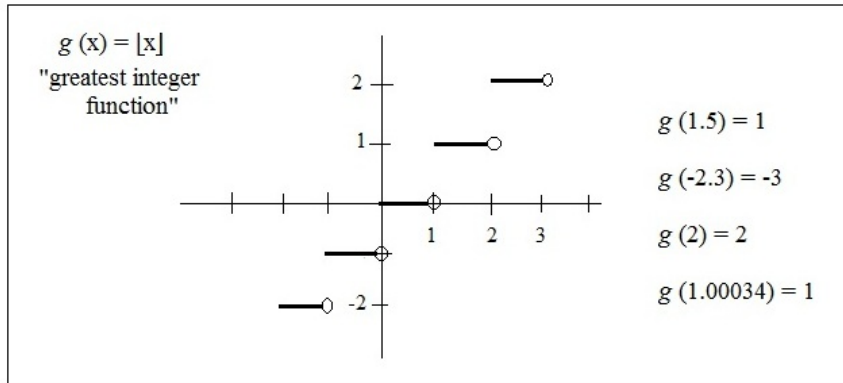
One more topic....

Greatest Integer Function

A function that maps a real number to the largest *previous* integer.

It shows the greatest integer that is not greater than the input.

The greatest integer function is commonly called the *floor function*.



Notation and symbols: There are several ways the greatest (or least) integer function may be expressed.

	Greatest integer value	Least integer value
Bracket with top (or bottom) cut off:	$\lfloor X \rfloor$	$\lceil X \rceil$
Alternate names:	floor function	ceiling function
Word form:	floor(x)	ceil(x)

Note: Instead of an open bracket (with the top cut off), the greatest integer function may use brackets in various forms:

open bracket	bracket	bold bracket	double bracket
$\lfloor x \rfloor$	$[x]$	$[x]$	$[[x]]$
$\lfloor 1.2 \rfloor = 1$	$[2.4] = 2$	$[-3.3] = -4$	$[[6.77]] = 6$

The **least integer function** or "ceiling function" maps a rational number to the next smallest integer.

$\text{ceil}(x)$ or $\text{ceiling}(x) = \lceil x \rceil$ is the smallest integer not less than x .

$$\lceil 4.33 \rceil = 5$$

$$\lceil 3. \rceil = 3$$

$$\lceil -7.89 \rceil = -7$$

Application of greatest integer function:

Example: The telephone company charges \$3.50 to connect and \$1 per minute for an international call.

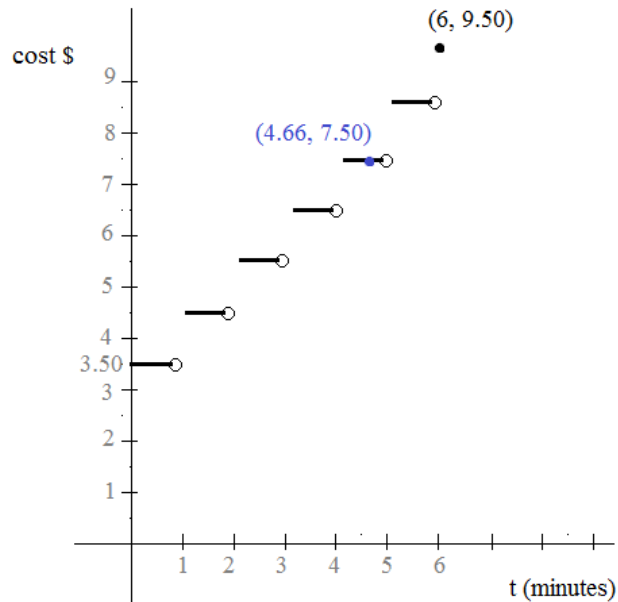
Write an equation that shows the cost of making an international call. (Use the greatest integer, or floor, function). Graph the function, representing calls of 0 to 6 minutes. How much would a 4 minute and 40 second call cost?

