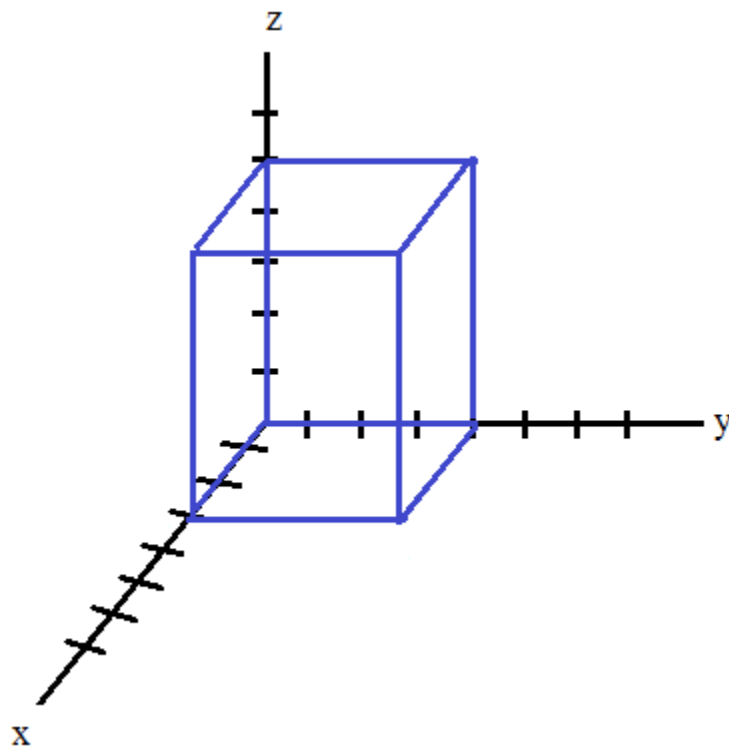# 3-Dimensional Space and Vectors

Notes, Examples, and Practice Quiz (with Answers)

Topics include 3-D coordinate plane, area, vector dot product and cross product, equation of a plane, and more.
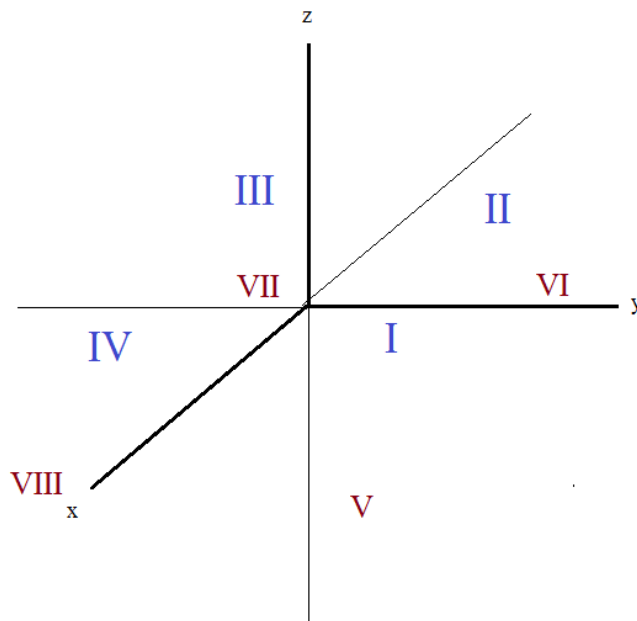
The xy-coordinate plane has 4 "quadrants"....

The xyz-coordinate space has 8 "octants"...

The four quadrants remain I, II, III, IV where z is positive...

Then, octants V, VI, VII, and VIII occur respectively where z is negative...

I, II, III, and IV are above the xy-plane

V, VI, VII, and VIII are below the xy-plane

Example: What Octant is the center of sphere $(x-4)^2 + (y+7)^2 + (z-3)^2 = 9$?

The center of the sphere is $(4, -7, 3)$

Looking at the x and y coordinates: $(4, -7)$ would lie in quadrant IV

Then, since the z coordinate (, , 3) is positive, the center lies in quadrant IV

(Note: if the coordinate were $(4, -7, -3)$, it would like in quadrant VIII)

Example: Find the equation of a sphere that passes through the following points:

(2, 1, 3)   (0, -1, 2)   (1, 4, -2)   (-1, 1, 3)

General Equation of a Sphere

$$x^2 + y^2 + z^2 + Dx + Ey + Fz + G = 0$$

Step 1: Utilize the general equation of a sphere

(2, 1, 3) $\implies$ $4 + 1 + 9 + 2D + 1E + 3F + G = 0$

$2D + 1E + 3F + G = -14$

(0, -1, 2) $\implies$ $0 + 1 + 4 + 0D + (-1)E + 2F + G = 0$

$-1E + 2F + G = -5$

(1, 4, -2) $\implies$ $1 + 16 + 4 + 1D + 4E + (-2)F + G = 0$

$1D + 4E - 2F + G = -21$

(-1, 1, 3) $1 + 1 + 9 + (-1)D + 1E + 3F + G = 0$

$-1D + 1E + 3F + G = -11$

Step 2: Solve system of equations

$2D + 1E + 3F + G = -14$

$-1E + 2F + G = -5$

$1D + 4E - 2F + G = -21$

$-1D + 1E + 3F + G = -11$

$D = -1$

$E = -43/13$

$F = -5/13$

$G = -98/13$

Step 3: Substitute and Simplify

$$x^2 + y^2 + z^2 - 1x - \frac{43}{13}y - \frac{5}{13}z = \frac{98}{13}$$

$\implies$ $13x^2 + 13y^2 + 13z^2 - 13x - 43y - 5z - 98 = 0$

To check answer, plug in each point...

Try (0, -1, 2) ----> 0 + 13 + 52 - 0 + 43 - 10 - 98 = 0 ✓

Example: Find the 3 traces of the sphere $(x - 5)^2 + (y + 3)^2 + (z + 1)^2 = 9$

The sphere with center (5, -3, -1) and radius 3

The xy-trace occurs when z = 0 -----> circle $(x - 5)^2 + (y + 3)^2 = 8$

The yz-trace occurs when x = 0 -----> no intersection $(y + 3)^2 + (z + 1)^2 = -16$

The xz-trace occurs when y = 0 -----> point $(x - 5)^2 + (z + 1)^2 = 0$

Example: What lengths of r would create a sphere that intercepts the xz and yz planes only?

$$(x - 3)^2 + (y + 4)^2 + (z - 8)^2 = r^2$$

$4 \leq r < 8$

if r < 8, the "z part" of the sphere won't reach the xy-plane...

Then, if r ≥ 4, the "x and y parts" will touch or cross the other planes.

Example: Find the area of the triangle whose vertices are

A (2, 3, -5)

B (-2, -2, 0)

C (3, 0, 6)

Method 1: Using vectors and cross product

Step 1: Select and identify 2 vectors that share the same initial or terminal point

$\overrightarrow{BA}$ (2, 3, -5) − (-2, -2, 0) -----> $< 4, 5, -5 >$ = v

$\overrightarrow{CA}$ (2, 3, -5) + (3, 0, 6) -----> $< -1, 3, -11 >$ = w

$\boxed{\text{Area} = \frac{1}{2} \| v \times w \|}$

Step 2: Find cross product

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & 5 & -5 \\ -1 & 3 & -11 \end{vmatrix} = \begin{vmatrix} 5 & -5 \\ 3 & -11 \end{vmatrix} i - \begin{vmatrix} 4 & -5 \\ -1 & -11 \end{vmatrix} j + \begin{vmatrix} 4 & 5 \\ -1 & 3 \end{vmatrix} k = -40i - (-49)j + 17k$$

$$\quad\quad\quad\quad\quad -40 \quad\quad\quad\quad -49 \quad\quad\quad\quad 17 \quad\quad\quad\quad \text{or } < -40, 49, 17 >$$

Step 3: Insert into area formula...

$$\text{Area} = \frac{1}{2} \| v \times w \| = \frac{1}{2} \| < -40, 49, 17 > \| = \frac{1}{2} \sqrt{4290} = \frac{1}{2} \cdot 65.5$$

