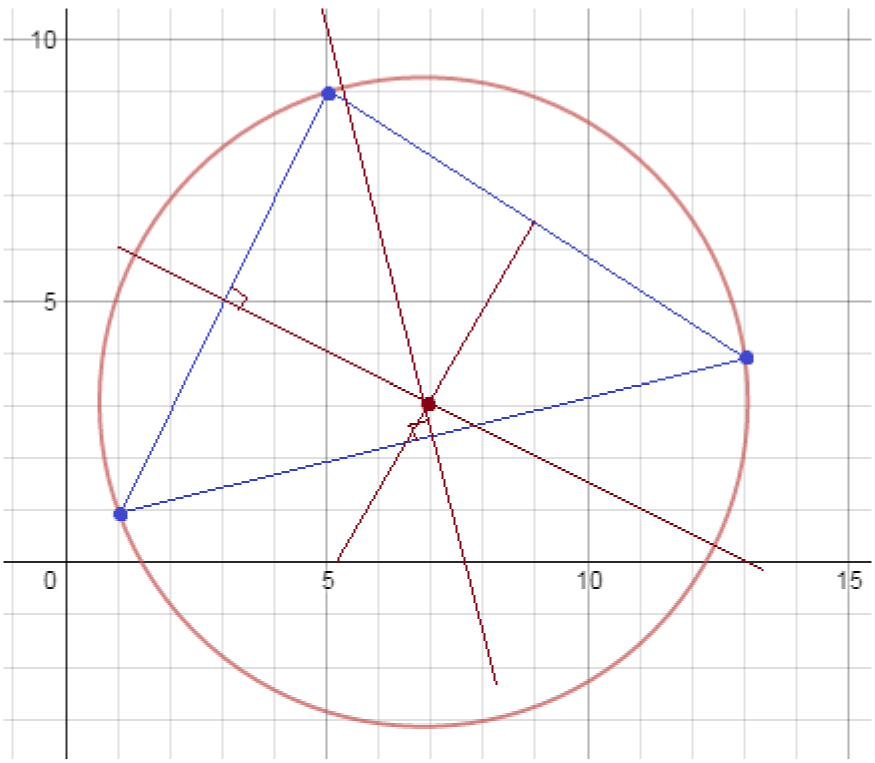
# Conics V: More Advanced

## Examples and Practice Test (with solutions)




*Including word problems, graphing, geometry and algebra applications, Pascal's Theorem and more…*

Mathplane.com

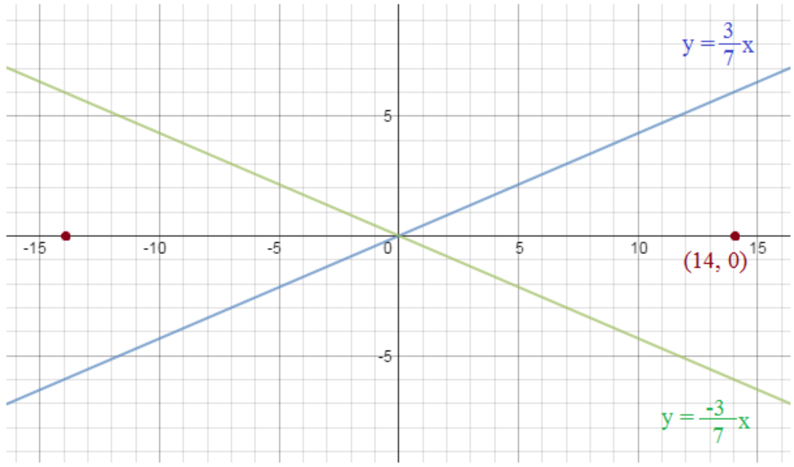
mathplane.com

*Example:* A hyperbola has asymptotes $y = \frac{3}{7}x$ and $y = \frac{-3}{7}x$

If one vertex is (14, 0), what is the equation of the hyperbola?

First, we notice the center is the origin (0, 0) --- (this is the intersection of the asymptotes)

Then, since one vertex is (14, 0), the other vertex must be (-14, 0) (vertices are equidistant from the center)




So, what are the "a" and "b" values (i.e. the "dimensions of the box")?

Since the asymptotes are $\frac{3}{7}$, $\frac{(rise)}{(run)}$ or $\frac{y}{x}$

the *ratio* of box sides must be 3:7

$$\frac{b}{a} = \frac{3}{7} \quad \text{or} \quad \frac{r_y}{r_x} = \frac{3}{7}$$

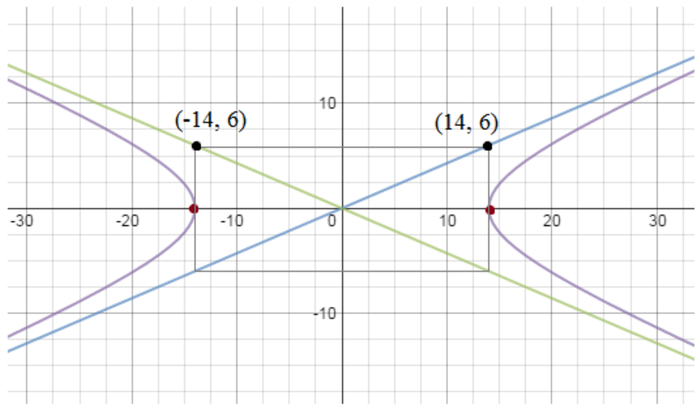
***But, what multiple of 3:7 ?

Since the vertex is (14, 0),

$$\frac{3}{7} = \frac{b}{14} \qquad b = 6$$

$$\frac{(x-0)^2}{14^2} - \frac{(y-0)^2}{6^2} = 1$$

$$\boxed{\frac{x^2}{196} - \frac{y^2}{36} = 1}$$




---

*Example:* Write the equation of a graph whose path of points moves so that the sum of the distances from F(−2, 4) and F(8, 4) is 24.

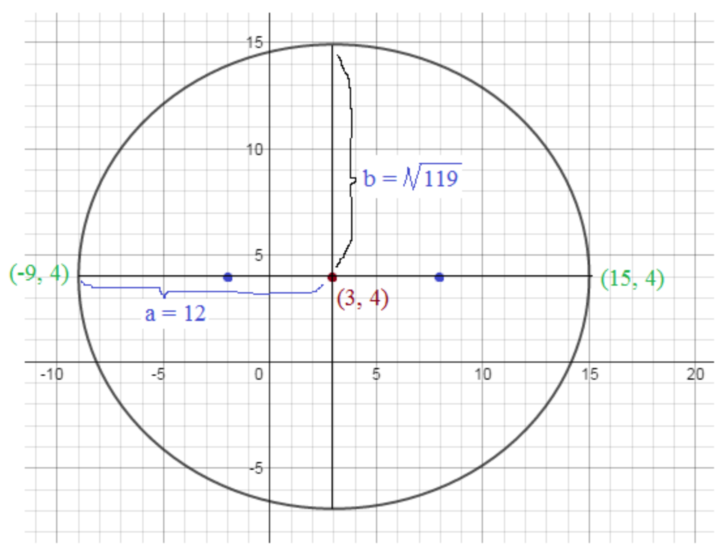
This path of points describes an ellipse...

First, we'll determine the center... (the midpoint between the two foci) (3, 4)

What are the vertices?

The foci have the same y-value, so this is a horizontal ellipse... And, since the sum of the distance is 24, that is the length of the major axis!

Therefore, the semi-major axis ("a" value) is 12




$a^2 - b^2 = c^2$

$12^2 - b^2 = 5^2$

$b = \sqrt{119}$

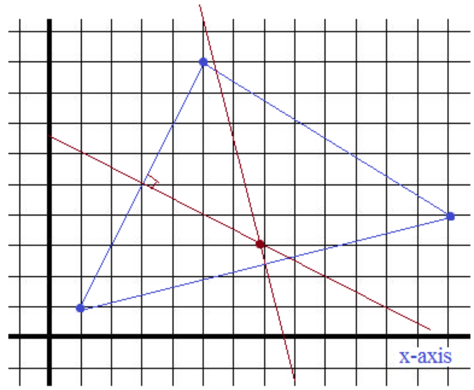
$$\boxed{\frac{(x-3)^2}{144} + \frac{(y-4)^2}{119} = 1}$$

*Example:* What is the equation of a circle containing points (1, 1), (5, 9), and (13, 4)?

Since we cannot assume that 2 of these points are endpoints of the same diameter, we must use another approach to find the radius and center...