$$\text{Cost} = \$3.50 + \lfloor t \rfloor \times (\$1)$$

Cost of
4 minute
40 second
call

$$= \$3.50 + \left\lfloor 4 \frac{2}{3} \right\rfloor \times (\$1)$$

$$= \$3.50 + 4(\$1) = \$7.50$$



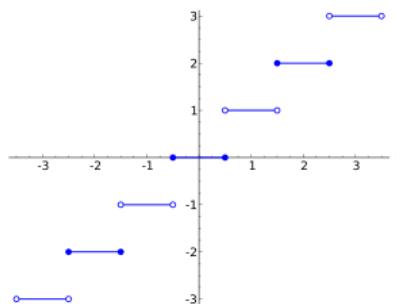
Nearest integer function: $\lceil X \rceil$

The nearest integer function is used in computer science.

Round the 1/2 numbers to the nearest even integer.

(or, round up, round down, round away from zero, or randomly round up or down)

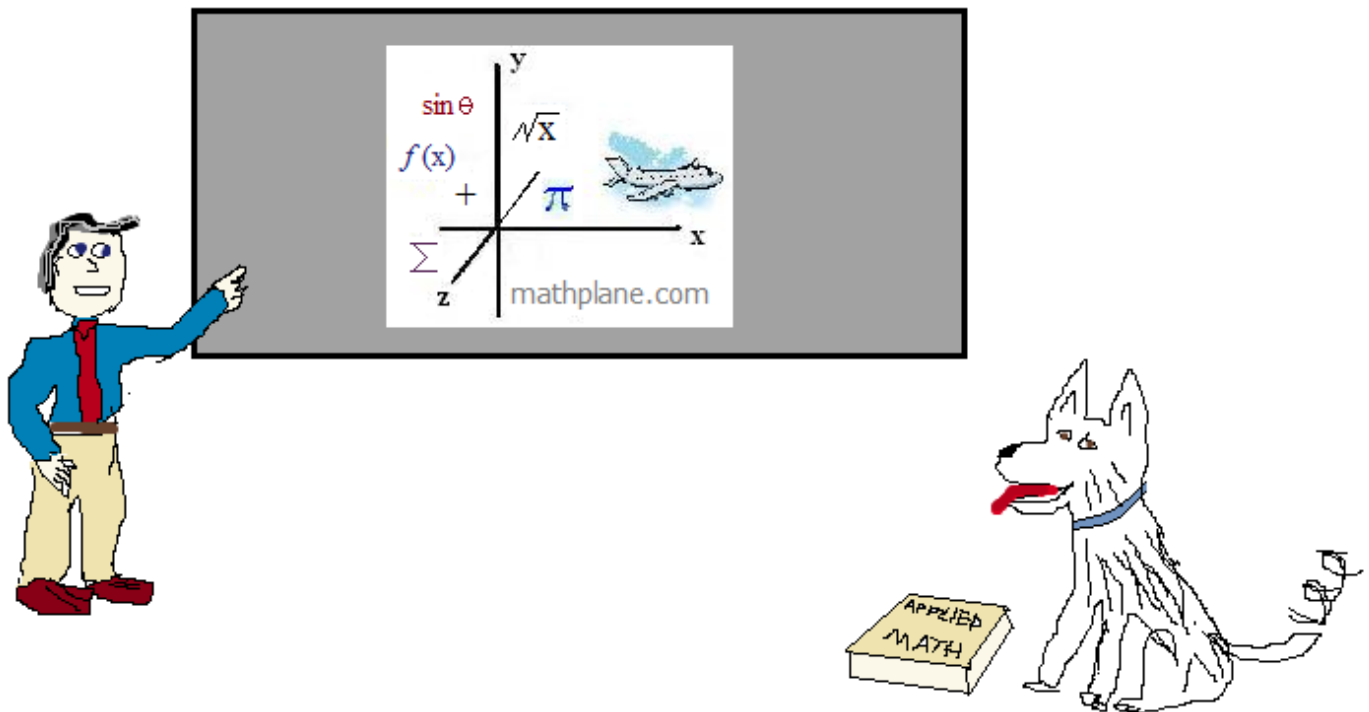
nint(x) or Round(x) $\lceil X \rceil$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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