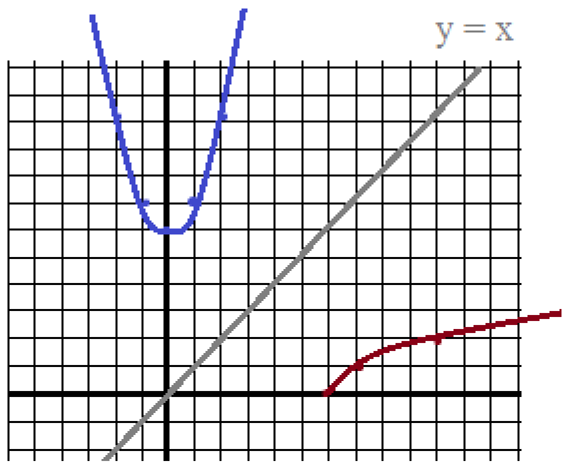
Inverse Functions

Practice questions (with solutions)



Includes graphing, finding inverses, symmetry, cryptography, and more…

Finding and Graphing Inverses

What is the inverse of X? It is <u>NOT</u> $\frac{1}{X}$ $\longrightarrow$ $\frac{1}{X}$ is the "reciprocal".

$\frac{1}{\text{"Something"}}$ is a Reciprocal..

Therefore,
"Something" times "its reciprocal" equals 1

$$7 \times \frac{1}{7} = 1$$

$$\cos\theta \cdot \sec\theta = 1$$

$$X^3 \cdot \frac{1}{X^3} = 1$$

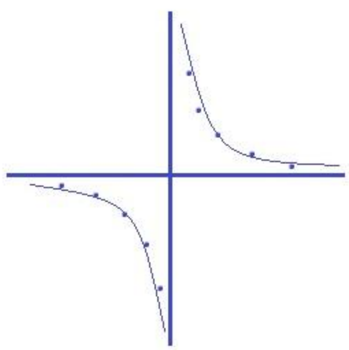
Reciprocal Examples

4	$\frac{1}{4}$
$\text{Sine}\,\theta$	$\frac{1}{\text{Sine}\,\theta} = \text{Csc}\,\theta$
$\sqrt{X}$	$\frac{1}{\sqrt{X}} = \frac{\sqrt{X}}{X}$
$\frac{2}{7}$	$\frac{7}{2}$
$\frac{bcd}{xyz}$	$\frac{1}{\frac{bcd}{xyz}} = \frac{xyz}{bcd}$

Graph of Reciprocal Function

$$f(x) = \frac{1}{x}$$

x	f (x)
-3	-1/3
-2	-1/2
-1	-1
0	undefined
1/3	3
1/2	2
1	1
2	1/2
3	1/3

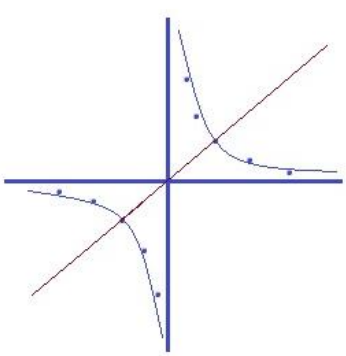


Note: Reciprocals of continuous functions often have "asymptotes" because 0 cannot be in the denominator.

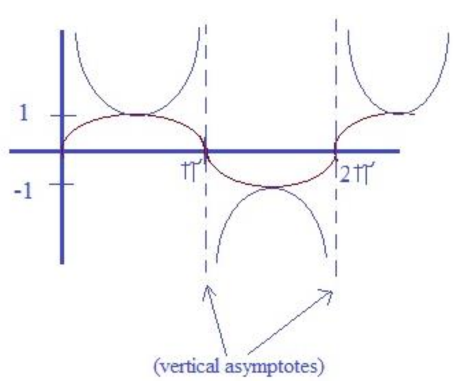
In this graph,
because there is no real value for f (0),
there is an asymptote at $x = 0$ (the y-axis)

Other Graphs to Compare

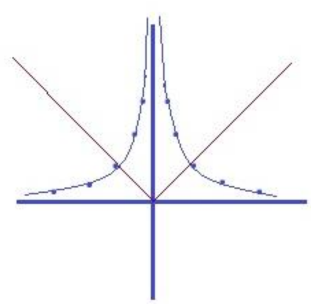
$f(x) = \frac{1}{x}$ $g(x) = x$



$f(x) = \csc x$ $g(x) = \sin x$



$f(x) = \frac{1}{|X|}$ $g(x) = |X|$



What is an inverse? It is the "reverse" of something.

Example 1: Relation A: (1, 2) (1, 6) (2, 7) (4, 4) (5, 8)

Find Inverse of Relation A:

("reverse" coordinates) (2, 1) (6, 1) (7, 2) (4, 4) (8, 5)

Example 2: $y = 6x - 3$

Find the inverse of y

$y = 6x - 3$

("reverse" or flip x and y) $x = 6y - 3$

$6y = x + 3$

(solve for the 'new' y) $y = \dfrac{(x+3)}{6}$

The inverse of $6x - 3$ is $\dfrac{(x+3)}{6}$

Example 3: $f(x) = 3x + 2$

Find $f^{-1}(x)$

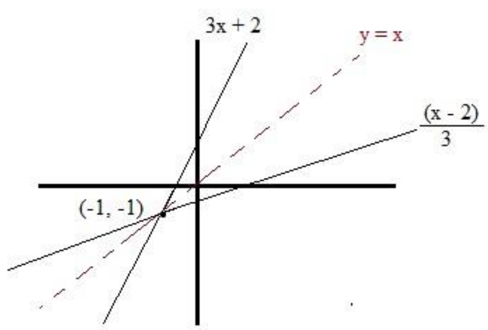
$y = 3x + 2$

(reverse x/y) $x = 3y + 2$

(solve for y) $y = \dfrac{x-2}{3}$

(replace y with f^{-1}) $f^{-1}(x) = \dfrac{x-2}{3}$

Graph $f(x)$ and $f^{-1}(x)$



Finding and Graphing Inverses (continued)

The inverse of Sine X is NOT $\dfrac{1}{\text{Sine X}}$

- The inverse of Sine X is Arcsin X or Sine^{-1} X
- The reciprocal of Sine X is $\dfrac{1}{\text{Sine X}}$ = Cosecant X

Checking your answer.

method 1: compare coordinates. If (x, y) is in f (x), then (y, x) must be in $f^{-1}(x)$

x	f (x)
-2	-4
-1	-1
0	2
1	5
2	8

x	$f^{-1}(x)$
-4	-2
-1	-1
0	-2/3
1	-1/3
2	0
3	1/3
5	1

method 2: In the graph, the line of symmetry between f (x) and $f^{-1}(x)$ is $y = x$

method 3: If f (x) and g (x) are inverses, then $f(g(x)) = x$ or $g(f(x)) = x$

$3\left(\dfrac{(x-2)}{3}\right) + 2 =$

$(x - 2) + 2 = x$ ✓

$\dfrac{(3x+2) - 2}{3} =$

$\dfrac{3x}{3} = x$ ✓

Finding and Graphing Inverses (continued)

Example 4 : Find the inverse of $y = |x - 1| + 4$

plot points and reverse:

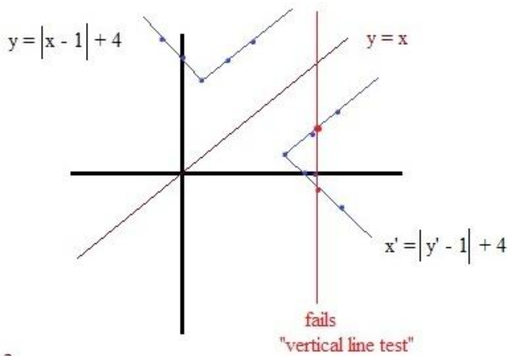
x	y
-2	7
-1	6
0	5
1	4
2	5
3	6
4	7
5	8

(inverse)

x' (y)	y' (x)
7	-2
6	-1
5	0
4	1
5	2
6	3
7	4
8	5

> **Review: Functions vs. Relations**
>
> Relation: a set containing pairs of related numbers.
>
> Function: a relation where for each X value there is only ONE Y value.
>
> (on the graph)
> A "vertical line test" can determine if the set of pairs is a function. (If you can draw a vertical line through 2 or more points, then it is NOT a function)
>
> Similarly, you can use the "horizontal line test" to determine if the inverse is a function. (If you can draw a horizontal line through 2 or more points, then the inverse is NOT a function)

graph:



function or relation?