$\boxed{\text{approx. } 32.75}$

Method 2: Using law of cosines, sines, and trig formulas

Step 1: Find lengths of sides (using distance formula)

side c (distance between A and B) $\sqrt{(2 - (-2))^2 + (3 - (-2))^2 + (-5 - 0)^2}$

$$\sqrt{16 + 25 + 25} = \sqrt{66}$$
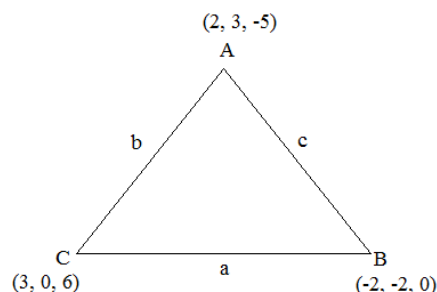
side a (distance between B and C) $\sqrt{(-2 - 3)^2 + (-2 - 0)^2 + (0 - 6)^2}$

$$\sqrt{25 + 4 + 36} = \sqrt{65}$$

side b (distance between C and A) $\sqrt{(3 - 2)^2 + (0 - 3)^2 + (6 - (-5))^2}$

$$\sqrt{1 + 9 + 121} = \sqrt{131}$$

(2, 3, -5)
A

b   c

C   a   B
(3, 0, 6)   (-2, -2, 0)

Step 2: Find length of 3rd side using law of cosines

$$c^2 = a^2 + b^2 - 2ab(\text{Cos}C)$$

$$66 = 65 + 131 - 2(\sqrt{65})(\sqrt{131})\text{Cos}C$$

$$\frac{-130}{-2\sqrt{8515}} = \text{Cos}C \quad\quad C = 45.22^\circ$$

$\boxed{\text{Area of triangle} = \frac{1}{2}ab\text{Sin}C}$

Step 3: Insert into area formula

$$\text{Area} = \frac{1}{2}\sqrt{65}\sqrt{131}\,\text{Sin}(45.22^\circ) = 32.75$$

Example: Find V x U. then, verify that the result is orthogonal to both V and U

U = < 3, 2, -8 >

V = < -1, 6, 7 >

note: V x U is NOT the same as U x V

$$V \times U = \begin{vmatrix} i & j & k \\ -1 & 6 & 7 \\ 3 & 2 & -8 \end{vmatrix} = \begin{vmatrix} 6 & 7 \\ 2 & -8 \end{vmatrix} i - \begin{vmatrix} -1 & 7 \\ 3 & -8 \end{vmatrix} j + \begin{vmatrix} -1 & 6 \\ 3 & 2 \end{vmatrix} k$$

note: the second group, the j component is (-)

(-48 - 14)i — (8 - 21)j + (-2 - 18)k

< -62, 13, -20 >

< -62, 13, -20 > · < 3, 2, -8 > = -186 + 26 + 160 = 0 ✓

< -62, 13, -20 > · < -1, 6, 7 > = 62 + 78 - 140 = 0 ✓

(since dot product equals zero, the vectors are orthogonal)

Example: Find equation of a plane that contains these points: (2, 3, -2) (3, 4, 2) (1, -1, 0)

We need the normal vector and a point... (we have 3 points to choose from)

Step 1: Find 2 vectors

A (2, 3, -2)
B (3, 4, 2)
$\overrightarrow{AB}$ < 1, 1, 4 >

A (2, 3, -2)
C (1, -1, 0)
$\overrightarrow{AC}$ < -1, -4, 2 >

Step 2: Use the cross product to find the normal vector
(the cross product is orthogonal to BOTH vectors)

$$\overrightarrow{N} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ -4 & 2 \end{vmatrix} i - \begin{vmatrix} 1 & 4 \\ -1 & 2 \end{vmatrix} j + \begin{vmatrix} 1 & 1 \\ -1 & -4 \end{vmatrix} k = 18i - 6j - 3k$$

Step 3: Plug your point into the normal vector

A(2, 3, -2)   18(2) - 6(3) - 3(-2) = 24   18x - 6y - 3z = 24

Step 4: Check the answer
(substitute each point into the equation of the plane!)

A(2, 3, -2) Obviously, this one will work: 18(2) - 6(3) - 3(-2) = 24 ✓

B(3, 4, 2)   18(3) - 6(4) - 3(2) = 54 - 24 - 6 = 24 ✓

C(1, -1, 0) 18(1) - 6(-1) - 3(0) = 18 + 6 - 0 = 24 ✓

Finding the equation of a plane (without using the cross product)

Example: Write the equation of the plane containing the points (-3, 5, -8) (-2, 3, -4) (6, 1, -3)

Step 1: find equation of 2 lines/vectors

(-3, 5, -8) to (-2, 3, -4) is < 1, -2, 4 >

(-3, 5, -8) to (6, 1, -3) is < 9, -4, 5 >

Step 2: find the normal vector

Since the normal vector is perpendicular to each vector above, the dot product will be 0.

Normal Vector < a, b, c > perpendicular to < 1, -2, 4 >   1a + -2b + 4c = 0

Normal Vector < a, b, c > perpendicular to < 9, -4, 5 >   9a + -4b + 5c = 0

To find a, b, c, we'll solve the system of equations....

Since there are 2 equations and 3 variables, there are multiple answers...
We'll pick one:

let a = 1:   1 + -2b + 4c = 0   ⟹   -2b + 4c = -1   4b - 8c = 2

              9 + -4b + 5c = 0   ⟹   -4b + 5c = -9 $\underline{-4b + 5c = -9}$

                                                              -3c = -7   c = 7/3

To check our work; we'll take the dot
product of the normal vector to each vector

< 1, -2, 4 > • < 6, 31, 14 > = 6 - 62 + 56 = 0 ✓

< 9, -4, 5 > • < 6, 31, 14 > = 54 - 124 + 70 = 0 ✓

1 + -2b + 4(7/3) = 0

-2b = -31/3   b = 31/6

The normal vector is < 1, 31/6, 7/3 >

Or, < 6, 31, 14 >

Step 3: Apply the equation of a plane....

Using the point (-3, 5, -8)....

6(x + 3) + 31(y - 5) + 14(z + 8) = 0

or

6x + 31y + 14z = 25

To check our work, we'll plug in all
3 points

(-3, 5, -8)   We used this point, so it works..

(-2, 3, -4)   6(-2) + 31(3) + 14(-4) = -12 + 93 - 56 = 25 ✓

(6, 1, -3)   6(6) + 31(1) + 14(-3) = 36 + 31 - 42 = 25 ✓

Finding the equation of a plane: 2 methods

Example: Find the equation of a plane containing the points $(2, 1, 3)$ $(1, 4, -2)$ and $(-1, 3, 5)$

Method 1: Using vectors, normal, and cross product

Step 1: Find identify 2 vectors from one of the points...

We'll use $(2, 1, 3)$

vector u from $(2, 1, 3)$ to $(1, 4, -2)$

$< -1, 3, -5 >$

vector v from $(2, 1, 3)$ to $(-1, 3, 5)$

$< -3, 2, 2 >$

Step 2: Determine the normal vector by taking cross product

$$u \times v = \begin{vmatrix} i & j & k \\ -1 & 3 & -5 \\ -3 & 2 & 2 \end{vmatrix} = i \begin{vmatrix} 3 & -5 \\ 2 & 2 \end{vmatrix} - j \begin{vmatrix} -1 & -5 \\ -3 & 2 \end{vmatrix} + k \begin{vmatrix} -1 & 3 \\ -3 & 2 \end{vmatrix}$$

$$16i + 17j + 7k$$

Note: since this is a normal vector, it should be orthogonal to vectors u and v!

To check, we'll take the dot product...

$< 16, 17, 7 > \cdot < -3, 2, 2 > = -48 + 34 + 14 = 0$ ✓

$< 16, 17, 7 > \cdot < -1, 3, -5 > = -16 + 51 - 35 = 0$ ✓

Step 3: Plug in the chosen point into the normal vector...

$(2, 1, 3)$ $16i + 17j + 7k$

$16(x - 2) + 17(y - 1) + 7(z - 3) = 0$

$\boxed{16x + 17y + 7z = 70}$

Method 2: Using the general form of a plane and system of equations

Step 1: Substitute each point into the equation of a plane

$$Ax + By + Cz + D = 0$$

$(2, 1, 3)$ ⟹ $2A + 1B + 3C + D = 0$

$(1, 4, -2)$ ⟹ $1A + 4B + (-2)C + D = 0$

$(-1, 3, 5)$ ⟹ $-1A + 3B + 5C + D = 0$

Step 2: Solve the system of equations

$$A = \frac{-8}{35}D$$

$$B = \frac{-17}{70}D \qquad D = D$$

$$C = \frac{-1}{10}D$$

Step 3: Clean up the equation

To get rid of the fractions, we'll let $D = 70$

$A = -16$ $B = -17$ $C = -7$ $D = 70$

$-16x - 17y - 7z + 70 = 0$

$\boxed{16x + 17y + 7z = 70}$

Note: To check our answer, we can plug in each point!

