Algebra Review 5

Practice Questions and Solutions

Topics include functions, graphing, solving, exponents, absolute values, factoring, and more.

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I. Interval Notation, Domain/Range, and Functions

Algebra Review 5

1) Write the following in Interval Notation

a) All real numbers

b) All negative numbers

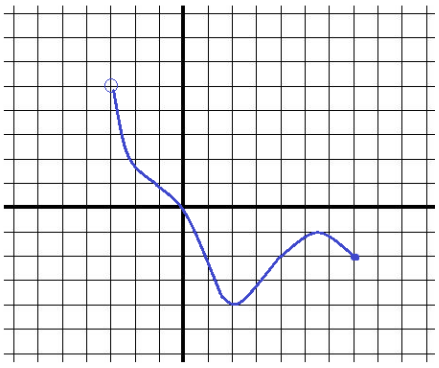
c) $x > 5$

d) $x \leq -2$

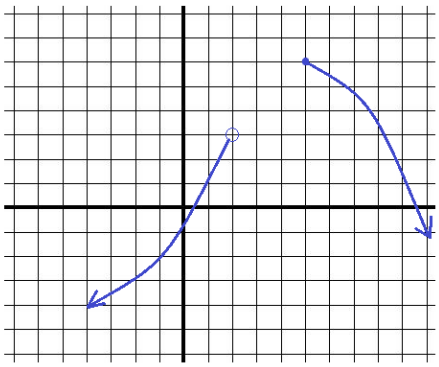
e) Rational numbers between -3 and 8

2) Identify the domain and range:

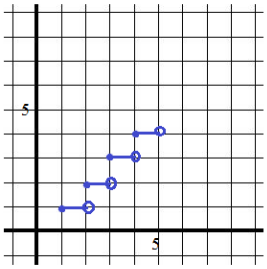
a)



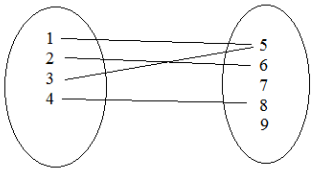
b)



c)



d)

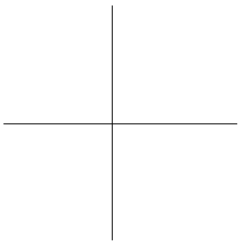


Is this a function?
If yes, is it one-to-one?

3) Fill in the table, identify the domain/range, and sketch...

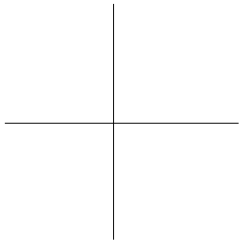
a) $f(x) = 2x^2 + 3$

x	$f(x)$
-2	
-1	
0	
1	
2	



b) $y = \pm\sqrt{x - 2} + 1$

x	y
0	
1	
2	
3	
6	
11	



4) Sketch a possible graph f with the following features:

f is an even function

Domain: [-7, 7]

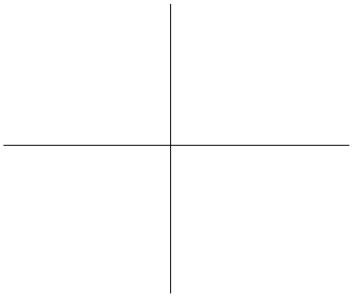
Range: [-5, 10]

$f(0) = -3$

$f(-4) = 1$

increasing on the interval $(0, 2) \cup (5, 7)$

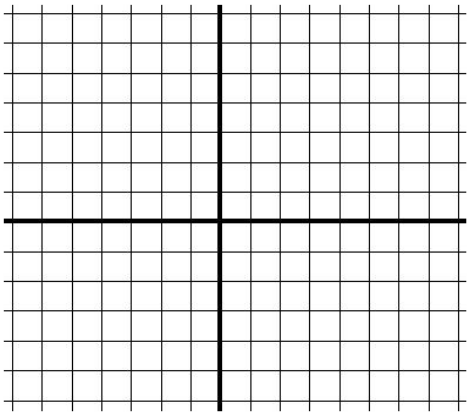
decreasing on the interval $(2, 5)$



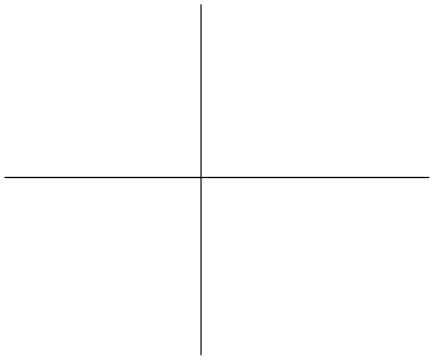
5) What is the domain of $\sqrt{\dfrac{x + 3}{x - 1}}$?