(equation) $y = |x - 1| + 4$

function? yes, it satisfies "vertical line test"
domain: all real numbers
range: $[4, \infty)$

(inverse) $x' = |y' - 1| + 4$

$x' - 4 = |y' - 1|$

$x' - 4 = \pm(y' - 1)$

function? no.
domain: $[4, \infty)$
range: all real numbers

Example 5: Find the inverse of $f(x) = 3x^2 + 1$

"Reverse" and solve:

$$y = 3x^2 + 1$$

$$x = 3y^2 + 1$$

$$3y^2 = x - 1$$

$$y = \sqrt{\frac{(x - 1)}{3}}$$

x	f (x)
-3	28
-2	13
-1	4
0	1
1	4
2	13
3	28

(inverse)

x	f ' (x)
(28)	(-3)
(13)	(-2)
(4)	(-1)
1	0
4	1
13	2
28	3

inverse function:

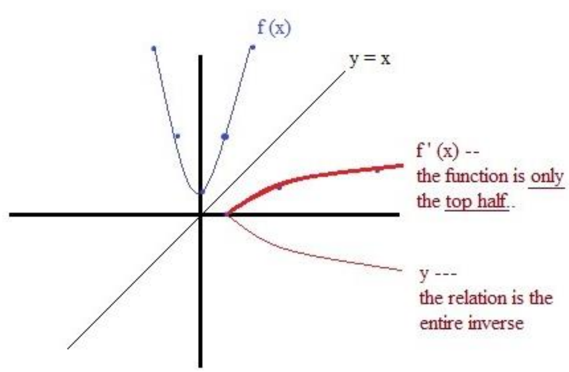
$$f'(x) = \sqrt{\frac{(x - 1)}{3}}$$

function: yes
domain: $[1, \infty)$
range: $[0, \infty)$

inverse relation:

$$y = \pm\sqrt{\frac{(x - 1)}{3}}$$

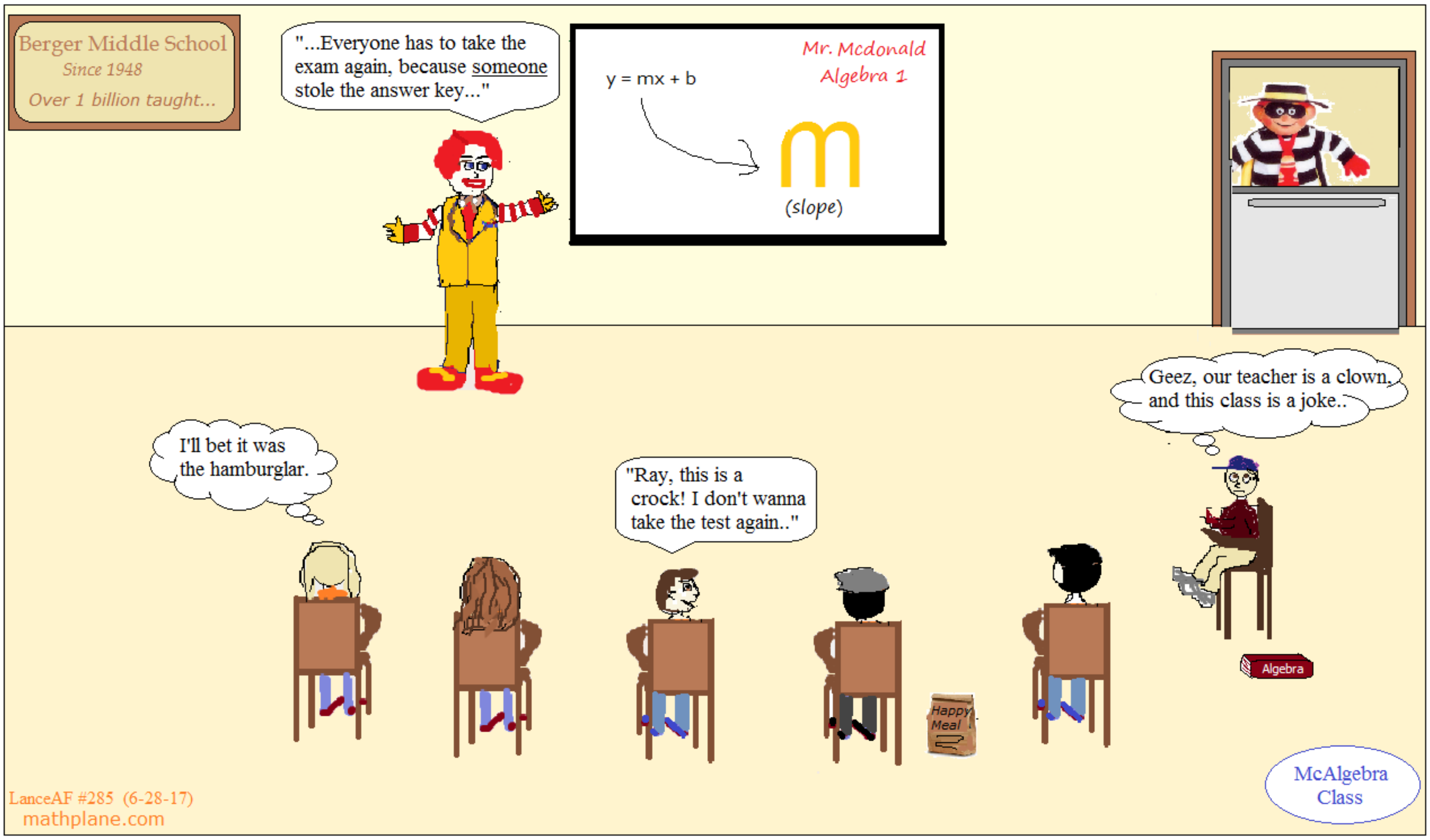
function: no
domain: $[1, \infty)$
range: all real numbers



check answer:

$$3\left(\sqrt{\frac{(x - 1)}{3}}\right)^2 + 1 = \qquad \sqrt{\frac{(3x^2 + 1) - 1}{3}} =$$

$$\frac{3(x - 1)}{3} + 1 = x \checkmark \qquad \sqrt{\frac{3x^2}{3}} = x \checkmark$$


Berger Middle School
Since 1948
Over 1 billion taught...
"...Everyone has to take the exam again, because someone stole the answer key..."
Mr. Mcdonald
Algebra 1
y = mx + b
m
(slope)
Geez, our teacher is a clown, and this class is a joke..
I'll bet it was the hamburglar.
"Ray, this is a crock! I don't wanna take the test again.."
Happy Meal
Algebra
McAlgebra Class
LanceAF #285 (6-28-17)
mathplane.com

Exercises-→

Domain, Range, and Inverse Functions

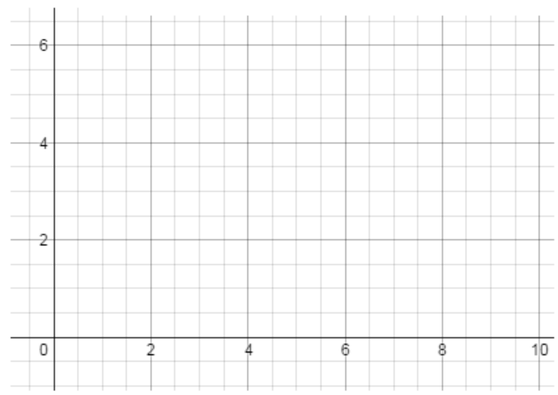
1) For the function $h(x) = \sqrt{3x - 4}$

a) find the inverse $h^{-1}(x)$

b) what is the domain of $h(x)$? the range of $h(x)$?