$(2, 1, 3)$ ⟹ $16(2) + 17(1) + 7(3) = 32 + 17 + 21 = 70$ ✓

$(1, 4, -2)$ ⟹ $16(1) + 17(4) + 7(-2) = 16 + 68 - 14 = 70$ ✓

$(-1, 3, 5)$ ⟹ $16(-1) + 17(3) + 7(5) = -16 + 51 + 35 = 70$ ✓

Example: Find the area of parallelogram ABCD whose vertices are

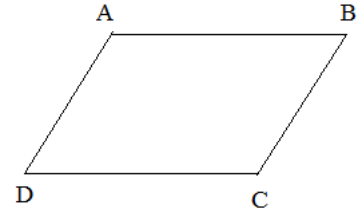   A (5, -6, 3)

       B (-2, -9, 8)

       C (2, -5, 5)

       D (9, -2, 0)

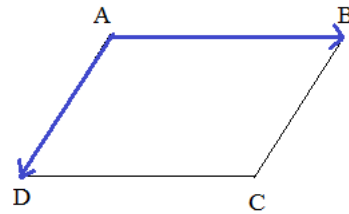Step 1: Sketch a diagram

Step 2: Find 2 vectors that share a common vertex...

$\overrightarrow{AB}$ < (-2 − 5) , (-9 − (-6)) , (8 − 3) >

       < -7, -3, 5 >

$\overrightarrow{AD}$ < (9 − 5) , (-2 − (-6)) , (0 − 3) >

       < 4, 4, -3 >

Step 3: Use Cross product

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ -7 & -3 & 5 \\ 4 & 4 & -3 \end{vmatrix} = \begin{vmatrix} -3 & 5 \\ 4 & -3 \end{vmatrix} i \; - \; \begin{vmatrix} -7 & 5 \\ 4 & -3 \end{vmatrix} j \; + \; \begin{vmatrix} -7 & -3 \\ 4 & 4 \end{vmatrix} k$$

                   -11i   −   1j   +   -16k   < -11, -1, -16 >

Step 4: Use Area formula for Parallelogram...

$$\left\| \overrightarrow{AB} \times \overrightarrow{AD} \right\| = \left\| <\text{-}11, \text{-}1, \text{-}16> \right\|$$

Area of parallelogram = ‖ u x v ‖

$$\sqrt{121 + 1 + 256} \; = \; \sqrt{378} \; = 3\sqrt{42} \; = 19.44$$
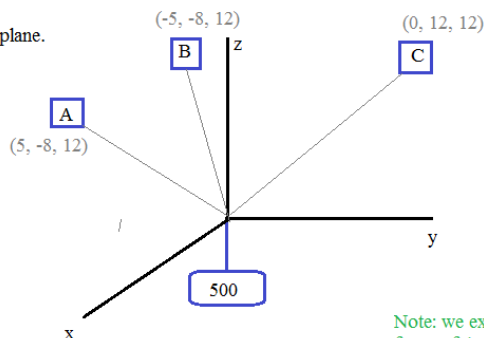
3-D vectors and force

Example: 3 pulleys are holding a 500 pound weight in place.
The position of the pulleys can be mapped onto a 3-d coordinate plane.

Pulley A   (5, -8, 12)

Pulley B   (-5, -8, 12)

Pulley C   (0, 12, 12)

Identify the force each pulley is exerting...

Note: we expect the force of A and B to be the same, since their angles to the weight are the same!

To solve, we must remember that a vector consists of a) direction and b) length..
THEN, we must account for the force in the magnitude of the vector!

First, let's find the unit vector of each pulley:

A: $\dfrac{<5, -8, 12>}{\sqrt{25 + 64 + 144}}$ B: $\dfrac{<-5, -8, 12>}{\sqrt{25 + 64 + 144}}$ C: $\dfrac{<0, 12, 12>}{\sqrt{0 + 144 + 144}}$

Since each pulley extends from the origin (0, 0, 0), the position is identical to the vector..

unit vector A: unit vector B: unit vector C:

$\left\langle \dfrac{5}{\sqrt{233}} \quad \dfrac{-8}{\sqrt{233}} \quad \dfrac{12}{\sqrt{233}} \right\rangle$ $\left\langle \dfrac{-5}{\sqrt{233}} \quad \dfrac{-8}{\sqrt{233}} \quad \dfrac{12}{\sqrt{233}} \right\rangle$ $\left\langle \dfrac{0}{12\sqrt{2}} \quad \dfrac{12}{12\sqrt{2}} \quad \dfrac{12}{12\sqrt{2}} \right\rangle$

Now, that we have the direction, we can add the force (magnitude) of each pulley!

$\|A\| \left\langle \dfrac{5}{\sqrt{233}} \quad \dfrac{-8}{\sqrt{233}} \quad \dfrac{12}{\sqrt{233}} \right\rangle + \|B\| \left\langle \dfrac{-5}{\sqrt{233}} \quad \dfrac{-8}{\sqrt{233}} \quad \dfrac{12}{\sqrt{233}} \right\rangle + \|C\| \left\langle \dfrac{0}{12\sqrt{2}} \quad \dfrac{12}{12\sqrt{2}} \quad \dfrac{12}{12\sqrt{2}} \right\rangle$

and, that total must equal the force of the 500 pound weight.. $<0, 0, 500>$

Note: the vector is $<0, 0, -500>$, BUT we want the <u>sum of all 4 vectors to equal 0</u>....
Therefore, the opposing weight is expressed as an opposite...

$\dfrac{5}{\sqrt{233}} \|A\| + \dfrac{-5}{\sqrt{233}} \|B\| + \dfrac{0}{12\sqrt{2}} \|C\| = 0$ The i vector components (x-direction)

$\dfrac{-8}{\sqrt{233}} \|A\| + \dfrac{-8}{\sqrt{233}} \|B\| + \dfrac{12}{12\sqrt{2}} \|C\| = 0$ The j vector components (y-direction)

$\dfrac{12}{\sqrt{233}} \|A\| + \dfrac{12}{\sqrt{233}} \|B\| + \dfrac{12}{12\sqrt{2}} \|C\| = 500$ The k vector components (z-direction)

Three linear equations, with 3 variables...

$\|A\| = 190.8 \quad \|B\| = 190.8 \quad \|C\| = 282.8$

$\dfrac{25\sqrt{233}}{2} \qquad \dfrac{25\sqrt{233}}{2} \qquad 200\sqrt{2}$

The answer seems reasonable..

If pulleys A, B, and C were directly above the weight, the force would be 500...
But, since they are pulling from an angle, the collective force required is greater than 500...

Distance between two planes

Example: find the distance between the planes

$$3x + 5y - z = 11$$

$$3(x - 2) + 5(y + 1) - 1(z + 7) = 0$$

Step 1: First, we check that the planes are parallel (and don't intersect).
Since the normal vectors are parallel, the planes are parallel.

Step 2: Pick points from each plane and connect them,
forming a vector u

We randomly pick a point that works in the first plane: (1, 1, -3)

and, we'll pick the shown point in the second plane: (2, -1, -7)

vector u $= < -1, 2, 4 >$

Step 3: Identify the normal vector n of each plane

The normal vector of each plane is $< a, b, c >$ n $= < 3, 5, -1 >$

Step 4: Apply the formula that "projects the vector u
onto the normal vector n"

$$|u \cdot n| = |(-3) + 10 + (-4)| = 3$$

$$|n| = \sqrt{3^2 + 5^2 + (-1)^2} = \sqrt{35}$$
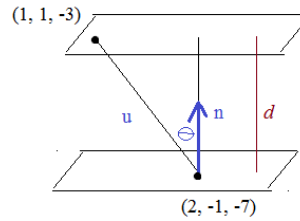
note:

$$\cos \ominus = \frac{d}{|u|}$$

$$\cos \ominus = \frac{u \cdot n}{|u| \cdot |n|}$$

distance $d = \dfrac{|u \cdot n|}{|n|}$

where u is a vector connecting the planes
and n is the normal vector of the parallel planes

distance $= \dfrac{3}{\sqrt{35}}$

(1, 1, -3)

(2, -1, -7)

u   n   d

Distance between two lines

Example: find the distance between the lines

$$x = 3t + 5 \qquad\qquad x = 3s + 1$$
$$y = -t + 2 \quad \text{and} \quad y = -s + 8$$
$$z = 2t - 3 \qquad\qquad z = 2s$$

Step 1: check that the planes have no intersection
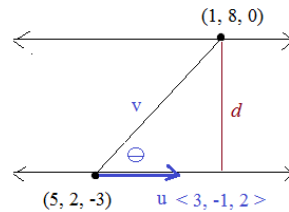(after all, if they intersect, then the distance is zero!)

