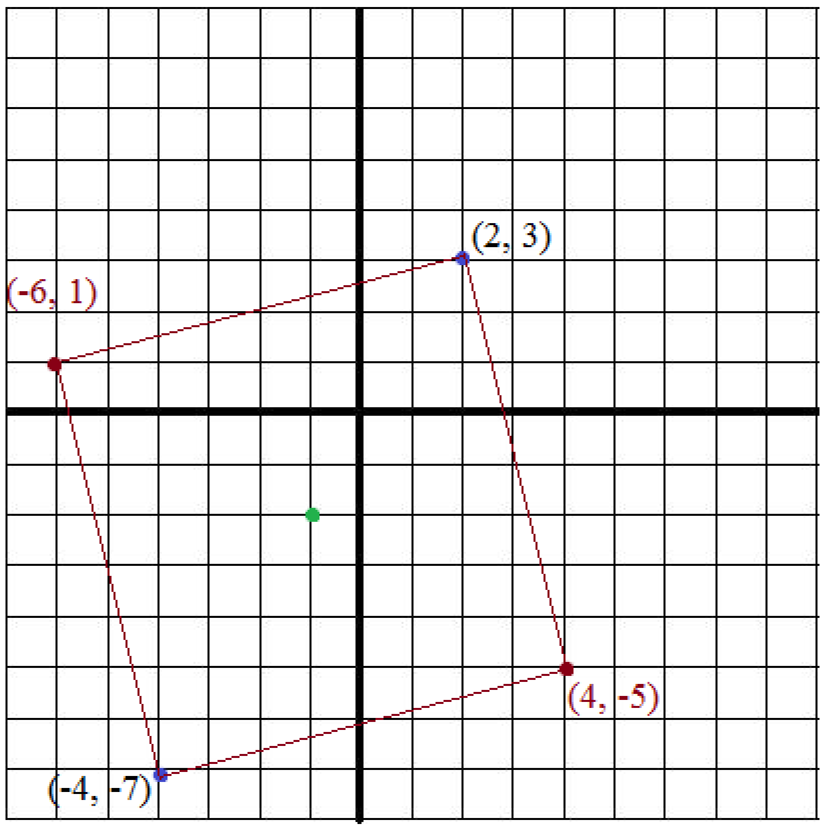
Coordinate Geometry 4 (Advanced)

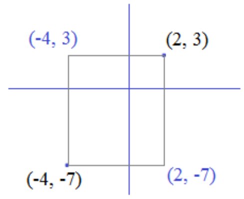


Topics include slope, reflection, centroid, circumcenter, altitudes, area, quadrilaterals, distance, and more.

Example: $(-4, -7)$ and $(2, 3)$ are opposite vertices of a square.

What are the other two vertices?

If this were are vertical square, this would be easy...



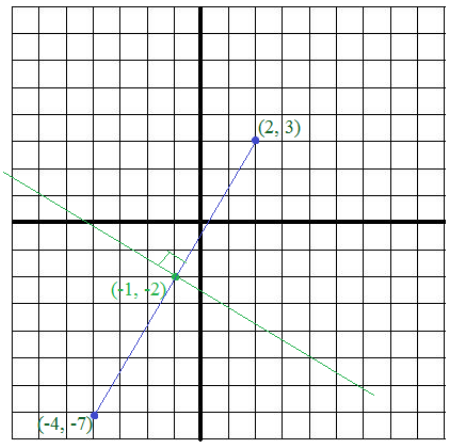
Unfortunately, that creates a rectangle...

This square will be 'tilted'...

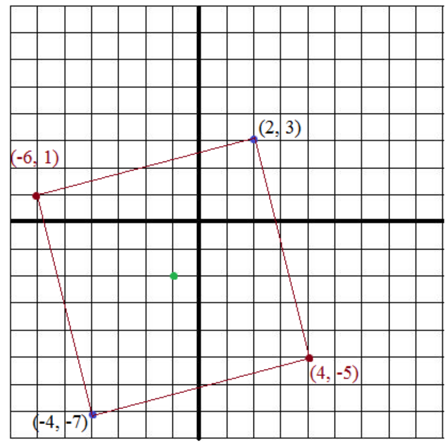
So, consider a square: the diagonals are perpendicular, congruent, and bisect each other...

slope of given diagonal: (2, 3) and (-4, -7) $\frac{-7-3}{-4-2} = \frac{5}{3}$

midpoint of given diagonal: (2, 3) and (-4, -7) $\left(\frac{2+(-4)}{2}, \frac{3+(-7)}{2}\right) = (-1, -2)$



slope of perpendicular line (the other diagonal) is the opposite reciprocal: $\frac{-3}{5}$



From the center of the square (-1, -2), the given vertices are "up 5, right 3" and "down 5, left 3"... Therefore, the other vertices will be the opposite!

("up 3, left 5" and down 3, right 5")

To check: length of each side $\sqrt{68}$

sides are perpendicular (slopes are opposite reciprocals)

Example: What is the distance between the lines $y = 3x + 4$ and $y = 3x - 7$?

The distance between to (parallel) lines is a segment that is perpendicular to both lines...

Step 1: find the equation of the perpendicular line.

Slope: Since the slope of each parallel line is 3, the slope of the perpendicular segment is -1/3 (opposite reciprocal)

Point: We know that the y-intercept will be (0, 4)

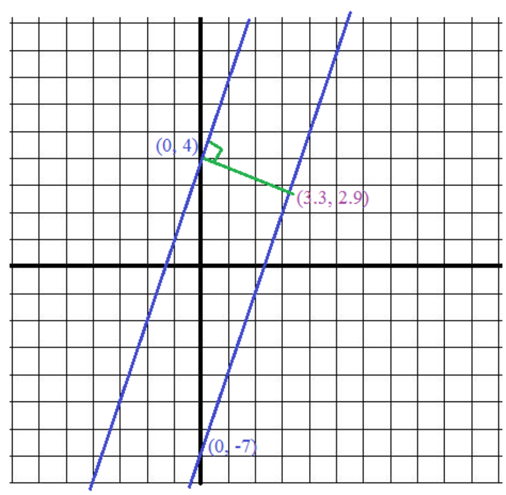
equation is $y = -1/3(x) + 4$

Step 2: find the intersection of the segment and other line.

$y = -1/3(x) + 4$
$y = 3(x) - 7$ $\Longrightarrow$ $-1/3(x) + 4 = 3(x) - 7$

$0 = -10/3(x) + 11$

$x = 3.3$ and, $y = 2.9$



Step 3: Find the distance between 2 points

(0, 4) and (3.3, 2.9)

distance $= \sqrt{(3.3 - 0)^2 + (2.9 - 4)^2}$

$= \sqrt{12.1} = 3.48$ (approx.)

Example: (0, 4) (-8, 10) and (-4, 2) are points on a circle.
Find the equation of the circle.

(Note: If we KNOW 2 of the points are endpoints of a diameter, then this is a rather straight-forward question. But, we cannot assume.)

Approach 1: Using Chords and bisectors.