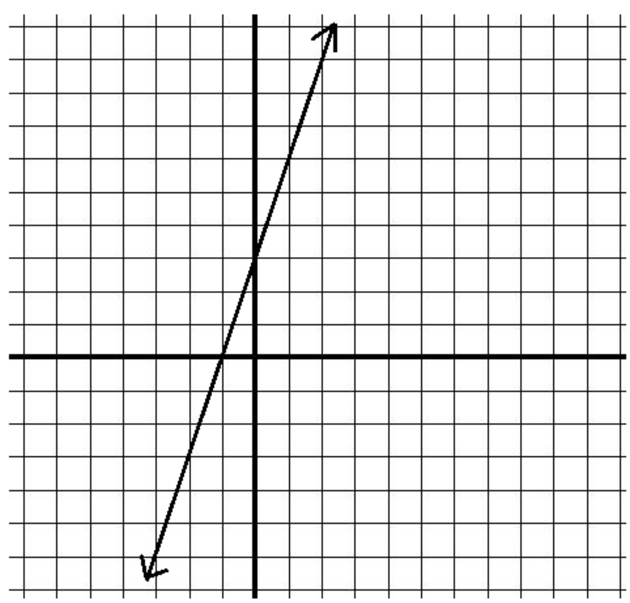
c) what is the domain of $h^{-1}(x)$? the range of $h^{-1}(x)$?

d) Graph the function $h(x)$, the inverse $h^{-1}(x)$, and the line y = x



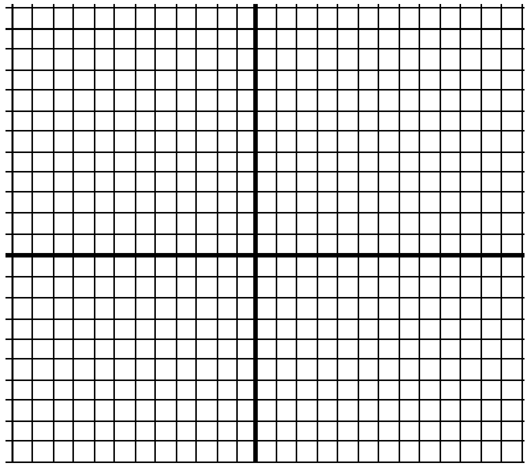
2) Graph the inverse:

Then, verify the results algebraically...



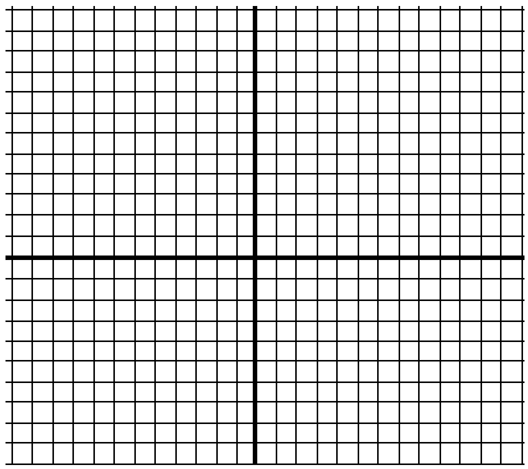
3) $g(x) = \sqrt[3]{(x-1)}$

a) Sketch the function $g(x)$



b) Find the inverse of $g(x)$

c) What is the domain and range of $g^{-1}(x)$?



d) Graph $-(g(x))$

4) If $f(x) = 5 - 2x$, what is $f^{-1}(3)$?

Domain, Range, and Inverse Functions

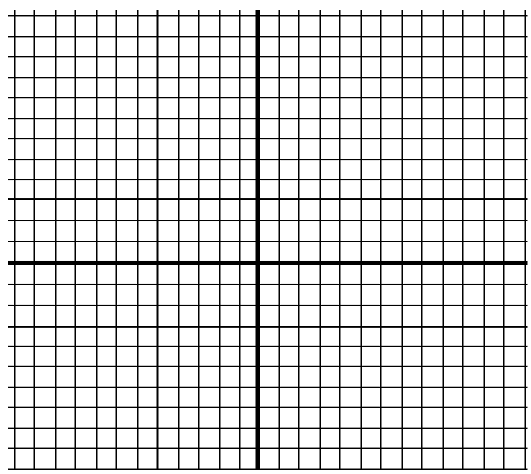
5) $f(x) = x^2 + 6$

a) Find the inverse $f^{-1}(x)$

b) Verify the inverse -- find $f(f^{-1}(x))$ and $f^{-1}(f(x))$

c) What is the domain and range of $f(x)$? Of $f^{-1}(x)$?
Are the "inverses" one-to-one?

d) Graph $f(x)$ and $f^{-1}(x)$



6) Fill in the blank

a)

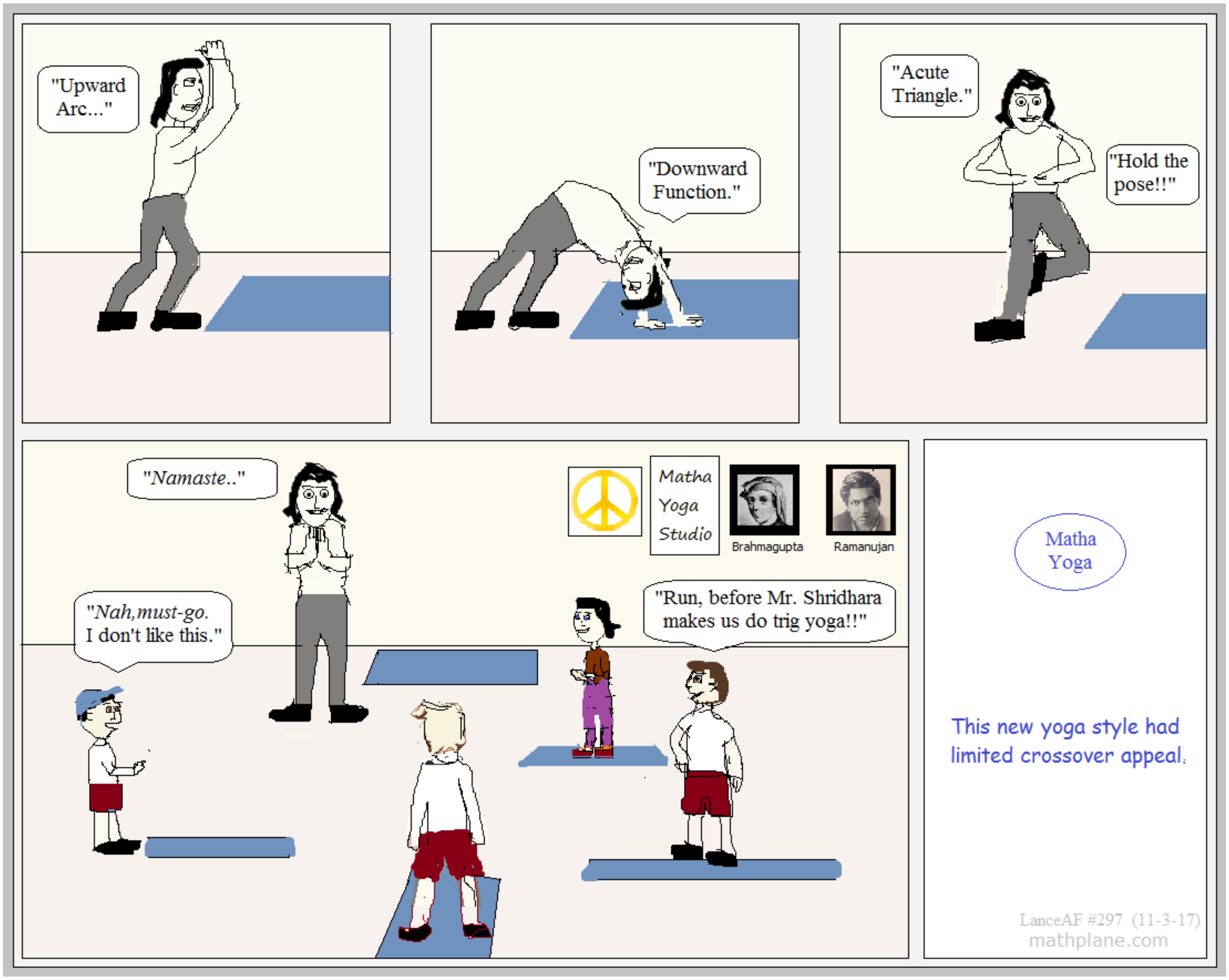
	$f(x)$	$f^{-1}(x)$
Domain	$(-\infty\ ,\ \infty\ )$	$[\ 8,\ 200\ ]$
Range	______________	______________
x-intercept	(5, 0)	(−2, 0)
y-intercept	______________	______________
additional point	(14, −1)	______________