This is easily checked, because the lines are
parallel (and not skew)...
They each have same direction vector $< 3, -1, 2 >$ ✓

Step 2: pick a point from each line and create a vector 'v'
connecting the points

We know (5, 2, -3) is on the first line;
and, (1, 8, 0) is on the second line

vector v $= < -4, 6, 3 >$

Step 3: find the vector orthogonal to the lines and the
created vector

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ -4 & 6 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 6 & 3 \end{vmatrix} i - \begin{vmatrix} 3 & 2 \\ -4 & 3 \end{vmatrix} j + \begin{vmatrix} 3 & -1 \\ -4 & 6 \end{vmatrix} k$$

$$< -15, -17, 14 >$$

note:

$$\sin \ominus = \frac{d}{|v|}$$

$$\sin \ominus = \frac{|u \times v|}{|u| \cdot |v|}$$

(1, 8, 0)

v   d

(5, 2, -3)   u $< 3, -1, 2 >$

Step 4: apply the formula that "projects the drawn vector 'v' in
the direction of the vector orthogonal to the lines"

distance $d = \dfrac{|u \times v|}{|u|}$

where u is the direction of a given line
and v is the vector connecting 2 points

$$|u \times v| = \text{magnitude: } \sqrt{(-15)^2 + (-17)^2 + (14)^2} = \sqrt{710}$$

$$|u| = \sqrt{(3)^2 + (-1)^2 + (2)^2} = \sqrt{14}$$

distance $= \dfrac{\sqrt{710}}{\sqrt{14}} = 7.12$

Distance of a point to a plane

Example: Find the distance from (2, -1, 5) to $3x + 2y - 2z - 7 = 0$

Method 1: using normal vector and vector from point to random point in the plane

    Step 1: Identify the normal vector n

        From the plane $3x + 2y - 2z - 7 = 0$,
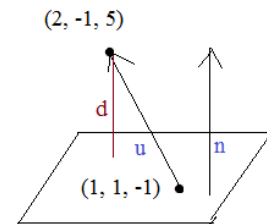   we know the normal vector n
        is
           $< 3, 2, -2 >$

    Step 2: choose a vector connecting the point to a place in the plane

        the point is (2, -1, 5)

        and, we can use (1, 1, -1) from the plane

        because $3(1) + 2(1) - 2(-1) - 7 = 0$

        The vector u from (1, 1, -1) to (2, -1, 5) is $< 1, -2, 6 >$

    Step 3: Project the vector u in the direction of n to find the distance....

distance $d = \dfrac{|u \cdot n|}{|n|}$

where u is a vector connecting the planes
and n is the normal vector of the parallel planes

$$d = \frac{\left|< 1, -2, 6 > \cdot < 3, 2, -2 >\right|}{\sqrt{3^2 + 2^2 + (-2)^2}} = \boxed{\frac{13}{\sqrt{17}}}$$

Method 2: Use distance from a point to a plane formula

The distance d from point $P_1(x_1, y_1, z_1)$

        to plane $Ax + By + Cz + D = 0$ is

$$d = \frac{\left|Ax_1 + By_1 + Cz_1 + D\right|}{\sqrt{A^2 + B^2 + C^2}}$$

Point (x, y, z) ----> (2, -1, 5)

coefficients A, B, C, D from plane -----> $3x + 2y - 2z - 7 = 0$

$$d = \frac{\left|3(2) + 2(-1) + -2(5) + -7\right|}{\sqrt{3^2 + 2^2 + (-2)^2}} = \frac{|-13|}{\sqrt{17}} = \boxed{\frac{13}{\sqrt{17}}}$$

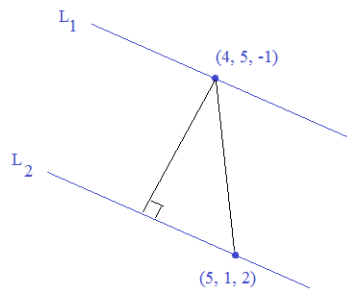*Example:* Find the distance between the following lines....

$$L_1 \begin{cases} x = 4 - 2t \\ y = 5 + 3t \\ z = -1 + t \end{cases} \qquad L_2 \begin{cases} x = 5 - 2s \\ y = 1 + 3s \\ z = 2 + s \end{cases}$$

Since the direction vectors are the same, $< -2, 3, 1 >$, these lines are parallel!

We need to find SOME POINT P that is closest to SOME point Q

$L_1 = 4 - 2t, \ 5 + 3t, \ -1 + t$

$L_2 = 5 - 2s, \ 1 + 3s, \ 2 + s$

$\overrightarrow{PQ} = < 1 - 2s + 2t, \ -4 + 3s - 3t, \ 3 + s - t >$

$\overrightarrow{PQ}$ is orthogonal to both lines...

So,

$\overrightarrow{PQ} \cdot L_1 = 0$

$\overrightarrow{PQ} \cdot < -2, 3, 1 > = 0$

$\overrightarrow{PQ} \cdot L_2 = 0$

$\overrightarrow{PQ} \cdot < -2, 3, 1 > = 0$

Possible sketch showing distance between points
$t = 0$ and $s = 0$

Both equations, since they are parallel, would be...

$< 1 - 2s + 2t, \ -4 + 3s - 3t, \ 3 + s - t > \ \cdot \ < -2, 3, 1 > = 0$

$\begin{array}{ccccccc} -2 + 4s - 4t & + & -12 + 9s - 9t & + & 3 + s - t & = 0 \end{array}$

$-11 + 14s - 14t = 0$

$\Longrightarrow \quad 14s - 14t = 11 \quad \Longleftarrow$

Now, we'll choose 2 points:

Let $s = 11/14$ and $t = 0$ because $14(11/14) - 14(0) = 11$

$$L_1 \begin{cases} x = 4 - 2t \\ y = 5 + 3t \\ z = -1 + t \end{cases} \qquad L_2 \begin{cases} x = 5 - 2s \\ y = 1 + 3s \\ z = 2 + s \end{cases}$$

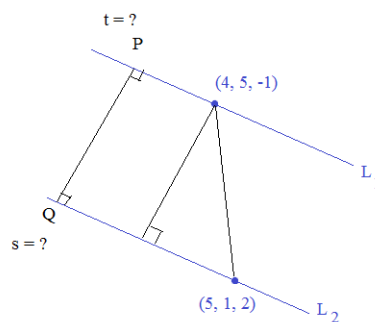distance between these two points:

$t = 0$: (4, 5, -1)

$s = 11/14$: (48/14, 47/14, 39/14)

$\text{distance} = \sqrt{(8/14)^2 + (23/14)^2 + (53/14)^2} = \sqrt{243/14} \quad \checkmark$

Here is a formula that applies the above structure:

$$\text{distance } d = \frac{| \, u \times v \, |}{| \, u \, |}$$

where u is the direction of a given line and v is the vector connecting 2 points

$u = < -2, 3, 1 >$

$v = < 1, -4, 3 >$

$u \times v = < 13, 7, 5 >$

$d = \dfrac{\sqrt{243}}{\sqrt{14}} \quad \checkmark$

Testing the limits of endurance,
these math figures will run on and on...

Practice Quiz-→

1) Three-Dimensional Coordinates
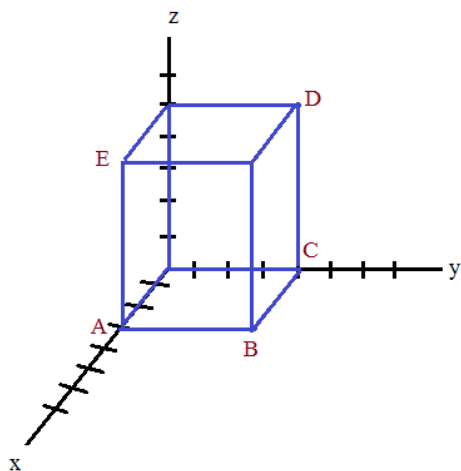
Identify the coordinates of the following points:

A:

B:

C:

D:

E:

Determine the midpoints of:

A and B

D and C

E and C

E and B

Find the distance between:

A and B

A and E

A and D

2) Spheres

   A) Find the center and radius of the following sphere:
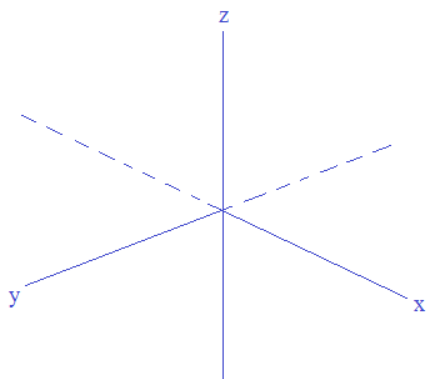
$$x^2 + y^2 + z^2 - 8y + 2z = 8$$

   Identify any 3 points that lie on the sphere:
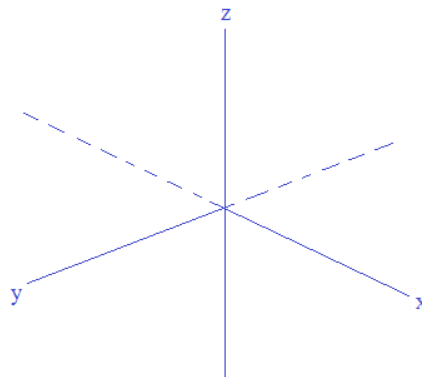
   Is the origin (0, 0, 0) inside the sphere?