Using chord 1: endpoints: (-4, 2) (-8, 10)

slope: -2

midpoint: (-6, 6)

therefore, the equation of the perpendicular bisector is

slope: 1/2 point: (-6, 6)

$y - 6 = 1/2(x + 6)$

Using chord 2: endpoints: (-4, 2) (0, 4)

slope: 1/2

midpoint: (-2, 3)

therefore, the equation of the perpendicular bisector is

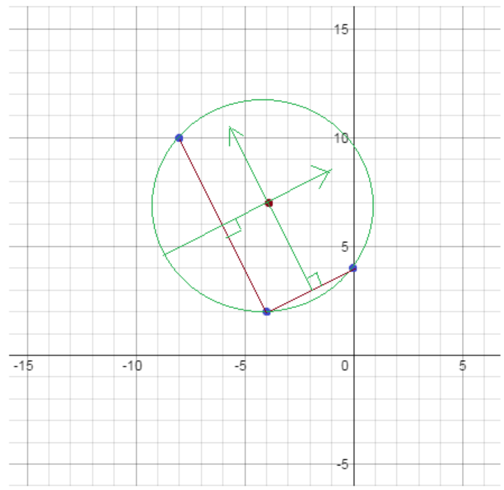
slope: -2 point: (-2, 3)

$y - 3 = -2(x + 2)$

Note: Since the chords are perpendicular, they form an inscribed right angle... Therefore, that right angle is inscribed in a semicircle (where the hypotenuse is the diameter of the circle)

(the perpendicular bisector of a chord will go through the center of the circle)

In the diagram is a sketch of the circle we're seeking..



the intersection of chord 1 and chord 2 will occur at the center...

$y - 6 = 1/2(x + 6) \quad \Longrightarrow \quad y = 1/2(x) + 9$

$y - 3 = -2(x + 2) \quad\quad\quad\quad y = -2x - 1$

$1/2(x) + 9 = -2x - 1 \qquad x = -4$

$5/2(x) = -10 \qquad y = 7$

Then, we see the radius is 5
(the distance from (-4, 7) to each of the 3 points)

$(x + 4)^2 + (y - 7)^2 = 25$

Approach 2: Using a system to solve algebraically

$$(x - h)^2 + (y - k)^2 = r^2 \quad \begin{cases} (0, 4) & (0 - h)^2 + (4 - k)^2 = r^2 \Longrightarrow h^2 + 16 - 8k + k^2 = r^2 \quad (1) \\ (-8, 10) & (-8 - h)^2 + (10 - k)^2 = r^2 \Longrightarrow 64 + 16h + h^2 + 100 - 20k + k^2 = r^2 \quad (2) \\ (-4\ 2) & (-4 - h)^2 + (2 - k)^2 = r^2 \Longrightarrow 16 + 8h + h^2 + 4 - 4k + k^2 = r^2 \quad (3) \end{cases}$$

set (1) and (2) equal to each other...

$$\cancel{h^2} + 16 - 8k + \cancel{k^2} = 64 + 16h + \cancel{h^2} + 100 - 20k + \cancel{k^2}$$

$$-148 = 16h - 12k \quad (4)$$

set (1) and (3) equal to each other..

$$\cancel{h^2} + \cancel{16} - 8k + \cancel{k^2} = \cancel{16} + 8h + \cancel{h^2} + 4 - 4k + \cancel{k^2}$$

$$-4 = 8h + 4k \quad (5)$$

solve the resulting system (4) and (5)...

$$\begin{aligned} -148 &= 16h - 12k \\ -4 &= 8h + 4k \end{aligned} \quad \Longrightarrow \quad \begin{aligned} -37 &= 4h - 3k \\ 2 &= -4h - 2k \\ \hline -35 &= -5k \end{aligned} \qquad k = 7 \quad \text{then, } h = -4$$

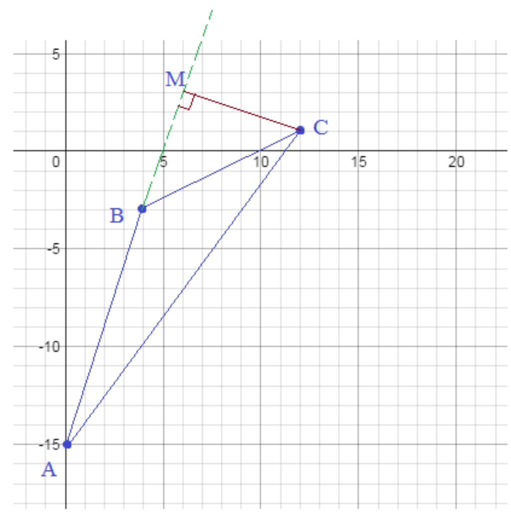
Example: On the coordinate plane, there is a triangle $\triangle ABC$, where the vertices are

A: (0 , -15)
B: (4, -3)
C: (12, 1)

What is the length of the altitude extended from C?

A quick sketch, and we see this is an obtuse triangle...

So, we extend side $\overline{AB}$ and draw the altitude.



To find the length $\overline{MC}$, we need to find the location of point M..

Equation of line $\overline{AB}$: slope: $\dfrac{-15-(-3)}{0-4} = 3$

y-intercept: (0, -15)

$\boxed{y = 3x - 15}$

Equation of line $\overline{MC}$: slope: (perpendicular to AB)

opposite reciprocal ---> -1/3

point: (12, 1)

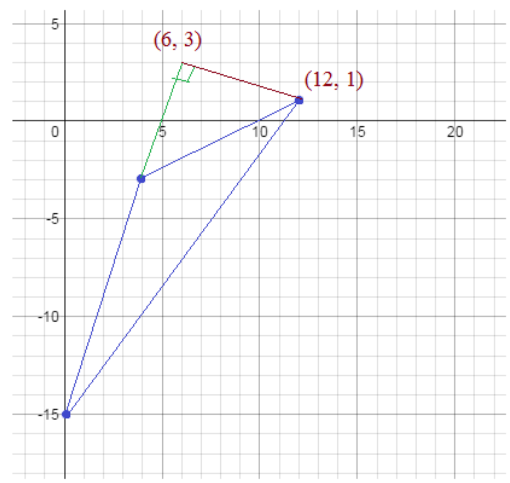
$y = -1/3(x) + b$

$1 = -1/3(12) + b$

$1 = -4 + b$

$b = 5$

$\boxed{y = -1/3(x) + 5}$



Intersection of line $\overline{AB}$ and $\overline{MC}$: $y = 3x - 15$

$y = -1/3(x) + 5$

$3x - 15 = -1/3(x) + 5$

$10/3(x) = 20$

$x = 6$ then, $y = 3(6) - 15$

$y = 3$

Therefore, the altitude extends from (12, 1) to (6, 3)

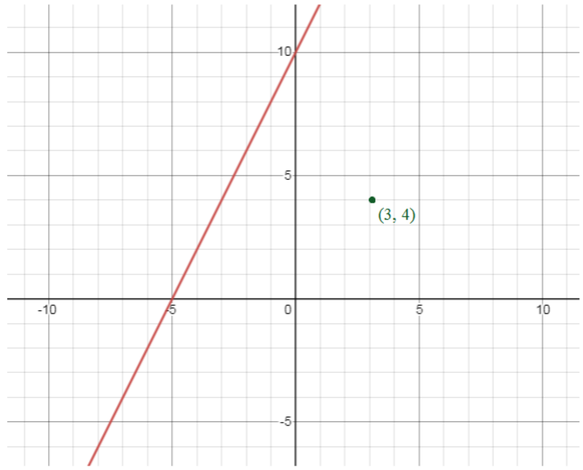
distance formula: $d = \sqrt{(12-6)^2 + (1-3)^2} = \sqrt{40} = \boxed{2\sqrt{10}}$

