Comparing Equations

Notes, Examples, and Practice Exercises

Topics include linear, exponential, quadratic best-fit models, charts, graphs, equations.

Comparing Equations: Exponential vs. Quadratic vs. Linear

Exponential Equations

$y = ab^x$

(x, y) are points on the curve

a is the initial value (y-intercept)
b is the growth factor

Example:

x	y		
2	18	36	x3
3	54	108	x3
4	162	324	x3
5	486	972	
6	1458		

$y = ab^x$

the growth factor is 3

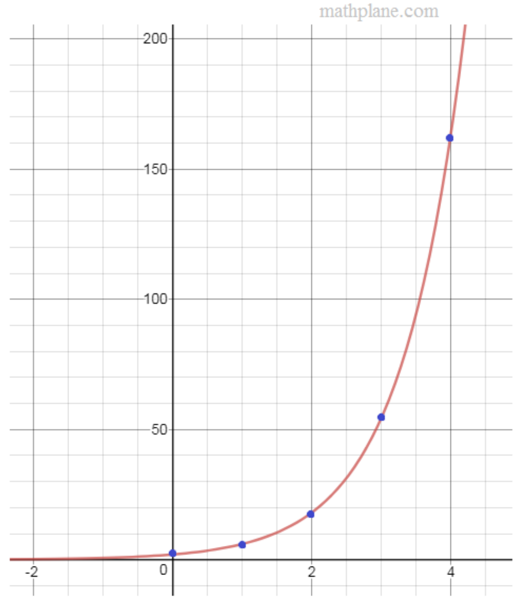
$y = a(3)^x$

then, pick a point to find a

$18 = a(3)^2$

$a = 2$

$\boxed{y = 2(3)^x}$



Quadratic Equations

$y = a(x - h)^2 + k$

(x, y) are points on the curve

(h, k) is the vertex
a determines the growth ("stretch")

Example:

x	y		
-1	-32	+14	-4
0	-18	+10	-4
1	-8	+6	-4
2	-2	+2	-4
3	0	-2	-4
4	-2	-6	
5	-8		

Recognize the symmetry!
the vertex is at (3, 0)

substitute (h, k) with the vertex (3, 0)

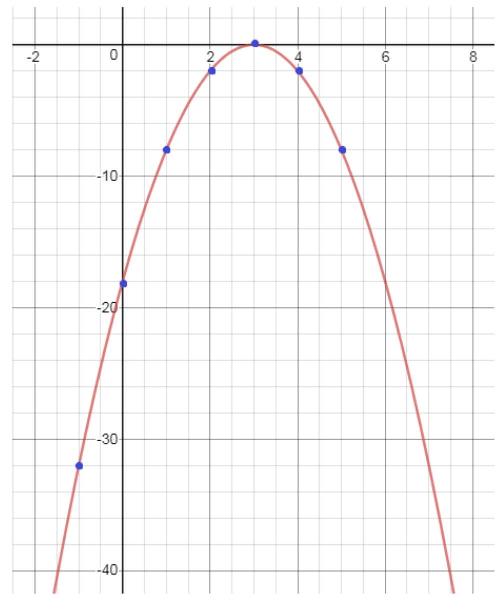
$y = a(x - 3)^2 + 0$

then, pick a point and substitute..

$-2 = a(4 - 3)^2 + 0$

$a = -2$

$\boxed{y = -2(x - 3)^2}$



Linear Equations

$y = mx + b$

(x, y) are points on the line

m is the slope
b is the y-intercept

Example:

x	y	
3	22	-5
4	17	-5
5	12	-5
6	7	-5
7	2	-5

Constant rate of change is linear

$y = mx + b$

slope $m = -5$

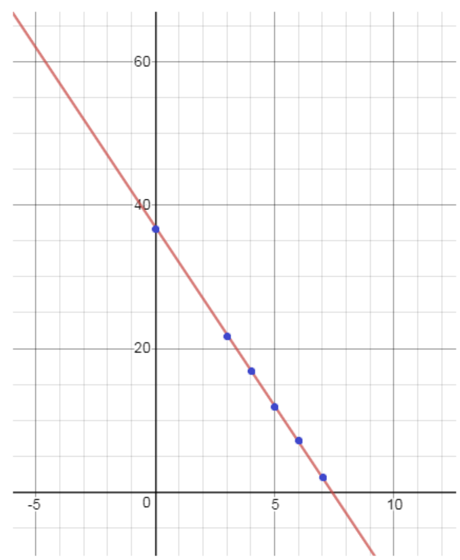
$y = -5x + b$

then, to find b (the y-intercept), substitute a point...

