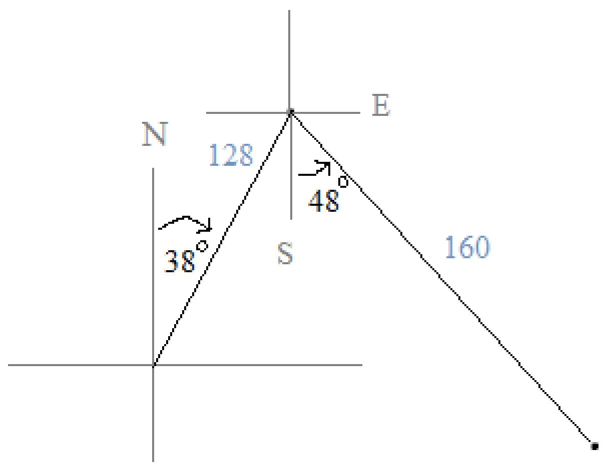
Trigonometry:

Navigation, Direction, and Bearings



Includes notes, terms, examples, and practice test (& solutions)

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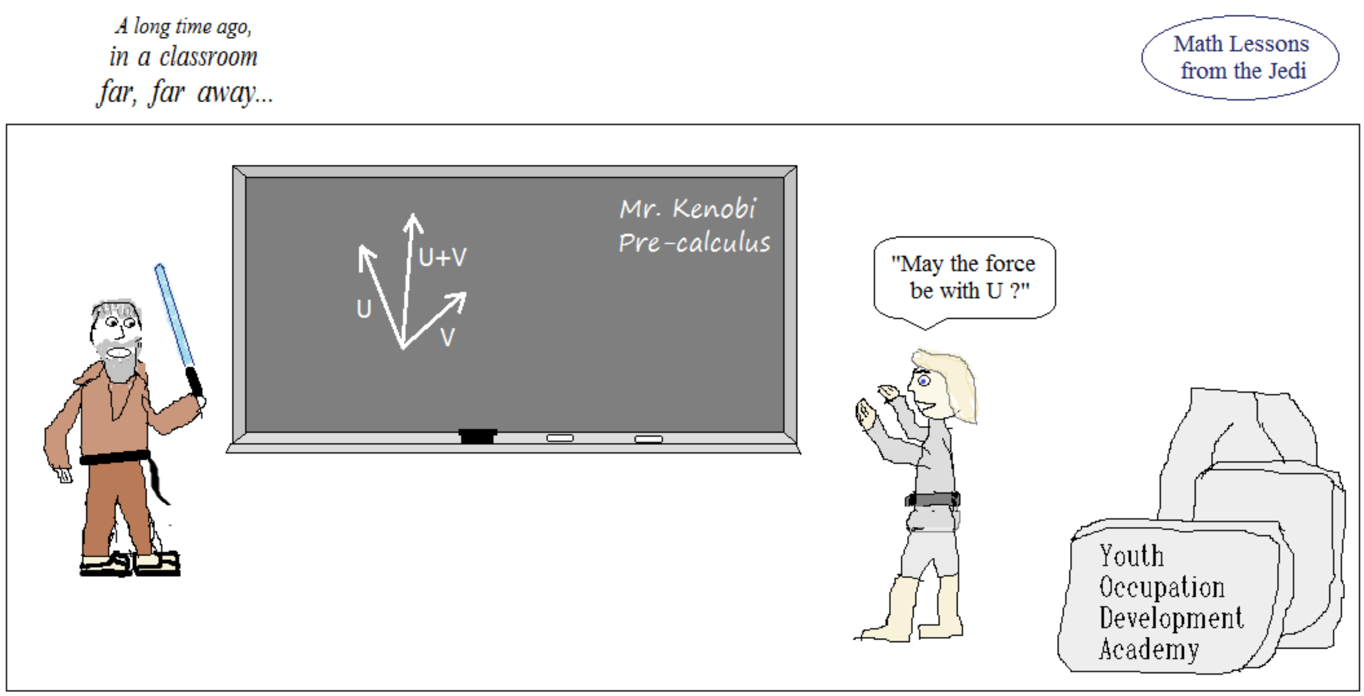


LanceAF #72 2-17-13
www.mathplane.com

Obi-Wan teaches Luke about resultant vectors and (the) force

Navigation

A useful application of trigonometry (and geometry) is in navigation and surveying.
The following sections contain notes, terms, and examples.

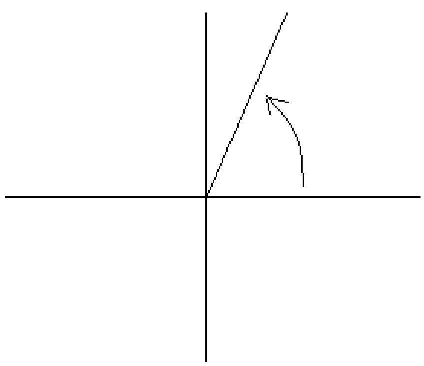
I. Which direction?

One source of confusion can be the direction of angles. Note the important difference!!

Geometry orientation
(coordinate plane)

67 degrees

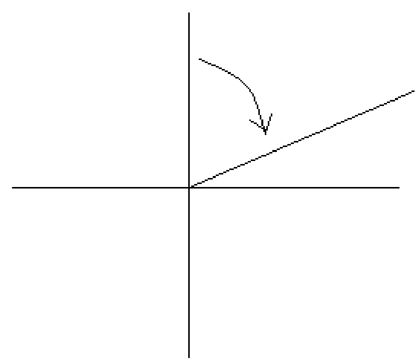
In standard position,
starts on x-axis and goes
counter-clockwise.



Navigation orientation

N67E

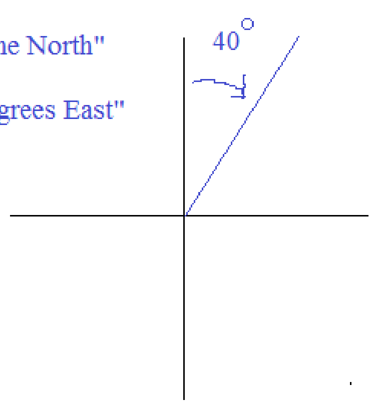
Starts on "y-axis" and
goes clockwise!