Using Geometry: the perpendicular bisectors will intersect at the circumcenter (i.e the center of a circle that circumscribes the triangle)

Standard form of a circle: $(x-h)^2 + (y-k)^2 = r^2$




The perpendicular bisector of (1, 1) and (5, 9)

midpoint: (3, 5)

slope of line: 2 slope of perpendicular bisector: $-1/2$

$$y - 5 = (-1/2)(x - 3)$$

$$y = \frac{-1}{2}x + \frac{13}{2}$$

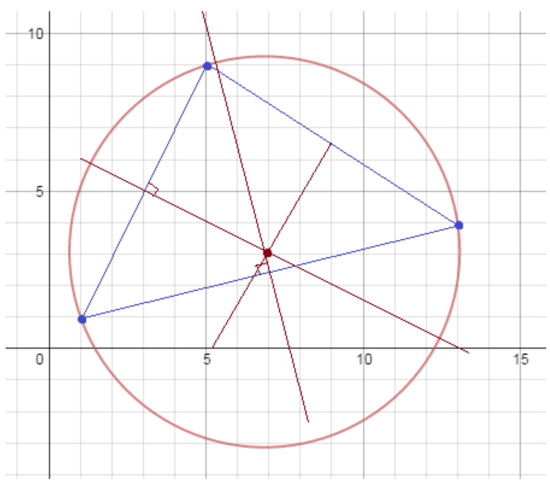
The perpendicular bisector of (1, 1) and (13, 4)

midpoint: (7, 5/2)

slope of line: 1/4 slope of perpendicular bisector: $-4$

$$y - 5/2 = (-4)(x - 7)$$

$$y = -4x + 30.5$$




the intersection of the perpendicular bisectors:

$$\frac{-1}{2}x + \frac{13}{2} = -4x + 30.5$$

$$3.5x = 24$$

$$x = 6.86 \qquad \text{then, } y = 3.07$$

center: (6.86, 3.07)

Finally, what is the distance from (6.86, 3.07) to each of the 3 points?

$$\text{distance} = \sqrt{(6.86 - 1)^2 + (3.07 - 1)^2} = 6.21$$

radius

circle: $(x - 6.86)^2 + (y - 3.07)^2 = 38.56$

---

*Example:* What is the vertex of the following equation?

$$y^2 - y - x + 6 = 0$$

Complete the square and transform to standard form

$$y^2 - y + \frac{1}{4} - x + 6 - \frac{1}{4} = 0$$

$$(y - \frac{1}{2})(y - \frac{1}{2}) - x + \frac{23}{4} = 0$$

$$-x = -(y - \frac{1}{2})^2 - \frac{23}{4}$$

$$x = (y - \frac{1}{2})^2 + \frac{23}{4}$$

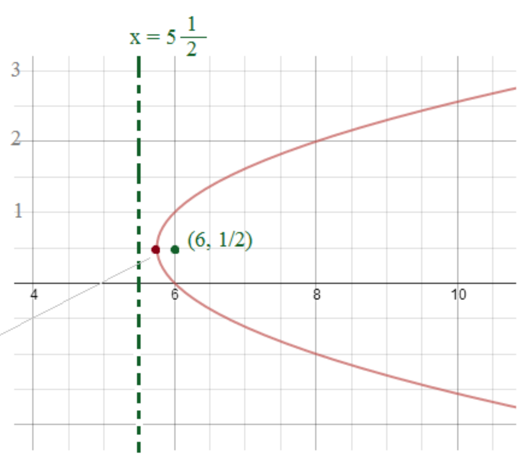
vertex: $(\frac{23}{4}, \frac{1}{2})$

$$p = \frac{1}{4a} \qquad p = \frac{1}{4}$$

horizontal parabola:

$x = a(y-k)^2 + h$

(h, k) is the vertex



*Example:* Find the domain and range of $2x^2 + y^2 + 8x - 2y = 11$

First, convert to standard form (by completing the square)

$$2x^2 + 8x \quad + y^2 - 2y \quad = 11$$

$$2(x^2 + 4x + 4) + y^2 - 2y + 1 = 11 + 2(4) + 1$$

$$2(x+2)^2 + (y-1)^2 = 20$$

$$\frac{(x+2)^2}{10} + \frac{(y-1)^2}{20} = 1$$

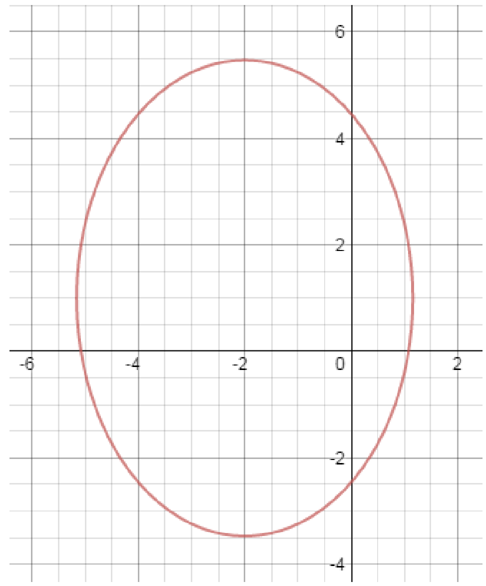
"a" $r_x = \sqrt{10}$

"b" $r_y = \sqrt{20}$

Center: (-2, 1)

so, vertices are $(-2, 1 + 2\sqrt{5})$
$(-2, 1 - 2\sqrt{5})$

co-vertices are $(-2 + \sqrt{10}, 1)$
$(-2 - \sqrt{10}, 1)$



The vertices are the maximum and minimum y values
and, the co-vertices are the maximum and minimum x values

Domain: $[-2 - \sqrt{10}, -2 + \sqrt{10}]$ or approximately $[-5.16, 1.16]$

Range: $[1 - 2\sqrt{5}, 1 + 2\sqrt{5}]$ or approximately $[-3.47, 5.47]$

---

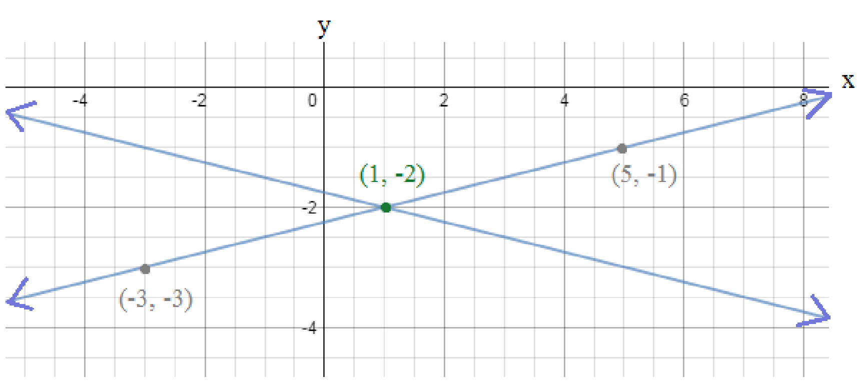
*Example:* Describe and graph the following conic: $16y^2 - x^2 + 2x + 64y + 63 = 0$

Complete the square to convert to standard form (hyperbola)

$$16(y^2 + 4y \quad) - (x^2 - 2x \quad) = -63$$

$$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -63 + 64 - 1$$

$$16(y+2)^2 - (x-1)^2 = 0$$ ← Degenerate conic! two intersecting lines (at point (1, -2))



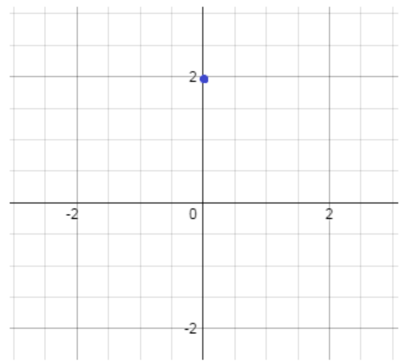
Degenerate Conics

What are they? Equations of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

*Example:* $x^2 + (y-2)^2 = 0$

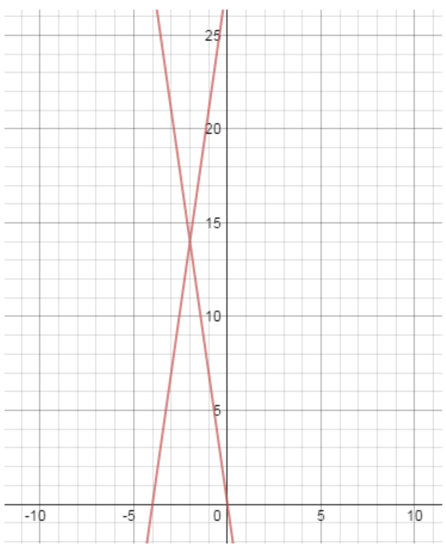
"Circle" with radius 0

It's a point at (0, 2)...




*Example:* $49x^2 - y^2 + 196x + 28y = 0$

"Hyperbola" consisting of just 2 intersecting lines...




$49(x^2 + 4x + 4\quad) - (y^2 - 28y + 196) = 0 + 196 - 196$

$49(x+2)^2 - (y-14)^2 = 0$

factoring difference of squares

$(7(x+2) + (y-14))(7(x+2) - (y+14)) = 0$

$(7(x+2) + (y-14)) = 0 \qquad y = -7x$

$(7(x+2) - (y-14)) = 0 \qquad y = 7x + 28$

They intersect at (-2, 14)

*Example:* $4x^2 + y^2 = -16$

This "ellipse" has no solutions.. Does not exist

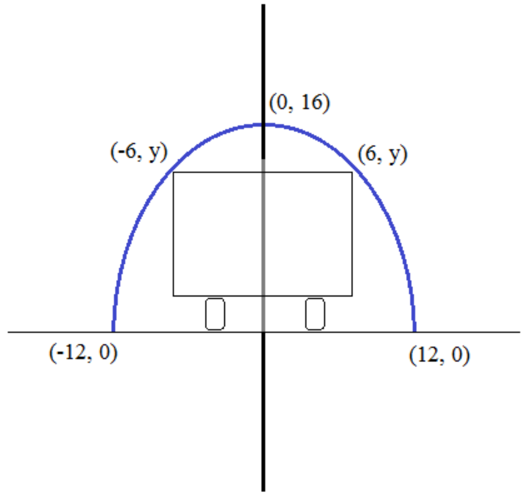
$\frac{x^2}{-4} + \frac{y^2}{-16} = 1$ "Corrupted Conic"

semi-axes are imaginary numbers!