$17 = -5(4) + b$

$b = 37$

$\boxed{y = -5x + 37}$



Comparing Equations: Exponential vs. Quadratic vs. Linear

Example A: A zombie can infect 10 people per hour..

Write a model describing the number of infected people..

A quick table of values:

t is the time (hours)

n is the number of infected people

t	n	
0	1	the zombie
1	11	the zombie and 10 people
2	21	
3	31	
4	41	

This is a linear model where the constant rate of change is 10, and the inital value is 1...

$y = mx + b \Rightarrow n = 10t + 1$

Example B: A zombie can infect 4 people per hour...
Then, anyone who is infected becomes a zombie and can infect others...

1

5 original 1 + 4 new infected

25 5 infected + 20 new infected

125 25 infected + 100 new infected

This is an exponential model where the growth factor is 5, and the initial value is 1...

$y = ab^x$ $\qquad y = 1(5)^t$

Example: Given the points (0, 3) and (2, 15), write linear, exponential, and quadratic models that pass through these points.

Linear: $y = mx + b$ the slope $m = \frac{15 - 3}{2 - 0} = 6$

and, the y-intercept is (0, 3)

$\boxed{y = 6x + 3}$

Exponential: $y = ab^x$ the initial value (a) is 3

because, if you substitute (0, 3)

$3 = ab^0$

$3 = a(1)$

$a = 3$

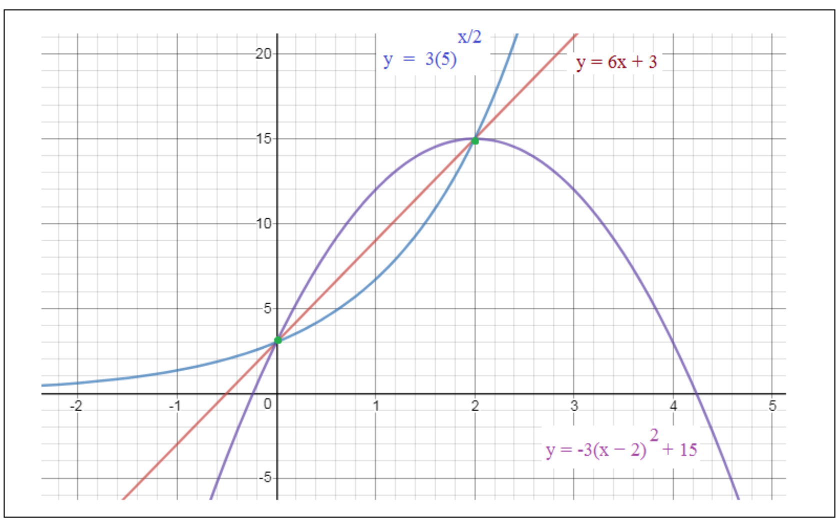
$y = 3b^x$

then, substitute (2, 15) to find b...

$15 = 3b^2$

$b = \sqrt{5}$

$\boxed{y = 3(5)^{x/2}}$



Quadratic: $y = a(x - h)^2 + k$ Since we only have 2 points, there are an infinite number of possiblities... Here are the 2 easiest:

assume vertex is (0, 3) with points (2, 15) and (-2, 15)

$y = a(x - 0)^2 + 3$

$15 = a(2 - 0)^2 + 3$

$15 = 4a + 3$

$a = 3$

$y = 3x^2 + 3$

assume vertex is (2, 15) with points (0, 3) and (4, 3)

$y = a(x - 2)^2 + 15$

$3 = a(0 - 2)^2 + 15$

$-12 = 4a$

$a = -3$

$\boxed{y = -3(x - 2)^2 + 15}$

Note: there could also be parabolas that open out to the left or right (as well as up and down)