II. Graphing

1) Graph $y < -x$

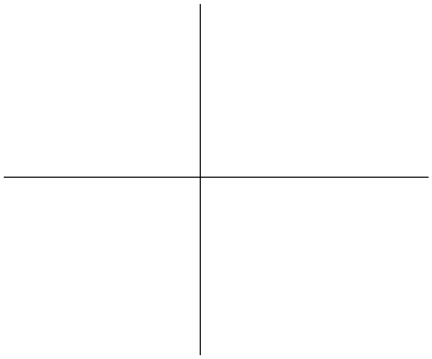


2) Graph a system with NO solutions

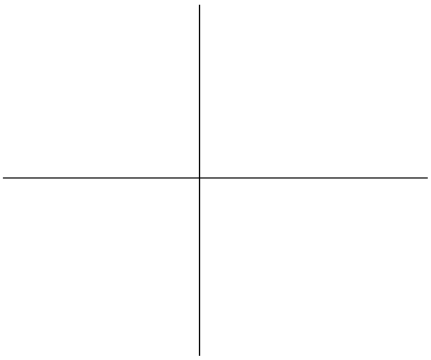


3) Graph $y = 2x + 5$, where the domain is $-3 \leq x \leq 3$

What is the range?

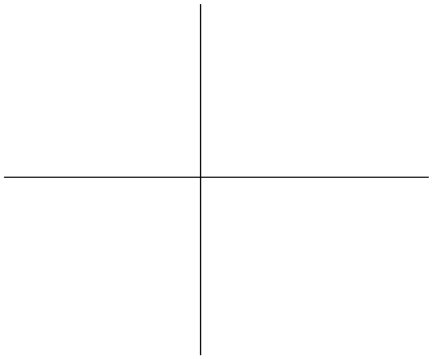


4) Sketch a function whose domain is $(-3, 6)$ and range $(1, 8]$



5) Solve graphically...

$$-|x| + 6 \geq |x + 4|$$



III. Exponents...

1) $7x^2 \cdot 7x^{-2} =$

a) 1

b) 49

c) $49x^{-4}$

d) $\frac{1}{49x^4}$

2) $(8^5)^x \cdot 8^{x^2} = 64^7$

What is x?

3) Fill in the boxes

$(6x^3y^{-4})(3xy)^{\square} = \frac{54x^{\square}}{\square^{\square}}$

4) $5(3x^{-4})^{-2}$

5) $2xy(4xy^3 - xy^3)$

6) $(-6)^0$ vs. -6^0

IV. Order of Operations ("PEMDAS" or "GEMDAS")

1) $6 - (-15 - -3) \div 2^2 =$

2) $2 + 3 \times 4 + 5 =$

3) $16 \div 8(2) =$

4) Which of these is 10?

a) $2 \times 11 - 1 \div 2$

b) $2 \cdot (12 - 2) \div 2$

c) $4/5 \times 50 \div 4$

d) $5^2 - 3 \cdot 5$

e) 2^5

5) $\sqrt{9 + 8(7 - 4) \div 3 \cdot 9}$

6) $12 \times 4 \div 3 \times 2$ vs. $(12 \times 4) \div (3 \times 2)$

7) $\frac{3}{8} + \frac{5}{8} \times 7$

8) $\frac{8 - 2(8 - 2)}{6 - 5(4)}$

V. Solve or Simplify

Algebra Review 5

1) $x^2 - 3x = 10x$

2) $3\sqrt{x} + 2\sqrt{x} = 20$

3) $x^2 + 12 + 43 - 2x = 90$

4) $x^2 - 6x = \frac{1}{2}x + 42$

5) $12^2 + x^2 = 13^2$

6) $2x + 4\frac{1}{2} = 1\frac{1}{4}$

7) $4 - 2(x - 5(3x - 7))$

8) $7 - (x^2 + 2)(x - 5)$

9) $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$

10) Solve for z: $xyz + 8 = z$

11) Solve for y: $ax + 4y + c = 9$

12) $|-3| + |3| + |x| = 4$

13) solve for p:

$i = p + prt$

14) $\frac{3x^2 + 21x - 24}{6x^2 - 6} =$

15) $\frac{y^2 - x^2}{y - x}$

VI. Inequalities

Solve and graph on a number line

1) $x^2 + 6x + 5 > 0$

2) $2x(x - 7)(x - 1) \geq 0$

3) $2|x - 7| + 2 < 8$

4) $-4 < -(x + 7) \leq 5$

If x is chosen from { -3, 0, 4, 6, 12 },

what is the probability x is a solution to $|x - 4| > 3$

a) 1/5

b) 2/5

c) 3/5

d) 4/5

e) none of the above

VII. Exponential Models

1) Growth or Decay?

a) $y = \left(\frac{1}{3}\right)6^{x}$

b) $y = 6\left(\frac{1}{5}\right)^{x}$

2) John has $2200 in an account that increases 7% annually...