Distance between a point and a line

The distance between line l $Ax + By + C = 0$ and point $P(x_1, y_1)$

$$d(P, l) = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Example: Find the distance from (3, 4) to the line $y = 2x + 10$



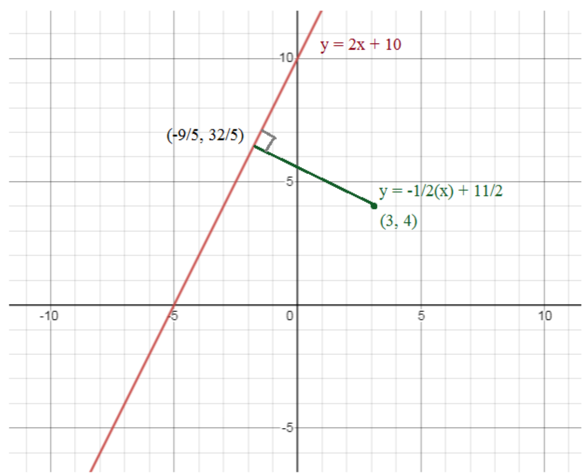
The point: (3, 4)

The line in general form: $2x - y + 10 = 0$

$A = 2$
$B = -1$
$C = 10$

$$\text{distance} = \frac{|2(3) + (-1)(4) + 10|}{\sqrt{2^2 + (-1)^2}} = \boxed{\frac{12}{\sqrt{5}}}$$

Now, let's check using coordinate geometry...



The slope of the line $y = 2x + 10$ is 2

Therefore, the straight distance from (3, 4) to the line will have a slope of -1/2
and, it will make a perpendicular line: $y - 4 = -1/2(x - 3)$

$y = -1/2(x) + 11/2$

We need to find where the lines intersect....

$y = 2x + 10$
$y = -1/2(x) + 11/2$

$2x + 10 = -1/2(x) + 11/2$

$5/2(x) = -9/2$

$5x = -9$

$x = -9/5$

$y = 2(-9/5) + 10$

$y = -18/5 + 50/5$

$y = 32/5$

The lines intersect at (-9/5, 32/5)

Finally, we can find the distance from (3, 4) to (-9/5, 32/5)

$$\sqrt{(-9/5 - 3)^2 + (32/5 - 4)^2}$$

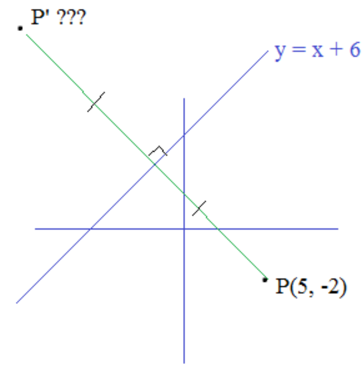
$$\sqrt{(-24/5)^2 + (12/5)^2}$$

$$\sqrt{\frac{720}{25}} = \boxed{\frac{12\sqrt{5}}{5}}$$ ✓

Example: If the point P(5, -2) is reflected over the line $y = x + 6$, what is the coordinate of P' ?

Step 1: Draw a quick sketch to estimate the result.

Notice, the distance from P to $y = x + 6$ is a straight (perpendicular) line segment
And, the identical distance from P' to $y = x + 6$ is a congruent line segment.



Step 2: Find the equation of the perpendicular line

To find the equation of a line, we need the slope and a point...

The point we'll use is (5, -2).

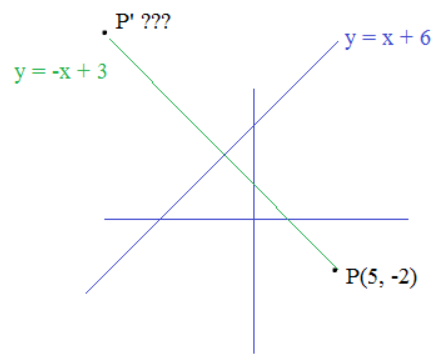
The slope of a perpendicular line is the opposite reciprocal.

(slope of $y = x + 6$ is 1)

The slope of line is -1

Equation of line: $y + 2 = -1(x - 5)$

$$y = -x + 3$$



Step 3: Find the intersection of the 2 lines

$y = x + 6$

$y = -x + 3$

using substitution: $x + 6 = -x + 3$

$2x = -3$

$x = -3/2 \qquad y = 9/2$

Step 4: Use the use the midpoint to determine the reflected point

P(5, -2) M(-3/2, 9/2) P'(x, y)

The distance from 5 to -3/2 is -13/2... So, the distance from -3/2 to x is -13/2: -8

The distance from -2 to 9/2 is 13/2... So, the distance from 9/2 to y is 13/2: 11

$\boxed{\text{P' is } (-8, 11)}$

Example: Is this a right triangle?

It looks like a right angle, but look at the coordinates!

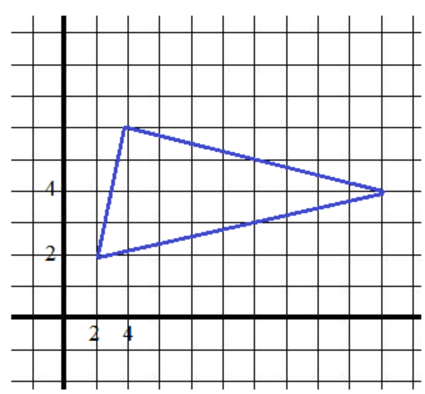
(2, 2) to (4, 6)

slope is 2

(4, 6) to (20, 4)

slope is -1/8

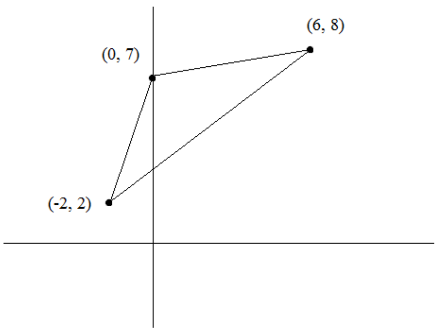
Since the slopes are not opposite reciprocals, this is NOT a right triangle!

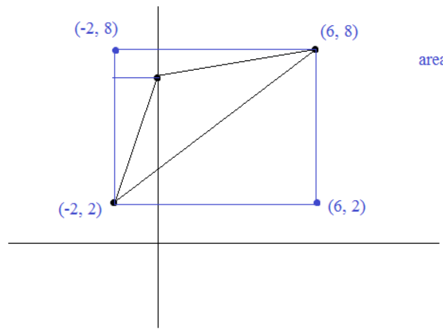


Example: The vertices of triangle ABC are A (-2, 2) B (0, 7) C (6, 8)
Find the area of the triangle.