Solving Systems of Non-linear Equations

Example: $2^{x+1} + 3^y = 11$

$2^x - 3^{y+1} = -26$

convert the exponents

$2^x 2^1 + 3^y = 11$

$2^x - 3^y 3^1 = -26$

solve using elimination method

$6 \cdot 2^x + 3 \cdot 3^y = 33$

$2^x - 3 \cdot 3^y = -26$

$7 \cdot 2^x = 7$

$\boxed{x = 0 \qquad y = 2}$

Example: Solve and graph: $y = x + 5$ $\quad y = \dfrac{4}{x+2}$

Substitution (set equal to each other) $\quad x + 5 = \dfrac{4}{x+2}$

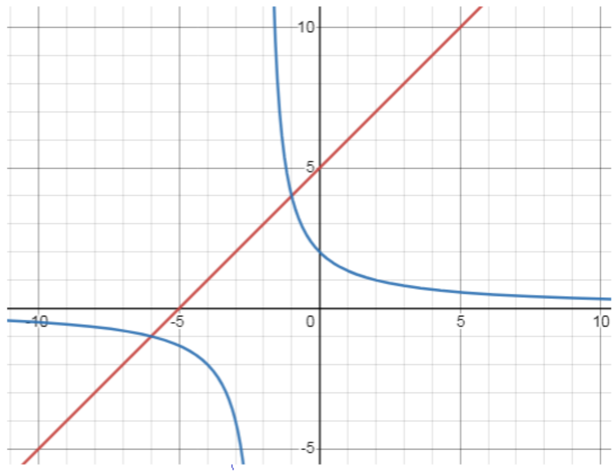
Cross-multiply $\quad x^2 + 7x + 10 = 4$

Factor, solve, and check... $\quad x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$

$x = -1, -6$

$\boxed{(-1, 4) \quad (-6, -1)}$



Example: For the system $y = x^2$ and $y = 4x + b$,

what values of b will give a) 1 solution b) no solutions c) 2 solutions ??

Solutions can be found by setting equations equal to each other (i.e. finding the intersections)..

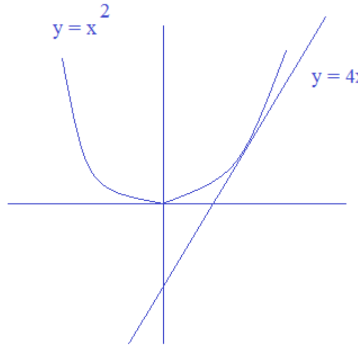
$x^2 = 4x + b \quad \Rightarrow \quad x^2 - 4x - b = 0$ (quadratic)

When discriminant equals 0, there is one solution...

$(-4)^2 - 4(1)(-b) = 0$

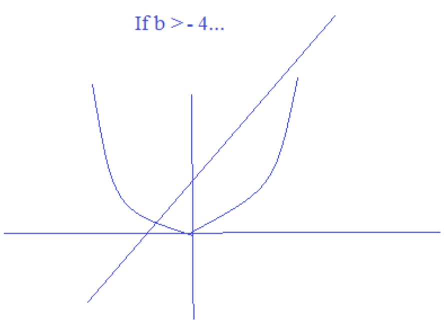
$16 + 4b = 0$

When b = -4, there is one solution...



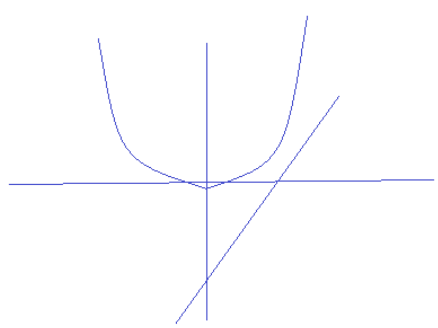
When discriminant is greater than 0, there are 2 solutions...

If b > - 4...



When discriminant is less than 0, there are no solutions...

If b < - 4, ...



Identify the best fit model for each table of data.
Write an equation to describe the model...

1)

x	-3	-2	-1	0	1	2	3
y	17	12	7	2	-3	-8	-13

2)

x	-3	-2	-1	0	1	2	3
y	20	11	4	-1	-4	-5	-4

3)

x	-3	-2	-1	0	1	2	3
y	8/9	4/9	2/9	1/9	1/18	1/36	1/72

4)

x	-3	-2	-1	0	1	2	3
y	-21	-16	-11	-6	-1	-6	-11

Identify the best fit model for each table of data.
Write an equation to describe the model...