*Example:* A tunnel, shaped as a semi-ellipse, is 16 feet high in the center and 24 feet wide...
If a truck (with a rectangular cargo hold) is 12 feet wide, what is the height of the tallest truck that could fit through the tunnel? (Assume the tunnel is one lane.)

Step 1: Set up the diagram

We set up the diagram with the center of the semi-ellipse at the origin...




Step 2: Find equation of ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where a is the semi-major axis
b is the semi-minor axis
and (h, k) is the center

a = 12
b = 16

center: (0, 0)

$$\frac{x^2}{144} + \frac{y^2}{256} = 1$$

Step 3: Find y-coordinate/height of 12 feet wide truck

Since the truck is 12 feet wide, the best fit would be down the middle, leaving 6 feet on each side....

$$\frac{6^2}{144} + \frac{y^2}{256} = 1 \qquad \frac{y^2}{256} = \frac{108}{144}$$

$$\frac{y^2}{256} = \frac{3}{4} \qquad y^2 = 192$$

$$y \approx 13.86$$

---

*Example:* An elliptical shaped sports stadium has a major axis of 800 feet and a minor axis of 500 feet.
The ends of the field are positioned at the foci of ellipse.
How long is the field?

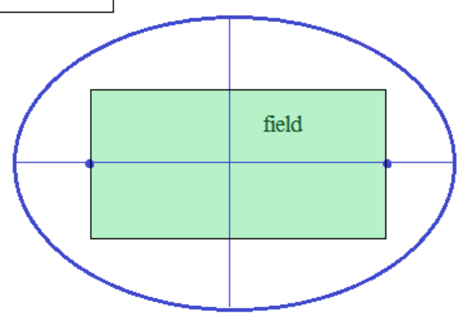
major semi-axis (a) = 400

minor semi-axis (b) = 250

foci (distance c from center): $c^2 = a^2 - b^2$

$c^2 = 160{,}000 - 62{,}500$

$c = 312.25$ (approximately)

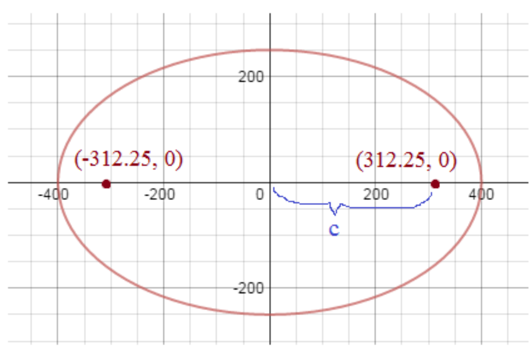


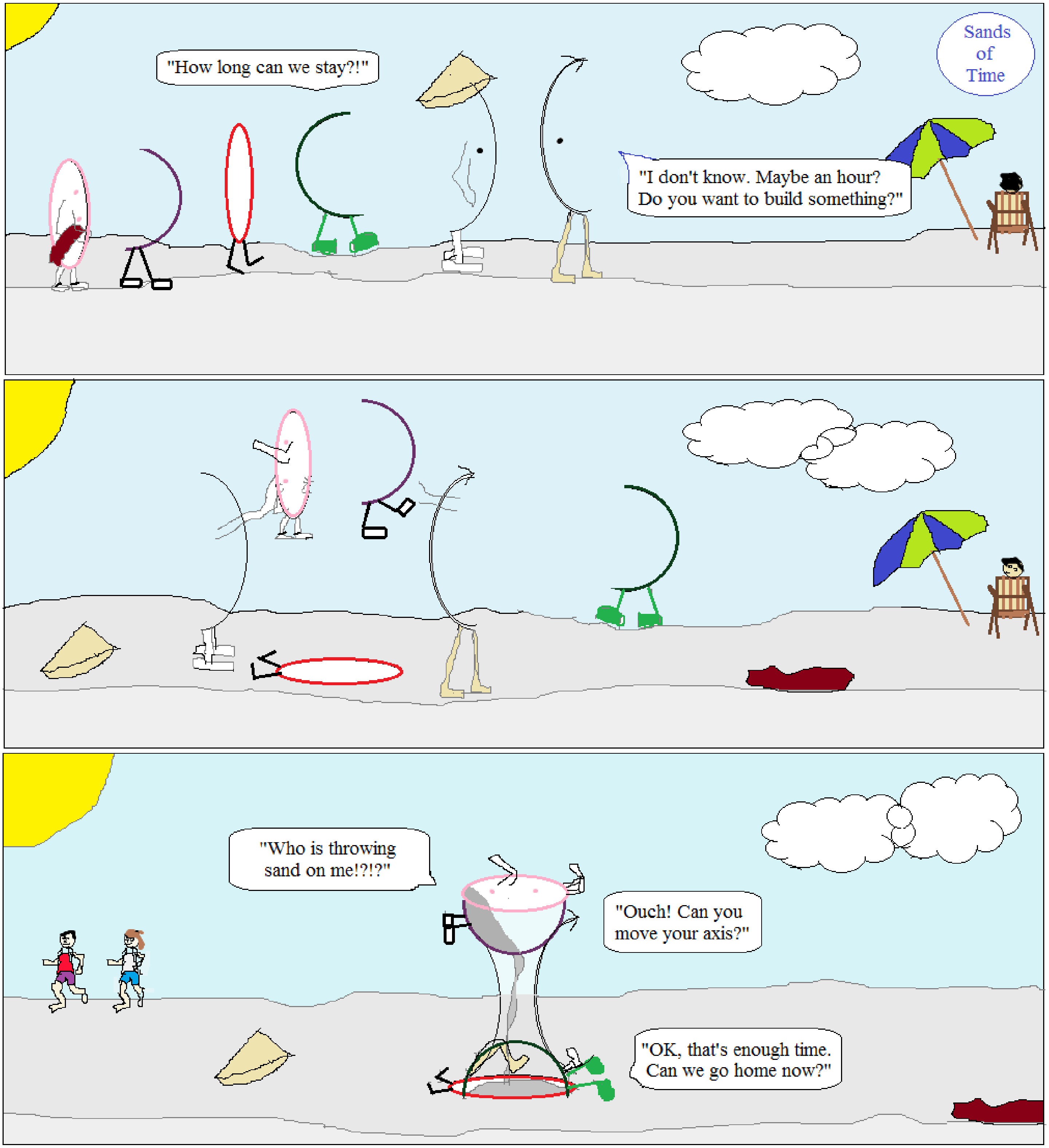

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{160000} + \frac{y^2}{62500} = 1$$

So, the distance between foci is

2 x 312.25 = 624.5 feet






Harry Jr. and the remaining Balance of Conicks spend an Hour at Glasse Beach...

LanceAF #151 (8-14-14)
mathplane.com

Practice Test-→

"Corrupted Conics": What is wrong with these?

Describe what is wrong with each conic:

1) Ellipse vertices: $(6, -1)$ $(6, 7)$

foci: $(6, -3)$ $(6, 10)$

2) Parabola vertex: $(8, -2)$

directrix: $y = -4$

focus: $(8, 1)$

3) Hyperbola $r_x = 10$ (semi-axis a) $r_y = 6$ (semi-axis b) $c = 4$

4) Circle $3x^2 + 12x + 3y^2 - 18y + 64 = 0$

5) Ellipse foci: (-2, 7) and (-2, -11)
major axis length: 26
co-vertices: (-8, 2) and (4, 2)

1) For the following equation, $16x^2 - 9y^2 - 96x + 288 = 0$

Conics V Practice Test

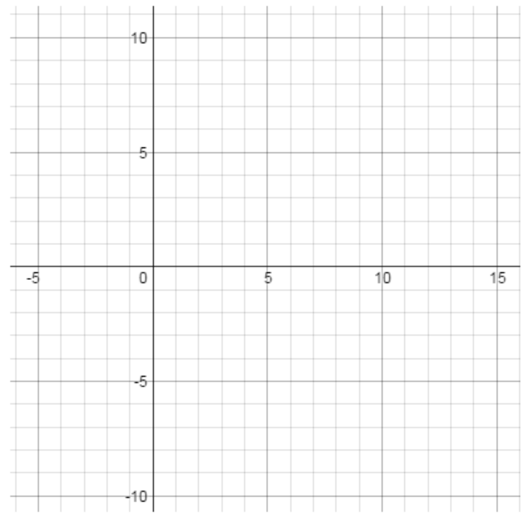
determine the Center:

Vertices:

Foci:

Asymptotes

Then, graph...



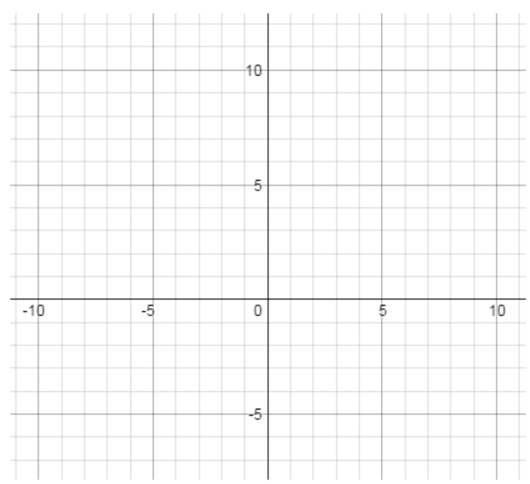

2) Find the center, focus, and directrix of the parabola: $y^2 = 4(x + 2y)$

Then, graph...

