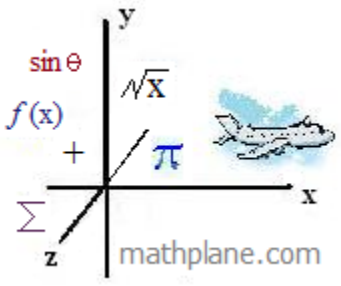
Algebra Review 2

Practice Exercises (with Solutions)

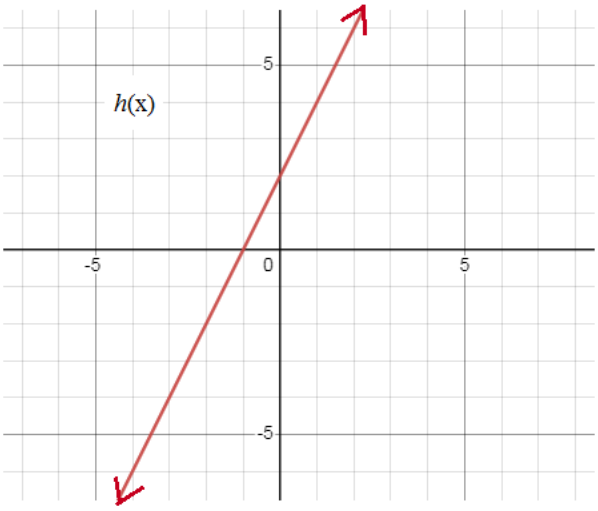
Topics include rational expressions, factoring, piecewise functions, domain & range, rate questions, inequalities, and more..



For the following functions, evaluate.

$f(x) = 3x^2 + 1$

x	0	1	2	3	4
$g(x)$	5	10	-1	2	-6



$f(1) =$

$g(1) =$

$h(1) =$

$(f + h)(3) =$

$(g + f)(2) =$

$(f \cdot g)(0) =$

$f(g(1)) =$

$g(f(0)) =$

$h(f(-3)) =$

$f(7) =$

$g(7) =$

$h(7) =$

$\frac{f}{g}(0) =$

$\frac{g}{h}(1) =$

$g \circ h\,(1) =$

$h \circ g\,(1) =$

If $f(x) = 28$, then x =

If $g(x) = 2$, then x =

If $h(x) = 4$, then x =

Functions and Notation Exercises

A) The operation ★ is defined by the function $a \bigstar b = 3a + b$ where a and b are real numbers.

1) Evaluate the following:

a) $7 \bigstar 4$ b) $6 \bigstar (-2)$ c) $5 \bigstar (2 \bigstar 1)$ d) $3(9 \bigstar 10)$

2) For what value of x would $x \bigstar x = 20$?

3) Is ★ commutative? Why or why not?

B) Function $f(x)$ represents the percentage of math students who get an A in their algebra class, where x is the years after 2010...

What do the following equations and inequalities represent?

1) $f(3) = 45$

2) $f(6) \geq 60$

3) $f(5) > f(8)$

4) $f(0) = f(7)$

What is the domain of $f(x)$? Range of $f(x)$?

C) Let x be a person.

$d(x)$ = The person's dog

$t(x)$ = Favorite toy

$f(x)$ = Best friend

Example: d(Lance) = Lance's dog ----> Norway the Husky

t(Norway) = Norway's favorite toy ---> ball

$f(d(\text{Lance}))$ = Lance's dog's best friend ----> Timber the husky

What do the following represent?

1) t(Linda) =

2) d(Leila) =

3) $t(f(\text{John}))$ =

4) f(Adam) =

Rational Expressions Quiz

Simplify:

1) $\frac{x}{3} + \frac{x}{5}$

2) $\frac{1}{a+4} + \frac{3}{a+4}$

3) $\frac{4}{x^2+4x+3} - \frac{1}{x+3}$

4) $\frac{3}{2x+6} + \frac{4}{6x+18}$

5) $\frac{7}{2d} - \frac{3}{2d}$

6) $\frac{2x}{x^2-1} - \frac{3}{x+1}$

7) $\frac{k-10}{20-2k}$

Solve:

1) $\frac{x}{x+2} + \frac{4}{x-2} = 1$

2) $\frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$

3) $\frac{1}{3s} = \frac{s}{2} - \frac{1}{6s}$

4) $\frac{x+2}{x+8} = \frac{x-2}{x+4}$

5) $1 - \frac{3}{z} = \frac{4}{z^2}$

6) $\frac{d}{3} + \frac{1}{2} = \frac{1}{3d}$

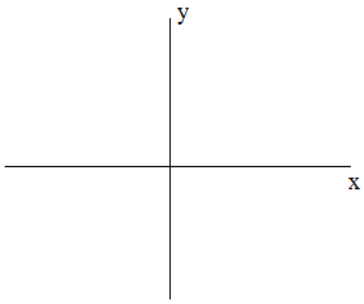
7) $\frac{1}{c-3} = \frac{c}{4}$

8) $\frac{5}{x-2} = \frac{5x+10}{x^2}$

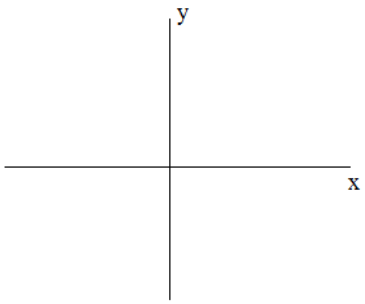
Algebra: Points and Lines

I. Determine (algebraically) if the given ordered pairs are solutions. Then, sketch the linear inequalities on a Cartesian plane. Plot the points to verify your answers!

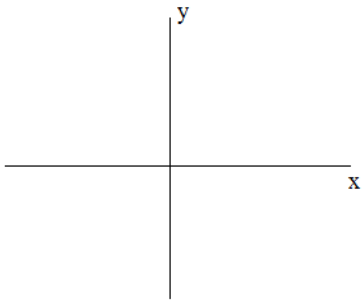
a) $y < -4x + 7$; $(4, -8)$ $(-2, 3)$



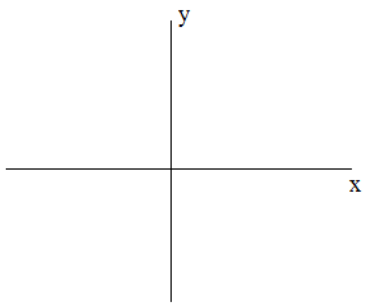
b) $x \leq -3$; $(0, 2)$ $(-3, 1)$



c) $10x + y > -1.5$; $(-1, 7)$ $(0, 4)$



d) $3y \geq 7$; $(4, 1)$ $(-3, 5)$



II. Answer following

A) Which of the following points lie on the line $3x - 10y = 12$?