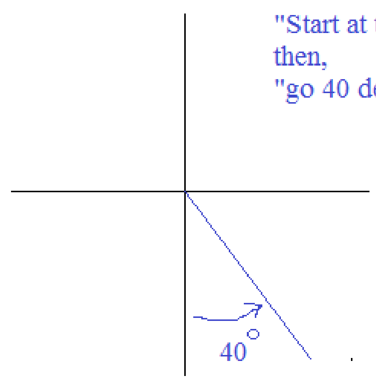


II. Compass Readings

N40E vs S40E

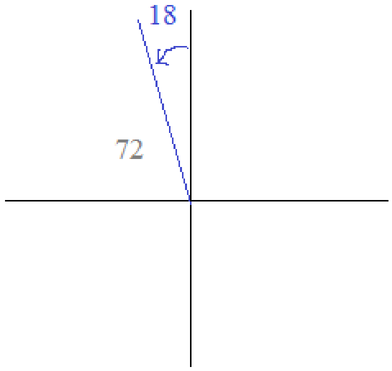
"Start at the North"
then,
"go 40 degrees East"

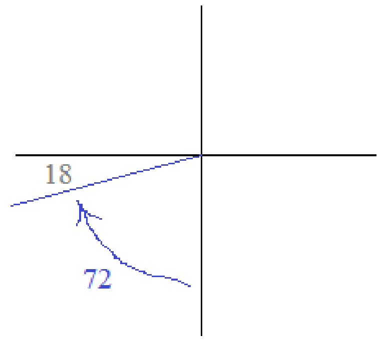




"Start at the South"
then,
"go 40 degrees East"

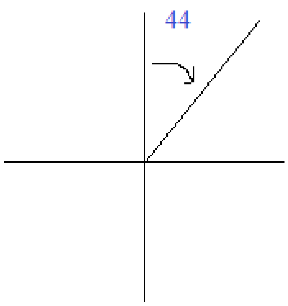
Question: Is N18W the same direction as S72W?

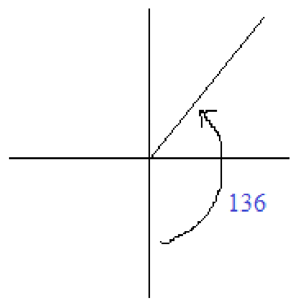




No, they are not the same direction...

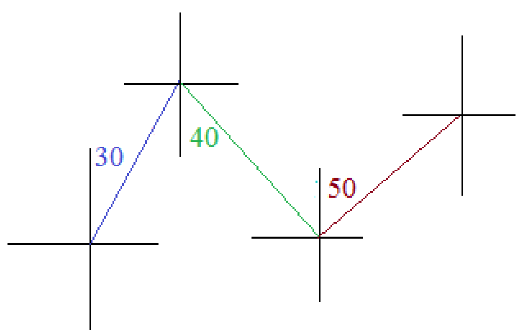
Express N44E in another way.





S136E

If an explorer walks N30E. then, continues S40E. Then, finally goes N50E.
Sketch a possible path.

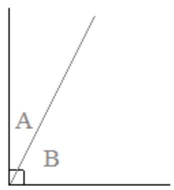


Every time the explorer changes direction, a new "reference grid" is drawn

III. Navigation Tools

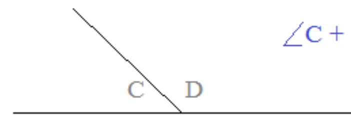
Geometry:

Complementary Angles



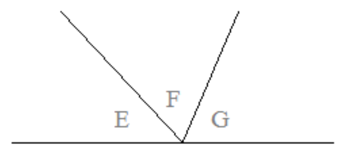
$\angle A + \angle B = 90$ degrees

Supplementary Angles



$\angle C + \angle D = 180$ degrees

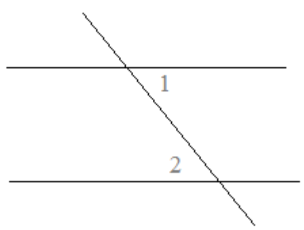
Addition Postulates



E, F, and G are adjacent angles

$\angle E + \angle F + \angle G = 180$ degrees

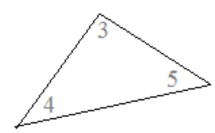
Alternate Interior Angles



If parallel lines are cut by a transversal, then alternate interior angles are congruent.

$\angle 1 = \angle 2$

Sum of Interior Angles of a Triangle is 180 degrees



$\angle 3 + \angle 4 + \angle 5 = 180^{\circ}$

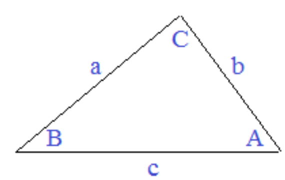
Trigonometry:

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

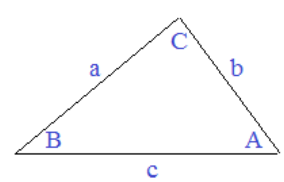
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$



Law of Sines

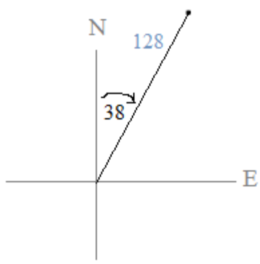
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

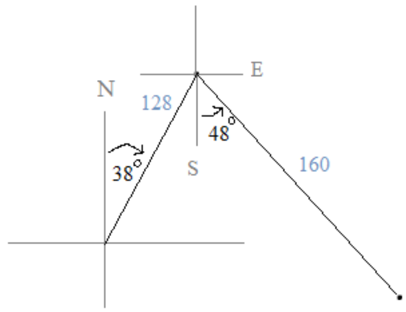


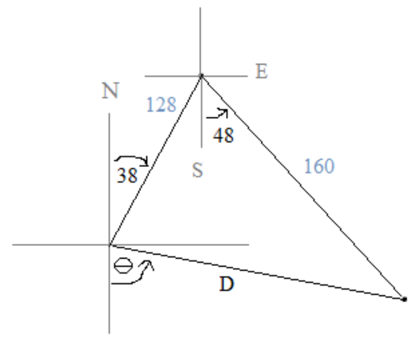
IV. *Example:* A hiker leaves his campsite and walks 128 meters N38E. Then, turns and walks 160 meters S48E.

a) What is the *distance* from the campsite?

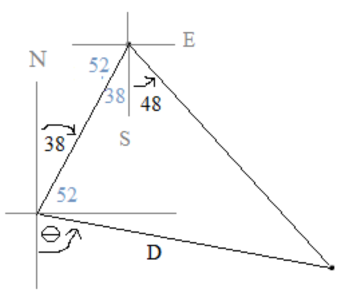
b) What is the *compass reading* from the starting point?





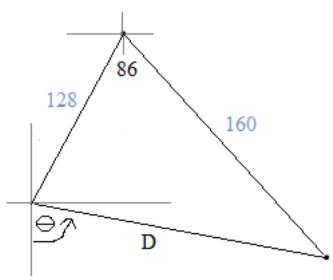


Use Geometry concepts to identify angles.
Then, use trig concepts to find side and compass reading.