Center:

Focus:

Directrix:



3) Determine the equation of an ellipse with these characteristics:

major axis = 12

minor axis = 8

foci are on the x-axis

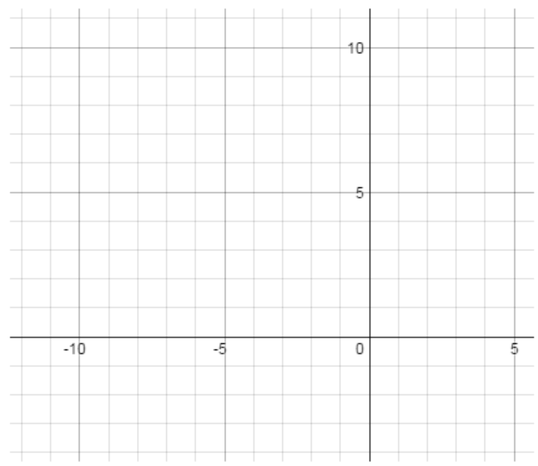
center is the origin

4) What is the length of the minor axis?

$$\frac{(x-2)^2}{25} + \frac{4(y+3)^2}{9} = 1$$

5) What are the x and y-intercepts of $(x+5)^2 + (y-4)^2 = 25$

Graph to confirm your answer!

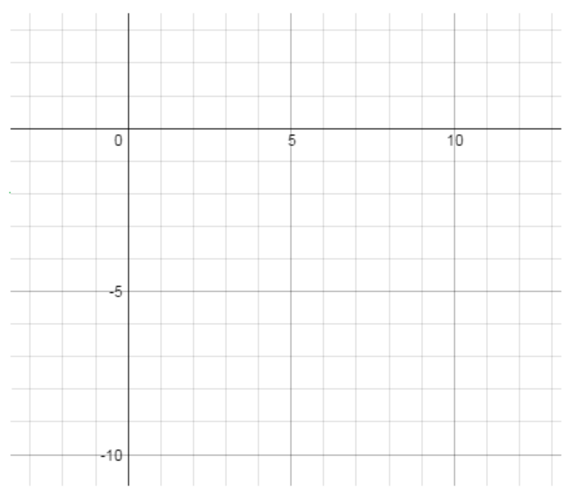



6) Find a circle with center (5, -3) that is

a) tangent to the x-axis

b) tangent to the y-axis

(optional: graph the two circles)

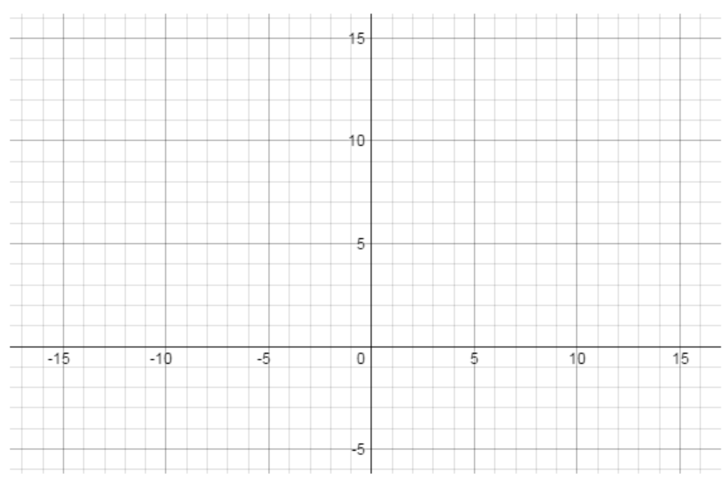


7) What is the equation of a circle containing points (2, 2), (8, 2), and (8, 12)?

8) Given: Focus: (-2, 3)

Directrix: y = -3

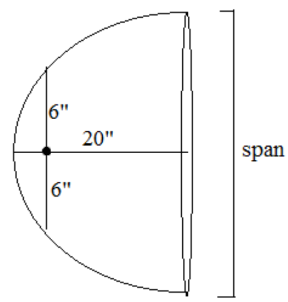
Find and the graph the equation of the parabola.



Conics V Practice Test

9) What is the equation(s) of a hyperbola with asymptotes $(y + 6) = \pm\frac{4}{3}(x - 2)$

10) A light has a parabolic reflector with focal diameter of 12".
If the depth of the reflector is 20" (from focus to edge), what is the width of the span?



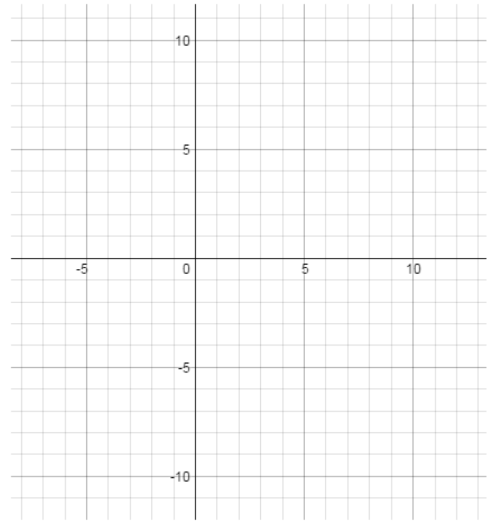

11) Find the equation of a circle with center (3, 6) and *tangent* to the line x + 3y = 30

Conics V Practice Test

12) Write the equation of the parabola with vertex at the origin, passes through (2, -4), and is

a) symmetric to the y-axis

b) symmetric to the x-axis




13) Write the equation of a graph whose path of points moves so that the sum of distances from F(4, 3) and F(4, −7) is 28

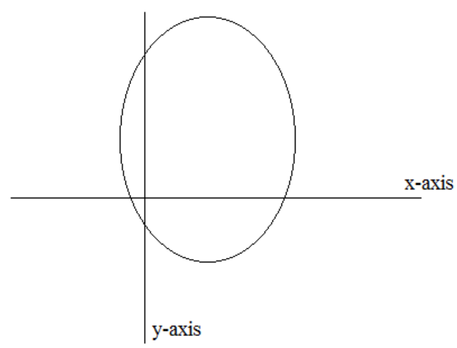
14) Which of the following is a possible equation for the graph? (Justify your answer)

a) $x^2 - 4x + 4y^2 - 40y = -100$

b) $4x^2 + 16x + y^2 + 10y + 40 = 0$

c) $4x^2 - 16x = -y^2 + 10y - 40$

d) $-16x + 4x^2 = y^2 - 10y + 40$




15) What are the domain and range of the conic?

$$\frac{(y+3)}{16} - \frac{(x-4)}{25} = 1$$

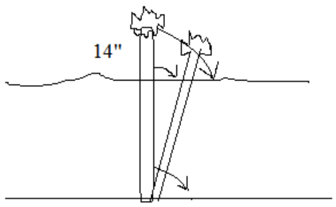
Conics Word Problems

1) Standing at one spot (focus) in a whispering gallery, 8 feet from the wall, a friend stands 100 feet away.
What is the length of the gallery?
What is the height of the gallery (at the middle)?

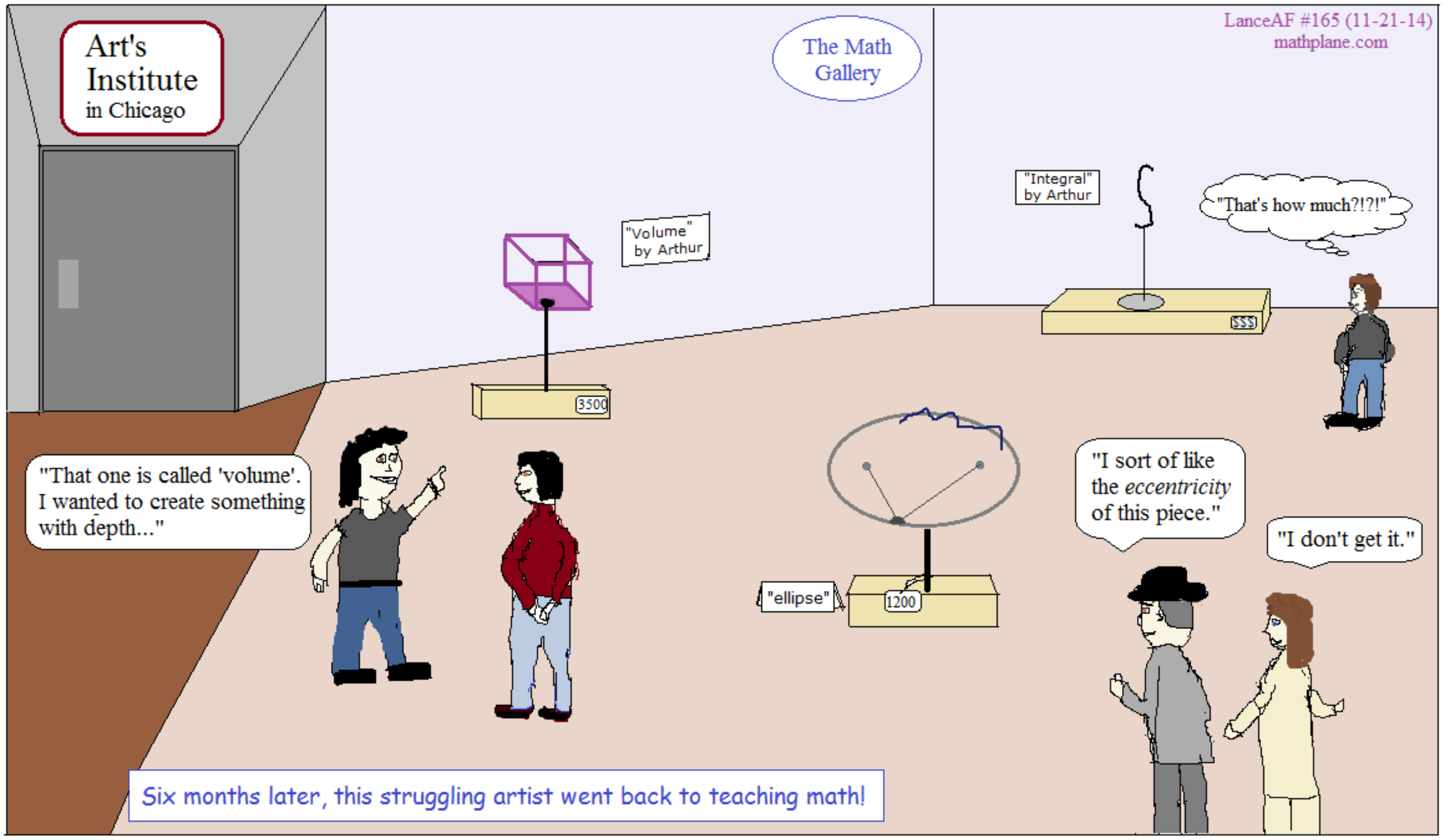
2) An oval (elliptical) racetrack is 80 meters long and 40 meters wide.
What is the width of the racetrack 10 meters from the side?