5)

x	-3	-2	-1	0	1	2	3
y	31	19	10	4	1	1	4

6)

x	-3	-2	-1	0	1	2	3
y	5	9	17	33	65	129	257

7)

x	-3	-2	-1	0	1	2	3
y	-18	-6	-2	0	2	6	18

8)

x	-3	-2	-1	0	1	2	3
y	4/3	11/6	7/3	17/6	10/3	23/6	13/3

9)

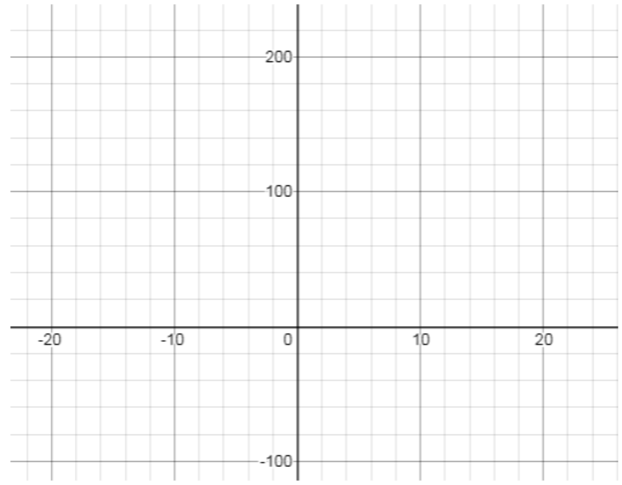
x	-3	-2	-1	0	1	2	3
y	-11	-4	1	4	5	4	1

System of Equations: Linear and Quadratic

Solve the following systems of equations. (Optional: Graph the systems showing the solutions)