3) Planes

A) Find the intercepts and sketch the following planes:

1) $x - 3y + 2z = 6$

2) $x + 2y + 3z = 6$

B) Find the equation of a plane that contains these points: (1, 2, 3)   (0, -4, -1)   (6, 1, 5)

4) Intersections and the 'trace'

A) The center of a sphere is (2, 4, 3).
If the radius is 3, describe the intersection with the

      1) xy-plane

      2) xz-plane

      3) yz-plane

B) $(x - 9)^2 + (y + 6)^2 + (z - 11)^2 = r^2$

   a) Find possible values of r that have NO trace in the xy, yz, and xz planes..

   b) Find possible values of r where the sphere intersects ONLY the xz plane..

   c) Describe the traces if r = 7

      1) xz trace

      2) yz trace

      3) xy trace

5) 3-Dimensional Vectors

A) What is u · v ?

u = 3i − 7j

v = 8j + 9k

B) What is v x u ?

C) Vector w lies on the yz-plane.
It has a magnitude of 6 and makes a 45-degree angle with the negative y-axis.

What is vector w?

D) Vector N lies on the xz-plane. The magnitude is 10.
If it's a 60-degree angle from the z-axis, what is the vector?

6) Vectors and Parametric Concepts

A) Write the parametric equation of a line that crosses the z-axis at z = 6 and crosses the xy-plane when x = 2 and y = 5.
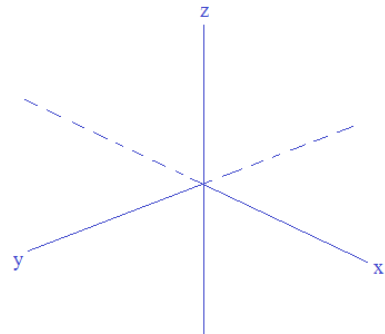
B) Are the vectors $<2, 3>$ and $<-4, -9>$ parallel, perpendicular, or neither?

C) Find *2 unit vectors* that are perpendicular to $j + 3k$ amd $i - 2j + 4k$.

D) What is the angle between vectors $\vec{v}$ and $\vec{w}$?

$\vec{v}$ = $< 1, 2, -5 >$

$\vec{w}$ = $< 0, 3, 3 >$

1) Given the points A $(3, 4, -1)$ and B $(7, -6, 5)$

      Find

            a) the midpoint   b) the distance between points

2) Find the intersection of the 2 lines

$$l \begin{cases} x = 4t + 1 \\ y = t + 3 \\ z = -2t + 8 \end{cases} \qquad m \begin{cases} x = 5s - 10 \\ y = -3s + 13 \\ z = 2s \end{cases}$$

3) What is the intersection of the planes

      $3x + 5y + z + 6 = 0$

      $2x - y + 3z + 1 = 0$

4) Find the distance from the point to the line

line: $x = 3t + 1$ point: $(4, 6, 7)$
        $y = t - 2$
        $z = 2t + 5$

5) Find the distance between the lines.

  $x = 3t + 1$ $x = 3s + 3$
  $y = t - 2$ $y = s + 5$
  $z = 2t + 5$ $z = 2s + 1$

6) What is the equation of the plane containing the points

   $(1, 4, -2)$ $(5, 0, 2)$ and $(3, -1, 3)$

A relationship of significant magnitude: Dot and Norm

Math
Romance
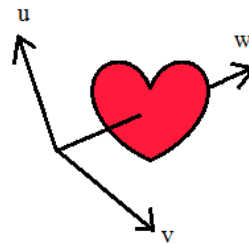II

I ♥
$\vec{U}$

Be my
$\vec{V}$alentine
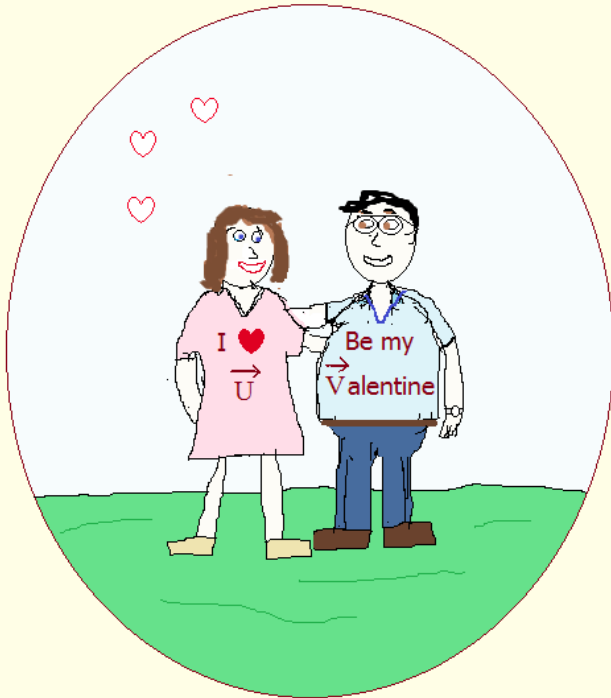
(Their embarrassed kids, *ike, jay,* and *kay,* were nowhere to be found...)

Solutions -→
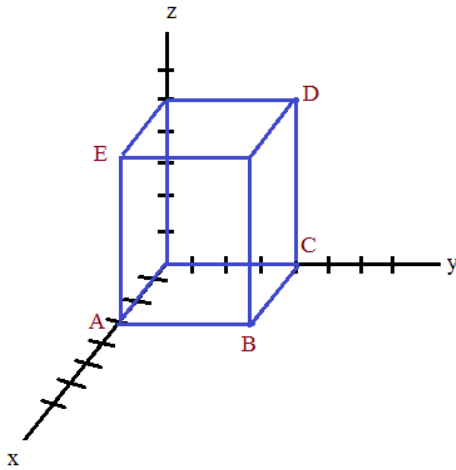
Identify the coordinates of the following points:

A: $(3, 0, 0)$

B: $(3, 4, 0)$

C: $(0, 4, 0)$

D: $(0, 4, 5)$

E: $(3, 0, 5)$

Determine the midpoints of:

A and B $x = \dfrac{3 + 3}{2}$ $y = \dfrac{0 + 4}{2}$ $z = \dfrac{0 + 0}{2}$ $(3, 2, 0)$

D and C $x = \dfrac{0 + 0}{2}$ $y = \dfrac{4 + 4}{2}$ $z = \dfrac{5 + 0}{2}$ $(0, 4, 2.5)$

E and C $(1.5, 2, 2.5)$ or $(3/2, 2, 5/2)$

E and B $(3, 2, 2.5)$

Find the distance between:

A and B $\sqrt{(3 - 3)^2 + (4 - 0)^2 + (0 - 0)^2} = 4$

A and E $\sqrt{(3 - 3)^2 + (0 - 0)^2 + (5 - 0)^2} = 5$

A and D $\sqrt{(3 - 0)^2 + (0 - 4)^2 + (0 - 5)^2} = \sqrt{50} = 5\sqrt{2}$

> NOTE: the similarity of the formulas for circles in a plane and spheres in space!