3) The ends of a sagging hammock are attached to 2 trees, each 60 inches above the ground.
The distance between the trees is 108 inches.
If the middle of the hammock is touching the ground, where is the hammock 30 inches above the ground?

4) A partially submerged plant has a flower 14 inches above the surface of the water.
If you pull the top of the stem 25 inches toward you, the rooted stem's flower touches the surface.
How deep in the water is the plant?




5) A parabolic shaped bridge has a maximum height of 20 feet and a span of 70 feet.
Find the height of the bridge 5 feet, 10 feet, and 25 feet from the center.


Art's
Institute
in Chicago
The Math
Gallery
LanceAF #165 (11-21-14)
mathplane.com
"Volume"
by Arthur
3500
"Integral"
by Arthur
"That's how much?!?!"
$$$
"That one is called 'volume'. I wanted to create something with depth..."
"ellipse"
1200
"I sort of like the *eccentricity* of this piece."
"I don't get it."
Six months later, this struggling artist went back to teaching math!

Solutions -→

"Corrupted Conics": What is wrong with these?

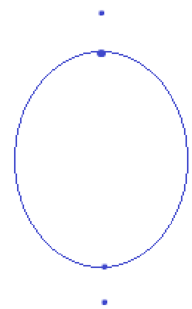
SOLUTIONS

Describe what is wrong with each conic:

1) Ellipse vertices: $(6, -1)$ $(6, 7)$

foci: $(6, -3)$ $(6, 10)$

The foci lie outside the vertices!



2) Parabola vertex: $(8, -2)$

directrix: $y = -4$

focus: $(8, 1)$

The distance (c) between the vertex and focus is 3 units...
and, the distance (c) between the vertex and the directrix is 2 units...

They must be equal!

3) Hyperbola $r_x = 10$ $r_y = 6$ $c = 4$

semi-axis a semi-axis b

$$c^2 = a^2 - b^2$$

$$= 100 - 36$$

$$c = 8$$

4) Circle $3x^2 + 12x + 3y^2 - 18y + 64 = 0$

$$3(x^2 + 4x + 4) + 3(y^2 - 6y + 9) = -64 + 12 + 27$$

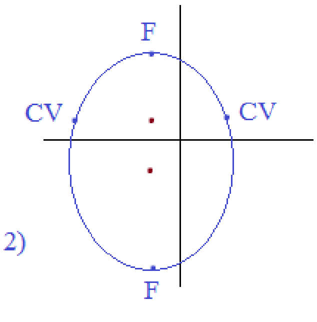
$$3(x + 2)^2 + 3(y - 3)^2 = -25$$

The radius is negative!

5) Ellipse foci: (-2, 7) and (-2, -11)
major axis length: 26
co-vertices: (-8, 2) and (4, 2)

The center is inconsistent!

the midpoint of the foci is (-2, -2)
but, the midpoint of the co-vertices is (-2, 2)



Conics V Practice Test

SOLUTIONS

1) For the following equation, $16x^2 - 9y^2 - 96x + 288 = 0$

determine the Center:

Vertices:

Foci:

Asymptotes

Then, graph...

Step 1: Convert to Standard Form (complete the square)

$$16x^2 - 96x \quad - 9y^2 = -288$$

$$16(x^2 - 6x + 9) - 9y^2 = -288 + 16(9)$$

$$16(x-3)^2 - 9y^2 = -144$$

$$\frac{9y^2}{144} - \frac{16(x-3)^2}{144} = 1$$

$$\boxed{\frac{y^2}{16} - \frac{(x-3)^2}{9} = 1}$$

Observation: The right side of equation is negative, so when we change to 1, it'll 'reverse' the hyperbola)

Step 2: Identify all the parts

Since y is positive, it is a 'vertical hyperbola' (faces up and down)

$b = \sqrt{16} = 4$ $\qquad h = 3$

$a = \sqrt{9} = 3$ $\qquad k = 0$

$c = \sqrt{a^2 + b^2} = 5$

Step 3: Answer

Center: (3, 0)

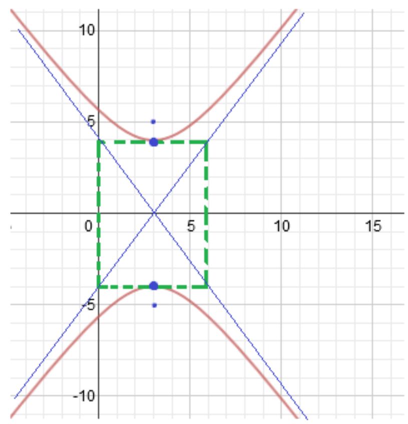
Vertices: 4 units above and below the center
(3, 4) (3, -4)

Foci: 5 units above and below the center
(3, 5) (3, -5)

Asymptotes: slope -- +b/a and -b/a 4/3 and -4/3
point -- use the center (3, 0)

equation of asymptote lines: $y - 0 = \frac{4}{3}(x-3)$ $\qquad y - 0 = \frac{-4}{3}(x-3)$

$$\boxed{y = \frac{4}{3}x - 4 \quad \text{and} \quad y = \frac{-4}{3}x - 4}$$




2) Find the center, focus, and directrix of the parabola: $y^2 = 4(x + 2y)$

Then, graph...

Center: (-4, 4)

Focus: (-3, 4)

Directrix: x = -5

Expand and change to standard form:

$$y^2 - 8y = 4x$$

$$y^2 - 8y + 16 = 4x + 16$$

$$(y-4)^2 = 4(x+4)$$

$4p = 4$

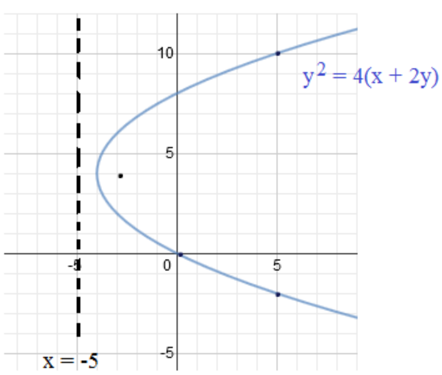
$p = 1$

horizontal parabola:

$(y-k)^2 = 4p(x-h)$

(h, k) is the vertex

Since x is positive, the parabola opens up to the right...




Quick check: (5, 10), (5, -2), (0, 0) are all points on the graph, and are points that algebraically fit in the equation..

3) Determine the equation of an ellipse with these characteristics:

SOLUTIONS



major axis = 12 — If major axis is 12, the a value is 6.

minor axis = 8 — If the minor axis is 8, the b value is 4

foci are on the x-axis — Since foci are on the x-axis, the ellipse is 'horizontal'

center is the origin

$$\frac{(x-0)^2}{6^2} + \frac{(y-0)^2}{4^2} = 1 \qquad \boxed{\frac{x^2}{36} + \frac{y^2}{16} = 1}$$

4) What is the length of the minor axis?

$$\frac{(x-2)^2}{25} + \frac{4(y+3)^2}{9} = 1$$

major axis

$$\frac{(y+3)^2}{\frac{9}{4}}$$

ANSWER: $\sqrt{\frac{9}{4}} \times 2 = \frac{3}{2} \times 2 = \boxed{3}$

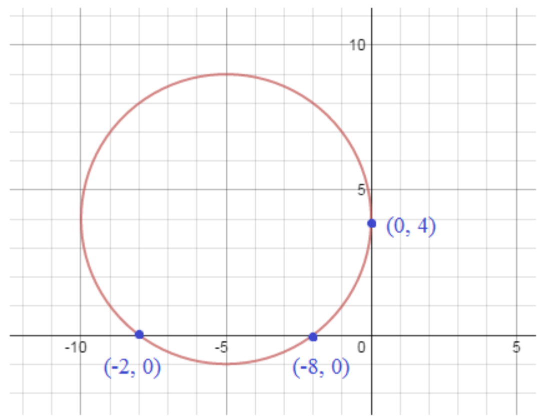
$R_y$ or semi-axis

5) What are the x and y-intercepts of $(x+5)^2 + (y-4)^2 = 25$

Graph to confirm your answer!