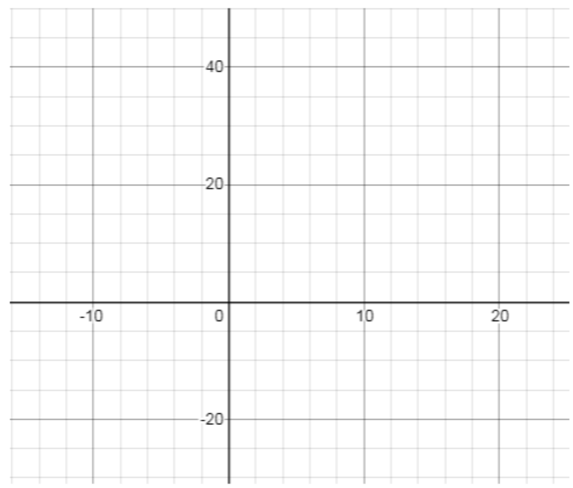
1) $y = x^2 + 6x - 16$

$y = 12x - 24$



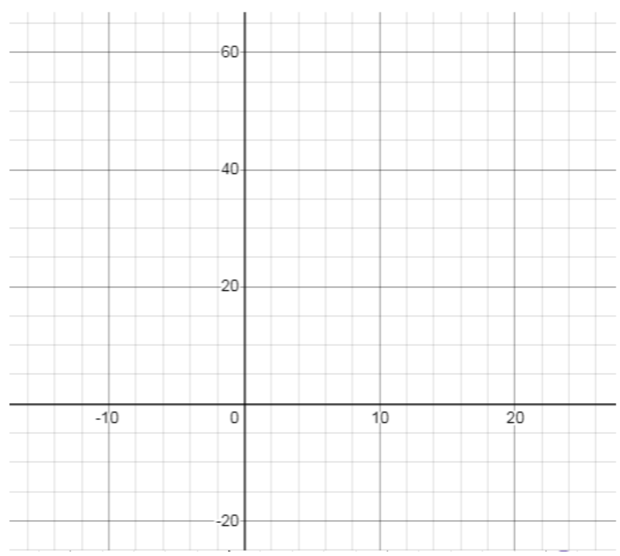
2) $y = -x^2 + 6x + 13$

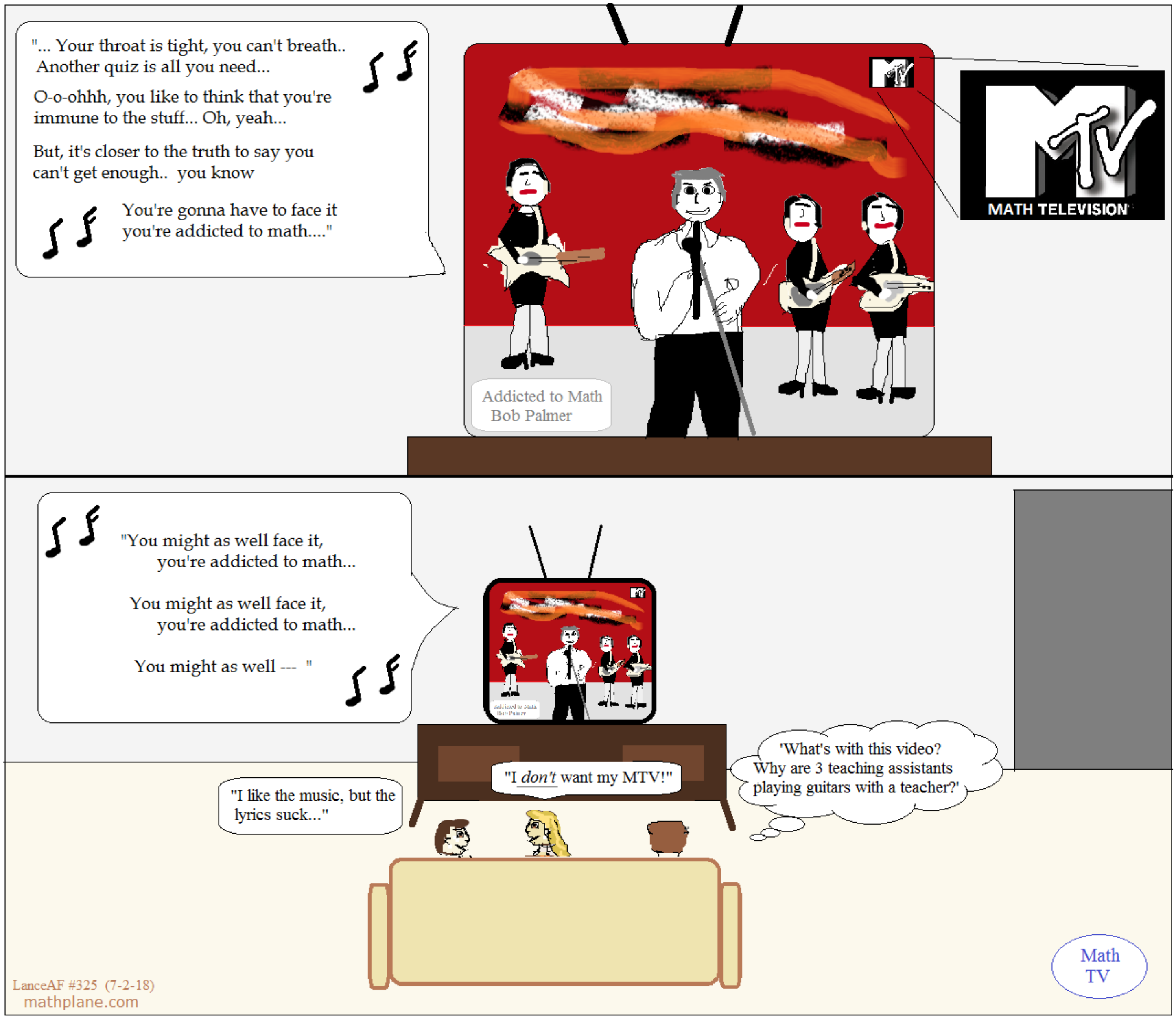
$y = -2x + 29$



3) $y = x^2 - 8x + 15$

$x + y = -3$




"... Your throat is tight, you can't breath.. Another quiz is all you need...
O-o-ohhh, you like to think that you're immune to the stuff... Oh, yeah...
But, it's closer to the truth to say you can't get enough.. you know
You're gonna have to face it you're addicted to math...."
MATH TELEVISION
Addicted to Math
Bob Palmer
"You might as well face it, you're addicted to math...
You might as well face it, you're addicted to math...
You might as well --- "
'What's with this video? Why are 3 teaching assistants playing guitars with a teacher?'
"I don't want my MTV!"
"I like the music, but the lyrics suck..."
LanceAF #325 (7-2-18)
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SOLUTIONS-→

Identify the best fit model for each table of data.
Write an equation to describe the model...