2) Spheres

A) Find the center and radius of the following sphere:

$$x^2 + y^2 + z^2 - 8y + 2z = 8$$

Complete the square to put general equation into standard form...

$$x^2 + y^2 - 8y + 16 + z^2 + 2z + 1 = 8 + 16 + 1$$

$$x^2 + (y - 4)^2 + (z + 1)^2 = 25$$

The center is $(0, 4, -1)$ and the radius is 5

Identify any 3 points that lie on the sphere:

Three easy points to find are any intercepts: If y and z are 0, $x^2 + (0)^2 + (0)^2 - 8(0) + 2(0) = 8$ $(\sqrt{8}, 0, 0)$ $(-\sqrt{8}, 0, 0)$

If x and y are 0, $(0)^2 + (0)^2 + z^2 - 8(0) + 2z = 8$

$$z^2 + 2z = 8$$

$$(z + 4)(z - 2) = 0$$

$(0, 0, 2)$ $(0, 0, -4)$

Is the origin $(0, 0, 0)$ inside the sphere?

Find radius --- that is the length from the center to the sphere...
Then, find distance from origin to point on the sphere..
If distance < radius, then origin is inside the sphere!

distance between origin and center of the sphere:

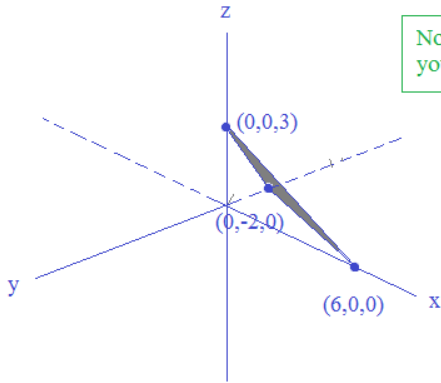$$\sqrt{(0 - 0)^2 + (4 - 0)^2 + (-1 - 0)^2} = \sqrt{17}$$

radius = 5   The origin is inside the sphere!

mathplane.com

3) Planes

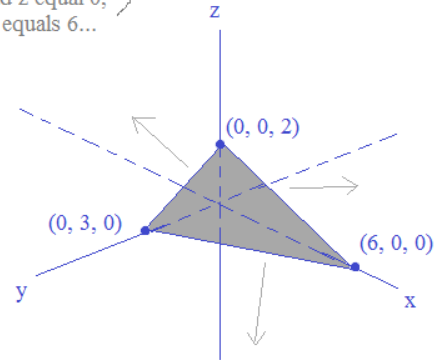A) Find the intercepts and sketch the following planes:

        x-intercept   y-intercept   z-intercept

1) $x - 3y + 2z = 6$ (6, 0, 0)   (0, -2, 0)   (0, 0, 3)   2) $x + 2y + 3z = 6$ (6, 0, 0)   (0, 3, 0)   (0, 0, 2)

If y and z equal 0, then x equals 6...

Note: Once you have 3 points, you can sketch the plane!

                                     A   B   C

B) Find the equation of a plane that contains these points: (1, 2, 3)   (0, -4, -1)   (6, 1, 5)

We need the normal vector and a point... (we have 3 points to choose from)

     Step 1: Find 2 vectors

$$\overrightarrow{AB} \quad < (0 - 1), \ (-4 - 2), \ (-1 - 3) > \ = \ < -1, -6, -4 >$$

$$\overrightarrow{AC} \quad < (6 - 1), \ (1 - 2), \ (5 - 3) > \ = \ < 5, -1, 2 >$$

     Step 2: Use the cross product to find the normal vector
            (the cross product is orthogonal to BOTH vectors)

$$\overrightarrow{N} \ = \ \overrightarrow{AB} \ X \ \overrightarrow{AC} \ = \ \begin{vmatrix} i & j & k \\ -1 & -6 & -4 \\ 5 & -1 & 2 \end{vmatrix} \ = \ \begin{vmatrix} -6 & -4 \\ -1 & 2 \end{vmatrix} i \ - \ \begin{vmatrix} -1 & -4 \\ 5 & 2 \end{vmatrix} j \ + \ \begin{vmatrix} -1 & -6 \\ 5 & -1 \end{vmatrix} k \ = \ -16i - 18j + 31k$$

     Step 3: Plug your point into the normal vector

       A(1, 2, 3) $-16(1) - 18(2) + 31(3) \ = \ 41$ $\boxed{-16x - 18y + 31z = 41}$

         To check: Does (0, -4, -1) lie in the plane? $-16(0) - 18(-4) + 31(-1) = 41$

                                                   $0 - (-72) - 31 \ = \ 41$

                                                          $41 \ = \ 41 \ \checkmark$

         Does (6, 1, 5) lie in the plane? $-16(6) - 18(1) + 31(5) \ = \ 41$

                                              $-96 - 18 + 155 \ = \ 41$

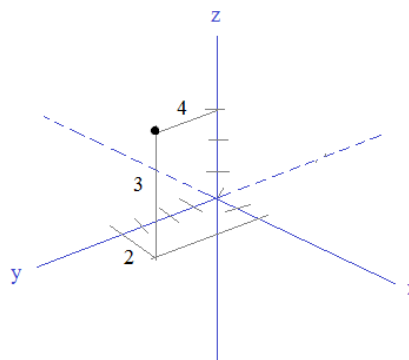 $41 = 41 \ \checkmark$

4) Intersections and the 'trace'　　　　　　　　SOLUTIONS

A) The center of a sphere is (2, 4, 3).
   If the radius is 3, describe the intersection with the

   1) xy-plane　　point　　(the sphere will touch (2, 4, 0) in
   　　　　　　　　　　　　 the xy-plane)

   2) xz-plane　　none　　since the center is 4 units away
   　　　　　　　　　　　　 from the xz-plane, a radius of
   　　　　　　　　　　　　 3 units will not reach....

   3) yz-plane　　circle

B) $(x - 9)^2 + (y + 6)^2 + (z - 11)^2 = r^2$

   a) Find possible values of r that have NO trace in the xy, yz, and xz planes..

   Sphere must not intersect either plane...
   Therefore, its radius must be less than all x, y, and z points...

   $$0 < r < 6$$

   (also, r must be positive length)

   b) Find possible values of r where the sphere intersects ONLY the xz plane..

   $$6 \leq r < 9$$

   must be long enough to intersect xz plane... BUT must be short enough
   not to intersect the other planes...

   c) Describe the traces if r = 7

   1) xz trace　　　　　　　　　　　　　　　2) yz trace

   for xz trace, y = 0　　　　　　　　　　NONE (if the radius is 7, then the sphere won't intersect the yz plane)

   $(x - 9)^2 + (0 + 6)^2 + (z - 11)^2 = 7^2$　　　$(0 - 9)^2 + (y + 6)^2 + (z - 11)^2 = 7^2$

   $(x - 9)^2 + 36 + (z - 11)^2 = 49$　　　　　$(y + 6)^2 + (z - 11)^2 = -32$　radius cannot be negative

   $(x - 9)^2 + (z - 11)^2 = 13$

   　　　　　　　　　　　　　　　　　　　3) xy trace

   a circle with center (9, 0, 11) and radius $\sqrt{13}$　　NONE (if the radius is 7, the sphere will not reach the xy plane)

   　　　　　　　　　　　　　　　　　　　$(x - 9)^2 + (y + 6)^2 + (0 - 11)^2 = 7^2$　(notice, the 'z distance' is
   　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　greater than the radius)

5) 3-Dimensional Vectors

$u = 3i - 7j$

$v = 8j + 9k$

A) What is u • v ?

$< 3, -7, 0 >$

$< 0, 8, 9 >$

$3 \cdot 0, \ -7 \cdot 8, \ 0 \cdot 9 \quad = 0 \ - 56 \ + \ 0 \ = \boxed{-56}$

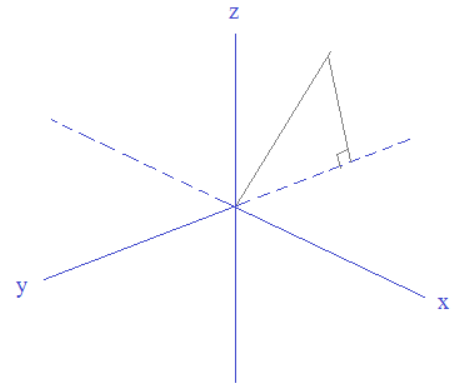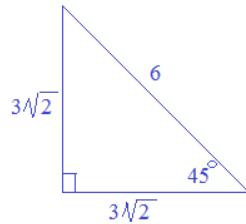B) What is v x u ?

note: v x u is not
equal to u x v!

$$\begin{vmatrix} i & j & k \\ 0 & 8 & 9 \\ 3 & -7 & 0 \end{vmatrix} = \begin{vmatrix} 8 & 9 \\ -7 & 0 \end{vmatrix} i - \begin{vmatrix} 0 & 9 \\ 3 & 0 \end{vmatrix} j + \begin{vmatrix} 0 & 8 \\ 3 & -7 \end{vmatrix} k = \boxed{63i + 27j - 24k}$$

negative

C) Vector w lies on the yz-plane.
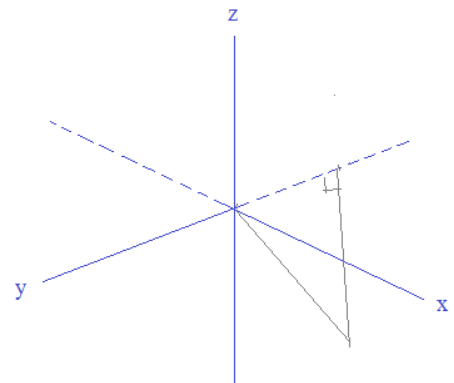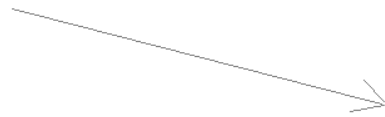It has a magnitude of 6 and makes a 45-degree angle with the negative y-axis.