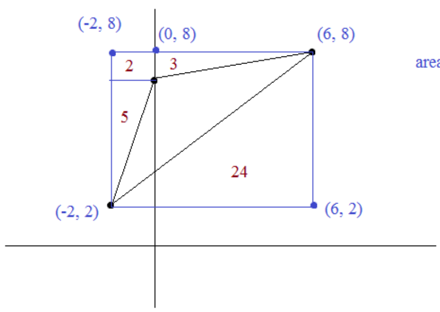
Coordinate Geometry and Area of an obtuse triangle

Method 1: Encasement





area of rectangle: 48

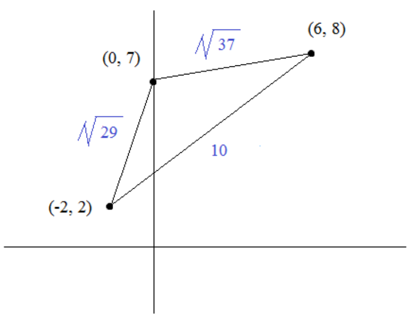


area of rectangle: 48

minus 34...

area is 14

Method 2: Hero's Formula

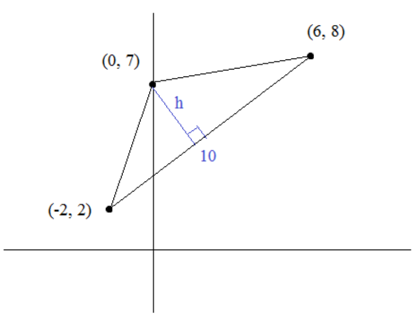


semiperimeter $= \dfrac{\sqrt{37} + \sqrt{29} + 10}{2} = s$ approximately 10.73

Area $= \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{10.73(4.65)(5.35)(.73)}$

$= 14$

Method 3: Finding the height...



Area $= \dfrac{1}{2}$ (base)(height)

We need to find the height...

line segment through (-2, 2) and (6, 8) $y - 2 = \dfrac{3}{4}(x + 2)$

slope of height is -4/3

line segment through (0, 7) $y = \dfrac{-4}{3}x + 7$

find the intersection using substitution:

$$\frac{-4}{3}x + 7 - 2 = \frac{3}{4}(x + 2)$$

$$\frac{-4}{3}x + 5 = \frac{3}{4}x + \frac{3}{2}$$

$$-16x + 60 = 9x + 18$$

$$42 = 25x \qquad y = \frac{-168}{75} + 7$$

$$x = \frac{42}{25} \qquad y = \frac{357}{75}$$

h = distance between (0, 7) and $\left(\dfrac{42}{25}, \dfrac{357}{75}\right)$

$$\sqrt{2.82 + 5.02} = 2.8$$

Area $= \dfrac{1}{2}$ (base)(height) $\Rightarrow$ $\dfrac{1}{2}(10)(2.8) = 14$

Example: Find the coordinates of the *circumcenter* of the triangle with vertices A(4, 12), B(14, 6), and C(-6, 2)

The *circumcenter* is the intersection of the *perpendicular bisectors*.....

To find the first perpendicular bisector:

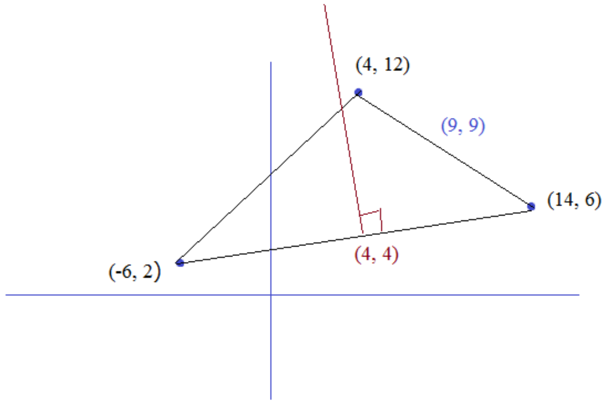
midpoint of $\overline{BC}$ ----> (4, 4)

we need the slope:

slope of $\overline{BC}$ $= \dfrac{4}{20} = \dfrac{1}{5}$

$\Longrightarrow$ slope of perp. bisector is -5

$$\boxed{\begin{array}{l} y - 4 = -5(x - 4) \\ y = -5x + 24 \end{array}}$$



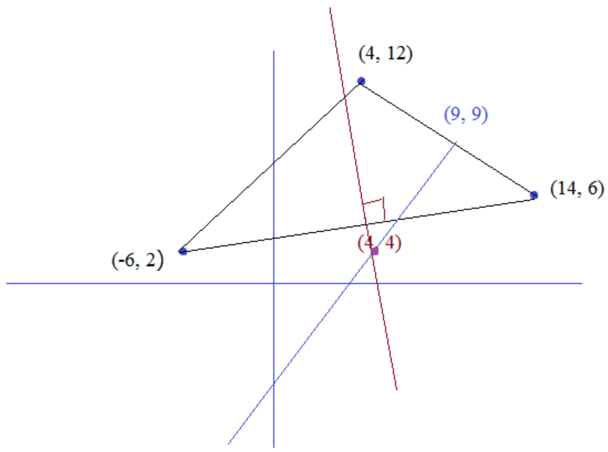
To find the next perpendicular bisector:

midpoint of AB -----> (9, 9)

slope of AB $= \dfrac{6}{-10} = \dfrac{-3}{5}$

$\Longrightarrow$ slope of perp. bisector is $\dfrac{5}{3}$

$$\boxed{\begin{array}{l} y - 9 = \dfrac{5}{3}(x - 9) \\ y = \dfrac{5}{3}x - 6 \end{array}}$$



Now, we can find the circumcenter
(the intersection of the perpendicular bisectors)