b)

	$f(x)$	$f^{-1}(x)$
Domain	$(-\infty\ ,\ \infty\ )$	______________
Range	$[11,\ \infty\ )$	______________
x-intercept	(4, 0)	______________
y-intercept	(0, 7)	______________
additional point	(7, 15)	______________

7) For the one-to-one function $f(x) = (x-3)^2 + 5$ where $x \le 3$

find $f^{-1}(x)$


"Upward Arc..."
"Downward Function."
"Acute Triangle."
"Hold the pose!!"
"Namaste.."
Matha Yoga Studio
Brahmagupta
Ramanujan
"Nah, must-go. I don't like this."
"Run, before Mr. Shridhara makes us do trig yoga!!"
Matha Yoga
This new yoga style had limited crossover appeal.
LanceAF #297 (11-3-17)
mathplane.com

SOLUTIONS -→

Domain, Range, and Inverse Functions

SOLUTIONS

1) For the function $h(x) = \sqrt{3x - 4}$

a) find the inverse $h^{-1}(x)$

for $y = \sqrt{3x - 4}$ switch the x and y...

$x = \sqrt{3y - 4}$ then, solve for y...

$x^2 = 3y - 4$

$3y = x^2 + 4$

$y = \dfrac{x^2 + 4}{3}$

$h^{-1}(x) = \dfrac{x^2 + 4}{3}$

where $x \geq 0$ ("restrict the domain" to make the functions 1 to 1)

b) what is the domain of $h(x)$? the range of $h(x)$?

(no negatives under a radical) domain: $x \geq \dfrac{4}{3}$ range: $h(x) \geq 0$

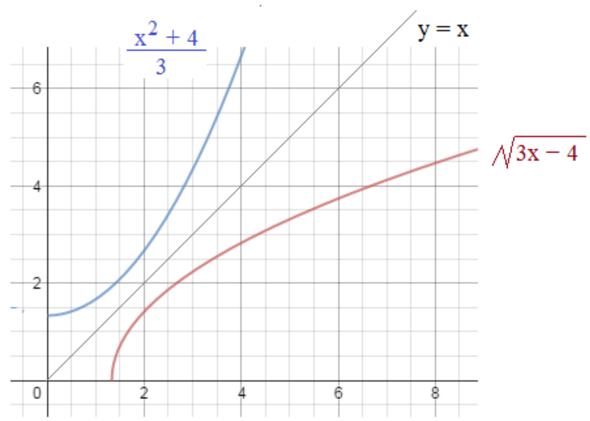
c) what is the domain of $h^{-1}(x)$? the range of $h^{-1}(x)$?

$h^{-1}(x) = \dfrac{x^2 + 4}{3}$ domain: $h(x) \geq 0$

where $x \geq 0$ range: $x \geq \dfrac{4}{3}$

Notice: the domain of $h(x)$ is the range of $h^{-1}(x)$
and, the range of $h(x)$ is the domain of $h^{-1}(x)$

d) Graph the function $h(x)$, the inverse $h^{-1}(x)$, and the line $y = x$



2) Graph the inverse.

Then, verify the results algebraically...

method 1: since it is a line, the inverse will be a line..
therefore, we need just 2 points!
---> pick two points and "flip the coordinates"..

(0, 3) ----> (3, 0)

(-2, -3) ----> (-3, -2)

then, draw a line throught the points...

method 2: the equation of the line is $\boxed{y = 3x + 3}$

find the inverse: $x = 3y + 3$ switch x and y

$3y = x - 3$ solve for y

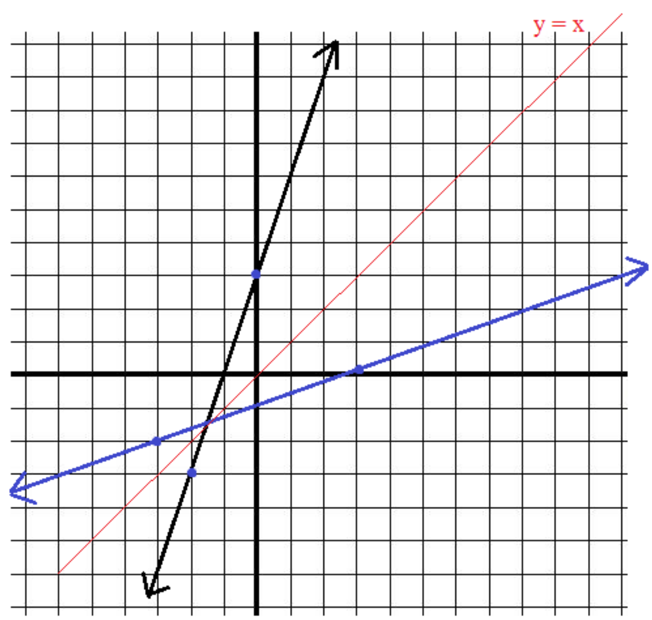
$y = \dfrac{x - 3}{3}$ $\boxed{y = \dfrac{1}{3}x - 1}$

assume line A: $f(x) = 3x + 3$

line B: $g(x) = \dfrac{1}{3}x - 1$

$f(g(x)) = 3(\dfrac{1}{3}x - 1) + 3$

$= x - 3 + 3 = x$ ✓



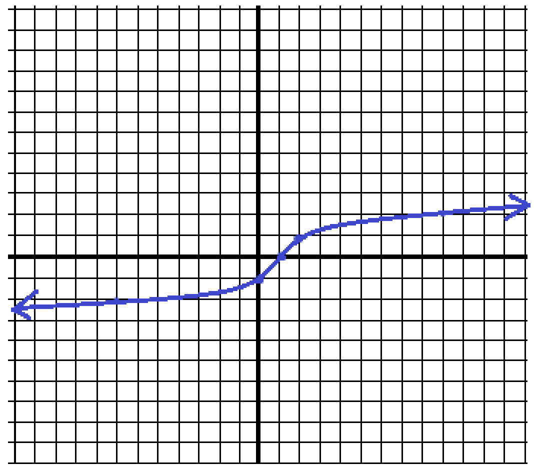
3) $g(x) = \sqrt[3]{(x-1)}$

SOLUTIONS

a) Sketch the function $g(x)$

note: this is $\sqrt[3]{x}$ shifted one unit to the right

x	g(x)
-26	-3
-7	-2
0	-1
1	0
2	1
9	2
28	3



b) Find the inverse of $g(x)$

$y = (x-1)^{\frac{1}{3}}$ write in exponential form; switch x and y

$x = (y-1)^{\frac{1}{3}}$ solve for y

$x^3 = y - 1$ $y = x^3 + 1$ $\longrightarrow$ $g^{-1}(x) = x^3 + 1$

c) What is the domain and range of $g^{-1}(x)$?