SOLUTIONS

1)

x	-3	-2	-1	0	1	2	3
y	17	12	7	2	-3	-8	-13

-5 -5 -5 -5 -5 -5

First set of differences are constant...
Therefore, it is *linear.*

Slope is -5
y-intercept is (0, 2)

$\boxed{y = -5x + 2}$

the sketch is *decreasing*

2)

x	-3	-2	-1	0	1	2	3
y	20	11	4	-1	-4	-5	-4

1st differences -9 -7 -5 -3 -1 +1

2nd differences +2 +2 +2 +2 +2

The second differences are constant
(plus, note the symmetry),
so this is a *quadratic*

the curve opens *upward*

vertex is (2, -5)

$y = a(x - 2)^2 - 5$

$\boxed{y = (x - 2)^2 - 5}$

$4 = a(-1 - 2)^2 - 5$

$a = 1$

3)

x	-3	-2	-1	0	1	2	3
y	8/9	4/9	2/9	1/9	1/18	1/36	1/72

x 1/2 x 1/2 x 1/2 x 1/2

$y = ab^x$

exponential equation

$a = 1/9$

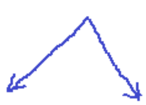
$b = 1/2$ (since it's less than 1, it is *decay or decreasing*)

$\boxed{y = (1/9)(1/2)^x}$

4)

x	-3	-2	-1	0	1	2	3
y	-21	-16	-11	-6	-1	-6	-11

+5 +5 +5 +5 -5 -5



The outputs have symmetry, AND they change at a
constant/linear rate... So, it's an

absolute value equation

$y = a|x - 1| - 1$

the sketch opens *downward*

$\boxed{y = -5|x - 1| - 1}$

Identify the best fit model for each table of data.
Write an equation to describe the model...

SOLUTIONS

5)

x	-3	-2	-1	0	1	2	3
y	31	19	10	4	1	1	4

1st diff. -12 -9 -6 -3 0 +3

2nd diff. +3 +3 +3 +3 +3

$\boxed{\textit{quadratic....}}$ symmetry and "2nd difference is constant"

We can see the y-intercept is (0, 4)...

$y = ax^2 + bx + c$ $c = 4$...

$\boxed{y = (3/2)x^2 - (9/2)x + 4}$

Using 2 other points: (2, 1) and (3, 4)...

$1 = a(2)^2 + b(2) + 4$ $-3 = 4a + 2b$

$4 = a(3)^2 + b(3) + 4$ $0 = 9a + 3b$

solve the system: $9 = -12a - 6b$
$0 = 18a + 6b$

$9 = 6a$ $a = 3/2$
$b = -9/2$

6)

x	-3	-2	-1	0	1	2	3
y	5	9	17	33	65	129	257

$y - 1 \Rightarrow$ 4 8 16 32 64 128 256 $\Leftarrow$ $\boxed{\textit{Exponential equation}}$

x2 x2 x2 x2 x2 x2

Using the new coordinates: (0, 32) (1, 64) (2. 128)

$y = ab^x$ $a = 32$ and $b = 2$

then, include the vertical shift:

$\boxed{y = 32(2)^x + 1}$

7)

x	-3	-2	-1	0	1	2	3
y	-18	-6	-2	0	2	6	18

1st diff +12 +4 +2 +2 +4 +12

2nd diff -8 -2 0 +2 +8

growth factor? x3 x3 x0 n/a x3 x3

$\boxed{\text{This does not model any of the following: linear, quadratic, exponential, absolute value...}}$

8)

x	-3	-2	-1	0	1	2	3
y	4/3	11/6	7/3	17/6	10/3	23/6	13/3

+1/2 +1/2 +1/2 +1/2 +1/2 +1/2

constant increase...