A brand new sports car costs $48,000, but it depreciates by 22% annually...

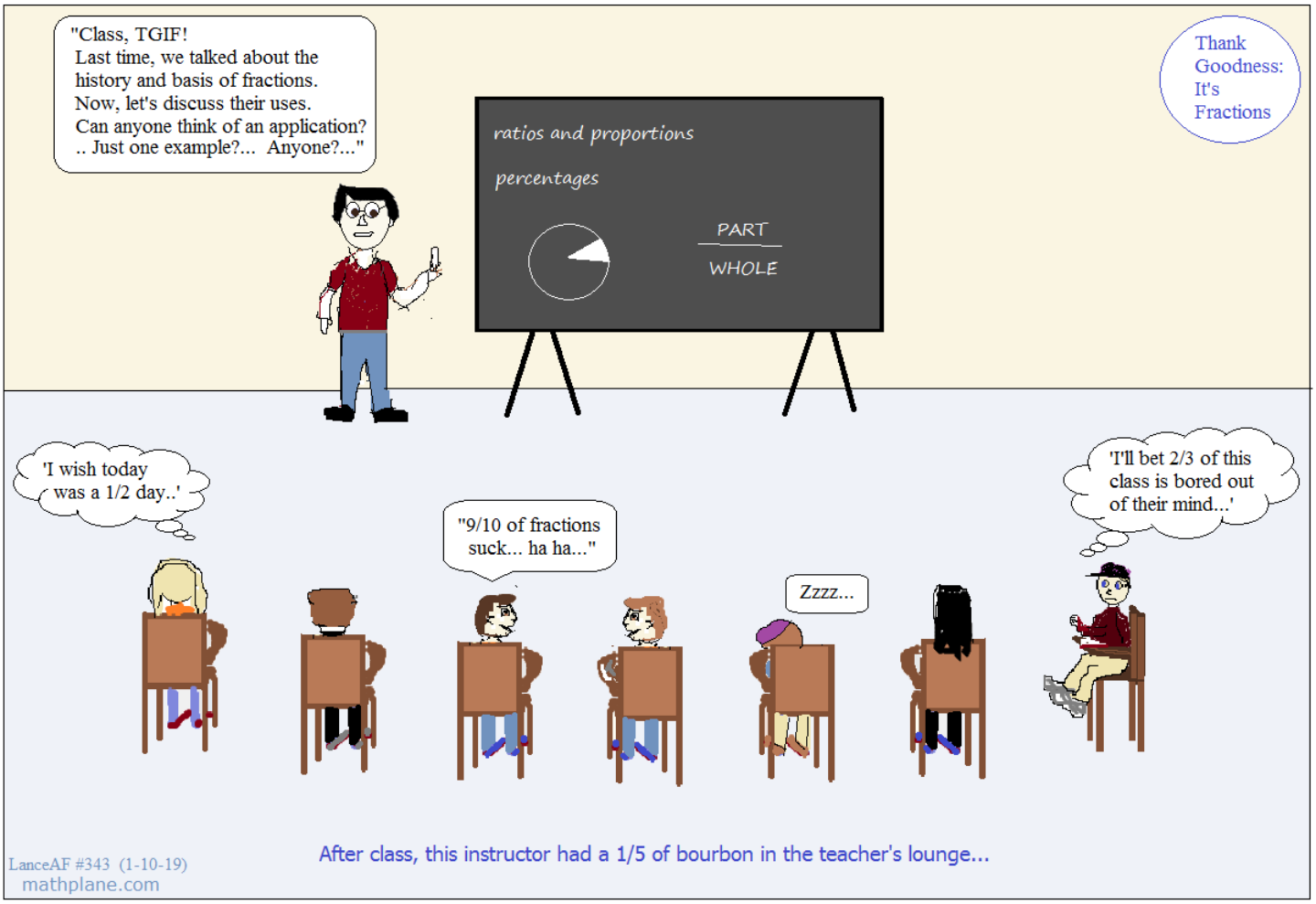
If John is willing to buy the sports car used, when would he be able to afford it?