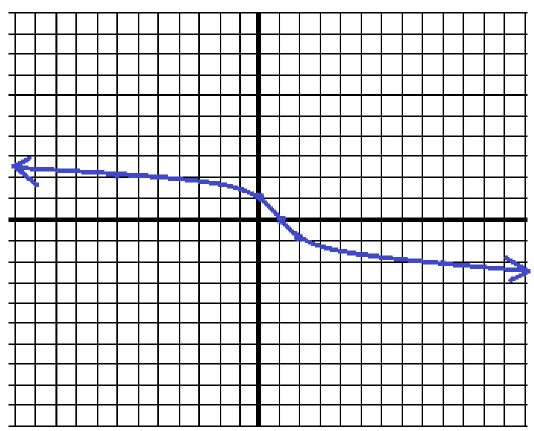
domain and range: all real numbers

d) Graph $-(g(x))$

$-g(x) = -\sqrt[3]{(x-1)}$

note: graph is 'opposite' image of above graph --- it is *reflected over the x-axis*

x	g(x)	−g(x)
-26	-3	3
-7	-2	2
0	-1	1
1	0	0
2	1	-1
9	2	-2
28	3	-3



4) If $f(x) = 5 - 2x$, what is $f^{-1}(3)$?

$5 - 2x = 3$ $x = 1$

$f(1) = 3$ So, the inverse (reverse the coordinate) is $(3, 1)$

answer: 1

Domain, Range, and Inverse Functions

SOLUTIONS

5) $f(x) = x^2 + 6$

a) Find the inverse $f^{-1}(x)$

$y = x^2 + 6$ (switch the x and y)

$x = y^2 + 6$ (solve for y)

$y^2 = x - 6$

$y = \sqrt{x-6} \longrightarrow f^{-1}(x) = \sqrt{x-6}$

note: since it is a function, the output is only $+\sqrt{\ }$ (and not $-$)

b) Verify the inverse -- find $f(f^{-1}(x))$ and $f^{-1}(f(x))$

$$f(\sqrt{x-6}) = (\sqrt{x-6})^2 + 6 = (x-6) + 6 = x \checkmark$$

$$f^{-1}(x^2+6) = \sqrt{(x^2+6)-6} = \sqrt{x^2+0} = x \checkmark$$

c) What is the domain and range of $f(x)$? Of $f^{-1}(x)$?
Are the "inverses" one-to-one?

$f(x) = x^2 + 6$

domain: all real numbers
range: $f(x) \geq 6$

since domain of $f(x)$ and range of $f^{-1}(x)$ are different, functions are not 1-to-1

$f^{-1}(x) = \sqrt{x-6}$

domain: $x \geq 6$ (if $x < 6$, then negative under the radical sign)

range: $y = f^{-1}(x) \geq 0$ (the opposites are omitted to preserve the function)

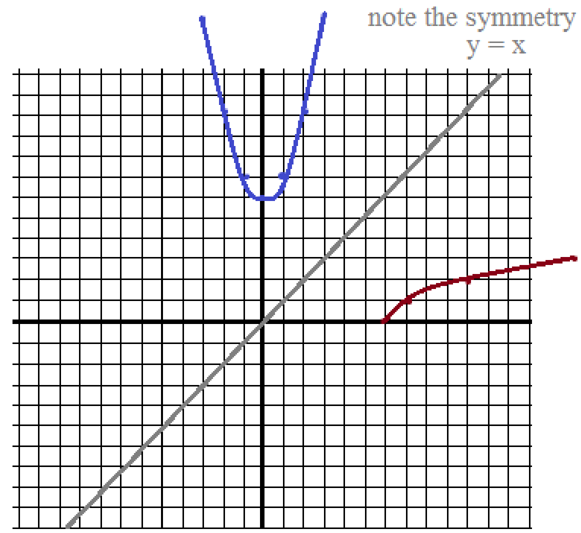
d) Graph $f(x)$ and $f^{-1}(x)$

x	f(x)
-3	15
-2	10
-1	7
0	6
1	7
2	10
3	15

x	$f^{-1}(x)$
~~15~~	~~-3~~
~~10~~	~~-2~~
~~7~~	~~-1~~
6	0
7	1
10	2
15	3
22	4

note: the ordered pairs are reversed!

note the symmetry
y = x



6) Fill in the blanks. (Assume the functions are 1 to 1.)

SOLUTIONS

a)

	$f(x)$	$f^{-1}(x)$
Domain	$(-\infty, \infty)$	$[8, 200]$
Range	$[8, 200]$	$(-\infty, \infty)$
x-intercept	(5, 0)	(−2, 0)
y-intercept	(0, -2)	(0, 5)
additional point	(14, −1)	(-1, 14)

Remember, the domain and range swap places..
(each individual point reflects over y = x)

b)

	$f(x)$	$f^{-1}(x)$
Domain	$(-\infty, \infty)$	$[11, \infty)$
Range	$[11, \infty)$	$(-\infty, \infty)$
x-intercept	(4, 0)	(7, 0)
y-intercept	(0, 7)	(0, 4)
additional point	(7, 15)	(15, 7)

7) For the one-to-one function $f(x) = (x-3)^2 + 5$ where $x \leq 3$

find $f^{-1}(x)$

domain of $f(x)$: $(-\infty, 3]$

range of $f(x)$: $[5, \infty)$

so, the domain of $f^{-1}(x)$: $[5, \infty)$

the range of $f^{-1}(x)$: $(-\infty, 3]$ $\Rightarrow$ must restrict the range to the negative values!

$$x = (y-3)^2 + 5$$

$$x - 5 = (y-3)^2$$

$$\pm\sqrt{x-5} = y - 3$$

$$y = \pm\sqrt{x-5} + 3$$

$$\boxed{y = -\sqrt{x-5} + 3}$$

Inverses Application: Cryptography

Suppose we want to send a secret message (using an algebraic function/code)

We could establish a 1-1 function for the translation...

Example: $f(x) = 3x + 7$ where x is a number representing a letter in the alphabet...

A = 1
B = 2
C = 3
etc...

If we want to send the letter A, we would find $f(1) = 3(1) + 7 = 10$ and send "10"

Then, how would the receiver decode the message?

The receiver would input the number into the inverse function!

$y = 3x + 7$ Find the inverse: $x = 3y + 7$

$3y = x - 7$

$y = \frac{x - 7}{3}$

To decode the message, use $f^{-1}(x) = \frac{x - 7}{3}$

$f^{-1}(10) = \frac{10 - 7}{3} = 1 \longrightarrow$ "A"

Again, this works effectively (accurately), because it's a 1-1 function...

a) If I want to send the message "help", what number sequence would I send?

h ---> 8 $f(8) = 31$

e ---> 5 $f(5) = 22$

l ---> 12 $f(12) = 43$

p ---> 16 $f(16) = 55$

31, 22, 43, 55

$f(x) = 3x + 7$

b) If I received a message with the sequence 46, 10, 67, 31, what would it be?