Parallel lines cut by transversal
(alternate interior angles)

complementary angles
(add up to 90 degrees)



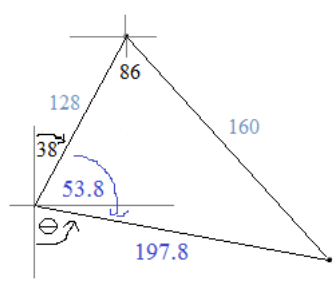
Law of cosines to find D:

$D^2 = 128^2 + 160^2 - 2(128)(160)\cos 86^\circ$

$D^2 = 41984 - 40960(\cos 86)$

$D^2 = 39127$

$D = 197.8$



Law of sines to find angle:

$\frac{\sin x}{160} = \frac{\sin 86}{197.8}$

$\sin x = \frac{160(\sin 86)}{197.8} = .807$

$x = 53.8$ degrees

Distance back to the campsite? **197.8 meters**

Compass reading from the campsite? $180 - 38 - 53.8 = \theta$

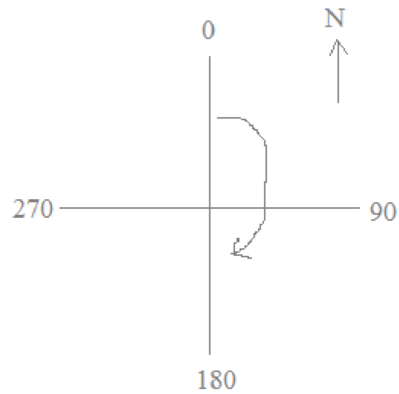
$\theta = 88.2^\circ$

S88.2E

V. "Bearings" in Trig Navigation

(Compass) Bearings are another way to express direction and orientation.

The direction begins on the "top of the y-axis" and moves clock-wise.

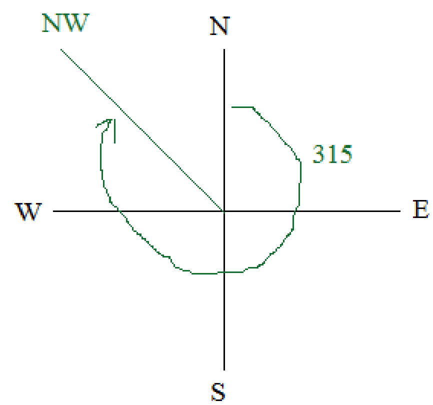


Examples:

Bearing of 0 is "due north"
Bearing of 90 is "due east"

What bearing corresponds to a Northwest direction?

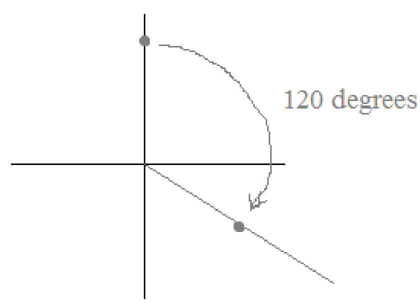
315 degrees



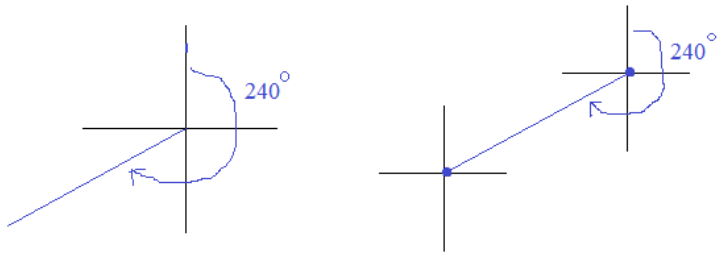
Example: Translate a bearing of 120 degrees

In other words, what is the corresponding compass direction?

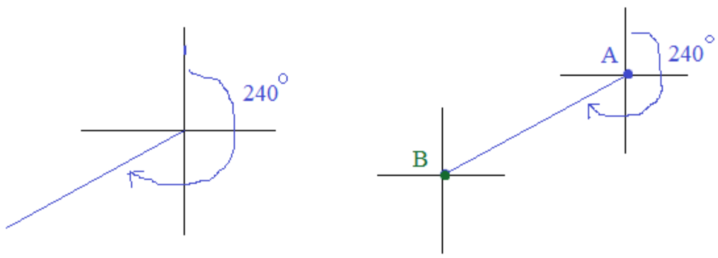
S60E



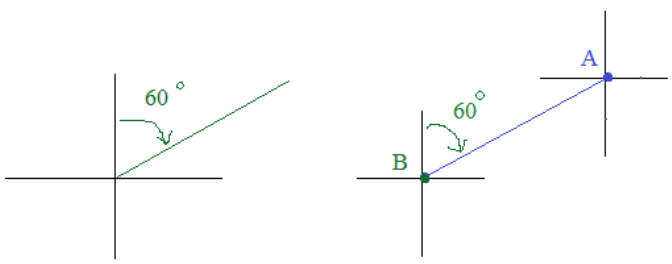
** A bearing is used to represent the direction from one point relative to another point.



**The bearing (measure) depends on the starting and ending point.



The bearing from A to B is 240.



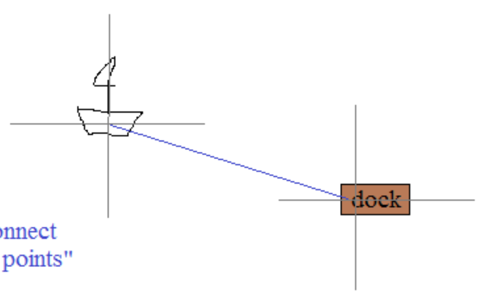
The bearing from B to A is 60.

Observation: the difference between the measures is 180°.

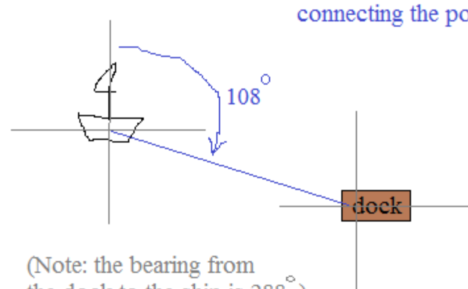
Example: What is the bearing from the ship to the dock?