3) What is the missing value in each table?
Can you identify each function?

x	f(x)
−3	3/2
−2	11/6
−1	13/6
0	5/2
1	

x	g(x)
−1	14
0	9
1	6
2	5
3	6
4	

x	h(x)
−2	-8/9
−1	-2/3
0	0
1	2
2	8
3	


"Class, TGIF!
Last time, we talked about the history and basis of fractions.
Now, let's discuss their uses.
Can anyone think of an application?
.. Just one example?... Anyone?..."
Thank Goodness: It's Fractions
ratios and proportions
percentages
PART
WHOLE
'I wish today was a 1/2 day..'
"9/10 of fractions suck... ha ha..."
Zzzz...
'I'll bet 2/3 of this class is bored out of their mind...'
LanceAF #343 (1-10-19)
mathplane.com
After class, this instructor had a 1/5 of bourbon in the teacher's lounge...

SOLUTIONS-→

SOLUTIONS

I. Interval Notation, Domain/Range, and Functions

1) Write the following in Interval Notation

a) All real numbers $(-\infty, \infty)$

b) All negative numbers $(-\infty, 0)$

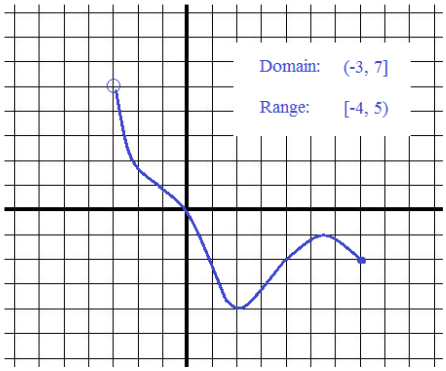
c) $x > 5$ $(5, \infty)$

d) $x \le -2$ $(-\infty, -2]$

e) Rational numbers between -3 and 8 (-3, 8)

2) Identify the domain and range:

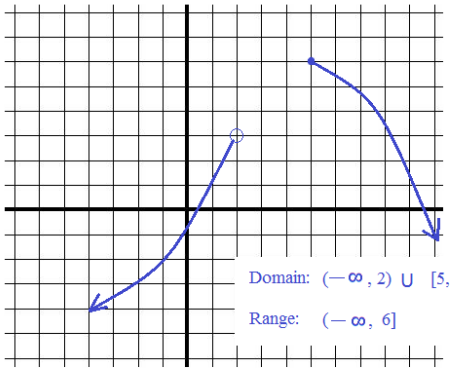
a)



Domain: (-3, 7]

Range: [-4, 5)

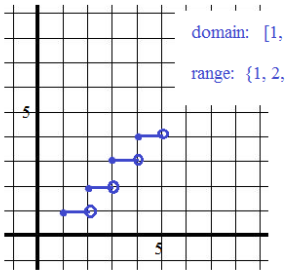
b)



Domain: $(-\infty, 2) \cup [5, \infty)$

Range: $(-\infty, 6]$

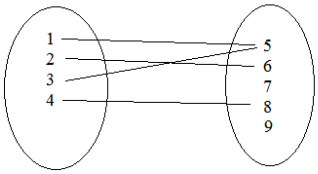
c)



domain: [1, 5)

range: {1, 2, 3, 4}

d)



domain: {1, 2, 3, 4}
range: {5, 6, 8}

Is this a function?
If yes, is it one-to-one?

Yes, function...
Not one-to-one

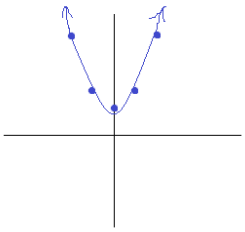
3) Fill in the table, identify the domain/range, and sketch...

a) $f(x) = 2x^2 + 3$

Domain: all real numbers

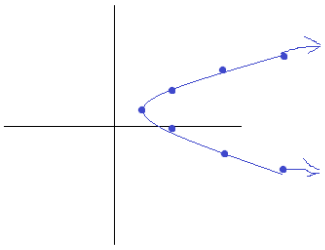
Range: $f(x) \ge 3$

x	f(x)
-2	11
-1	5
0	3
1	5
2	11



b) $y = \pm\sqrt{x - 2} + 1$

x	y
0	DNE
1	Does Not Exist
2	1
3	2 or 0
6	3 or -1
11	4 or -2



4) Sketch a possible graph f with the following features:

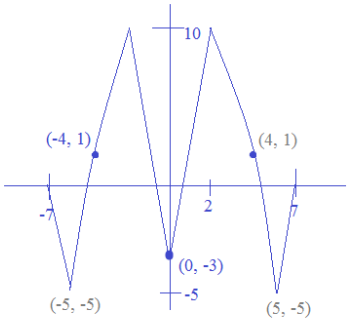
f is an even function "even" -- symmetry over y-axis