$(4, 0)$ $(1, -1)$ $(10, 2)$ $(14, 3)$

B) Which of the following points does the line $y = 2x + 6$ pass through?

$(4, 14)$ $(-3, 0)$ $(52, 110)$ $(-32, -58)$

Linear Algebra Review Questions

I. Determine if the line segments are parallel, perpendicular, or neither

1) $\overline{RS}$ R(2, 2) S(4, 6)
   $\overline{TV}$ T(7, 3) V(9, 2)

2) $\overline{EF}$ E(7, 11) F(8, 15)
   $\overline{LN}$ L(-1, 5) N(1, 11)

3) $\overline{GH}$ G(76, 2) H(44, -30)
   $\overline{JK}$ J(0, 0) K(7, 7)

4) $\overline{AB}$ A(3, 1) B(3, 4)
   $\overline{CD}$ C(1, 7) D(-3, 7)

5) $\overline{MN}$ M(3, 4) N(8, 4)
   $\overline{OP}$ O(1, 7) P(-1, 7)

II. Which of the following pairs are <u>perpendicular</u>?

1) $y - \frac{1}{2}x = 0$
   $y - 2x = -1$

2) $y = -x + 7$
   $y - x = 20$

3) $x = -1$
   $y = 3$

4) $3x + 6y = 11$
   $y - 6 = 2(x + \frac{1}{2})$

5) $y + 3 = -3(x + 4)$
   $y = 3x - 5$

Factoring Review: 5 examples

Quadratic Standard Form

$Ax^2 + Bx + C$

1) $25 - 16y^2$

2) $4t^2 - 13t - 12$

3) $2y^6 - 32y^2$

4) $6x^2 + x - 12$

5) $y^3 - 2y^2 - y + 2$

Rate Questions

1) John leaves his house at 8:00 am, walking East at a speed of 5 miles/hour.
At noon, his brother leaves the house, and rides a bike in the same direction at 15 miles/hour.

a) What time do the brothers meet?

b) How far from home are they?

2) Train 1 and Train 2 are 300 miles apart.
At 9:00 am, Train 1 departs station A, heading North on the track at 60 mph.
At 10:00 am, Train 2 departs station B, heading South on a parallel track at 40 mph.

a) When will the Trains pass each other?

b) How far from station A are the trains when they meet?

3) Trains 3 and 4 sit at station C. At noon, train 3 departs, going West at 60 m/h.
At 3:00 train 4 departs, going East at 70 m/h.

a) At 7:00, how far apart are the trains?

b) When will the trains be 1200 miles apart?

Rate Questions

4) A canoe goes up stream in 7 hours. Then, turns around and goes back downstream in 5 hours.
If the rowers can paddle 20 km/hour in still water,

a) What is the rate of the stream?

b) How far did the canoe travel?

5) Bill can run 50 km in the same time that Joe can run 40 km.
If Bill runs 2 km/hour faster than Joe, what are their running rates?

6) Alex bikes at a rate of 12 miles per hour. Tim bikes at a rate of 10 miles per hour.
If Alex leaves school at 2:00 pm, and Tim leaves at 3:00 pm, when does Tim catch Alex?

Specific Solution Sets

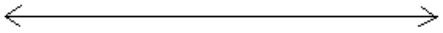
Find the solution set. Then, graph on a number line.

1) $4x - 3 < 9$ $x \in$ {Real Numbers}

2) $|x - 2| > 5$ $x \in$ {Integers}

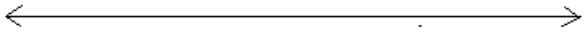
3) $|x + 4| \leq 4$ $x \in$ {Integers}

4) $2x + 7 < 18$ $x \in$ {Whole Numbers}



5) $|3 - 4x| < 9$ $x \in$ {Positive Numbers}

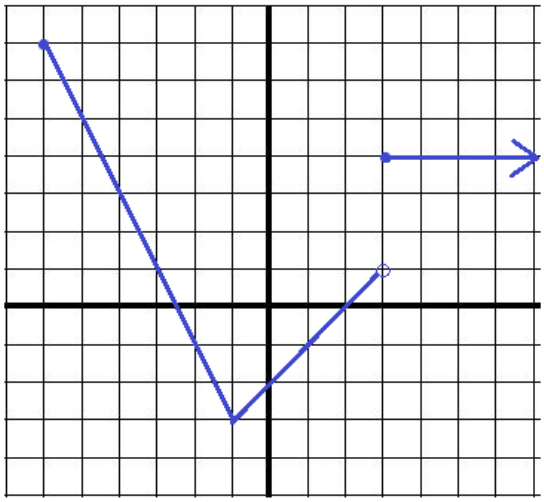
6) $\frac{(x + 2)(x - 5)}{(x + 1)} \leq 0$ $x \in$ {Real Numbers}



Piecewise Functions, Domain and Range

Find the domain and range. Then, write the equation.

1)

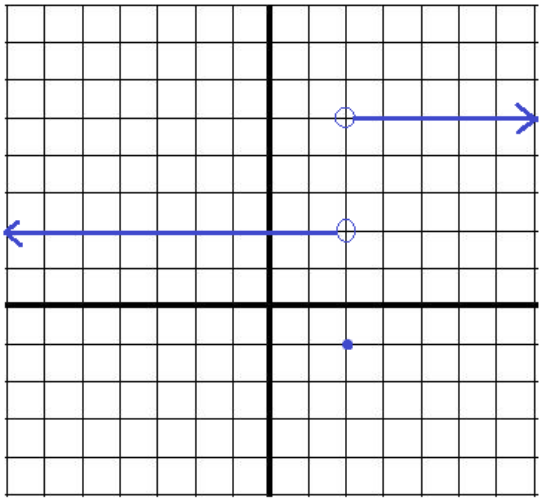


Domain:

Range:

Function:

2)

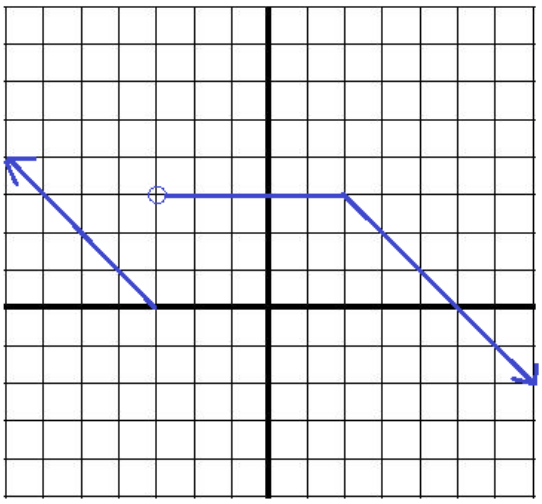


Domain:

Range:

Function:

3)

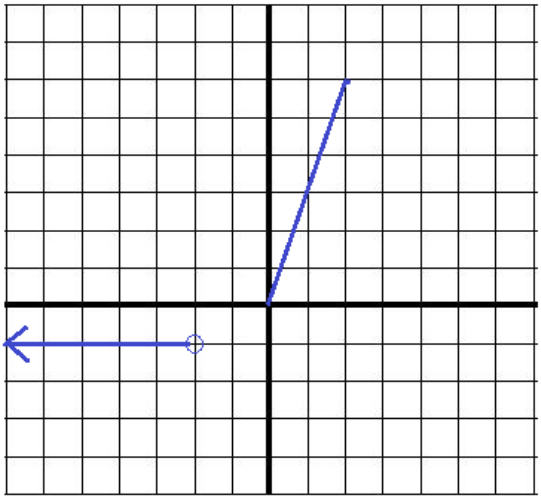


Domain:

Range:

Function:

4)



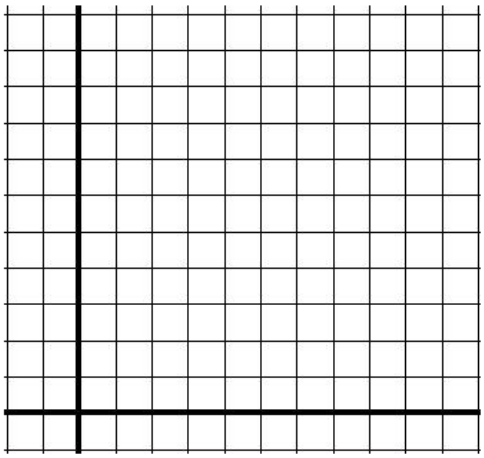
Domain:

Range:

Function:

Algebra Graphing Review

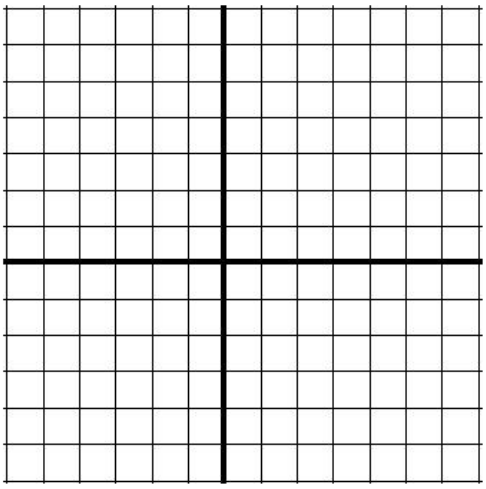
1) John receives a $60 gift card to the movie theater. If evening shows are $10 and matinees are $6, how many movie tickets can he afford? (graph the inequality)



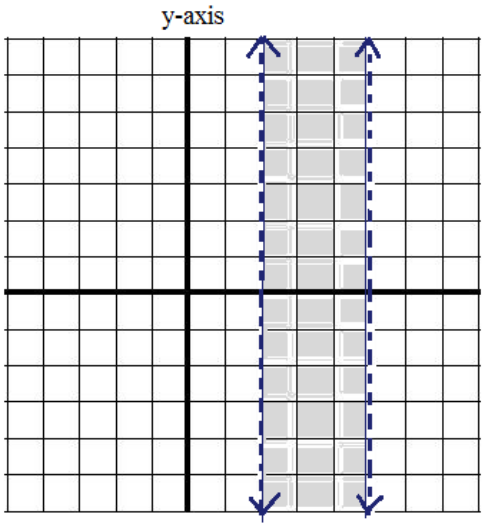
2)

$$f(x) = \begin{cases} -x + 2 & \text{if } 0 \le x < 2 \\ -x + 4 & \text{if } 2 \le x \le 5 \end{cases}$$

Graph the function (for the interval [0, 5])



3)



Write equations that describe the system of inequalities (on the left)

LanceAF #111 11/7/13
mathplane.com

Skip got a disappointing C+ on the math assignment...
(but, he did see 2 awesome reruns of Magnum P.I.)

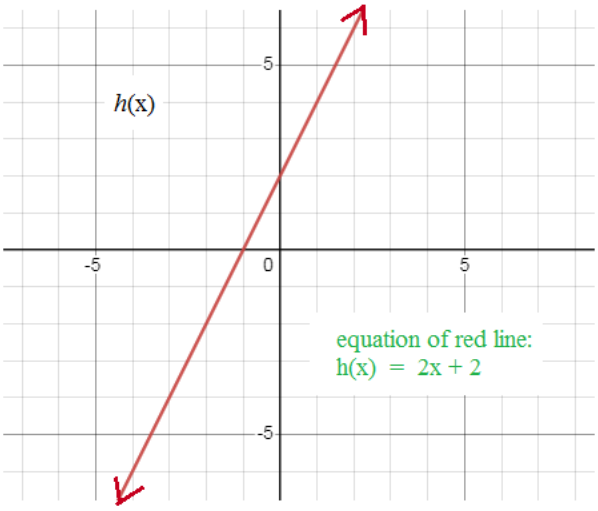
SOLUTIONS

For the following functions, evaluate.

SOLUTIONS

$f(x) = 3x^2 + 1$

x	0	1	2	3	4
$g(x)$	5	10	-1	2	-6



$f(1) =$ 4 plug into equation

$g(1) =$ 10 find in the chart

$h(1) =$ 4 corresponds to (1, 4) on the graph

$f(7) =$ 148

$g(7) =$ does not exist

$h(7) =$ 16

$(f+h)(3) =$ 28 + 8 = 36

$(g+f)(2) =$ -1 + 13 = 12

$(f \cdot g)(0) =$ 1 x 5 = 5

$\frac{f}{g}(0) = \frac{1}{5}$

$\frac{g}{h}(1) = \frac{10}{4}$

$f(g(1)) =$ g(1) = 10 then, f(10) = 301

$g(f(0)) =$ f(0) = 1 then, g(1) = 10

$h(f(-3)) =$ f(-3) = 28 then, h(28) = 58

equation of red line is y = 2x + 2

$g \circ h\,(1) =$ h(1) = 4 then, g(4) = -6

$h \circ g\,(1) =$ g(1) = 10 then, h(10) = 22

If $f(x) = 28$, then x = 3 or -3 set $3x^2 + 1 = 28$

If $g(x) = 2$, then x = 3 g(3) = 2

If $h(x) = 4$, then x = 1 (1, 4) on the graph

Functions and Notation Exercises

SOLUTIONS

A) The operation ★ is defined by the function $a \bigstar b = 3a + b$ where a and b are real numbers.