Step 1: Set up grids

Step 2: "Connect the points"



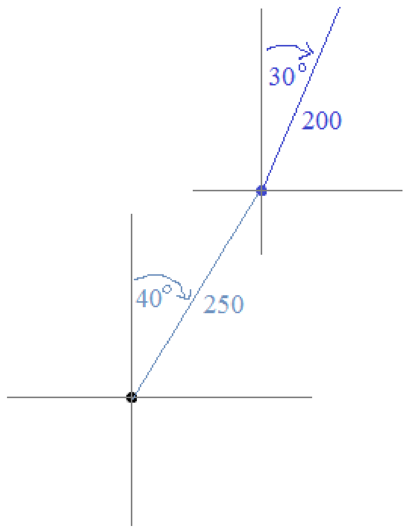
Step 3: Measure the angle from due north to the line connecting the points



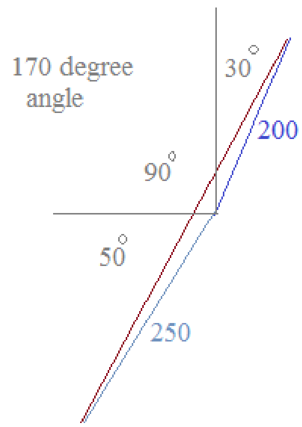
(Note: the bearing from the dock to the ship is 288°)

Example: A plane travels 250 miles at a bearing of 40 degrees.
Then, it turns and travels 200 miles at a bearing of 30 degrees.
How far has it traveled?

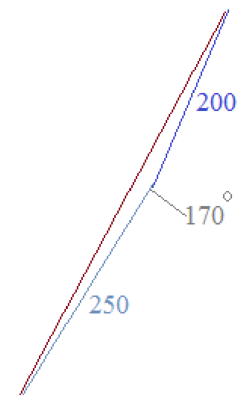
Sketch and label the path of the plane.



The distance is represented by the 3rd side of a triangle!



Use geometry and isolate the triangle.



Since we have 2 sides and the included angle,
we can use law of cosines to find the opposite side: $c^2 = a^2 + b^2 - 2ab(\cos C)$

$$(\text{distance})^2 = (250)^2 + (200)^2 - 2(250)(200)\cos(170^\circ)$$

$$= 62500 + 40000 - 100000(-.9848)$$

$$= 102500 + 98480 = 200{,}980$$

Since $(\text{distance})^2 = 200{,}980$

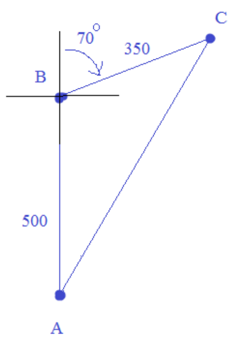
distance the plane traveled is 448.3 miles

448 is a bit under (250 + 200), so
the answer seems reasonable. ✓

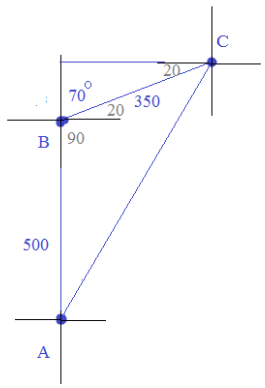
Example: A plane with One-Stop Airways flies due North 500 miles from airport A to airport B. Then, the plane continues at a directional bearing of N70E from airport B to airport C for another 350 miles.

If a plane with Non-Stop Airways flew directly from airport A to airport C, how far would it travel? What direction would it travel?

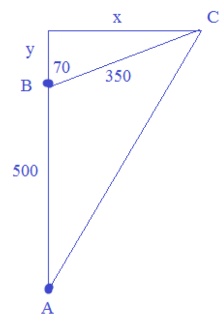
Step 1: Draw a diagram...



Step 2: use geometry to label triangles



Step 3: Use Geometry and Trigonometry to find direction and distance



$\cos(B) = \cos(70) = \dfrac{y}{350}$ $\quad y = 119.7$

$\sin(B) = \sin(70) = \dfrac{x}{350}$ $\quad x = 328.9$

If $x = 328.9$, $y = 119.7$, and $\overline{AB} = 500$, then using the Pythagorean Theorem,

$\overline{AC}^2 = 328.9^2 + 619.7^2$

$\overline{AC} = 701.57$

$\tan(A) = \dfrac{x}{\overline{AB} + y} = \dfrac{328.9}{500 + 119.7}$

$A = 27.96°$

direction of non-stop airplane: N27.96E

Express answer: using law of cosines! $\quad c^2 = a^2 + b^2 - 2ab(\cos C)$

$d^2 = 350^2 + 500^2 - 2(350)(500)\cos(110°)$

$d = 701.58$ miles

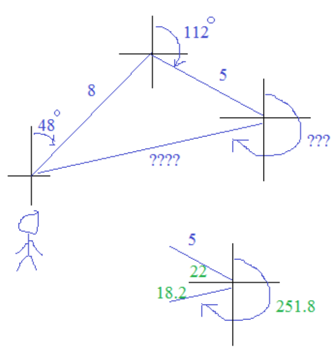
using law of sines! $\quad \dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b}$

$\dfrac{\sin(A)}{350} = \dfrac{\sin(110)}{701.57}$

$\sin(A) = .4688$

$A = 27.95°$

Example: A hiker leaves camp and walks 8 miles at a bearing of 48 degrees. After a 45 minute break, the hiker continues walking 5 more miles at a bearing of 112 degrees. If he were to hike directly back to camp, how far must he travel? In what direction?



Using law of cosines: $d^2 = 5^2 + 8^2 + 2(5)(8)\cos(116)$

$d = 11.14$

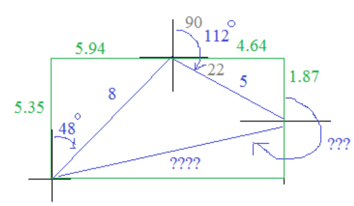
Using law of sines: $\dfrac{\sin(116)}{11.14} = \dfrac{\sin(X)}{8}$

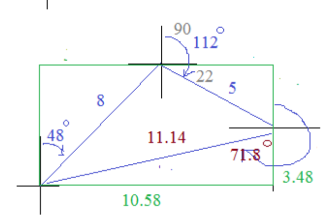
$X = 40.2$ degrees

since $X = 40.2$ and top 1/2 of angle is 22 degrees, the bottom 1/2 of angle is 18.2 degrees...

therefore, bearing is 251.8 degrees...

Pythagorean Theorem: $3.48^2 + 10.58^2 = 11.14^2$





Study Break:
Math Snacks

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (t), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

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Try the Practice Quiz…

Trigonometry quiz: Navigation

1) A ship leaves port and travels 20 miles at a bearing of N32E. Then, another ship leaves the port and travels 28 miles at a bearing S42E. What is the distance between the two ships?