Domain: [-7, 7]

Range: [-5, 10]

$f(0) = -3$ increasing on the interval $(0, 2) \cup (5, 7)$

$f(-4) = 1$ decreasing on the interval $(2, 5)$



(one possible answer)

5) What is the domain of $\sqrt{\frac{x+3}{x-1}}$

$x \ne 1$

$\frac{x+3}{x-1} \ge 0$

$x \ge -3$

but, not equal to 1

Two main rules: no negatives under the radical
AND
no zero in the denominator

Domain: $[-3, 1) \cup (1, \infty)$

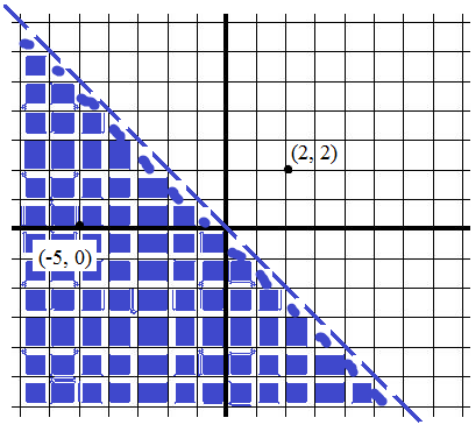
II. Graphing

SOLUTIONS

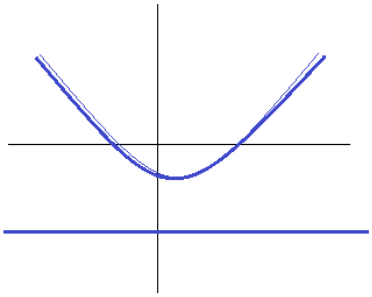
1) Graph $y < -x$

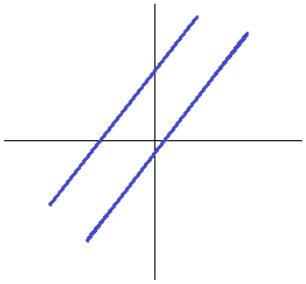
dashed line

At (2, 2), the inequality is false
At (-5, 0), the inequality is true



2) Graph a system with NO solutions

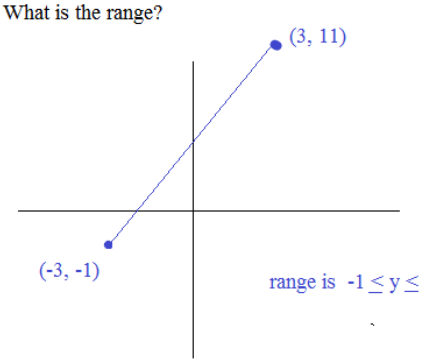




Here are 2 possiblities...

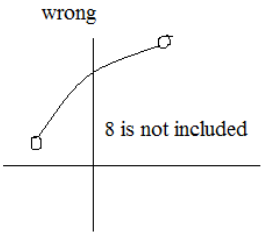
3) Graph $y = 2x + 5$, where the domain is $-3 \leq x \leq 3$

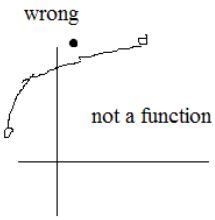
What is the range?

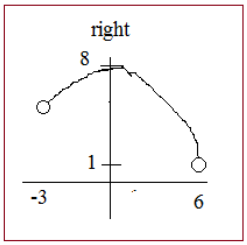


range is $-1 \leq y \leq 11$

4) Sketch a function whose domain is (-3, 6) and range (1, 8]







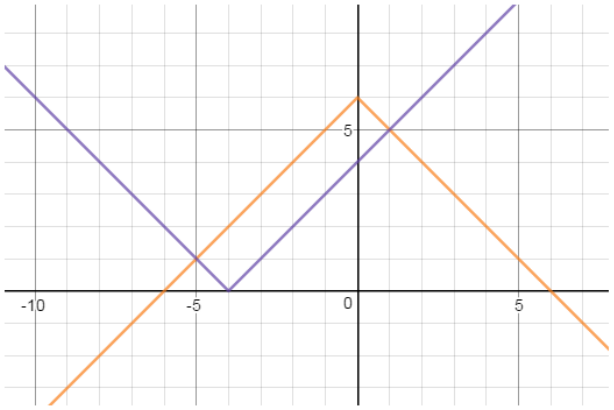
5) Solve graphically...

$$-|x| + 6 \geq |x + 4|$$

Graphically, we can see the intersections occur at x = -5 and 1

The orange graph is above the purple graph on the interval (-5, 1)

[-5, 1] (note: a quick check at x = 0, shows the equation is correct!)