Since both terms are squared, positive and have the same coefficient, this is standard form of a circle...

center: (-5, 4) radius: 5




To find x-intercept(s), set y = 0 and solve

$(x+5)^2 + (0-4)^2 = 25 \qquad (x+5)^2 = 9$

$x + 5 = \pm 3$

$\boxed{(-2, 0) \text{ and } (-8, 0)}$

To find y-intercept(s), set x = 0 and solve

$(0+5)^2 + (y-4)^2 = 25 \qquad (y-4)^2 = 0$

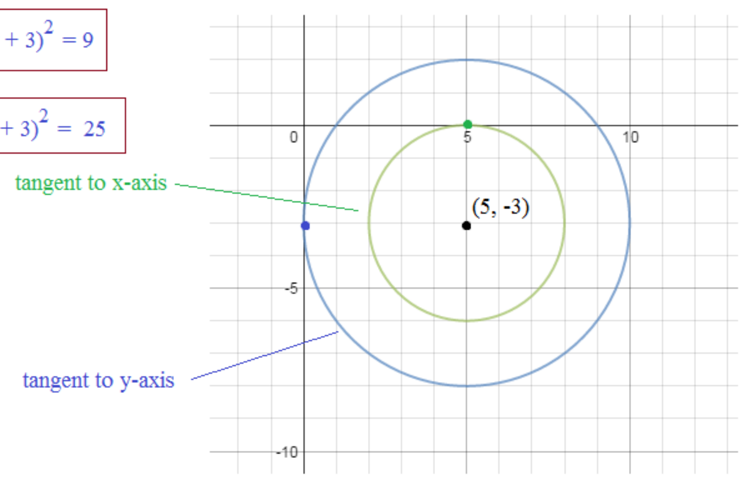
$\boxed{(0, 4)}$

6) Find a circle with center (5, -3) that is

a) tangent to the x-axis — radius will be 3 — $\boxed{(x-5)^2 + (y+3)^2 = 9}$

b) tangent to the y-axis — radius will be 5 — $\boxed{(x-5)^2 + (y+3)^2 = 25}$

(optional: graph the two circles)



SOLUTIONS



7) What is the equation of a circle containing points (2, 2), (8, 2), and (8, 12)?

Since we cannot assume that 2 of these points are endpoints of a diameter, we must use another approach to find the radius and center....

Using Geometry: the perpendicular bisectors will intersect at the circumcenter (i.e the center of a circle that circumscribes the triangle)

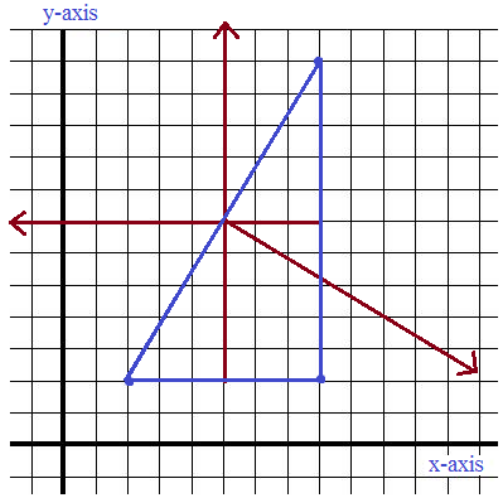
Since the 3 points form a right triangle, finding the perpendicular bisectors is rather easy...

(2, 2) and (8, 2) ---> $x = 5$

(8, 2) and (8, 12) ---> $y = 7$

and, the 3rd ⊥ bisector will go through the same intersection (5, 7)

also, slope of 3rd side is $10/6 = 5/3$...
so, perp. bisector slope is $-3/5$
therefore, equation is $(y - 7) = -3/5(x - 5)$




The intersection of the 3 lines is (5, 7).
This is the orthocenter; the center of a circle that circumscribes the triangle...

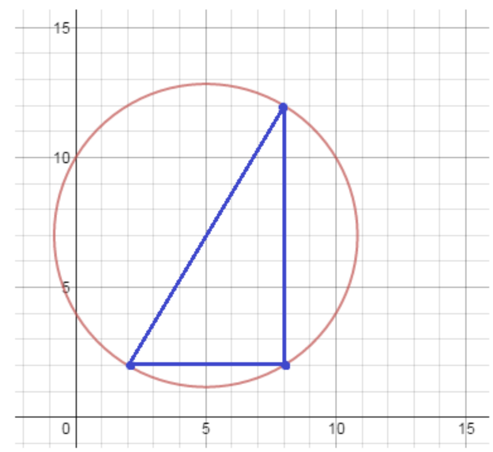
Standard form of a circle: $(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + (y - 7)^2 = 34$

The radius of the circle is the distance from (5, 7) to any point:

$$\text{radius} = \sqrt{(5 - 2)^2 + (7 - 2)^2}$$

$$= \sqrt{34}$$




8) Given: Focus: (-2, 3)

Directrix: $y = -3$

Find and the graph the equation of the parabola.

vertex:
midpoint between focus and directrix is
(-2, 0)

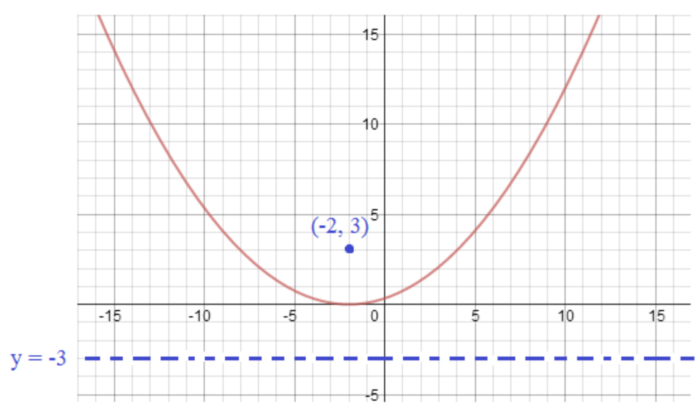
'p' value:
distance from vertex to focus is 3 units

$$a = \frac{1}{4p} = \frac{1}{12}$$

$y = \frac{1}{12}(x + 2)^2$

or

$(x + 2)^2 = 12y$



SOLUTIONS

9) What is the equation(s) of a hyperbola with asymptotes $(y + 6) = \pm\frac{4}{3}(x - 2)$

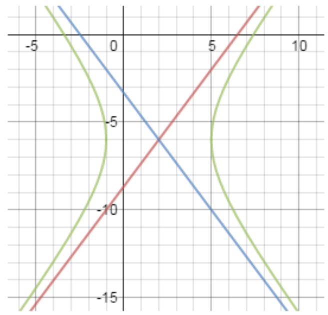
Intersection of lines is center (2, -6)

There are 2 answers!

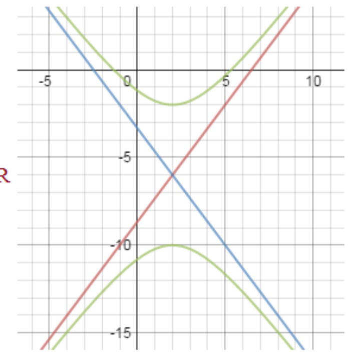
$$\frac{(x-2)^2}{9} - \frac{(y+6)^2}{16} = 1$$

OR

$$\frac{(y+6)^2}{16} - \frac{(x-2)^2}{9} = 1$$




OR




10) A light has a parabolic reflector with focal diameter of 12".
If the depth of the reflector is 20" (from focus to edge), what is the width of the span?

Since focal diameter is 12", the 'p' value is 3"

focal diameter = 4p

If we map the reflector onto an xy coordinate plane,

vertex: (0, 0)

focus: (3, 0)

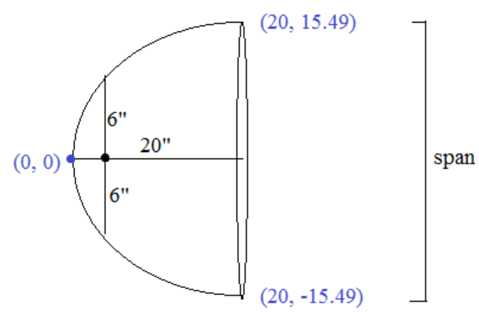
$$x = \frac{1}{12}y^2$$

After finding the equation, we plug in (20, y) to find the span...

$$20 = \frac{1}{12}y^2$$

$y = \pm\sqrt{240}$ or approx. 15.49 and -15.49

so, the span is 30.98




11) Find the equation of a circle with center (3, 6) and *tangent* to the line x + 3y = 30

For the equation of a circle, we need the center and the radius...
The center is (3, 6)... what is the radius?

We need to find the distance from the center to any point on the circle...

(from geometry), we know a line tangent is perpendicular to the radius...

slope of x + 3y = 30 is -1/3...
therefore, the slope of the radius is 3 (opposite reciprocal)

What is the equation of the radius:

slope 3 and through (3, 6) $\quad y - 6 = 3(x - 3)$

$$y = 3x - 3$$

Then, where does the radius intersect the tangent line?