What is vector w?

$\boxed{< 0, \ -3\sqrt{2}, \ 3\sqrt{2} >}$

or,

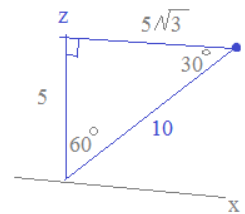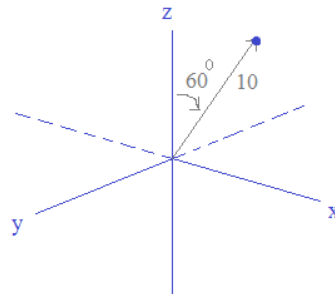$\boxed{< 0, \ -3\sqrt{2}, \ -3\sqrt{2} >}$

D) Vector N lies on the xz-plane. The magnitude is 10.
If it's a 60-degree angle from the z-axis, what is the vector?

$\boxed{< 5\sqrt{3}, \ 0, \ 5 >}$ or $\boxed{< -5\sqrt{3}, \ 0, \ 5 >}$

A) Write the parametric equation of a line that crosses the z-axis at z = 6 and crosses the xy-plane when x = 2 and y = 5.

Since the line crosses the z-axis at z = 6, we know (0, 0, 6) is a point on the line...

And, since it crosses the xy-plane when x = 2 and y = 5, we know (2, 5, 0) is a point on the line..

The direction of the line: $(2, 5, 0) - (0, 0, 6) = <2, 5, -6>$
     (slope)

Then, using a point on the line, (2, 5, 0), the parametric equation is

$$x = 2 + 2t$$
$$y = 5 + 5t$$
$$z = -6t$$

Quick check: if t = 0, then (x, y, z) = (2, 5, 0) ✔

and, if t = -1, then (x, y, z) = (0, 0, 6) ✔

Both points fit in the parametric equation, and therefore lie on the line..

B) Are the vectors $<2, 3>$ and $<-4, -9>$ parallel, perpendicular, or neither?

If the vectors are perpendicular, then the dot product equals zero..

Dot product: $(2 \times -4)(3 \times -9) = -35$ Not perpendicular

If the vectors have the same "slope", direction, then they are parallel..
(i.e. do they differ by a scalar value?)

$\frac{3}{2} \neq \frac{-9}{-4}$ Not parallel

There is no value of n where

$n <2, 3> = <-4, -9>$

The vectors are neither....

C) Find *2 unit vectors* that are perpendicular to $j + 3k$ amd $i - 2j + 4k$.

To find perpendicular vector(s) or normal(s), use the cross product...

$$\begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ 1 & -2 & 4 \end{vmatrix} = i \begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix} - j \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} + k \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 10i + 3j - 1k$$

To check: (the dot product of perpendicular/orthogonal vectors is 0)

$<0, 1, 3> \cdot <10, 3, -1> = 0 + 3 - 3 = 0$ ✔

$<1, -2, 4> \cdot <10, 3, -1> = 10 - 6 - 4 = 0$ ✔

Then, to find the unit vector...

the length of $<10, 3, -1> = \sqrt{10^2 + 3^2 + (-1)^2} = \sqrt{110}$

so, unit vector is

$$\frac{1}{\sqrt{110}}\left(10i + 3j - 1k\right) = \boxed{\frac{\sqrt{110}}{11}i + \frac{3\sqrt{110}}{110}j - \frac{\sqrt{110}}{110}k}$$

Note: the unit vector has a length of 1

then, another unit vector that is perpendicular has the same length but is going the opposite direction....

$$\boxed{\frac{-\sqrt{110}}{11}i - \frac{3\sqrt{110}}{110}j + \frac{\sqrt{110}}{110}k}$$

D) What is the angle between vectors $\vec{v}$ and $\vec{w}$?

$\vec{v} = <1, 2, -5>$

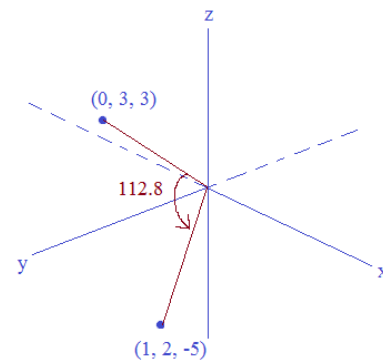$\vec{w} = <0, 3, 3>$

$v \cdot w = (1 \times 0) + (2 \times 3) + (-5 \times 3) = -9$

$|v| = \sqrt{1^2 + 2^2 + (-5)^2} = \sqrt{30}$

$|w| = \sqrt{0^2 + 3^2 + 3^2} = 3\sqrt{2}$

$\cos\Theta = \frac{v \cdot w}{|v||w|}$

$\cos\Theta = \frac{-9}{\sqrt{30} \cdot 3\sqrt{2}} = \frac{-9}{6\sqrt{15}}$

$$\boxed{\Theta = 112.8^\circ}$$

(0, 3, 3)

112.8

(1, 2, -5)

1) Given the points   A $(3, 4, -1)$ and B $(7, -6, 5)$

 Find

a) the midpoint

$$\left\langle \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right\rangle$$

$$\frac{3 + 7}{2} \quad \frac{4 + (-6)}{2} \quad \frac{(-1) + 5}{2}$$

$$(5, -1, 2)$$

b) the distance between points

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3 - 7)^2 + (4 - (-6))^2 + (-1 - 5)^2}$$

$$d = \sqrt{16 + 100 + 36} \quad = \quad \sqrt{152} \quad = \quad 2\sqrt{38}$$

Note: the formulas are same as 2-D midpoint and distance formulas. We just add the z-coordinate)

2) Find the intersection of the 2 lines

$$l \begin{cases} x = 4t + 1 \\ y = t + 3 \\ z = -2t + 8 \end{cases} \qquad m \begin{cases} x = 5s - 10 \\ y = -3s + 13 \\ z = 2s \end{cases}$$

Note: the direction of l is $<4, 1, -2>$ and the direction of m is $<5, -3, 2>$ so, the lines are not parallel. There is an intersection..

Substitution: set x's, y's, and z's equal to each other..

$$4t + 1 = 5s - 10 \implies 5s - 4t = 11$$
$$t + 3 = -3s + 13 \implies 3s + t = 10$$
$$-2t + 8 = 2s$$

solve system:

$$5s - 4t = 11$$
$$12s + 4t = 40$$
$$\overline{}$$
$$17s = 51 \qquad s = 3$$
$$t = 1$$

plug in s and t.....

$$x = 4(1) + 1 = 5$$
$$y = (1) + 3 = 4$$
$$z = -2(1) + 8 = 6$$

$$x = 5(3) + 10 = 5$$
$$y = -3(3) + 13 = 4$$
$$z = 2(3) = 6$$

$$(5, 4, 6)$$

3) What is the intersection of the planes

Note: the normal vectors are $<3, 5, 1>$ and $<2, -1, 3>$ so, the planes are not parallel.. they will intersect at a line...

$$3x + 5y + z + 6 = 0$$
$$2x - y + 3z + 1 = 0$$

solve the system:

$$3x + 5y + z = -6$$
$$10x - 5y + 15z = -5$$
$$\overline{}$$
$$13x + 16z = -11$$

Now, we'll create a parametric equation:

let $x = t$

$$13t + 16z = -11$$
$$16z = -13t - 11$$
$$z = \frac{-13}{16}t - \frac{11}{16}$$

$$x = t$$
$$y = -\frac{7}{16}t - \frac{17}{16}$$
$$z = \frac{-13}{16}t - \frac{11}{16}$$

then, use one of the original planes to find y...