Algebra Review 5

III. Exponents...

SOLUTIONS

1) $7x^2 \cdot 7x^{-2} =$

a) 1

b) 49

c) $49x^{-4}$

d) $\frac{1}{49x^4}$

$49x^0 = 49$

2) $(8^5)^x \cdot 8^{x^2} = 64^7$

What is x?

$8^{5x + x^2} = 8^{14}$

$x^2 + 5x - 14 = 0$

$(x + 7)(x - 2) = 0$

$x = 2$ or -7

3) Fill in the boxes

$(6x^3 y^{-4})(3xy)^{\boxed{2}} = \frac{54x^{\boxed{5}}}{y^{\boxed{2}}}$

must be 2, because $6 \times 3^2 = 54$

4) $5(3x^{-4})^{-2}$

$\frac{5x^8}{9}$

5) $2xy(4xy^3 - xy^3)$

$2xy(3xy^3) = 6x^2y^4$

6) $(-6)^0$ vs. -6^0

1 vs $-1(6)^0 = -1$

IV. Order of Operations ("PEMDAS" or "GEMDAS")

1) $6 - (-15 - -3) \div 2^2 =$

$6 - (-12) / 4 = 9$

2) $2 + 3 \times 4 + 5 =$

$2 + 12 + 5 = 19$

3) $16 \div 8(2) =$

$16 \div 8 \times 2 = 4$

4) Which of these is 10?

a) $2 \times 11 - 1 \div 2$ 21 1/2

b) $2 \cdot (12 - 2) \div 2$ 10

c) $4/5 \times 50 \div 4$ 10

d) $5^2 - 3 \cdot 5$ 10

e) 2^5 32

5) $\sqrt{9 + 8(7 - 4) \div 3 \cdot 9}$

$\sqrt{9 + 8 \times 3 \div 3 \times 9}$

$\sqrt{9 + 72} = 9$

6) $12 \times 4 \div 3 \times 2$ vs. $(12 \times 4) \div (3 \times 2)$

$12 \times 4 = 48$

$48 / 3 = 16$

$16 \times 2 = 32$

$48 / 6 = 8$

7) $\frac{3}{8} + \frac{5}{8} \times 7$

multiply: $\frac{3}{8} + \frac{35}{8}$

add: $\frac{38}{8}$

8) $\frac{8 - 2(8 - 2)}{6 - 5(4)}$

$\frac{8 - 12}{6 - 20} = \frac{2}{7}$

V. Solve or Simplify

SOLUTIONS

Algebra Review 5

1) $x^2 - 3x = 10x$

$x^2 - 13x = 0$

$x(x - 13) = 0$

$x = 13$ or 0

2) $3\sqrt{x} + 2\sqrt{x} = 20$

$5\sqrt{x} = 20$

$\sqrt{x} = 4$

$x = 16$

3) $x^2 + 12 + 43 - 2x = 90$

$x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

$x = -5, 7$

4) $x^2 - 6x = \frac{1}{2}x + 42$

$2x^2 - 13x - 84 = 0$

$(2x - 21)(x + 4) = 0$

$x = 21/2$ or -4

5) $12^2 + x^2 = 13^2$

$x^2 = 169 - 144$

$x = 5$ or -5

6) $2x + 4\frac{1}{2} = 1\frac{1}{4}$

method 1: combine fractions

$2x + \frac{9}{2} = \frac{5}{4}$

$2x = \frac{-13}{4}$

method 2: get rid of fractions

$8x + 18 = 5$

$8x = -13$

$x = -13/8$

$B^2 - 11B + 24$

7) $4 - 2(x - 5(3x - 7))$

$4 - 2(x - 15x + 35)$

$4 - 2x + 30x - 70$

$28x - 66$

8) $7 - (x^2 + 2)(x - 5)$

$7 - (x^3 - 5x^2 + 2x - 10)$

$-x^3 + 5x^2 - 2x + 17$

9) $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$

$\langle (x^2 - 2x) - 8 \rangle \langle (x^2 - 2x) - 3 \rangle$

$(x - 4)(x + 2)(x - 3)(x + 1)$

10) Solve for z: $xyz + 8 = z$

$xyz - z = -8$

$z(xy - 1) = -8$

$z = \frac{-8}{(xy - 1)}$

11) Solve for y: $ax + 4y + c = 9$

$4y = 9 - c - ax$

$y = \frac{9 - c - ax}{4}$

12) $|-3| + |3| + |x| = 4$

$3 + 3 + |x| = 4$

$|x| = -2$

no solution!