1) Evaluate the following:

a) $7 \bigstar 4$

$3(7) + 4 =$ **25**

b) $6 \bigstar (-2)$

$3(6) + (-2) =$ **16**

c) $5 \bigstar (2 \bigstar 1)$

$5 \bigstar [\, 3(2) + 1]$

$5 \bigstar 7$

$3(5) + 7 =$ **22**

d) $3(9 \bigstar 10)$

$3 [\, 3(9) + 10 \,]$

$3 [37] =$ **111**

2) For what value of x would $x \bigstar x = 20$?

$x \bigstar x = 20 \Rightarrow 3x + x = 20$

$4x = 20$

x = 5

3) Is ★ commutative? Why or why not?

Commutative property implies you can 'reverse the order' -- ex: $2 + 5 = 5 + 2$ TRUE
$3 - 6 = 6 - 3$ FALSE

$a \bigstar b = 3a + b$

$b \bigstar a = 3b + a$

Not commutative (unless a = b)

B) Function $f(x)$ represents the percentage of math students who get an A in their algebra class, where x is the years after 2010...

What do the following equations and inequalities represent?

1) $f(3) = 45$ — 45% of students got an A in 2013

2) $f(6) \geq 60$ — At least 60% of students got an A in 2016

3) $f(5) > f(8)$ — More students got A's in 2015 than in 2018

4) $f(0) = f(7)$ — The same number of students got A's in 2010 as in 2017

What is the domain of $f(x)$? Range of $f(x)$?

Domain: $0 \leq x \leq 13$ (today is 2023)

Range: $0 \leq f(x) \leq 100$ (0 percent to 100 percent)

(NOTE: if years before 2010 are included, then domain is $x \leq 13$)

i.e. $x = -20$ for 1990.

C) Let x be a person.

$d(x)$ = The person's dog

$t(x)$ = Favorite toy

$f(x)$ = Best friend

Example: d(Lance) = Lance's dog ----> Norway the Husky

t(Norway) = Norway's favorite toy ---> ball

$f(d(\text{Lance}))$ = Lance's dog's best friend ----> Timber the husky

What do the following represent?

1) t(Linda) = Linda's favorite toy

2) d(Leila) = Leila's dog

3) $t(f(\text{John}))$ = John's best friend's favorite toy

4) f(Adam) = Adam's best friend

Rational Expressions Quiz

SOLUTIONS

Simplify:

1) $\frac{x}{3} + \frac{x}{5}$

$\frac{5x}{15} + \frac{3x}{15} =$

$\boxed{\frac{8x}{15}}$

2) $\frac{1}{a+4} + \frac{3}{a+4}$

$\boxed{\frac{4}{a+4}}$

3) $\frac{4}{x^2+4x+3} - \frac{1}{x+3}$

$\frac{4}{(x+1)(x+3)} - \frac{1}{(x+3)} =$

$\frac{4}{(x+1)(x+3)} - \frac{(x+1)}{(x+1)(x+3)} =$

$\boxed{\frac{3-x}{(x+1)(x+3)}}$

4) $\frac{3}{2x+6} + \frac{4}{6x+18}$

$\frac{3}{2(x+3)} + \frac{4}{6(x+3)} =$

$\frac{9}{6(x+3)} + \frac{4}{6(x+3)} =$

$\boxed{\frac{13}{6(x+3)}}$

5) $\frac{7}{2d} - \frac{3}{2d}$

$\frac{4}{2d} =$

$\boxed{\frac{2}{d}}$

6) $\frac{2x}{x^2-1} - \frac{3}{x+1}$

$\frac{2x}{(x+1)(x-1)} - \frac{3}{(x+1)} =$

$\frac{2x}{(x+1)(x-1)} - \frac{3(x-1)}{(x+1)(x-1)} =$

$\frac{-x+3}{(x+1)(x-1)} = \boxed{\frac{3-x}{x^2-1}}$

7) $\frac{k-10}{20-2k}$

$\frac{k-10}{-2(k-10)} =$

$\boxed{-\frac{1}{2}}$

Solve: (Plug in to check solutions)

1) $\frac{x}{x+2} + \frac{4}{x-2} = 1$

$\frac{x(x-2)}{(x+2)(x-2)} + \frac{4(x+2)}{(x-2)(x+2)} = 1$

$\frac{x^2-2x+4x+8}{x^2-4} = \frac{1}{1}$

$x^2 - 4 = x^2 + 2x + 8$

$\boxed{x = -6}$

$\frac{(-6)}{(-6)+2} + \frac{4}{(-6)-2}$

$\frac{3}{2} + \frac{4}{-8} = 1$ ✓

2) $\frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$

Use common denominator

$\frac{6}{6x} + \frac{3}{6x} = \frac{x}{6x}$

$\frac{9}{6x} = \frac{x}{6x}$

$\boxed{x = 9}$

$\frac{1}{(9)} + \frac{1}{2(9)} = \frac{1}{6}$

$\frac{2}{18} + \frac{1}{18} = \frac{3}{18}$ ✓

3) $\frac{1}{3s} = \frac{s}{2} - \frac{1}{6s}$

multiply by 6s

$2 = 3s^2 - 1$

$3s^2 = 3$

$\boxed{s = 1, -1}$

$\frac{1}{3(1)} = \frac{(1)}{2} - \frac{1}{6(1)}$

$\frac{1}{3} = \frac{1}{2} - \frac{1}{6}$ ✓

$\frac{1}{3(-1)} = \frac{(-1)}{2} - \frac{1}{6(-1)}$

$\frac{-1}{3} = \frac{-1}{2} + \frac{1}{6}$ ✓

4) $\frac{x+2}{x+8} = \frac{x-2}{x+4}$

cross multiply

$x^2 + 2x + 4x + 8 = x^2 + 8x - 2x - 16$

$6x + 8 = 6x - 16$

No solution!