$f^{-1}(46) = 13$ ---> m

$f^{-1}(10) = 1$ ---> a

$f^{-1}(67) = 20$ ---> t

$f^{-1}(31) = 8$ ---> h

m, a, t, h

$f^{-1}(x) = \frac{x - 7}{3}$

Finding Inverse Functions

Example: Find the inverse of $f(x) = \dfrac{2x+5}{x-7}$

let $f(x) = y$ $\qquad y = \dfrac{2x+5}{x-7}$

"flip" the x and y $\qquad x = \dfrac{2y+5}{y-7}$

solve for y

$$x(y-7) = 2y+5$$

$$xy - 7x = 2y + 5$$

$$xy - 2y = 5 + 7x$$

$$y(x-2) = 7x + 5$$

$$y = \frac{7x+5}{x-2}$$

change back to function notation $\qquad \boxed{f^{-1}(x) = \dfrac{7x+5}{x-2}}$

To check the answer: $f(f^{-1}(x)) = x$ or $f \circ f^{-1} = x$

$$\frac{2\left(\dfrac{7x+5}{x-2}\right)+5}{\left(\dfrac{7x+5}{x-2}\right)-7} = \frac{\dfrac{14x+10 + 5x-10}{x-2}}{\dfrac{7x+5-7x+14}{x-2}} = \frac{19x}{19} = x \checkmark$$

Example: Find the inverse of $y = \dfrac{1+3x}{5-2x}$

"flip/change the x and y" $\qquad x = \dfrac{1+3y}{5-2y}$

"solve for y" (cross multiply and simplify)

$$5x - 2xy = 1 + 3y$$

$$5x - 1 = 3y + 2xy$$

$$5x - 1 = y(3 + 2x)$$

$$\boxed{y = \frac{5x-1}{2x+3}}$$

Check:

$$\frac{1 + 3\left(\dfrac{5x-1}{2x+3}\right)}{5 - 2\left(\dfrac{5x-1}{2x+3}\right)} = \frac{\dfrac{2x+3+15x-3}{2x+3}}{\dfrac{10x+15-10x+2}{2x+3}} = \frac{17x}{17} = x \checkmark$$

Example: Find the inverse: $y = x^2 + 2x$

$x = y^2 + 2y$

$0 = y^2 + 2y - x$

Use quadratic formula to find what y equals...

$y = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-x)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{4 + 4x}}{2}$

$= \boxed{-1 \pm \sqrt{1 + x}}$

Example: $y = x^2 + 8x + 5$

complete the square

$y + 16 = x^2 + 8x + 16 + 5$

$y + 16 = (x + 4)^2 + 5$

$y = (x + 4)^2 - 11$

find inverse

$x = (y + 4)^2 - 11$

$x + 11 = (y + 4)^2$

$y + 4 = \pm\sqrt{x + 11}$

$\boxed{y = \pm\sqrt{x + 11} - 4}$

Example: $y = x^2 + 7x + 12$

$x = y^2 + 7y + 12$

$0 = y^2 + 7y + (12 - x)$

a b c

quadratic formula

$y = \dfrac{-7 \pm \sqrt{49 - (4)(1)(12 - x)}}{2}$

$y = \dfrac{-7 \pm \sqrt{49 - 48 + 4x}}{2}$

$y = \dfrac{-7}{2} \pm \dfrac{\sqrt{1 + 4x}}{2}$

$y = \dfrac{-7}{2} \pm \dfrac{\sqrt{4 \cdot (1/4 + x)}}{2}$

$y = \dfrac{-7}{2} \pm \dfrac{2\sqrt{(1/4 + x)}}{2}$

$\boxed{y = \dfrac{-7}{2} \pm \sqrt{(1/4 + x)}}$

random check: if $x = 1$, then $y = 20$

if $x = -8$, then $y = 20$

inverse: if $x = 20$, then $y = 1$ (or, -8)

Example: For the 1-1 function $f(x) = 2x^3 + 1$

$f^{-1}(x) =$

$f \circ f^{-1}(128) =$

$y = 2x^3 + 1$

$x = 2y^3 + 1$

$\frac{x-1}{2} = y^3$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}}$

If $f(x)$ and $f^{-1}(x)$ are inverses, then the composite is x

therefore, $f \circ f^{-1}(128) = \boxed{128}$

Example: Find the inverse of $\sqrt{x+7} - 3$

$y = \sqrt{x+7} - 3$

flip variables

$x = \sqrt{y+7} - 3$

solve for y

$x + 3 = \sqrt{y+7}$

$(x+3)^2 = y + 7$

$y = (x+3)^2 - 7$

Now we must restrict the domain!

$f(x) = \sqrt{x+7} - 3$ domain: $[-7, +\infty)$ range: $[-3, +\infty)$

$f^{-1}(x) = (x+3)^2 - 7$ domain: $[-3, +\infty)$ range: $[-7, +\infty)$

domain and range switch!

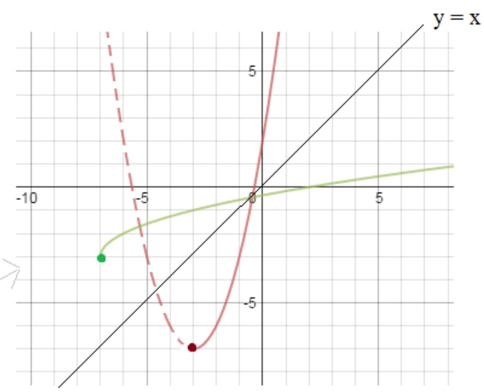
Therefore, the inverse is $(x+3)^2 - 7$ where $x \geq -3$

Note: the inverses reflect over y = x

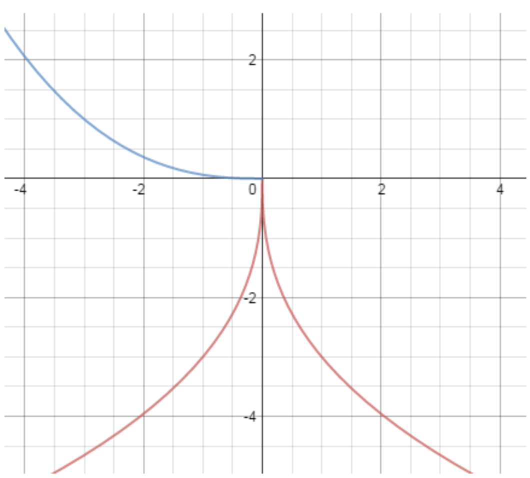
Also, $f(f^{-1}(x)) = x$

or

$f^{-1}(f(x)) = x$



Example: $y = -3x^{\frac{2}{5}}$ Graph and find the inverse



Since this graph fails the "horizontal line test", the inverse is not a function...

$x = -3y^{\frac{2}{5}}$

$\frac{-1}{3}x = y^{\frac{2}{5}}$

$\left(\frac{-1}{3}x\right)^{\frac{5}{2}} = y$

To find inverse, flip the variables, then solve for y...

NOTE: the inverse does not exist for positive numbers!

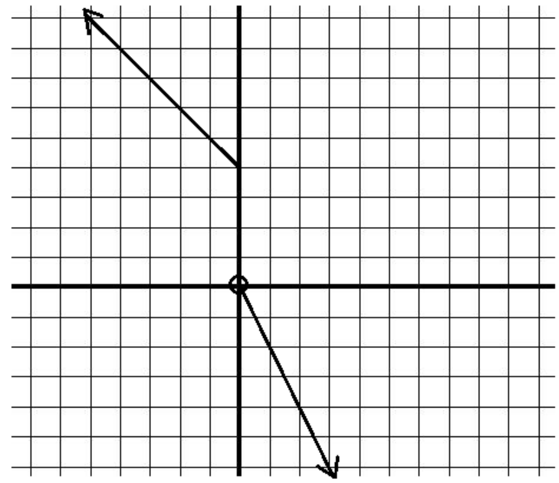
ex: if x = 1, then $y = (-1/3)^{\frac{5}{2}}$ $\boxed{\text{DNE}}$

Therefore, the inverse does not exist UNLESS you restrict the domain...

(to $x \geq 0$)

Write the inverse of the piecewise function

$$f(x) = \begin{cases} -x + 4 & \text{if } x \le 0 \\ -2x & \text{if } x > 0 \end{cases}$$



Note: the function satisfies the vertical and horizontal line tests, so it is a 1-to-1 function..

What is $f(3)$?

$f(-3)$?

$f^{-1}(6)$?

ANSWERS...