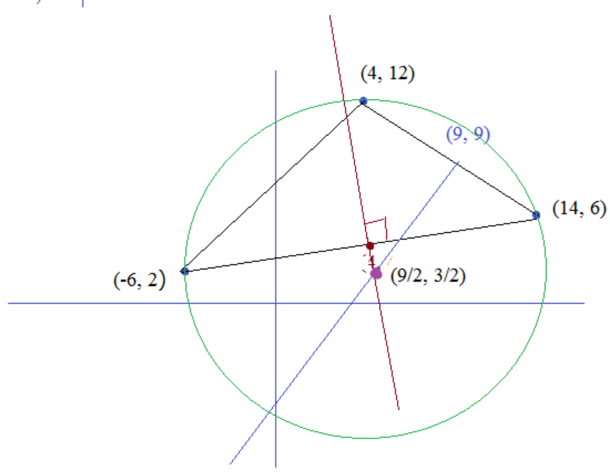
$$y = -5x + 24$$

$$y = \frac{5}{3}x - 6$$

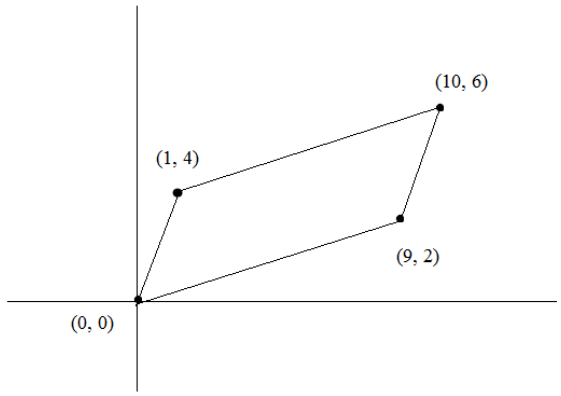
$$-5x + 24 = \frac{5}{3}x - 6$$

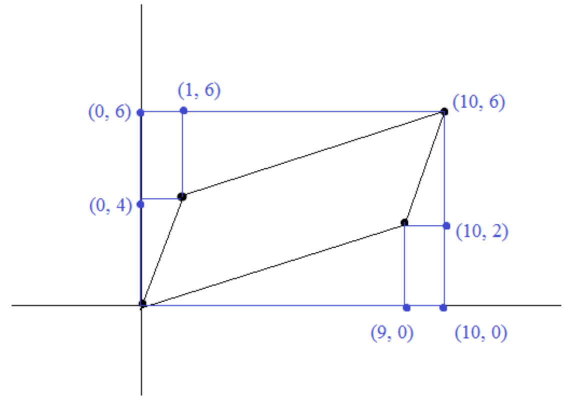
$$30 = \frac{20}{3}x$$

$$\boxed{\begin{array}{l} x = \dfrac{90}{20} = 9/2 \\ y = 9/3 = 3/2 \end{array}}$$

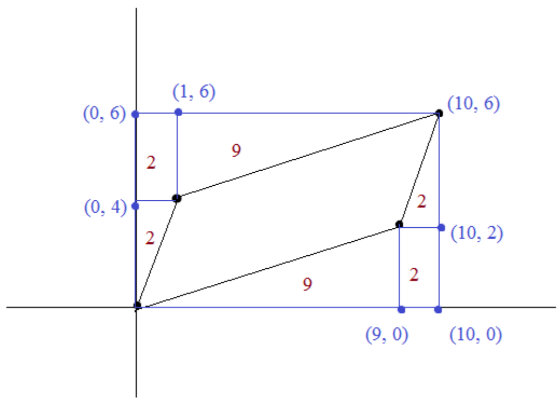


Example: Find the area of the quadrilateral with vertices A (0, 0) B (1, 4) C (9, 2) D (10, 6)





METHOD 1: Encasement

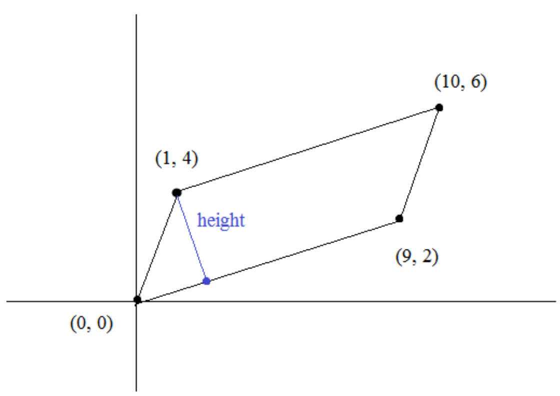


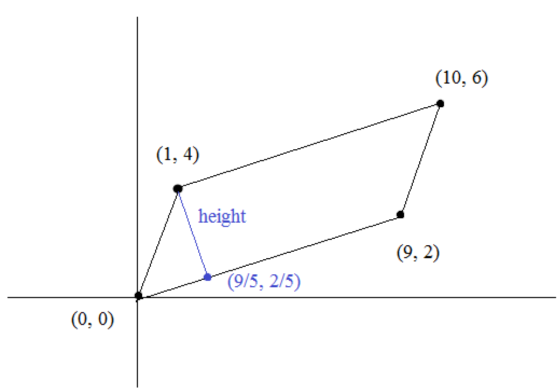
area of rectangle: 6 x 10 = 60

area of cut out pieces: 26

area of parallelogram: 34 ✓

METHOD 2: Find the base and height





length of base: distance between (1, 4) and (10, 6)

$\sqrt{85}$

then, to find the height...

equation of base: $y = \frac{2}{9}x$

equation of line through height: $y - 4 = \frac{-9}{2}(x - 1)$

find intersection using substitution:

$$\frac{2}{9}x - 4 = \frac{-9}{2}(x - 1)$$

(multiply by 18)

$4x - 72 = -81(x - 1)$

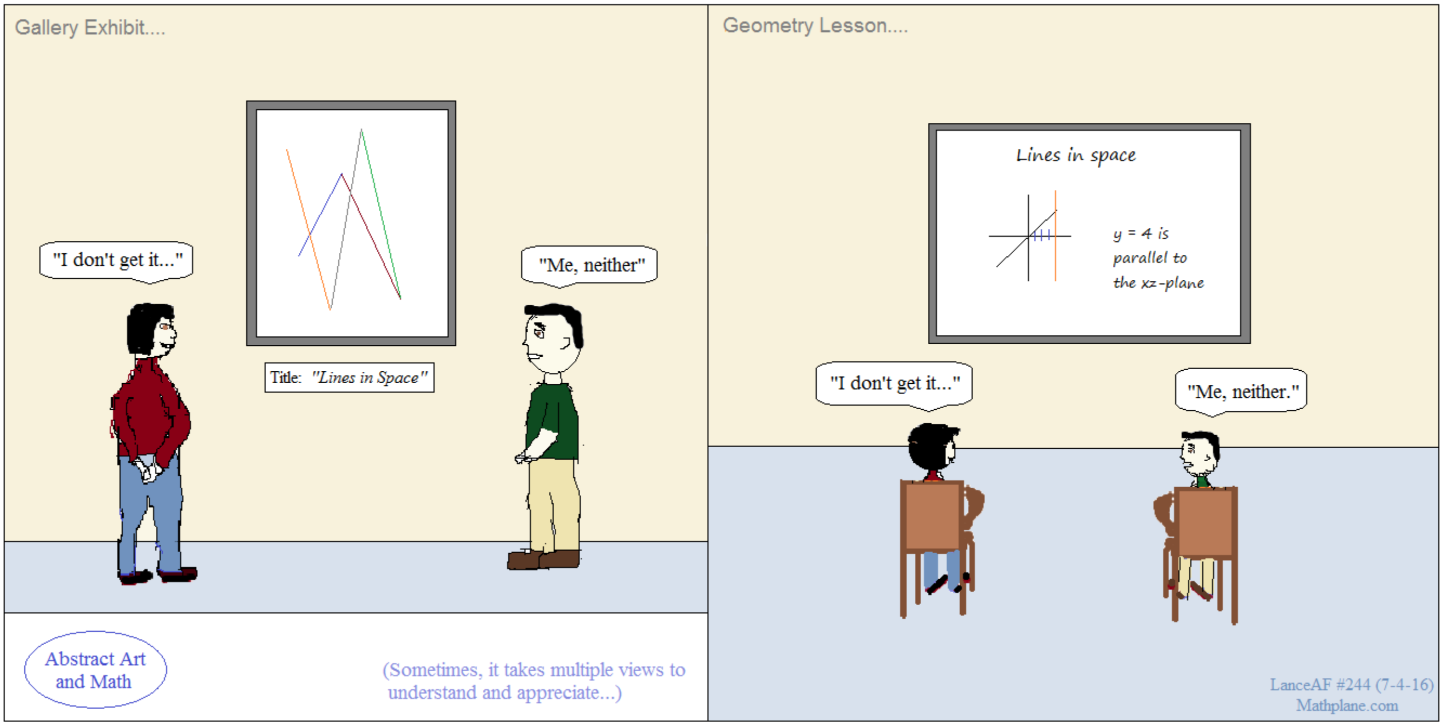
$85x = 153$

$x = 153/85$ $\qquad$ $y = \frac{2}{9}(9/5) = 2/5$

$= 9/5$

Area of parallelogram : (base)(height)