5) $1 - \frac{3}{z} = \frac{4}{z^2}$

multiply by z^2

$z^2 - 3z = 4$

$z^2 - 3z - 4 = 0$

$(z - 4)(z + 1) = 0$

$\boxed{z = 4, -1}$

$1 - \frac{3}{(4)} = \frac{4}{(4)^2}$

$1 - 3/4 = 1/4$ ✓

$1 - \frac{3}{(-1)} = \frac{4}{(-1)^2}$

$1 - (-3) = 4/1$ ✓

6) $\frac{d}{3} + \frac{1}{2} = \frac{1}{3d}$

multiply by 6d

$2d^2 + 3d = 2$

$2d^2 + 3d - 2 = 0$

$(2d - 1)(d + 2) = 0$

$\boxed{d = 1/2, -2}$

$\frac{(-2)}{3} + \frac{1}{2} = \frac{1}{3(-2)}$

$\frac{-4}{6} + \frac{3}{6} = \frac{1}{-6}$ ✓

$\frac{(1/2)}{3} + \frac{1}{2} = \frac{1}{3(1/2)}$

$\frac{1}{6} + \frac{3}{6} = \frac{2}{3}$ ✓

7) $\frac{1}{c-3} = \frac{c}{4}$

cross multiply

$c^2 - 3c = 4$

$c^2 - 3c - 4 = 0$

$(c - 4)(c + 1) = 0$

$\boxed{c = 4, -1}$

$\frac{1}{(4)-3} = \frac{(4)}{4}$ ✓

$\frac{1}{(-1)-3} = \frac{(-1)}{4}$ ✓

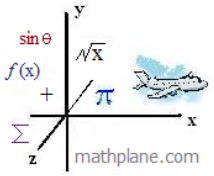
8) $\frac{5}{x-2} = \frac{5x+10}{x^2}$

cross multiply

$5x^2 = 5x^2 - 10x + 10x - 20$

$5x^2 = 5x^2 - 20$

No Solution!



Algebra: Points and Lines

SOLUTIONS

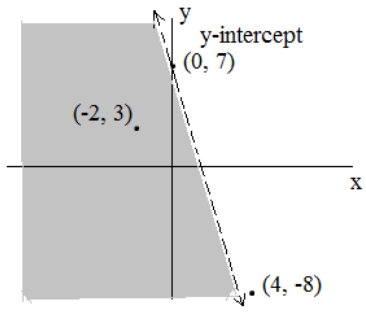
I. Determine (algebraically) if the given ordered pairs are solutions. Then, sketch the linear inequalities on a Cartesian plane. Plot the points to verify your answers!

a) $y < -4x + 7$; (4, −8) (−2, 3)

Test each point:

(4, -8): $(-8) < -4(4) + 7$
$-8 < -9$
NOT a solution

(-2, 3): $(3) < -4(-2) + 7$
$3 < 15$
SOLUTION

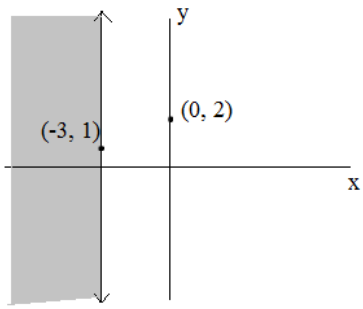


b) $x \leq -3$; (0, 2) (−3, 1)

Test each point:

(0, 2): $0 \leq -3$
NOT a solution

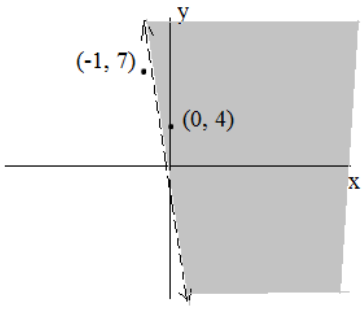
(-3, 1): $-3 \leq -3$
SOLUTION



c) $10x + y > -1.5$; (−1, 7) (0, 4)

(-1, 7): $10(-1) + (7) > -1.5$
$-3 \not> -1.5$
NOT a solution

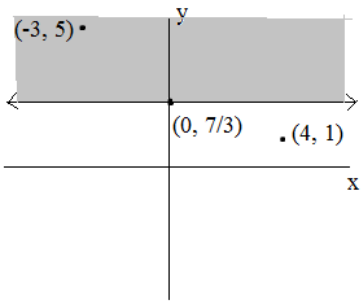
(0, 4): $10(0) + 4 > -1.5$
$4 > -1.5$
SOLUTION



d) $3y \geq 7$; (4, 1) (−3, 5)

(4, 1): $3(1) \geq 7$
$3 \not\geq 7$
NOT a solution

(-3, 5): $3(5) \geq 7$
$15 \geq 7$
SOLUTION



II. Answer following

A) Which of the following points lie on the line $3x - 10y = 12$?

(4, 0) (1, −1) (10, 2) (14, 3)

$3(4) - 10(0) = 12$ YES

$3(1) - 10(-1) = 13$ NO $\neq 12$

$3(10) - 10(2) = 10$ NO $\neq 12$

$3(14) - 10(3) = 12$ YES

B) Which of the following points does the line $y = 2x + 6$ pass through?

(4, 14) (−3, 0) (52, 110) (−32, − 58)

ALL OF THEM!!

$(14) = 2(4) + 6$
$14 = 14$

$(0) = 2(-3) + 6$
$0 = 0$

$(110) = 2(52) + 6$
$110 = 110$

$(-58) = 2(-32) + 6$
$-58 = -58$

Linear Algebra Review Questions

SOLUTIONS

slope = $\frac{\text{"rise"}}{\text{"run"}} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$

parallel lines: slopes are equal

perpendicular lines: opposite reciprocals

I. Determine if the line segments are parallel, perpendicular, or neither

1) $\overline{RS}$ R(2, 2) S(4, 6) slope = 2
$\overline{TV}$ T(7, 3) V(9, 2) slope = $\frac{-1}{2}$ perpendicular

2) $\overline{EF}$ E(7, 11) F(8, 15) slope = 4
$\overline{LN}$ L(-1, 5) N(1, 11) slope = 3 neither

3) $\overline{GH}$ G(76, 2) H(44, -30) slope = 32/32 = 1
$\overline{JK}$ J(0, 0) K(7, 7) slope = 7/7 = 1 parallel

4) $\overline{AB}$ A(3, 1) B(3, 4) slope is undefined ("no slope") ----> vertical line segment
$\overline{CD}$ C(1, 7) D(-3, 7) slope is 0 ----> horizontal line segment perpendicular

5) $\overline{MN}$ M(3, 4) N(8, 4) slope is 0 (horizontal)
$\overline{OP}$ O(1, 7) P(-1, 7) slope is 0 (horizontal) parallel

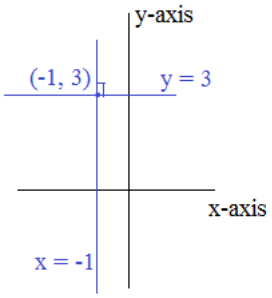
II. Which of the following pairs are perpendicular?

The slopes of perpendicular lines are opposite reciprocals..