13) solve for p:

$i = p + prt$

$i = p(1 + rt)$

$\frac{i}{(1 + rt)} = p$

14) $\frac{3x^2 + 21x - 24}{6x^2 - 6} = \frac{3(x^2 + 7x - 8)}{6(x^2 - 1)}$

$\frac{3(x + 8)(x - 1)}{6(x + 1)(x - 1)}$

and, $x \neq 1$ or -1 $\quad \frac{x + 8}{2(x + 1)}$

15) $\frac{y^2 - x^2}{y - x}$

$\frac{y^2 - x^2}{y - x} = \frac{(y + x)(y - x)}{(y - x)} = y + x$

$\frac{y^2 - x^2}{y - x} \neq \frac{y - x}{1}$

Note: you cannot cancel parts of sums/differences.
Instead, you must cancel factors...

Algebra Review 5

VI. Inequalities

SOLUTIONS

Solve and graph on a number line

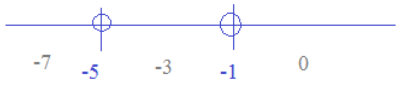
1) $x^2 + 6x + 5 > 0$

test regions...

$(x + 5)(x + 1) > 0$?

set equal to zero... $(x + 5)(x + 1) = 0$

critical values are $x = -1$ and -5



true false true

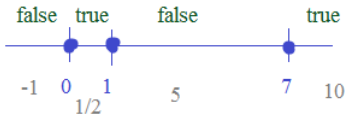
$x < -5$ or $x > -1$

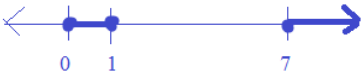
2) $2x(x - 7)(x - 1) \geq 0$

test regions...

The equation is already factored...

critical values are 0, 7, 1





$[0, 1] \cup [7, \infty)$

3) $2|x - 7| + 2 < 8$

solve to find critical values...

test regions...

$2|x - 7| = 6$

$|x - 7| = 3$

$x = 4$ or 10

false true false

$(4, 10)$ or $4 < x < 10$

4) $-4 < -(x + 7) \leq 5$

separate the equations...

$-4 < -(x + 7)$ and $-(x + 7) \leq 5$

$x < -3$ $x \geq -12$

$-12 \leq x < -3$

If x is chosen from { -3, 0, 4, 6, 12 },

what is the probability x is a solution to $|x - 4| > 3$?

a) 1/5

b) 2/5

c) 3/5

d) 4/5

e) none of the above

$|x - 4| = 3$

$x = 1$ or 7

3 out of 5 are solutions....

-3, 0, 4, 6, 12

VII. Exponential Models

Algebra Review 5

SOLUTIONS

1) Growth or Decay?

a) $y = \left(\frac{1}{3}\right)6^x$ growth $y = ab^x$ $b = 6 > 1$

b) $y = 6\left(\frac{1}{5}\right)^x$ decay $y = ab^x$ $b = 1/5 < 1$

2) John has $2200 in an account that increases 7% annually...

A brand new sports car costs $48,000, but it depreciates by 22% annually...

If John is willing to buy the sports car used, when would he be able to afford it?

Exponential Model of John's account...

7% growth

t = time in years

initial value: $2200

$A_J = 2200(1.07)^t$

Exponential Model of the Sport's car's value...

22% decay

t = time in years

intial value: $48,000

$A_C = 48{,}000(.78)^t$

When are the values equal?

$2200(1.07)^t = 48000(.78)^t$

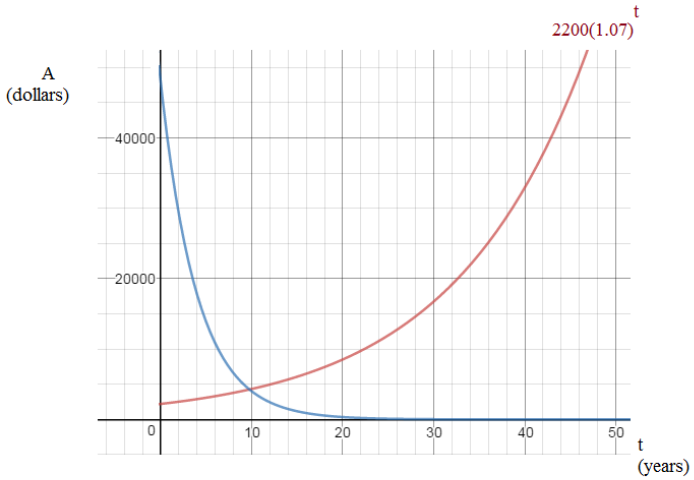
$(1.07)^t = 21.8182(.78)^t$

$(1.37179)^t = 21.8182$

$\log_{(1.37179)} 21.8182 = t$ $\quad \frac{\log(21.8182)}{\log(1.37179)} = t$

$t = 9.752$ (approximately)

John would have to wait almost 10 years before he could buy that car!



the intersection is when John has enough money to buy the sports car...