2) What direction has a bearing of 225 degrees?

3) A racer runs 5 miles north, 2 miles west, 3 miles north, and 4 miles west.
How far is he from the starting line?

4) A math explorer leaves his home base and travels in the direction N 70° W. He travels 30 miles and reaches the rest station. The next week, he travels 50 miles in the direction N 10° E, reaching his destination.

a) Find the distance between the home base and the destination.

b) Find the bearing from the final destination back to the home base.

5) If the bearing from A to B is 115 degrees, what is the bearing from B to A?

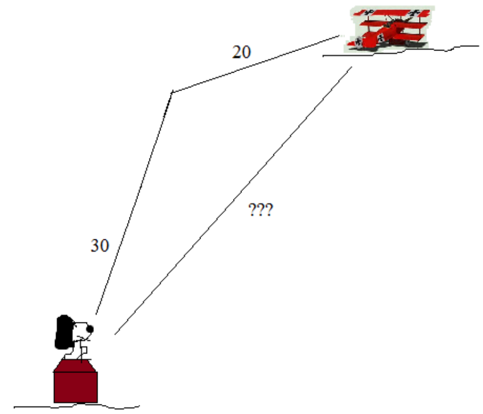
6) Two planes simultaneously take off from an airport. A plane flies 400 miles/hour at a bearing of 70 degrees. And, B plane flies 500 miles at a bearing of 300 degrees. How far apart are they after three hours?

7) On a map, Maytown is due south of Davidville and southeast of Vicksburg.
If Maytown is 40 miles from Davidville, and Maytown is 55 miles from Vicksburg, and Vicksburg is 45 miles from Davidville,
what *bearing* is required to get from Maytown to Vicksburg?

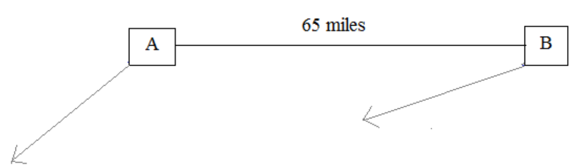
8) Airports A and B are 400 miles apart.
Maverick flies northwest from airport A to airport C.
From C, he flies 350 miles at a heading (bearing) of 215 degrees to airport B.
How far is airport C from A?

9) The Red Baron leaves Peanuts Airfield at noon and travels N25°E for 30 miles.
Then, the Baron turns and flies another 20 miles in the N72°E direction before landing at Woodstock Airfield to refuel.

At 1:00 PM, Snoopy leaves Peanuts Airfield to pursue the Baron.
How far, and in which direction, should Snoopy travel to get to Woodstock as fast as possible?



10) Car 1 leaves Station A and goes 40 miles/hour at S45 W.
Simultaneously, 65 miles due East, Car 2 leaves Station B and travels S70 W.
Two hours later, Car 2 collides with Car 1!!
Assuming each car was going straight at their constant speeds, how fast was Car 2 traveling?



11) A boat leaves the harbor for an island 30 miles due east.
The ocean current is pulling in a northeast direction at 8 miles per hour.
How fast and in which direction should the boat travel to reach the island in 2 hours?

12) Nav the Gator crawls 8 miles through the swamp in a S70E direction.
After a snack, Nav turns and continues 5 miles heading S10W..
If Nav wants to return directly to his home, how far and which direction should he crawl?

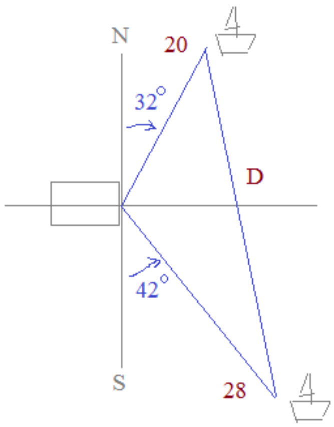
13) An airplane is flying at a bearing of 60° with an air speed of 350 miles per hour.
There is a 40 mile-per-hour wind speed with a bearing of 110°.
What is the ground speed of the airplane?
What is the course of the airplane?

14) A plane is traveling at 400 miles per hour in a N25W direction.
Its ground speed and direction are 440 miles per hour and N10E respectively.
What is the speed and direction of the wind?

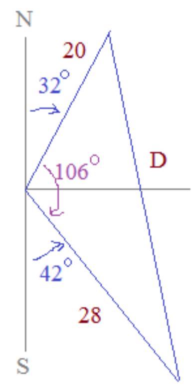
SOLUTIONS

1) A ship leaves port and travels 20 miles at a bearing of N32E. Then, another ship leaves the port and travels 28 miles at a bearing S42E. What is the distance between the two ships?

Step 1: Draw a picture



Step 2: Identify triangle and formula



**Use law of cosines to find the distance!

Step 3: Solve

$$D^2 = 20^2 + 28^2 - 2(20)(28)\cos 106$$

$$D^2 = 1184 - 1120(\cos 106)$$

$$D^2 = 1184 - 1120(-.2756)$$

$$D^2 = 1492.7$$

distance D = 38.6 miles

Step 4: Quick check..

38 miles seems reasonable (compared to given distances)

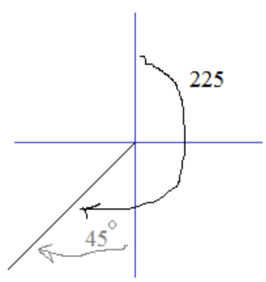
law of sines: $\frac{\sin 106^\circ}{38.6} = \frac{\sin A}{20} = \frac{\sin B}{28}$

$A = 29.87^\circ$

$B = 44.21^\circ$

$$\begin{array}{r} 29.87 \\ 44.21 \\ +\ 106 \\ \hline \text{approx. } 180 \end{array}$$

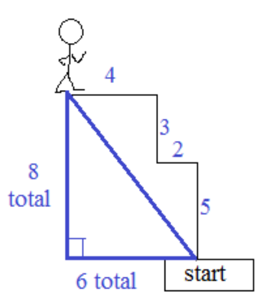
2) What direction has a bearing of 225 degrees?



S45W

or directly Southwest

3) A racer runs 5 miles north, 2 miles west, 3 miles north, and 4 miles west.
How far is he from the starting line?

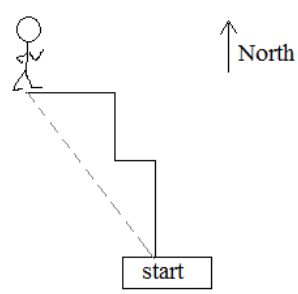


right triangle:

6 - 8 - x

x = 10
("pythagorean triplet")

The racer is 10 miles from the starting line.