1) $y - \frac{1}{2}x = 0$ $y = \frac{1}{2}x + 0$ slope = 1/2
$y - 2x = -1$ $y = 2x - 1$ slope = 2 NO (slopes are reciprocals, but not opposites)

2) $y = -x + 7$ slope = -1
$y - x = 20$ $y = x + 20$ slope = 1 YES

3) $x = -1$ vertical line
$y = 3$ horizontal line YES



4) $3x + 6y = 11$ $6y = -3x + 11$ $y = -\frac{1}{2}x + \frac{11}{6}$ slope = -1/2
$y - 6 = 2(x + \frac{1}{2})$ (point slope form) slope = 2 YES

5) $y + 3 = -3(x + 4)$ slope = -3
$y = 3x - 5$ slope = 3 NO

Factoring Review: 5 examples

SOLUTIONS

Quadratic Standard Form

$Ax^2 + Bx + C$

1) $25 - 16y^2$ (difference of squares) $a^2 - b^2 = (a + b)(a - b)$

$a = 5$
$b = 4y$

$(5 + 4y)(5 - 4y)$

2) $4t^2 - 13t - 12$ (divide and regroup)

$AC = 4(-12) = -48$
$B = -13$
What multiplies to -48 and adds to -13? 3 and -16

-13t
/ \
$4t^2 + -16t + 3t - 12$ divide
$(4t^2 - 16t) + (3t - 12)$ factor
$4t(t - 4) + 3(t - 4)$ regroup

$(4t + 3)(t - 4)$

3) $2y^6 - 32y^2$ (GCF and difference of squares)

$2y^2 (y^4 - 16)$
$2y^2 (y^2 + 4)(y^2 - 4)$

$2y^2 (y^2 + 4)(y + 2)(y - 2)$

4) $6x^2 + x - 12$

What multiplies to -72 and adds to 1? 9 and -8

$A = 6$
$B = 1$
$C = -12$

$AC = -72$
$B = 1$

$6x^2 + 9x - 8x - 12$ divide
$(6x^2 + 9x) + (-8x - 12)$ factor
$3x(2x + 3) + -4(2x + 3)$ regroup

$(3x - 4)(2x + 3)$

5) $y^3 - 2y^2 - y + 2$ (separate, factor, regroup)

$y^3 - y \quad -2y^2 + 2$
$y(y^2 - 1) - 2(y^2 - 1)$
$(y - 2)(y^2 - 1)$ (difference of squares)

$(y - 2)(y + 1)(y - 1)$

SOLUTIONS

Rate Questions

1) John leaves his house at 8:00 am, walking East at a speed of 5 miles/hour.
At noon, his brother leaves the house, and rides a bike in the same direction at 15 miles/hour.

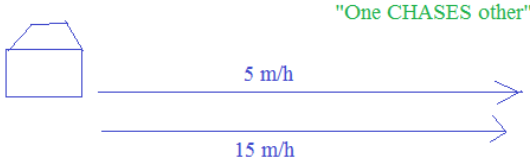
a) What time do the brothers meet?

they meet at 2:00 pm

b) How far from home are they?

if t = 6, then d = 30 miles

"One CHASES other"



distance = rate x time

John d = 5 m/h (t)

brother d = 15 m/h (t − 4)

When will the distances be the same?

5 m/h (t) = 15 m/h (t − 4)

5t = 15t − 60

t = 6

2) Train 1 and Train 2 are 300 miles apart.
At 9:00 am, Train 1 departs station A, heading North on the track at 60 mph.
At 10:00 am, Train 2 departs station B, heading South on a parallel track at 40 mph.

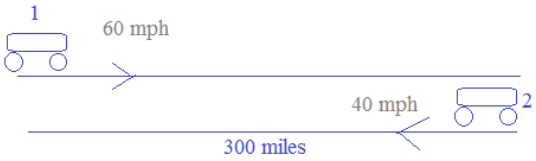
a) When will the Trains pass each other?

12:24 pm

b) How far from station A are the trains when they meet?

the trains are 204 miles from station A

"Trains go TOWARD each other"



d_1 = 60 mph (t)

d_2 = 40 mph (t - 1) because train 2 left one hour later

distance = rate x time

$d_1 + d_2 = 300$

60t + 40t − 40 = 300

100t = 340

t = 3.4 hours

3.4 hours ----> 3 hours, 24 minutes

@ t = 3.4, d_1 = 204 miles

d_2 = 96 miles..

3) Trains 3 and 4 sit at station C. At noon, train 3 departs, going West at 60 m/h.
At 3:00 train 4 departs, going East at 70 m/h.

a) At 7:00, how far apart are the trains?

700 miles

b) When will the trains be 1200 miles apart?

10:51 pm

train 3: distance = 60 m/h (7 hours) = 420 miles West

train 4: distance = 70 m/h (4 hours) = 280 miles East

trains are 700 miles apart...

distance = rate x time

distance of 3 + distance of 4 = 1200 miles

60 m/h (time) + 70 m/h (time − 3 hours) = 1200 miles

60t + 70t − 210 = 1200

130t = 1410

t = 10.846 hours... ----> 10 hours and 51 minutes

SOLUTIONS

Rate Questions

4) A canoe goes up stream in 7 hours. Then, turns around and goes back downstream in 5 hours.
If the rowers can paddle 20 km/hour in still water,

a) What is the rate of the stream?

3 1/3 km/hour

b) How far did the canoe travel?

233 1/3 km
(total up and down the river)

distance = rate x time

up stream distance = (20 km/hr − rate of stream)(7 hours)

down stream distance = (20 km/hr + rate of stream)(5 hours)

since distances are the same, $140 - 7S = 100 + 5S$

$40 = 12S$

$S = 3.33$ km/hour

If $S = 10/3$ km/hour,

then distance = $(20 - 10/3)(7) = 116\ 2/3$ km

or distance = $(20 + 10/3)(5) = 116\ 2/3$ km

5) Bill can run 50 km in the same time that Joe can run 40 km.
If Bill runs 2 km/hour faster than Joe, what are their running rates?

distance = rate x time

Bill $50 = (r + 2)(t)$ $\quad 50 = rt + 2t$

Joe $40 = (r)(t)$ $\quad 40 = rt$

$50 = 40 + 2t$

$t = 5$

Since $t = 5$, $r = 8$

Joe runs at 8 km/hour
Bill runs at 10 km/hour

6) Alex bikes at a rate of 12 miles per hour. Tim bikes at a rate of 10 miles per hour.
If Alex leaves school at 2:00 pm, and Tim leaves at 3:00 pm, when does Tim catch Alex?

distance = rate x time

Alex: distance = $12(t)$

Tim: distance = $10(t - 1)$

If they meet, the distance will be the same..

$12t = 10t - 10$

$t = -5$ Since time cannot be negative, Tim never catches Alex!!

Specific Solution Sets

SOLUTIONS

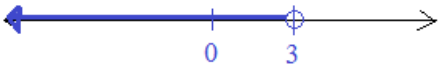
Find the solution set. Then, graph on a number line.