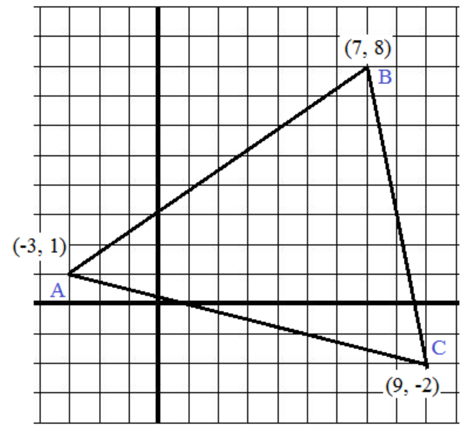
$\sqrt{85}\ (3.7) = 34$ ✓



Practice Quiz-→

A) Find lines that include the following (from triangle ABC):

1) The median from A to $\overline{BC}$
(write as a linear equation in *point-slope form*)



2) The Altitude from B to $\overline{AC}$
(write as a linear equation in *standard form*)

3) The Perpendicular Bisector of $\overline{BC}$
(write the linear equation *slope intercept form*)

B) Given: Right triangle SMR with altitude $\overline{MP}$
and horizontal hypotenuse $\overline{SR}$

M: (3, 4) S: (-5, -1)

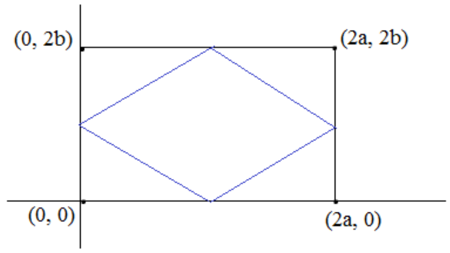
Find: Coordinate R

	T	R	I
C) Find the coordinate of the circumcenter from the triangle TRI	(0, 2)	(6, -4)	(8, 4)

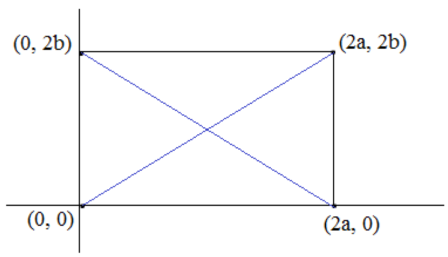
	X	Y	Z
D) Find the coordinate of the centroid in the triangle XYZ	(1, 0)	(7, 15)	(13, 0)

	A	B	C
E) Find the coordinate of the orthocenter of the triangle ABC	(-1, 4)	(6, 7)	(9, 0)

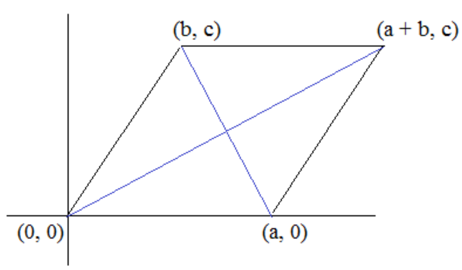
Prove: The connected midpoints of a rectangle form a parallelogram.



Prove: The diagonals of a rectangle bisect each other.



Prove: The diagonals of a rhombus are perpendicular to each other.



1) If point P (8, 12) is reflected over the line $y = 3x + 8$,

then what is the coordinate of point P' ?

2) If point Q (-7, -1) is reflected over the line $2x + 5y = 10$,

then what is the coordinate of point Q'?

Coordinate Geometry: Reflecting point over line

3) If Point B (-8, 4) is reflected over the line $y = -1$,

then what is the coordinate of point B' ?

4) ***Challenge

If point A (-3, 5) is reflected over the y-axis and THEN reflected over the line $y = -4x - 6$,

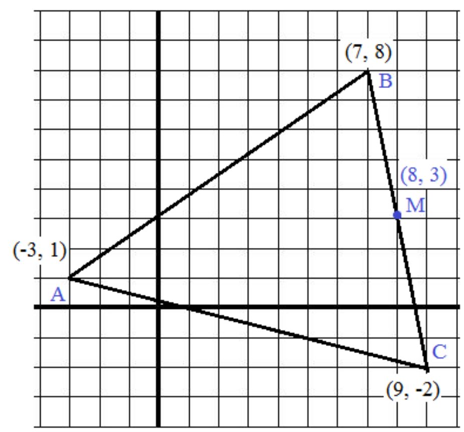
where does the point land?


LanceAF #305 (1-4-18)
mathplane.com
Evil Reflections
"Mirror, mirror on the wall -- who's the fairest teacher of them all?"
Doc excelled...
And, Happy and Bashful didn't care...
But, the rest suffered in this math class...
"Notice the lines of symmetry."
Reflections
E
V
I
L
Grimm Hall
'I hate this teacher. She's such a witch.'
"Pssst.. Hey, Sleepy. Wake up!"
"Dopey, I think Ms. Queen has it out for me. My grade is terrible..."
"Your grade?!?! Snow White, you should see mine."
Geometry

SOLUTIONS-→

A) Find lines that include the following (from triangle ABC):

SOLUTIONS



1) The median from A to $\overline{BC}$
(write as a linear equation in *point-slope form*)

To express the equation of a line, we need the slope and a point:

Point: A -- (-3, 1)
Slope: the slope going through A and the midpoint of $\overline{BC}$

Midpoint M $= \left(\frac{7+9}{2}, \frac{8+(-2)}{2} \right) = (8, 3)$

Slope of line going through A and M: $\frac{3 - 1}{8 - (-3)} = \frac{2}{11}$

$$\boxed{y - 1 = \frac{2}{11}(x + 3) \quad \text{or} \quad y - 3 = \frac{2}{11}(x - 8)}$$

2) The Altitude from B to $\overline{AC}$
(write as a linear equation in *standard form*)

We need a point and the slope...