$\boxed{\textit{Linear Equation}}$

$y = mx + b$

the y-intercept (b) is (0, 17/6)

the linear growth/slope is $m = 1/2$

$$\boxed{y = \frac{1}{2}x + \frac{17}{6}}$$

9)

x	-3	-2	-1	0	1	2	3
y	-11	-4	1	4	5	4	1

1st difference +7 +5 +3 +1 -1 -3

2nd difference -2 -2 -2 -2 -2

There is symmetry, so it's either a quadratic, absolute value, or none..
Since the first difference is not constant, it can't be absolute value...
And,
since the 2nd difference is constant, $\boxed{\text{it's a } \textit{quadratic}!}$

The vertex is at (1, 5) Then, using (0, 4) -----> $4 = a(0 - 1)^2 + 5$

$a = -1$

$\boxed{y = -1(x - 1)^2 + 5}$

Solve the following systems of equations. (Optional: Graph the systems showing the solutions)

System of Equations: Linear and Quadratic

SOLUTIONS

1) $y = x^2 + 6x - 16$

$y = 12x - 24$

set equations equal to each other...

$x^2 + 6x - 16 = 12x - 24$

collect terms...

$x^2 - 6x + 8 = 0$

factor...

$(x - 2)(x - 4) = 0$

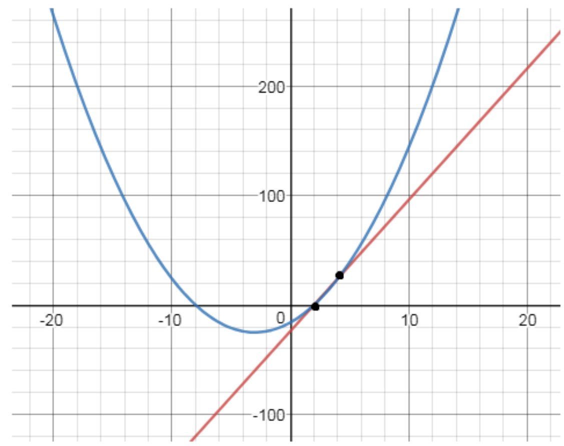
$x = 2$ and 4

if $x = 2$, then $y = 12(2) - 24 = 0$

$\boxed{(2, 0)}$

if $x = 4$, then $y = 12(4) - 24 = 24$

$\boxed{(4, 24)}$



2) $y = -x^2 + 6x + 13$

$y = -2x + 29$

$-x^2 + 6x + 13 = -2x + 29$

$0 = x^2 - 8x + 16$

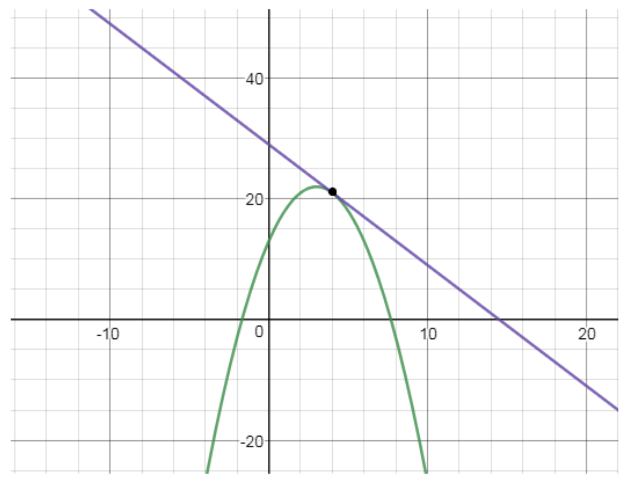
$0 = (x - 4)(x - 4)$

$x = 4$

then, $y = -2(4) + 29 = 21$

only 1 solution!!