1) $4x - 3 < 9$ $x \in$ {Real Numbers}

$4x < 12$

$x < 3$

{x < 3}



2) $|x - 2| > 5$ $x \in$ {Integers}

find "critical values"

$x - 2 = 5$ $x - 2 = -5$

$x = 7$ $x = -3$

-5: $|-5 - 2| > 5$ ✓

0: $|0 - 2| \not> 5$ ✗

10: $|10 - 2| > 5$ ✓

"open circles"... then, test regions.... INTEGERS only!

{8, 9, 10, ...} and {-4, -5, -6, ...}

3) $|x + 4| \leq 4$ $x \in$ {Integers}

$x + 4 \leq 4$ and $x + 4 \geq -4$

$x \leq 0$ and $x \geq -8$

AND an integer!

{-8, -7, ... , -2, -1, 0}

4) $2x + 7 < 18$ $x \in$ {Whole Numbers}

$2x < 11$

$x < 5.5$ AND must be whole number

{0, 1, 2, 3, 4, 5}

5) $|3 - 4x| < 9$ $x \in$ {Positive Numbers}

$3 - 4x = 9$ $3 - 4x = -9$

$-4x = 6$ $-4x = -12$

$x = -3/2$ $x = 3$

(critical values)

test regions and eliminate negative numbers!

{0 < x < 3}

6) $\frac{(x + 2)(x - 5)}{(x + 1)} \leq 0$ $x \in$ {Real Numbers}

critical values and asymptote:

-2, -1, 5

test regions: -3: negative

-1.5: positive

0: negative

7: positive

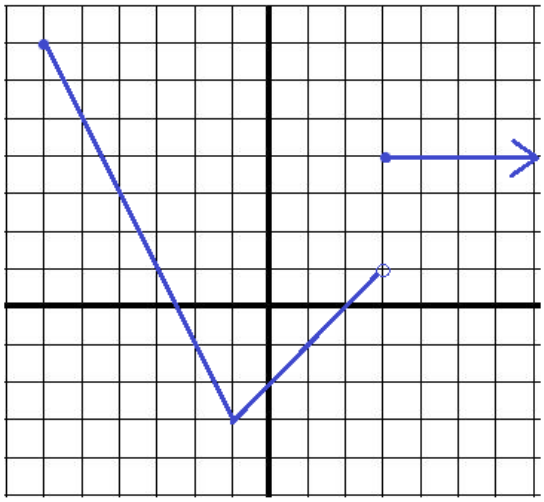
interval notation: $(-\infty, -2] \cup (-1, 5]$

Piecewise Functions, Domain and Range

SOLUTIONS

Find the domain and range. Then, write the equation.

1)

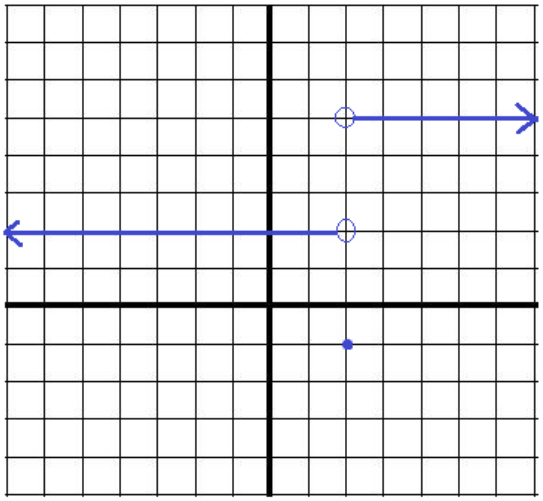


Domain: $[-6, \infty)$

Range: $[-3, 7]$

Function:
$$f(x) = \begin{cases} -2x - 5 & \text{if } -6 \le x < -1 \\ x - 2 & \text{if } -1 \le x < 3 \\ 4 & \text{if } x \ge 3 \end{cases}$$

2)

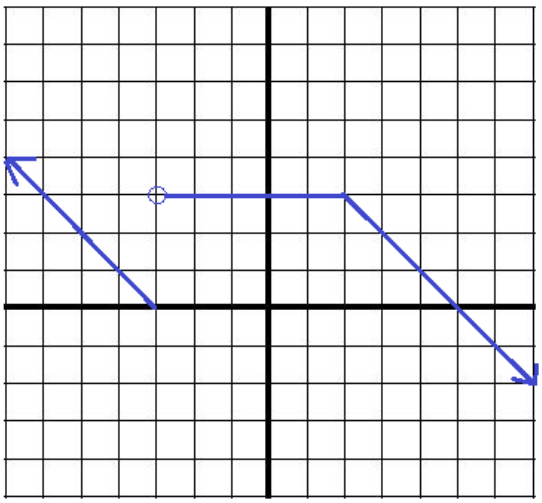


Domain: all real numbers $(-\infty, \infty)$

Range: $\{-1, 2, 5\}$

Function:
$$f(x) = \begin{cases} 2 & \text{if } x > 2 \\ -1 & \text{if } x = 2 \\ 5 & \text{if } x > 2 \end{cases}$$

3)

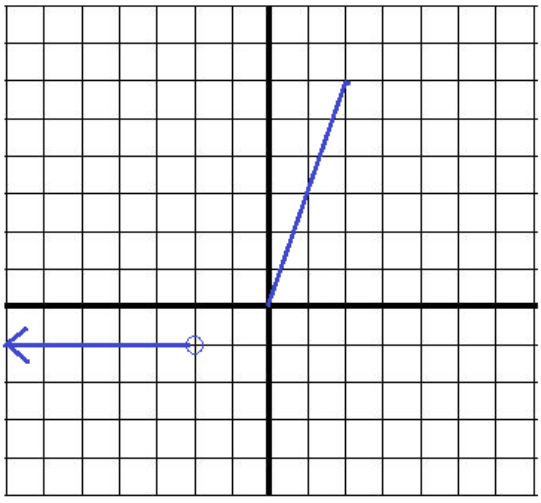


Domain: all real numbers $(-\infty, \infty)$

Range: all real numbers $(-\infty, \infty)$

Function:
$$f(x) = \begin{cases} -x - 3 & \text{if } x \le -3 \\ 3 & \text{if } -3 < x < 2 \\ -x + 5 & \text{if } x \ge 2 \end{cases}$$

4)



Domain: $(-\infty, -2) \cup [0, 2]$

Range: $\{x \mid 0 < x < 6, \ x = -1\}$

Function:
$$f(x) = \begin{cases} -1 & \text{if } x < -2 \\ 3x & \text{if } x \ge 0 \end{cases}$$

Algebra Graphing Review

1) John receives a $60 gift card to the movie theater. If evening shows are $10 and matinees are $6, how many movie tickets can he afford? (graph the inequality)