$$3x + 5y + z + 6 = 0$$
$$3t + 5y + \frac{-13}{16}t - \frac{11}{16} + 6 = 0$$
$$5y + \frac{35}{16}t + \frac{85}{16} = 0$$
$$y = -\frac{7}{16}t - \frac{17}{16}$$

Let's check a point: assume $t = 0$, $x = 0$
$$y = -17/16$$
$$z = -11/16$$
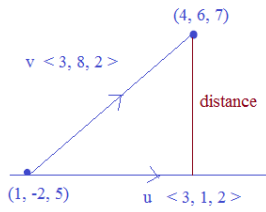
does $(0, -17/16, -11/16)$ lie in both planes?

$$3(0) + 5(-17/16) + (-11/16) + 6 = 0$$
$$0 + -85/16 + -11/16 + 6 = 0 \checkmark$$

$$2(0) - (-17/16) + 3(-11/16) + 1 = 0$$
$$0 + 17/16 - 33/16 + 1 = 0 \checkmark$$

4) Find the distance from the point to the line

SOLUTIONS

3-D Coordinate Geometry

line: $x = 3t + 1$ point: (4, 6, 7)

$y = t - 2$

$z = 2t + 5$

(4, 6, 7)

v < 3, 8, 2 >

distance

(1, -2, 5)   u < 3, 1, 2 >

$$\text{distance} = \frac{|u \times v|}{|u|}$$

$u = <3, 1, 2>$ u is direction of line

$v = <3, 8, 2>$ v is the direction from the point to some point on the line

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 3 & 8 & 2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 2 \\ 8 & 2 \end{vmatrix} i - \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} j + \begin{vmatrix} 3 & 1 \\ 3 & 8 \end{vmatrix} k = -14i - 0j + 21k$$

$\Rightarrow$ length is $\sqrt{(-14)^2 + (21)^2} = \sqrt{637}$

length of u is $\sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}$
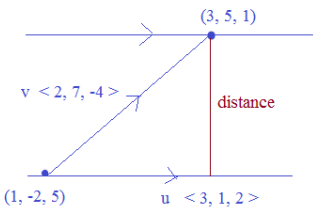
$$\text{distance} = \frac{\sqrt{637}}{\sqrt{14}}$$

approx. 6.75

5) Find the distance between the lines.

$x = 3t + 1$ $x = 3s + 3$
$y = t - 2$ $y = s + 5$
$z = 2t + 5$ $z = 2s + 1$

(3, 5, 1)

v < 2, 7, -4 >   distance

(1, -2, 5)   u < 3, 1, 2 >

$$\text{distance} = \frac{|u \times v|}{|u|}$$

Note: the lines are parallel.. each vector is < 3, 1, 2 >
if they weren't parallel, then the distance between them would be 0 (i.e. the intersection point)

$u < 3, 1, 2 >$ $|u| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}$

$v < 2, 7, -4 >$

$$u \times v = \begin{vmatrix} 1 & 2 \\ 7 & -4 \end{vmatrix} i - \begin{vmatrix} 3 & 2 \\ 2 & -4 \end{vmatrix} j + \begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix} k = < -18, 16, 19 >$$

$$|u \times v| = \sqrt{(-18)^2 + (16)^2 + (19)^2} = \sqrt{941}$$

$$\text{distance} = \frac{\sqrt{941}}{\sqrt{14}} = 8.2 \text{ (approx.)}$$

6) What is the equation of the plane containing the points

(1, 4, -2) (5, 0, 2) and (3, -1, 3)

Note: 3 non-collinear points determine a plane
or, 2 distinct lines

For a plane, we need the normal vector and a point...

To find the normal vector, we'll take the cross product of 2 vectors in the plane...

vector 1: using (1, 4, -2) to (5, 0, 2)   < 4, -4, 4 >

vector 2: using (1, 4, -2) to (3, -1, 3)   < 2, -5, 5 >

Note: we choose 2 vectors that start from the same point...

cross product of vectors

$$\text{normal vector} = \begin{vmatrix} i & j & k \\ 4 & -4 & 4 \\ 2 & -5 & 5 \end{vmatrix} = \begin{vmatrix} -4 & 4 \\ -5 & 5 \end{vmatrix} i - \begin{vmatrix} 4 & 4 \\ 2 & 5 \end{vmatrix} j + \begin{vmatrix} 4 & -4 \\ 2 & -5 \end{vmatrix} k \Rightarrow 0i - 12j - 12k \quad < 0, -12, -12 >$$

The normal vector is perpendicular to the vectors lying in the plane..
Let's check our answer:

< 0, -12, -12 > • < 4, -4, 4 > = 0 + 48 - 48 = 0 ✓

dot products equal 0

< 0, -12, -12 > • < 2, -5, 5 > = 0 + 60 -60 = 0 ✓

Equation of a plane: $Ax + By + Cz + D = 0$

To find D, we'll plug in a point: (5, 0, 2)

and, the normal vector: < 0, -12, -12 >

$0(5) + -12(0) + -12(2) + D = 0$

$D = 24$

$-12y - 12z + 24 = 0$
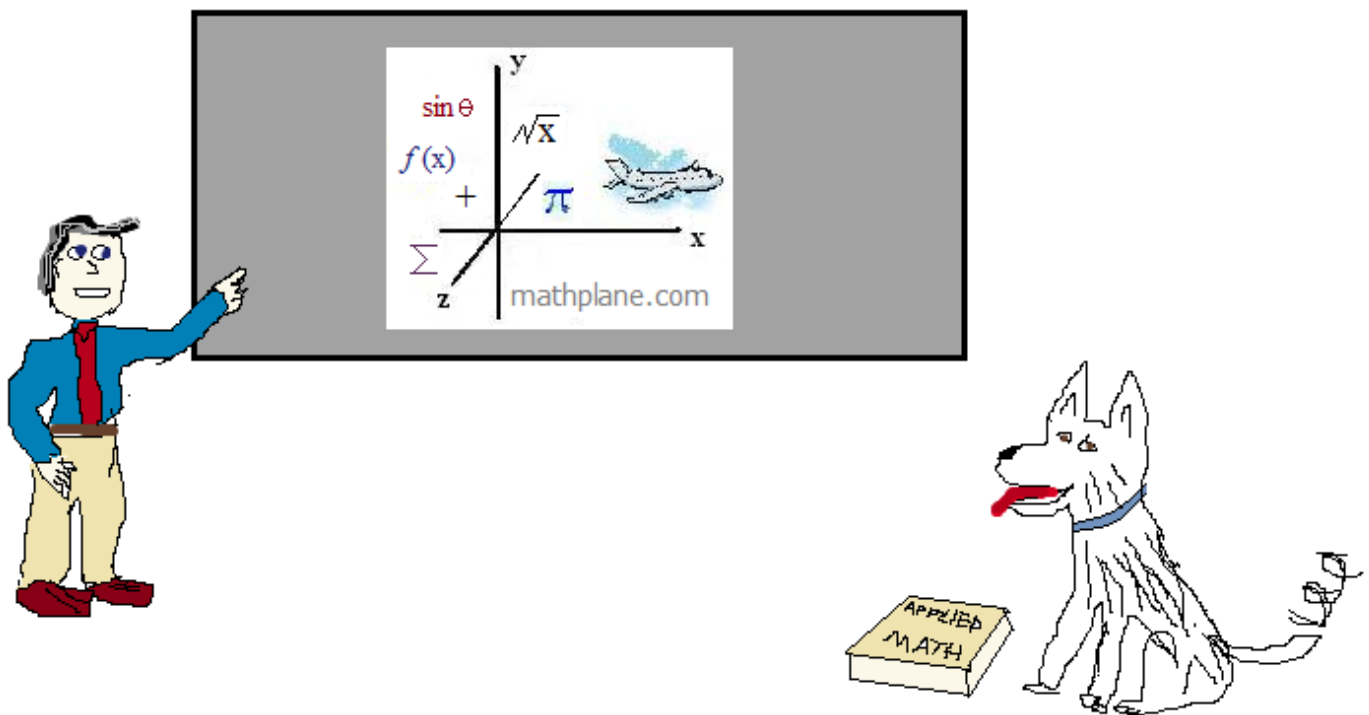
To check our answer, we'll plug in the 3 points:

(5, 0, 2)   -12(0) - 12(2) + 24 = 0 ✓

(1, 4, -2)   -12(4) - 12(-2) + 24 = 0 ✓

(3, -1, 3)   -12(-1) - 12(3) + 24 = 0 ✓

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Good luck.

Also, find Mathplane material at Teacherspayteachers and TES

Or our Mathplane.ORG site.