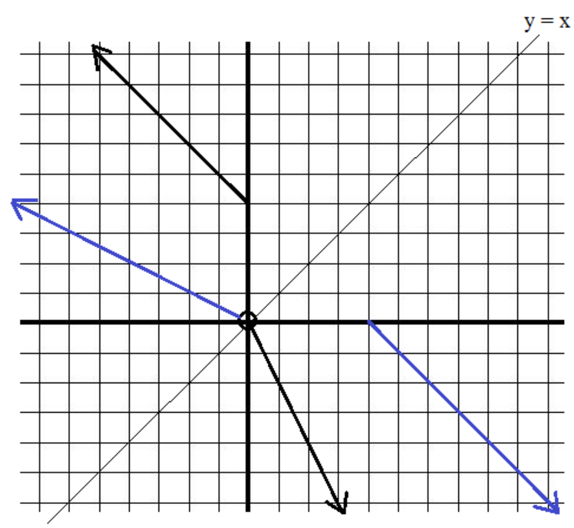
Take the first half of the piecewise function... $y = -x + 4$

"flip the x and y" $x = -y + 4$

then, solve for y... $y = -x + 4$

since the domain is $(-\infty, 0]$ and range is $[4, \infty)$

the inverse domain is $[4, \infty)$ and the inverse range is $(-\infty, 0]$



Now, take the second half of the piecewise function... $y = -2x$

"flip the x and y" $x = -2y$

then, solve for y... $y = -\frac{1}{2}x$

since the domain is $(0, \infty)$ and range is $(-\infty, 0)$

the inverse domain is $(-\infty, 0)$ and the inverse range is $(0, \infty)$

$$f^{-1}(x) = \begin{cases} -\frac{1}{2}x & \text{if } x < 0 \\ -x + 4 & \text{if } x \ge 4 \end{cases}$$

What is $f(3)$? Using the graph or the functions themselves....

$f(-3)$? $f(3) = -6$

$f^{-1}(6)$? $f(-3) = 7$

$f^{-1}(6) = -2$ note: $f(-2) = 6$

Inverses Exercises

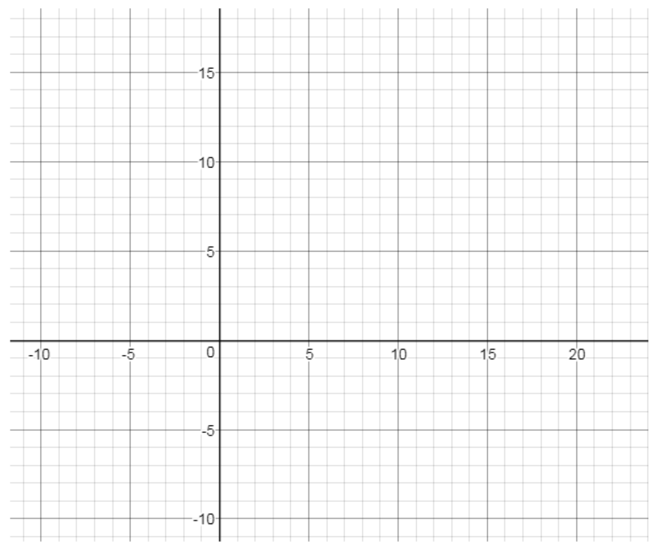
1) Find the inverse function.... (You may need to restrict the domain.)

$f(x) = 1 + \sqrt{x + 1}$

2) $f(x) = 3x + 7$

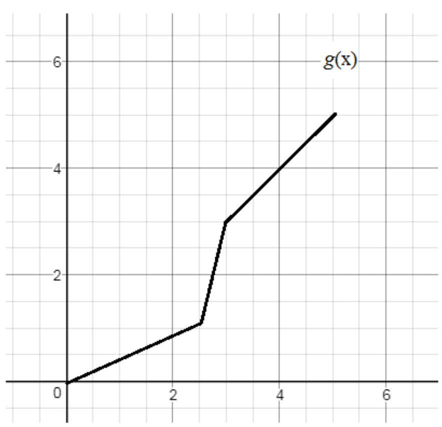
What is $f^{-1}(16)$?

Sketch $f(x)$ and its inverse...



3) Given the graph of $g(x)$,

sketch $g^{-1}(x)$



4) Find the inverse: $y = -5^{x+2} - 7$

5) Find the inverse of the quadratic $y = x^2 + 6x + 9$

6) $f(x) = (x-2)^2 + 3$ for $x \le 2$

find $f^{-1}(x)$ where $f(x)$ and $f^{-1}(x)$ are one-to-one...

SOLUTIONS

1) Find the inverse function.... (You may need to restrict the domain.)

$f(x) = 1 + \sqrt{x+1}$

"switch the x's and y's"

$x = 1 + \sqrt{y+1}$

then, solve for y

$x - 1 = \sqrt{y+1}$

$(x-1)^2 = y + 1$

$y = (x-1)^2 - 1$

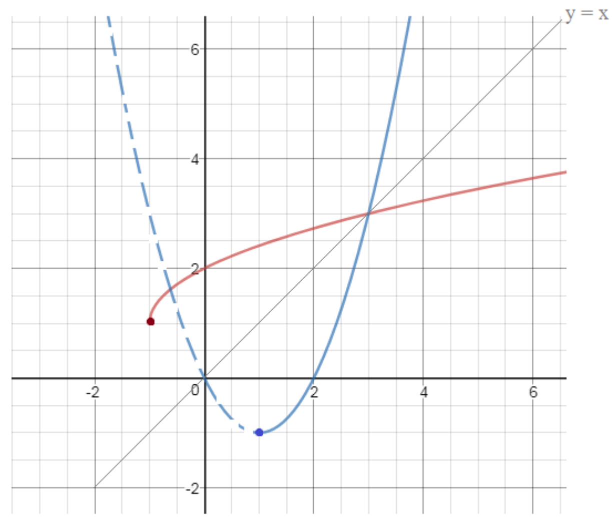
Since the range of $f(x)$ is $[1, \infty)$

the domain of $f^{-1}(x)$ is also $[1, \infty)$

restrict the domain...

$f^{-1}(x) = (x-1)^2 - 1$

where $x \geq 1$



2) $f(x) = 3x + 7$

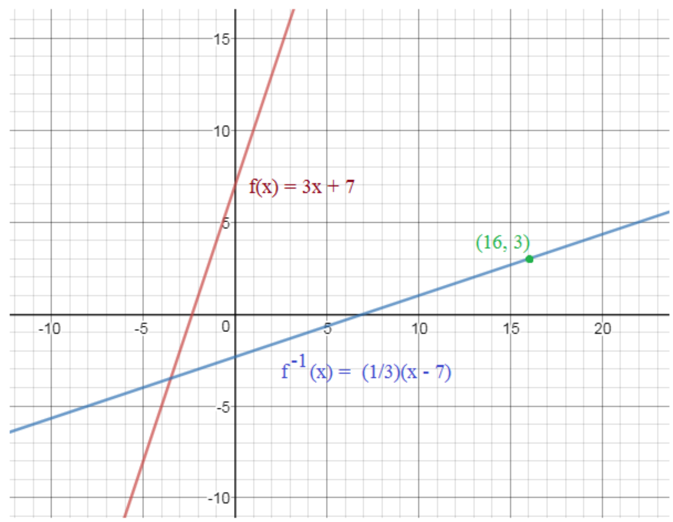
What is $f^{-1}(16)$?

$f(a) = 16$ when $f^{-1}(16) = a$

if $f(a) = 16$, then $3a + 7 = 16$...

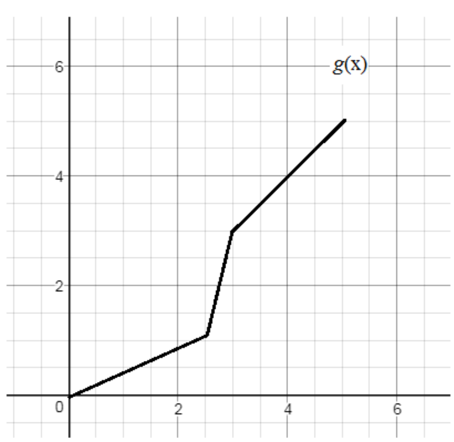
so, $\boxed{a = 3}$

Sketch f(x) and its inverse...