$y = 3x - 3$

$x + 3y = 30$

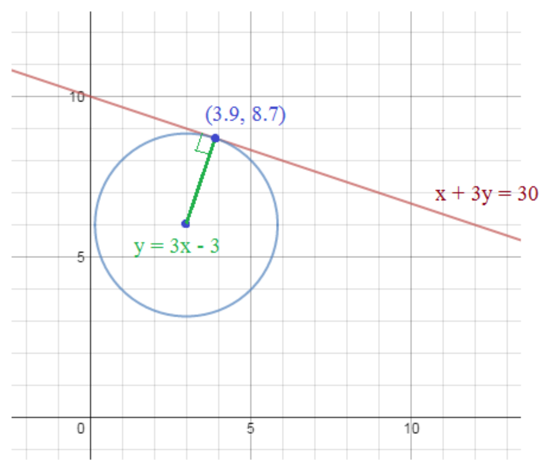
$x + 3(3x - 3) = 30 \qquad 10x = 39$

$x = 39/10 \qquad y = 87/10$

Finally, find the length of the radius:

(radius) $\quad d = \sqrt{(3 - (39/10))^2 + (6 - (87/10))^2}$

$$r^2 = \frac{81}{100} + \frac{729}{100} = \frac{81}{10}$$




$$(x-3)^2 + (y-6)^2 = \frac{81}{10}$$

SOLUTIONS

Conics V Practice Test

12) Write the equation of the parabola with vertex at the origin, passes through (2, -4), and is

a) symmetric to the y-axis

$(h, k) = (0, 0)$

$y = a(x - h)^2 + k$

$-4 = a(2 - 0)^2 + 0$

$a = -1$

$x^2 = -1y$

or

$y = -x^2$

and, for graphing:

$a = 4p$

or

$a = 1/4p$

$p = -1/4$

b) symmetric to the x-axis

$(h, k) = (0, 0)$

$x = a(y - k)^2 + h$

$2 = a(-4 - 0)^2 + 0$

$a = \frac{1}{8}$

$x = \frac{1}{8}(y)^2$

or

$8x = y^2$

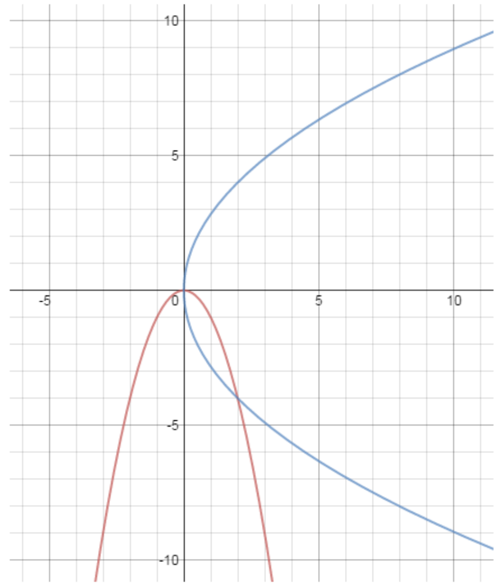
and, for graphing:

$a = (1/4)p$

or

$a = 4p$

$p = 2$




13) Write the equation of a graph whose path of points moves so that the sum of distances from F(4, 3) and F(4, −7) is 28

foci are 'vertical' -- lie on x = 4, so it's a vertical ellipse

center is (4, -2) (midpoint of the foci)

since sum of distances is 28, the major axis is 28...
and, the semi-major axis is 14...

$R_y^2 - R_x^2 = c^2 \qquad a^2 - b^2 = c^2$

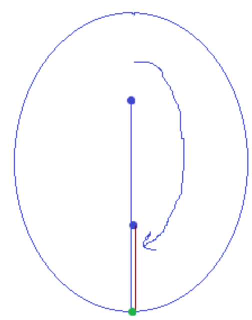
where c is the distance from the center to each focus...

$196 - b^2 = 5^2$

$\frac{(x-4)^2}{b^2} + \frac{(y+2)^2}{196}$

$\frac{(x-4)^2}{171} + \frac{(y+2)^2}{196} = 1$

Notice: the sum of the distances from each focus to any point on the ellipse is the length of the major axis!



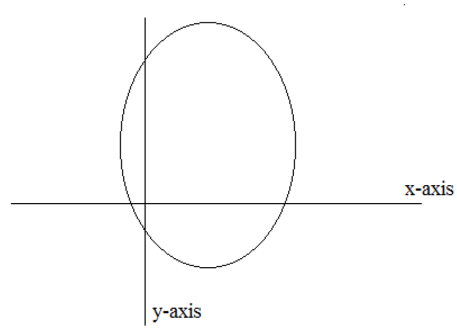
14) Which of the following is a possible equation for the graph?

a) $x^2 - 4x + 4y^2 - 40y = -100$ Not possible: center will be below x-axis...

b) $4x^2 + 16x + y^2 + 10y + 40 = 0$ Not possible.. After completing the square, it will be apparent that the center has a negative x term

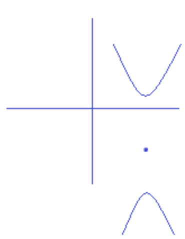
c) $4x^2 - 16x = -y^2 + 10y - 40$

d) $-16x + 4x^2 = y^2 - 10y + 40$ This will be a hyperbola




15) What are the domain and range of the conic?

$$\frac{(y+3)}{16} - \frac{(x-4)}{25} = 1$$



domain: all reals

range:
(neg. infinity, -7] ∪ [1, infinity)

Conics Word Problems

SOLUTIONS

1) Standing at one spot (focus) in a whispering gallery, 8 feet from the wall, a friend stands 100 feet away.
What is the length of the gallery?
What is the height of the gallery (at the middle)?

In the model, we'll use (0, 0) as the center (h, k)

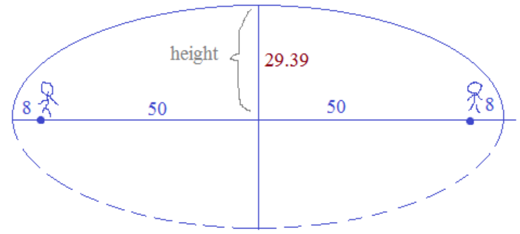
And, we can see from the diagram that the major axis is 116 feet, and the semi-axis (a) is 58 feet...

$$\frac{(x-0)^2}{58^2} + \frac{(y-0)^2}{b^2} = 1$$

Since we know $a^2 - b^2 = c^2$
where c is the distance from center to focus,

$$3364 - b^2 = 50^2$$

$$b = 29.39$$




$$\frac{x^2}{3364} + \frac{y^2}{864} = 1$$

2) An oval (elliptical) racetrack is 80 meters long and 40 meters wide.
What is the width of the racetrack 10 meters from the side?

Using a sketch with the center at (0, 0), the model of the ellipse is

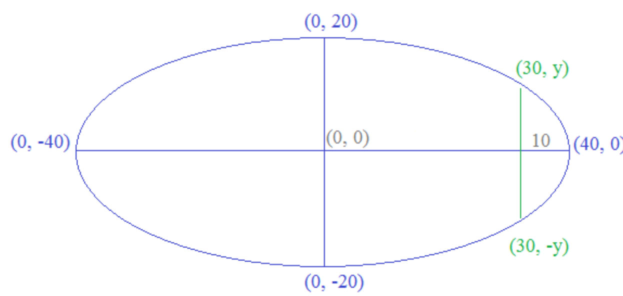
$$\frac{x^2}{1600} + \frac{y^2}{400} = 1$$

To find the width 10 meters from the side, we substitute the point(s) (30, y)

$$\frac{900}{1600} + \frac{y^2}{400} = 1$$

$$\frac{y^2}{400} = \frac{700}{1600}$$

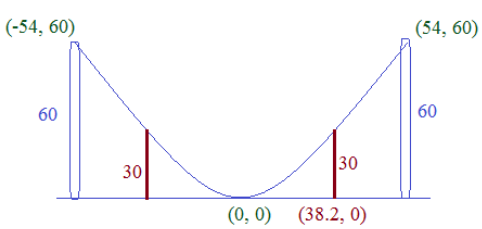
$$y = 13.23 \text{ and } -13.23$$




Therefore, the width of the ellipse is the distance from (30, 13.23) and (30, -13.23)

approximately 26.46 meters

3) The ends of a sagging hammock are attached to 2 trees, each 60 inches above the ground.
The distance between the trees is 108 inches.
If the middle of the hammock is touching the ground, where is the hammock 30 inches above the ground?




This is a 'vertical parabola'...

$$(x-h)^2 = a(y-k)$$

$$(54-0)^2 = a(60-0)$$

$$2916 = 60a$$

$$a = 48.6$$

The parabolic model is

$$x^2 = 48.6y$$

$$\text{or,} \quad y = \frac{1}{48.6}x^2$$

(note: a quick check, all 3 points fit in the model)

The hammock is 30 inches above the ground when y = 30

$$x^2 = 48.6(30)$$

$$x^2 = 1458$$

$$x = +/- \ 38.18$$

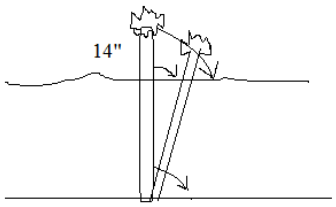
The hammock is 30 inches above ground approx. 21.82 inches from either tree..

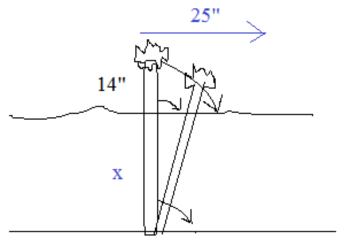
Conics Word Problems

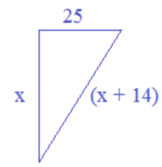
SOLUTIONS

4) A partially submerged plant has a flower 14 inches above the surface of the water.
If you pull the top of the stem 25 inches toward you, the rooted stem's flower touches the surface.
How deep in the water is the plant?