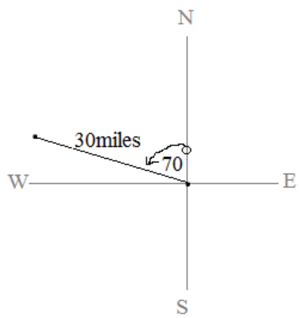
4) A math explorer leaves his home base and travels in the direction N 70°W. He travels 30 miles and reaches the rest station. The next week, he travels 50 miles in the direction N 10°E, reaching his destination.

a) Find the distance between the home base and the destination.

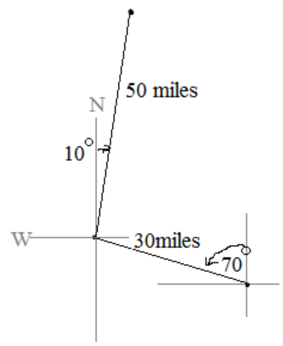
b) Find the bearing from the final destination back to the home base.

Step 1: Draw a picture

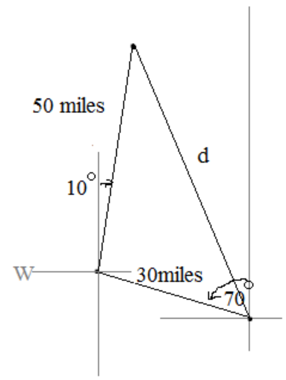
(Home to Rest station)



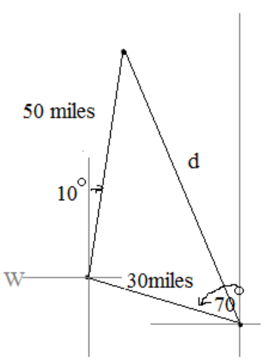
(Rest Station to Destination)



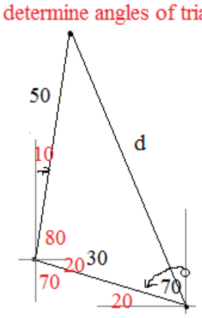
(Destination to Home)

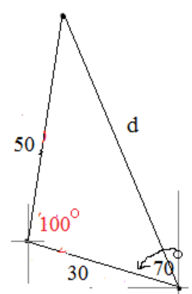


Step 2: Extract the triangle and find distance d



Use geometry relations to determine angles of triangle





Law of Cosines to find d

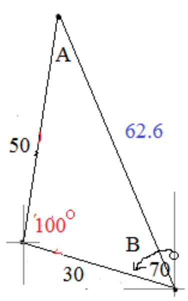
$a^2 = b^2 + c^2 - 2bc(\cos A)$

$d^2 = (30)^2 + (50)^2 - 2(50)(30)\cos 100^\circ$

$d^2 = 900 + 2500 - 3000(-.174) = 3922$

$d \cong 62.6$ miles

Step 3: Fill in triangle with angle measurements and find bearing



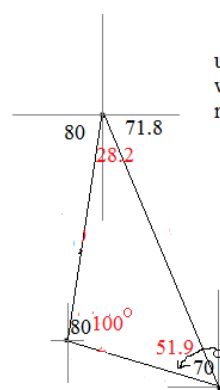
Law of Sines to find A and B:

$\frac{\sin 100^\circ}{62.6} = \frac{\sin A}{30}$ $\qquad$ $\frac{\sin 100^\circ}{62.6} = \frac{\sin B}{50}$

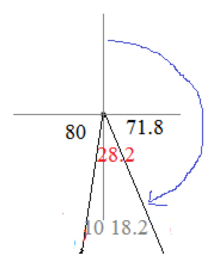
$\frac{.985}{62.6} = \frac{\sin A}{30}$ $\qquad$ $\frac{.985}{62.6} = \frac{\sin B}{50}$

$\sin A = .472$ $\qquad$ $\sin B = .787$

$A \cong 28.2^\circ$ $\qquad$ $B \cong 51.9^\circ$



using geometry rules, we can determine relevant angles

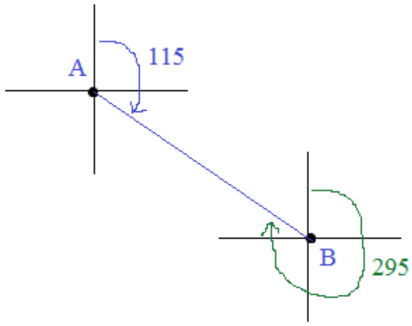


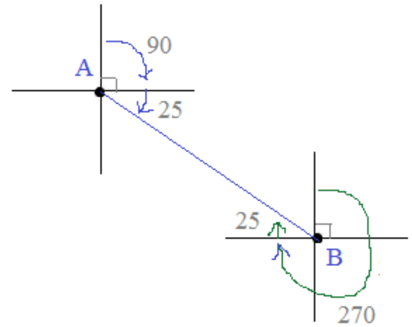
The direction is N161.8E
or S18.2E

Or, the bearing is 161.8

Note: Using horizontal and vertical axes maintain consistent bearings and help determine angle measurements.

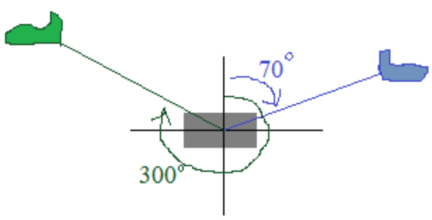
5) If the bearing from A to B is 115 degrees, what is the bearing from B to A?





6) Two planes simultaneously take off from an airport. A plane flies 400 miles/hour at a bearing of 70 degrees. And, B plane flies 500 miles at a bearing of 300 degrees. How far apart are they after three hours?

Sketch a diagram:



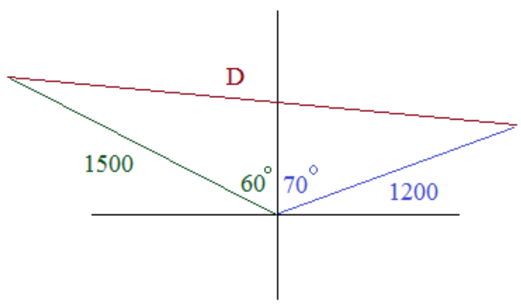
In 3 hours,

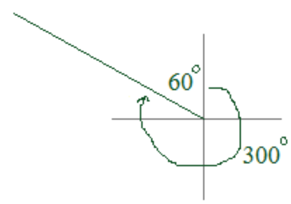
A Plane flies 1200 miles

B Plane flies 1500 miles

(Sides of the triangle)

Use Geometry to label the parts:





Use Trigonometry to find Distance:

Law of Cosines: $D^2 = (1500)^2 + (1200)^2 - 2(1500)(1200)\text{Cos}130^\circ$

$D^2 = 2{,}250{,}000 + 1{,}440{,}000 - (-2{,}314{,}035)$

$D^2 = 6{,}004{,}035$

square root:

Distance = 2450.3 miles

7) On a map, Maytown is due south of Davidville and southeast of Vicksburg.
If Maytown is 40 miles from Davidville, and Maytown is 55 miles from Vicksburg, and Vicksburg is 45 miles from Davidville,
what *bearing* is required to get from Maytown to Vicksburg?