Let E = # of evening shows M = # of matinee shows

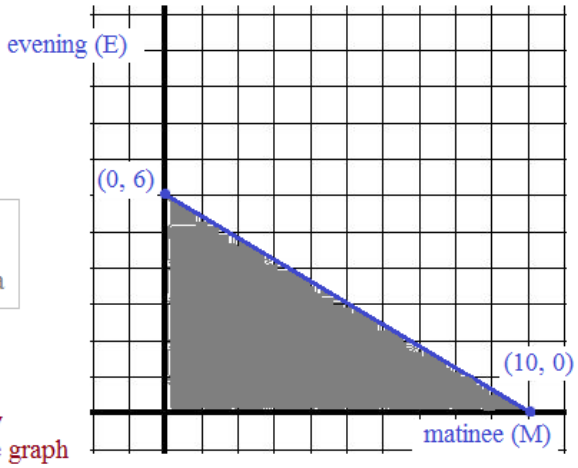
The inequality is $6(M) + $10(E) ≤ $60

money spent on matinees money spent on evening shows

Since this is a linear equation, we can find 2 points and then draw a line through both points...

If John only goes to matinees: (10, 0)
If John only goes to evening shows: (0, 6)

John can afford any combination of tickets in the gray shaded area



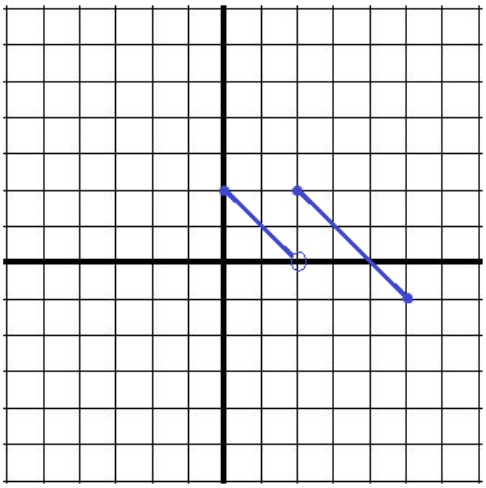
domain: $0 \le M \le 10$ (cannot have negative tickets)
range: $0 \le E \le 6$

Note: we're assuming John can buy partial tickets... If he can't, then the graph would only consist of whole numbers.

2)

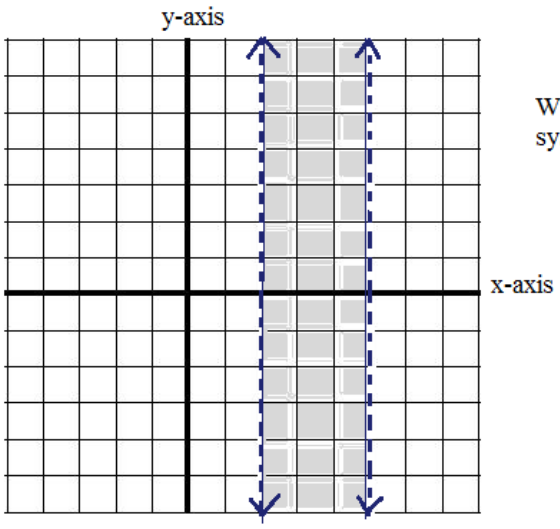
$$f(x) = \begin{cases} -x + 2 & \text{if } 0 \le x < 2 \\ -x + 4 & \text{if } 2 \le x \le 5 \end{cases}$$

Graph the function (for the interval [0, 5])



both parts of the piecewise function have the same slope (-1)

3)



Write equations that describe the system of inequalities (on the left)

dashed lines: < or >
solid lines: ≤ or ≥

the vertical line on the left: x = 2
Since it's shaded on the right, the inequality is x > 2

the vertical line on the right: x = 5
Since it's shaded on the left, the inequality is x < 5

The system of inequalities is the intersection:

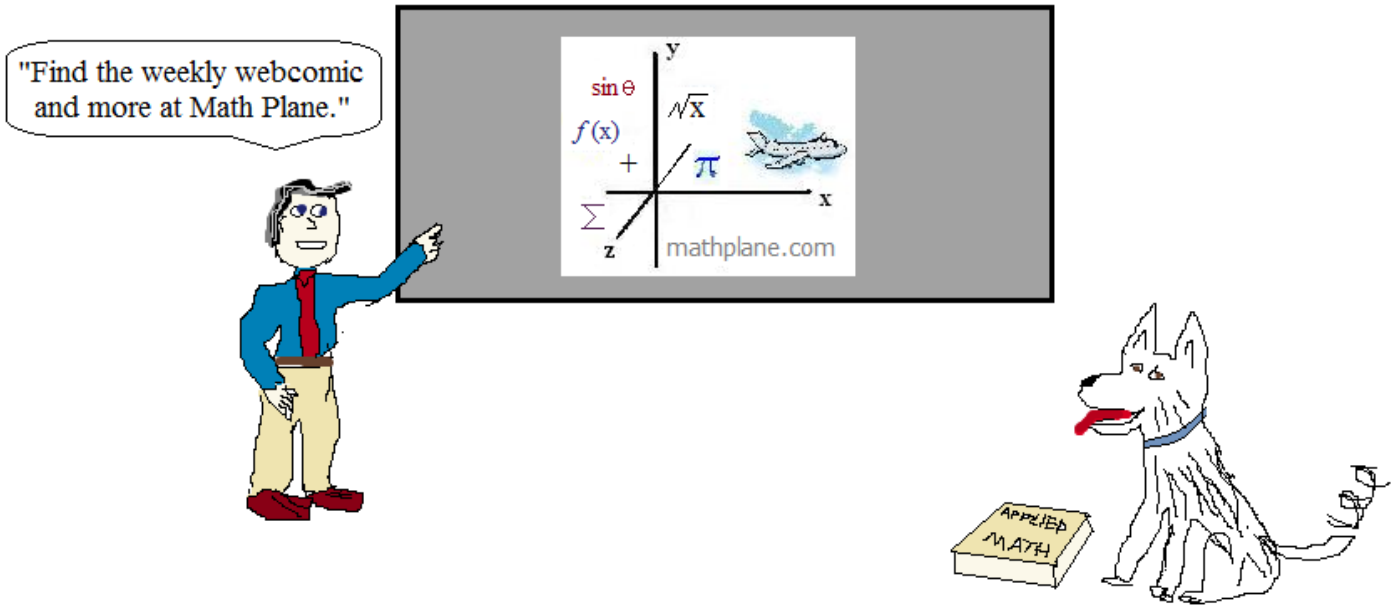
$2 < x < 5$

Thanks for visiting the site. (Hope it helped!)

Check out other Algebra Review exercises at mathplane.com.

If you have questions, suggestions, or requests, let us know.

Thanks!



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