Point: B (7, 8)
Slope: *perpendicular* to $\overline{AC}$

slope of $\overline{AC}$ is $\frac{1 - (-2)}{-3 - 9} = \frac{3}{-12}$

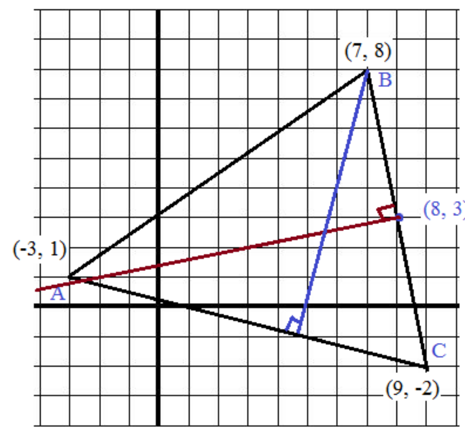
slope of line perpendicular to $\overline{AC}$ is 4 (opposite reciprocal)

Altitude line:

$y - 8 = 4(x - 7)$

$y - 8 = 4x - 28$

$\boxed{4x - y = 20}$



3) The Perpendicular Bisector of $\overline{BC}$
(write the linear equation *slope intercept form*)

Need the midpoint of $\overline{BC}$
and the slope of a line perpendicular to $\overline{BC}$

Midpoint of $\overline{BC}$ = (8, 3) (found in question 1))

slope of $\overline{BC}$ $= \frac{8 - (-2)}{7 - 9} = -5$

slope of line perpendicular to $\overline{BC}$ = 1/5

linear equation of perpendicular bisector:

$y - 3 = 1/5(x - 8)$

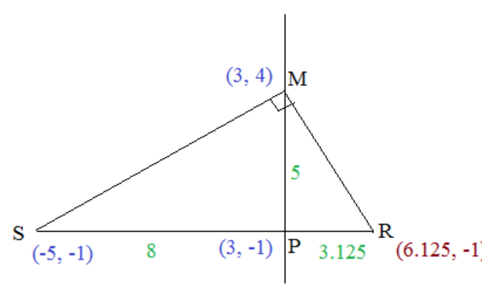
$y - 3 = 1/5x - 8/5$

$\boxed{y = 1/5x + 7/5}$

B) Given: Right triangle SMR with altitude $\overline{MP}$
and horizontal hypotenuse $\overline{SR}$

M: (3, 4) S: (-5, -1)

Find: Coordinate R



Since MP is an altitude, it is perpendicular to SR...
If SR is horizontal, then MP is vertical and P is (3, -1)

Length of MP is 5 and SP is 8

"Altitude to Hypotenuse": MP is the geometric mean of PR and SP

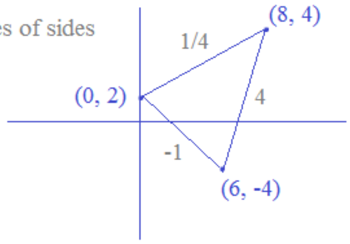
$\frac{8}{5} = \frac{5}{PR}$ $\quad$ 8(PR) = 25 $\quad$ PR = 3.125

Therefore, $\boxed{\text{R is } (6.125, -1)}$

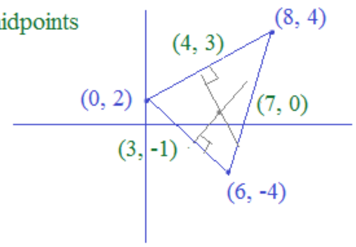
C) Find the coordinate of the circumcenter from the triangle TRI

T	R	I
(0, 2)	(6, -4)	(8, 4)

slopes of sides



midpoints



find 2 perpendicular bisectors...

$\overline{TR}$ perp. bisector: slope: 1 point: (3, -1)

$$y + 1 = 1(x - 3)$$

$\overline{TI}$ perp. bisector: slope: -4 point: (4, 3)

$$y - 3 = -4(x - 4)$$

and, their intersection...

$$y = x - 4 \qquad y = -4x + 19$$

$$x - 4 = -4x + 19$$

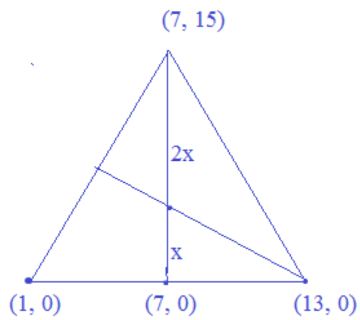
$$5x = 23 \qquad \boxed{\begin{array}{l} x = 4.6 \\ y = .6 \end{array}}$$

D) Find the coordinate of the <u>centroid</u> in the triangle XYZ

X	Y	Z
(1, 0)	(7, 15)	(13, 0)

Isosceles triangle

$\boxed{\text{Centroid is } (7, 5)}$



Centroid is 2/3 distance from each vertex!

distance from (7, 0) to (7, 15) is 15

$x + 2x = 15$

$x = 5$

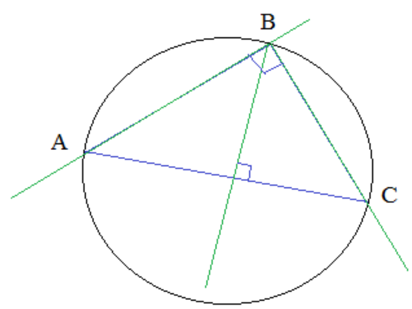
E) Find the coordinate of the orthocenter of the triangle ABC

A	B	C
(-1, 4)	(6, 7)	(9, 0)

$\overline{AB}$ --- slope is 3/7
$\overline{BC}$ --- slope of -7/3

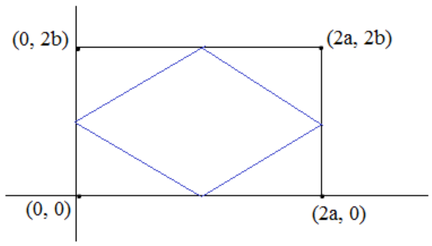
Since $\overline{AB} \perp \overline{BC}$, it is a right triangle..
Therefore, orthocenter is at the vertex B

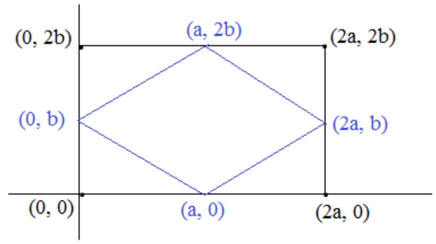
$\boxed{(6, 7)}$



Inscribed right triangle in a semicircle...

Prove: The connected midpoints of a rectangle form a parallelogram.





Using the midpoint formula, we can identify all the midpoints...

Then, find all the slopes...

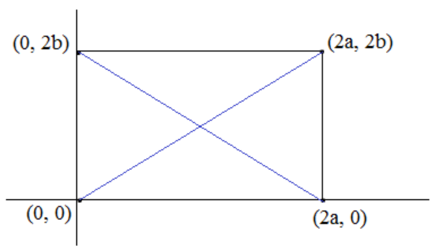
(0, b) and (a, 2b): $\frac{2b - b}{a - 0} = b/a$ (a, 2b) and (2a, b): $\frac{b - 2b}{2a - a} = \text{-}b/a$

(a, 0) and (2a, b): $\frac{b - 0}{2a - a} = b/a$ (0, b) and (a, 0): $\frac{b - 0}{0 - a} = \text{-}b/a$

Since the opposite sides have the same slopes, they are parallel lines...

Since opposite sides are parallel, then it's a parallelogram...

Prove: The diagonals of a rectangle bisect each other.