SOLUTIONS

Use law of cosines:

$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

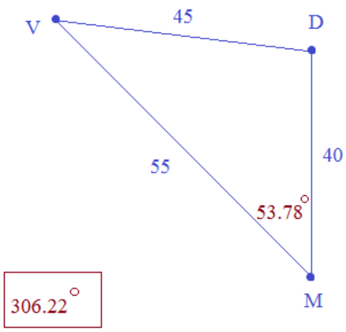
$$45^2 = 55^2 + 40^2 - 2(55)(40)\cos(M)$$

$$2025 = 3025 + 1600 - 4400(\cos M)$$

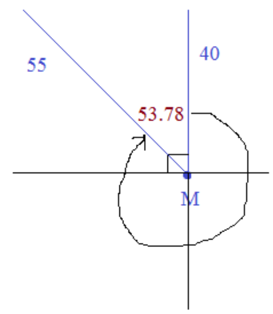
$$-2600 = -4400(\cos M)$$

$$.5909 = \cos M$$

$$M = 53.78$$



$360 - 53.78 =$ **306.22°**



8) Airports A and B are 400 miles apart.
Maverick flies northwest from airport A to airport C.
From C, he flies 350 miles at a heading (bearing) of 215 degrees to airport B.
How far is airport C from A?

Use law of cosines:

$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$400^2 = 350^2 + b^2 - 2(350)(b)\cos(80^\circ)$$

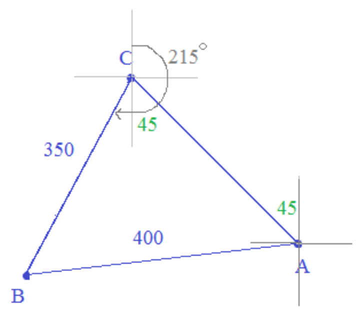
$$160000 = 122500 + b^2 - 121.55b$$

$$b^2 - 121.55b - 37500 = 0$$

$b = \sout{-142.2}$ or 263.7

since angle isn't negative,

263.7°



or, law of sines

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin(80)}{400} = \frac{\sin A}{350}$$

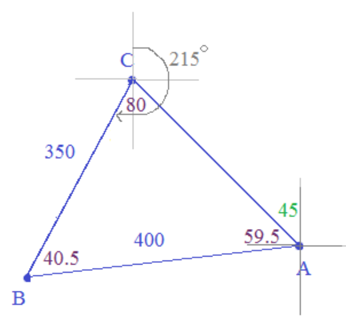
$A = 59.5$ degrees

Since sum of angles in triangle is 180:

$$A + B + C = 180$$

$$59.5 + B + 80 = 180$$

$$B = 40.5$$



$$b^2 = 350^2 + 400^2 - 2(350)(400)\cos(40.5)$$

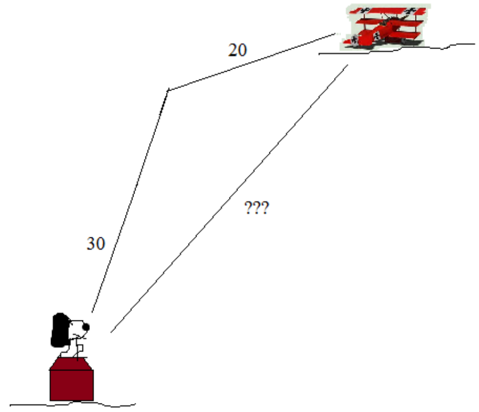
$$= 122500 + 160000 - 280000\cos(40.5)$$

$=$ **263.8°**

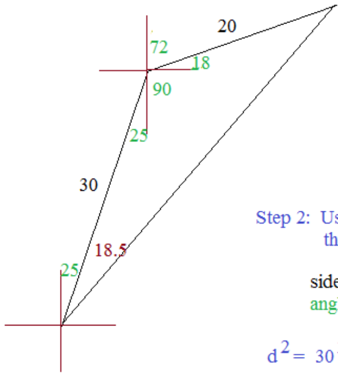
9) The Red Baron leaves Peanuts Airfield at noon and travels N25°E for 30 miles.
Then, the Baron turns and flies another 20 miles in the N72°E direction before landing at Woodstock Airfield to refuel.

At 1:00 PM, Snoopy leaves Peanuts Airfield to pursue the Baron.
How far, and in which direction, should Snoopy travel to get to Woodstock as fast as possible?

SOLUTIONS



Step 1: Using navigation, construct a digram (triangle)



Step 2: Use Law of Cosines to determine the distance Snoopy must travel...

sides are 20 and 30 miles
angle is 25 + 90 + 18 = 133 degrees

$$d^2 = 30^2 + 20^2 - 2(30)(20)\text{Cos}(133^\circ)$$

$$= 1300 - 1200(-.682)$$

$$d = \boxed{46 \text{ miles (approx)}}$$

Step 3: using law of sines and angle addition, determine the direction...

$$\frac{\sin(C)}{20} = \frac{\sin(133)}{46}$$

C = 18.5 degrees

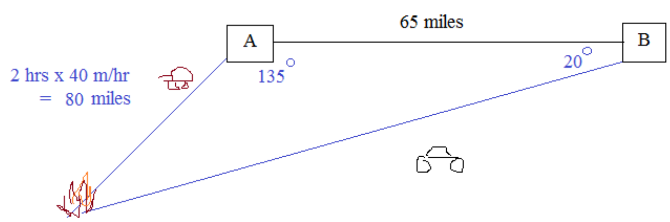
Then, 18.5 + 25 = 43.5 degrees...

So, $\boxed{N43.5^\circ E}$

10) Car 1 leaves Station A and goes 40 miles/hour at S45 W.
Simultaneously, 65 miles due East, Car 2 leaves Station B and travels S70 W.
Two hours later, Car 2 collides with Car 1!!
Assuming each car was going straight at their constant speeds, how fast was Car 2 traveling?

Step 1: Construct and label a digram

Step 2: Recognize the triangle and apply law of cosines



$$d^2 = 65^2 + 80^2 - 2(65)(80)\text{Cos}(135^\circ)$$

the distance (d) traveled by Car 2 is 134.1 miles...