3) What is the missing value in each table?
Can you identify each function?

x	$f(x)$
−3	3/2
−2	11/6
−1	13/6
0	5/2
1	17/6

This has the pattern of an arithmetic sequence.

(linear equation with slope +1/3)

$f(1) = 17/6$

$f(x) = \frac{1}{3}x + \frac{5}{2}$

x	$g(x)$
−1	14
0	9
1	6
2	5
3	6
4	9

$g(4) = 9$

This is a quadratic, so it has vertical symmetry

$g(x) = (x-2)^2 + 5$

x	$h(x)$
−2	-8/9
−1	-2/3
0	0
1	2
2	8
3	26

If you shift the function UP 1 unit, you see a pattern!

(-2, 1/9) (-1, 1/3) (0, 1) (1, 3) (2, 9)

Notice, the common ratio is 3!

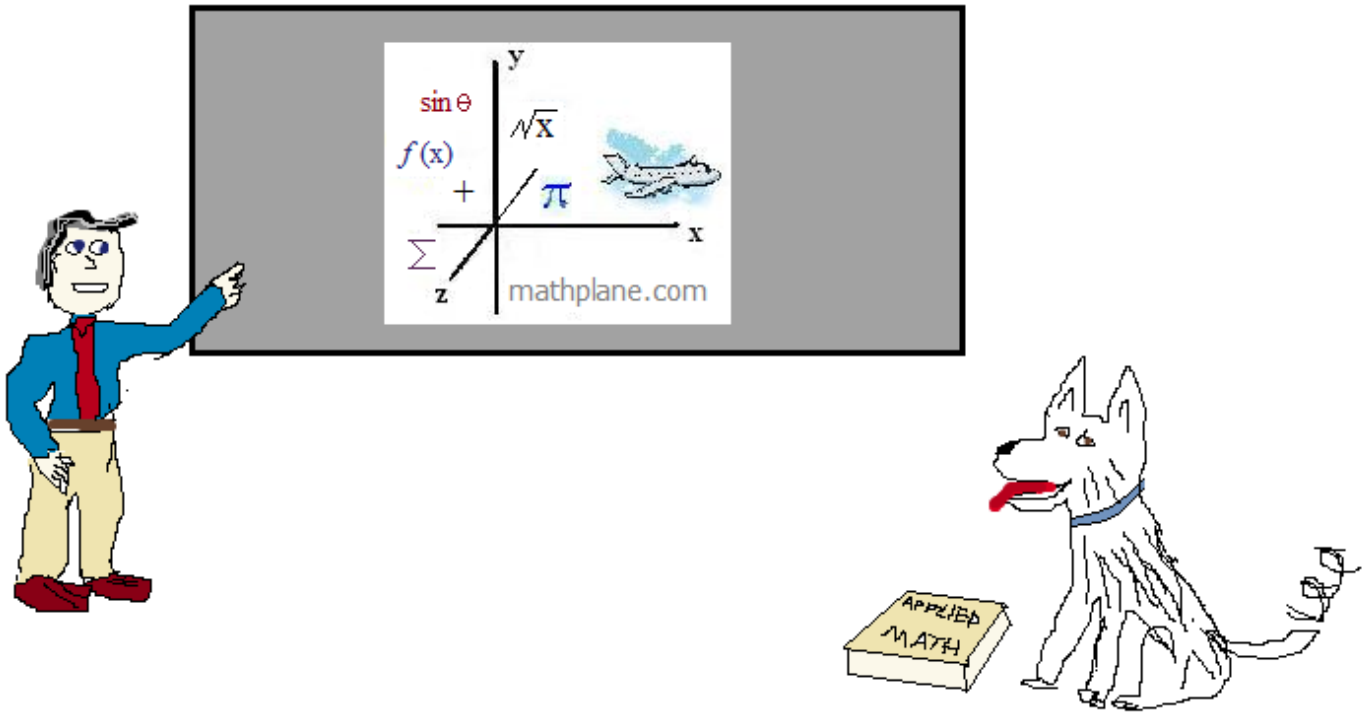
This is an exponential function:

$h(x) = 3^x - 1$ $\quad h(3) = 26$

Thanks for visiting! (Hope the review helped.)

If you have questions, suggestions, or requests, let us know.

Cheers



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