$\boxed{(4, 21)}$



3) $y = x^2 - 8x + 15$

$x + y = -3$

$x^2 - 8x + 15 = -x - 3$

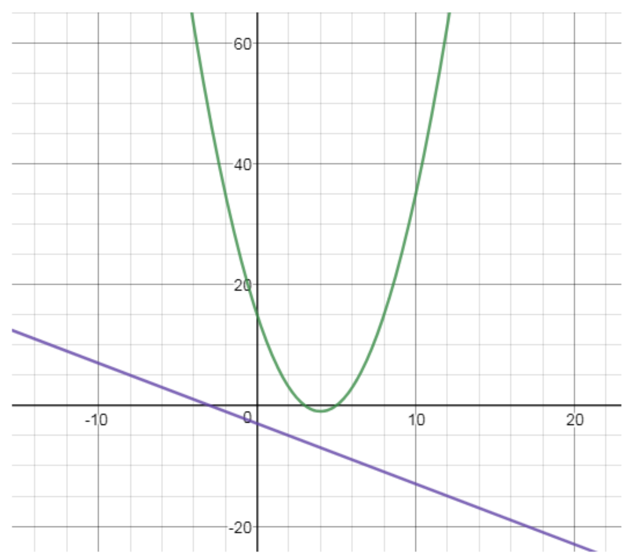
$x^2 - 7x + 18 = 0$

the discriminant is $b^2 - 4ac$

$(-7)^2 - 4(1)(18) = -23$

since the discriminant is less than zero, there are

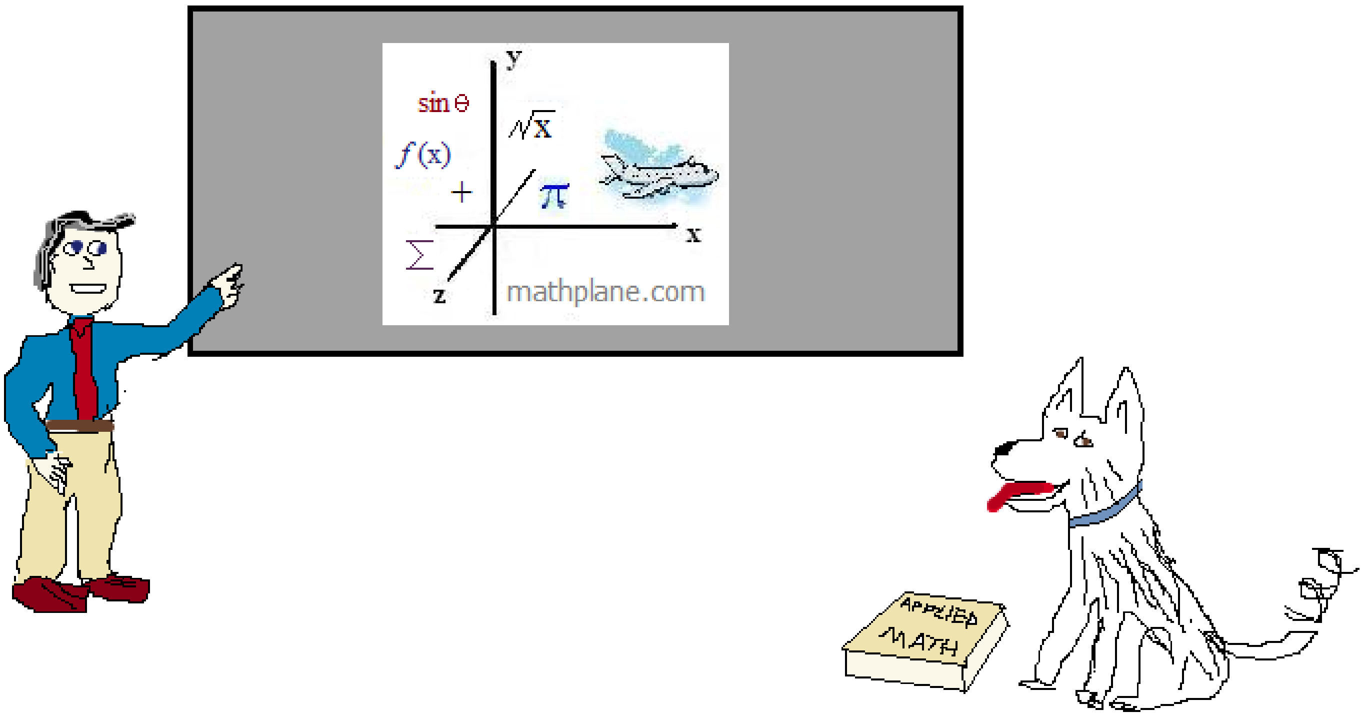
$\boxed{\text{No real solutions!}}$



Thanks for visiting. (hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.



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