Step 3: Apply to distance = rate x time to answer the question...

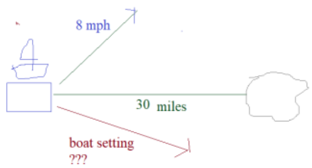
Since the driver drove for 2 hours, the speed of Car 2

is approx $\boxed{\text{67 miles/hour...}}$

SOLUTIONS

11) A boat leaves the harbor for an island 30 miles due east.
The ocean current is pulling in a northeast direction at 8 miles per hour.
How fast and in which direction should the boat travel to reach the island in 2 hours?

Step 1: Draw a sketch



Since it is a 2-hour journey, we'll set the vectors to that timeframe...

Wind: 16 miles in a northeast direction (or 45 degree geometry angle)

Destination: 30 miles due east...

Boat setting: ????

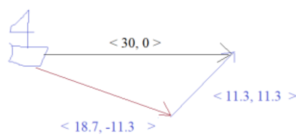
Step 2: Set up the equation and convert to vector form

Boat setting + Current change = Destination

< ????, ???? > + < $16\cos(45^\circ)$, $16\sin(45^\circ)$ > = < 30, 0 >

< ????, ???? > + < 11.3, 11.3 > = < 30, 0 >

The boat setting vector should be < 18.7, -11.3 >



Step 3: Solve and answer questions

The magnitude represents the distance the boat must travel...

$\sqrt{18.7^2 + (-11.3)^2}$ = 21.8 miles

It's a 2 hour trip...

Therefore, the boat must travel 10.9 miles per hour!

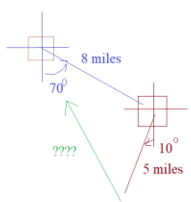
$\tan^{-1}\left(\frac{-11.3}{18.7}\right)$ = -31 degrees

The direction the boat should set is

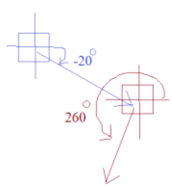
S59E or a bearing of 121°

12) Nav the Gator crawls 8 miles through the swamp in a S70E direction.
After a snack, Nav turns and continues 5 miles heading S10W..
If Nav wants to return directly to his home, how far and which direction should he crawl?

Step 1: Draw a sketch



(Identify the standard position angles used to convert to vectors)



Step 2: Set up the equation and convert to vector form

First leg + Second leg = Total travel

< $8\cos(-20^\circ)$, $8\sin(-20^\circ)$ > + < $5\cos(260^\circ)$, $5\sin(260^\circ)$ > = < ????, ???? >

< 7.5, -2.7 > + < -.87, -4.9 > = < 6.63, -7.6 >

Step 3: Solve and answer questions

To find the distance back home, we need to find the length of the resultant vector...

$\sqrt{6.63^2 + (-7.6)^2}$ = 10.1 miles

The direction of the vector is

$\tan^{-1}\left(\frac{-7.6}{6.63}\right)$ = -49 degrees....

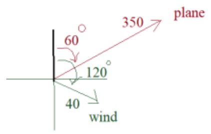
Now, since the gator must go back, it must travel in the opposite direction: 131 degrees...

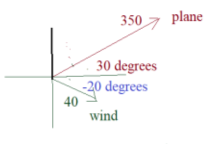
N41W

SOLUTIONS

13) An airplane is flying at a bearing of 60° with an air speed of 350 miles per hour.
There is a 40 mile-per-hour wind speed with a bearing of 110°.
What is the ground speed of the airplane?
What is the course of the airplane?

Step 1: Draw a sketch





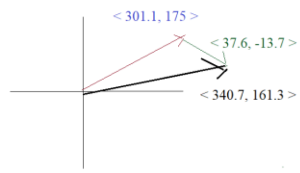
Step 2: Set up the equation and convert to vector form

Air speed + Wind speed = Ground speed

$< 350\cos(30^\circ), 350\sin(30^\circ) > + < 40\cos(-20^\circ), 40\sin(-20^\circ) > = < i, j >$

(NOTE: we must convert the bearings into geometry angle measures; then, we can transform into vector form!)

$< 303.1, 175 > + < 37.6, -13.7 > = < 340.7, 161.3 >$



Step 3: Solve and answer questions

To find the ground speed, we'll take the magnitude of the vector...

$\sqrt{340.7^2 + 161.3^2}$ = 377 miles per hour

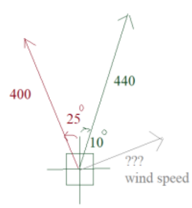
To find the course of the plane, we'll find the angle and convert into bearings...

$\tan^{-1}\left(\frac{161.3}{340.7}\right) = 25.33^\circ$ Looking at the diagram, this is a reasonable answer.

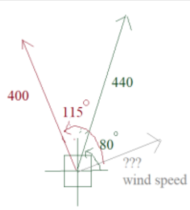
The bearing is $90 - 25.33 = 64.67^\circ$

14) A plane is traveling at 400 miles per hour in a N25W direction.
Its ground speed and direction are 440 miles per hour and N10E respectively.
What is the speed and direction of the wind?

Step 1: Draw a sketch



N25W is equivalent to 115 angle degrees
N10E is equivalent to 80 angle degrees



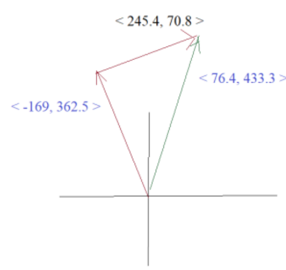
Step 2: Set up the equation and convert to vector form

Air speed + Wind speed = Ground speed

$< 400\cos(115^\circ), 400\sin(115^\circ) > + < ???, ??? > = < 440\cos(80^\circ), 440\sin(80^\circ) >$

$< -169, 362.5 > + < ???, ??? > = < 76.4, 433.3 >$

Wind vector: < 245.4, 70.8 >



Step 3: Solve and answer questions

magnitude of vector < 245.4, 70.8 >

will give wind speed..

$\sqrt{245.4^2 + 70.8^2}$ = 255 miles per hour

$\tan^{-1}\left(\frac{70.8}{245.4}\right) = 16.1^\circ$

is the angle of the wind vector...

The navigation direction is N73.9E

Thanks for checking out this packet. Hope it was a helpful resource!

If you have questions, suggestions, or a request, let us know.

Best always,

Lance

Mathplane.com and Mathplane.org for Mobile

(**Appreciate your support. All proceeds go to site maintenance and treats for my dog, Oscar!)