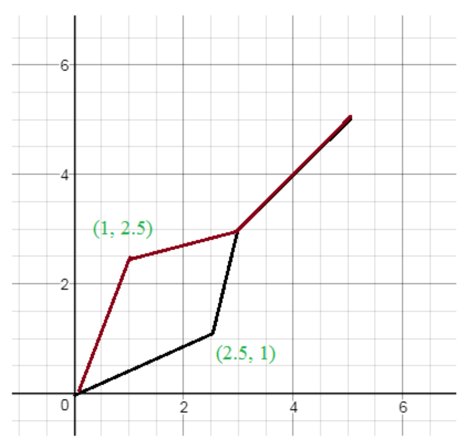


3) Given the graph of $g(x)$,

sketch $g^{-1}(x)$



("flip the x and y coordinates")



Inverses Exercises

SOLUTIONS

4) Find the inverse: $y = -5^{x+2} - 7$

$x = -5^{y+2} - 7$

$x + 7 = -5^{y+2}$

$-(x + 7) = 5^{y+2}$

$\log_5 (-x - 7) = y + 2$

$$\boxed{y = \log_5 (-x - 7) - 2}$$

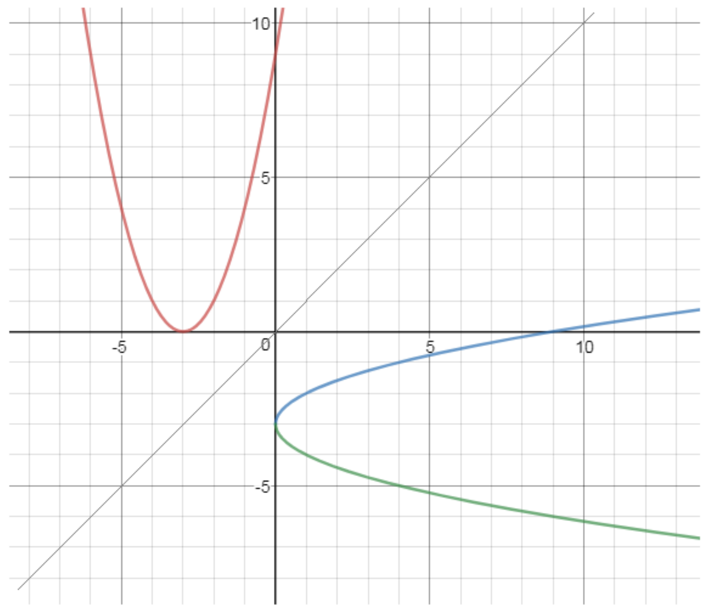
5) Find the inverse of the quadratic $y = x^2 + 6x + 9$

$x = y^2 + 6y + 9$

$x = (y + 3)^2$ perfect square

$\pm\sqrt{x} = y + 3$

$$\boxed{y = -3 \pm\sqrt{x}}$$



6) $f(x) = (x - 2)^2 + 3$ for $x \le 2$

find $f^{-1}(x)$ where $f(x)$ and $f^{-1}(x)$ are one-to-one...

f(x) domain: $x \le 2$

range: $x \ge 3$

$x = (y - 2)^2 + 3$

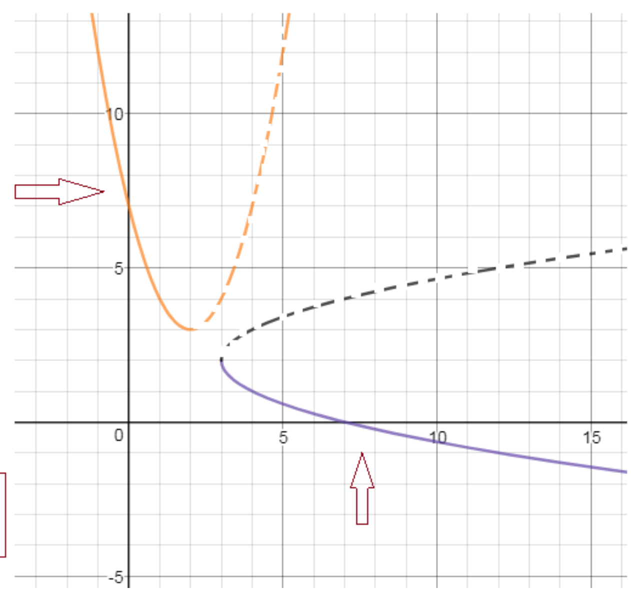
$x - 3 = (y - 2)^2$

$\pm\sqrt{x - 3} = (y - 2)$

$y = 2 \pm\sqrt{x - 3}$

domain must be $x \ge 3$
and
range must be $x \le 2$

so, $\boxed{y = 2 - \sqrt{x - 3}}$



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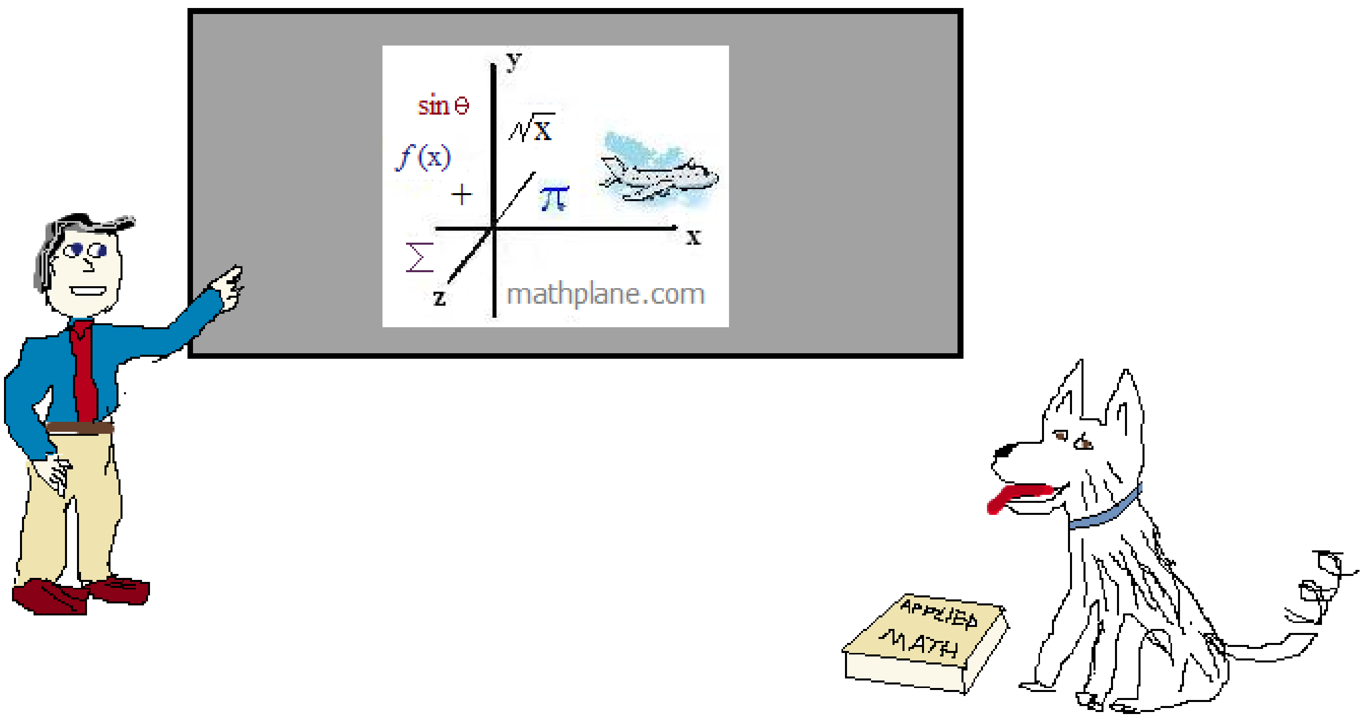
Thanks for visiting! (Hope it helps)

If you have questions, suggestions, or requests, let us know.

Cheers



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