(circle application)




$$x^2 + 25^2 = (x + 14)^2$$

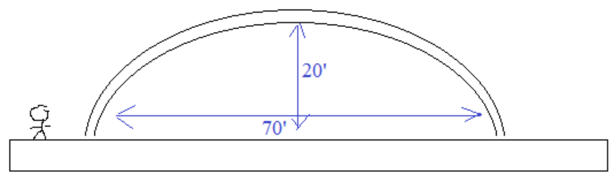
$$x^2 + 625 = x^2 + 28x + 196$$

$$429 = 28x$$

x = 15.32 inches

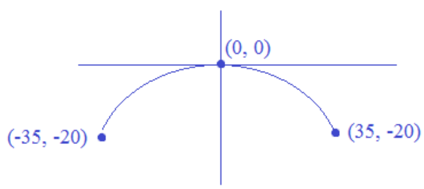
5) A parabolic shaped bridge has a maximum height of 20 feet and a span of 70 feet.
Find the height of the bridge 5 feet, 10 feet, and 25 feet from the center.

Step 1: Draw a diagram




Step 2: Set up the parabola model

Since this is a vertical parabola, the standard form is $(x - h)^2 = a(y - k)$




$$(35 - 0)^2 = a(-20 - 0)$$

$$1225 = -20a$$

$$a = -61.25$$

$$x^2 = -61.25(y)$$

(to check, plug in the points (0, 0), (35, -20), (-35, -20))

Step 3: Answer the questions

5 feet from center ---> $x = 5$ (or, $x = -5$) $(5)^2 = -61.25(y)$ $y = -.41$ so, the height is 19.59

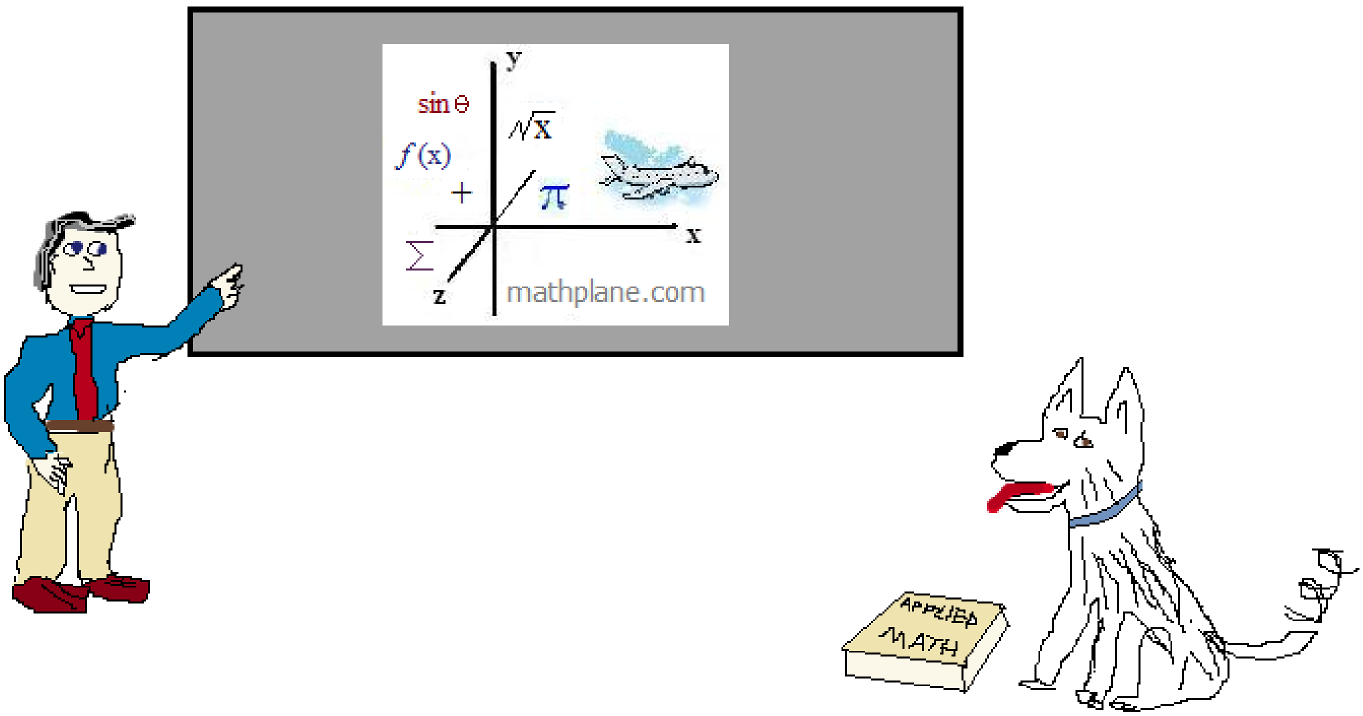
10 feet from center ---> $x = 10$ (or, $x = -10$) $(10)^2 = -61.25(y)$ $y = -1.63$ so, the height is 18.37

25 feet from center ---> $x = 25$ (or, $x = -25$) $(25)^2 = -61.25(y)$ $y = -10.2$ so, the height is 9.8

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.




Also, at Mathplane.ORG

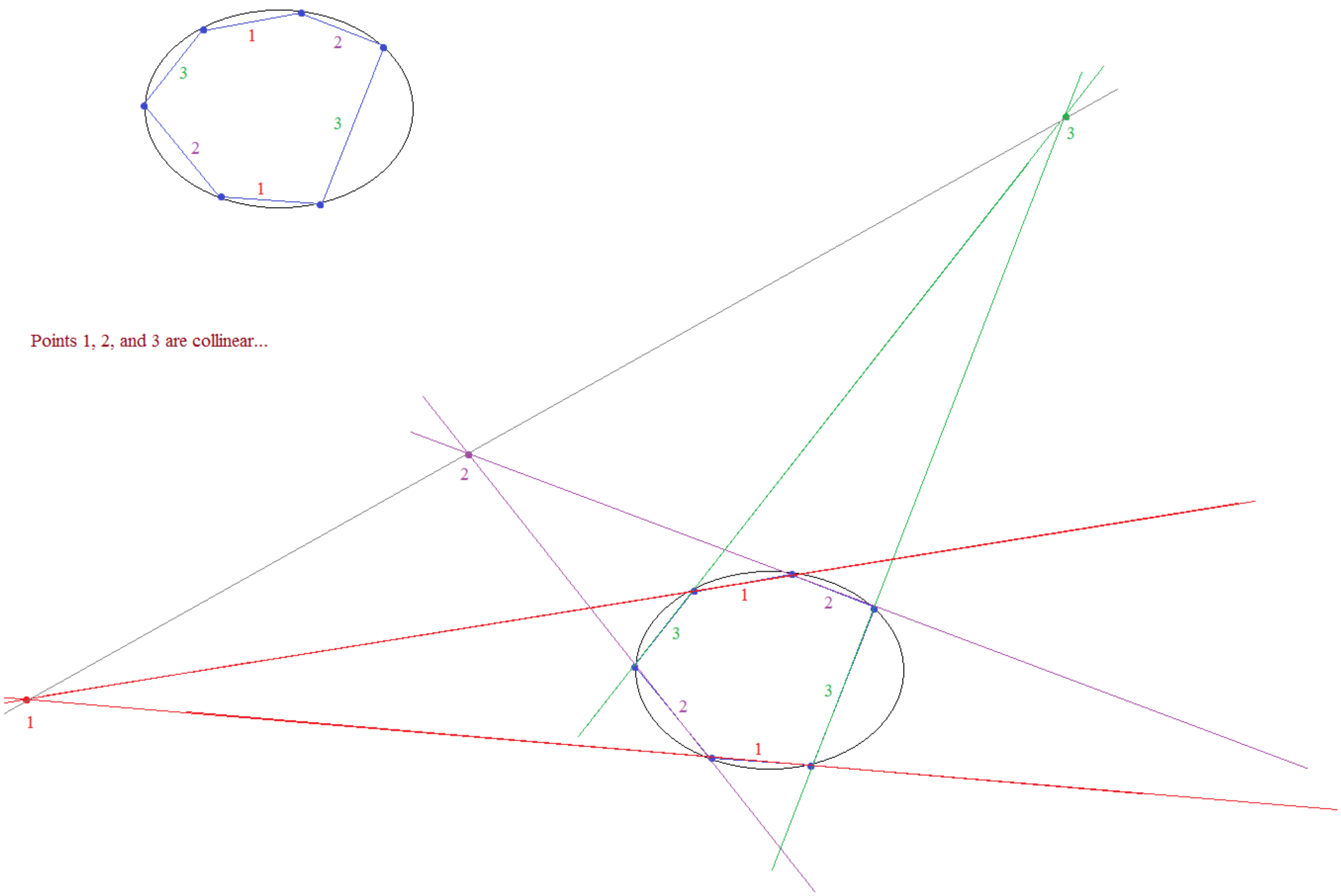
and our store at TeachersPayTeachers

Pascal's Theorem

If 6 random points are picked on a conic and joined by segments to make a hexagon,
then the 3 pairs of opposite sides meet in 3 points...

Those 3 points form a line, "The Pascal Line of a Hexagon"

*Example:*




Points 1, 2, and 3 are collinear...