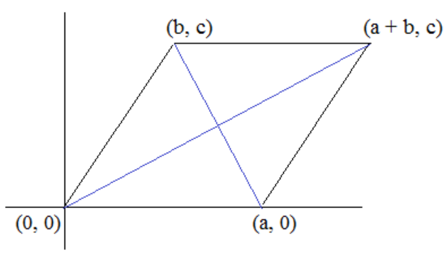


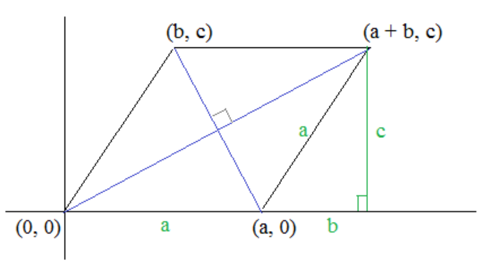
The midpoint of (0, 2b) and (2a, 0) is (a, b)

The midpoint of (0, 0) and (2a, 2b) is (a, b)

Since each diagonal has the same midpoint, then they bisect each other.

Prove: The diagonals of a rhombus are perpendicular to each other.





The slope of the diagonals:

(0, 0) and (a + b, c): $\frac{c - 0}{a + b - 0} = \frac{c}{a + b}$ (1)

(a, 0) and (b, c): $\frac{c - 0}{b - a} = \frac{c}{b - a}$ (2)

$b^2 + c^2 = a^2$ (because all sides of rhombus are congruent and equal the length a)

$c^2 = a^2 - b^2$

$c^2 = (a + b)(a - b)$

$(a + b) = \frac{c^2}{(a - b)}$ or $(a - b) = \frac{c^2}{(a + b)}$

(1) $\frac{c}{a + b} = \frac{c}{\frac{c^2}{(a - b)}} = \frac{(a - b)}{c}$

Opposite reciprocals!

(2) $\frac{c}{b - a} = \frac{\text{-}c}{a - b}$

Since the slopes of the diagonals are opposite reciprocals, the diagonals are perpendicular.

Coordinate Geometry: Reflecting point over line

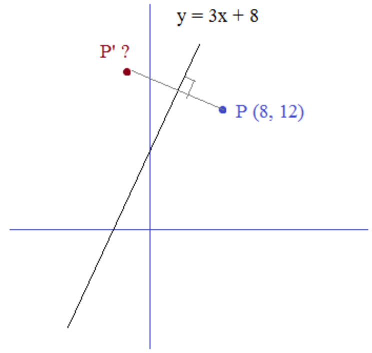
mathplane.com

SOLUTIONS

1) If point P (8, 12) is reflected over the line $y = 3x + 8$,

then what is the coordinate of point P' ?

ANSWER: (-4, 16)



Step 1: Draw quick sketch to get estimate

Step 2: Find equation of "reflection line segment"

We know the slope of the line $y = 3x + 8$ is 3

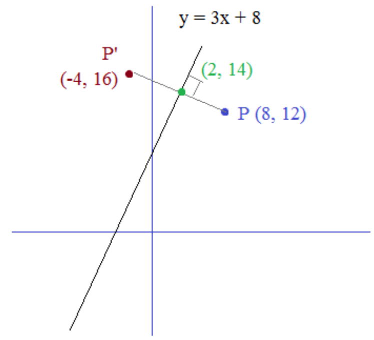
Therefore, the slope of the reflection line (perpendicular) is

-1/3

Since it goes through (8, 12), the equation of the line is

$$y - 12 = \frac{-1}{3}(x - 8)$$

Step 3: Find the intersection of the "reflection line" and original line



$$y = 3x + 8$$

$$y - 12 = \frac{-1}{3}(x - 8)$$

Using substitution:

$$(3x + 8) - 12 = \frac{-1}{3}(x - 8)$$

$$3x - 4 = \frac{-1}{3}(x - 8)$$

$$9x - 12 = -1(x - 8)$$

$$10x = 20$$

$x = 2$ then, $y = 14$

Step 4: Utilize the midpoint formula

Recognizing that the intersection (2, 14) is the midpoint of P and P',

(x - 6, y + 2)		(x - 6, y + 2)		
(8, 12)	⇨	(2, 14)	⇨	(-4, 16)
P		Midpoint		P'

Midpoint of P and P' is M

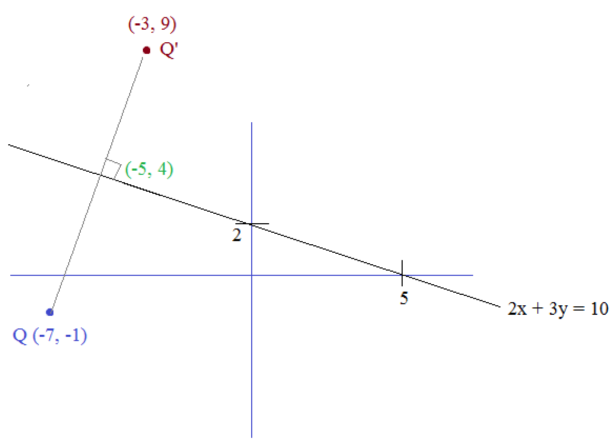
$$\left(\frac{8 + x}{2}, \frac{y + 12}{2}\right) = (2, 14) \qquad \frac{8 + x}{2} = 2 \qquad x = -4$$

$$\frac{y + 12}{2} = 14 \qquad y = 16$$

2) If point Q (-7, -1) is reflected over the line $2x + 5y = 10$,

then what is the coordinate of point Q'?

ANSWER: (-3, 9)



Step 1: Draw quick sketch to get estimate

Step 2: Find equation of "reflection line segment"

Find the slope of line: $2x + 5y = 10$

$$y = \frac{-2}{5}x + 2 \qquad \text{slope is } -2/3$$

Therefore, the slope of the perpendicular reflection line segment is 5/2

Since it must pass through (-7, -1), the equaiton of the line is

$$y + 1 = \frac{5}{2}(x + 7)$$

Step 3: Find the intersection of the "reflection line" and original line

$$y + 1 = \frac{5}{2}(x + 7)$$

$$\boxed{2x + 5y = 10}$$

$$2y + 2 = 5x + 35$$

$$\boxed{-5x + 2y = 33}$$

$$10x + 25y = 50$$

$$-10x + 4y = 66$$

$$29y = 116$$

$y = 4$ so, $x = -5$

Step 4: Utilize the midpoint formula

Since endpoint Q (-7, -1) to midpoint (-5, 4)

is $x + 2$ and $y + 5$...

then, midpoint (-5, 4) to endpoint Q'

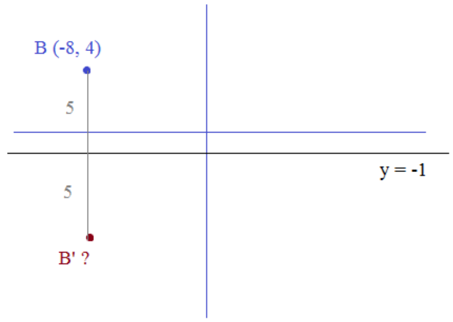
is $\underset{x}{-5 + 2}$ and $\underset{y}{4 + 5}$ ------> (-3, 9)

SOLUTIONS

3) If Point B (-8, 4) is reflected over the line $y = -1$,

then what is the coordinate of point B' ?

Step 1: Sketch the diagram



Step 2: Find distance from point to line..

***Since this is a horizontal line of reflection (and the slope is 0), the direction of the point will be directly down!
(i.e the slope from B to B' is undefined)

the distance from (-8, 4) to $y = -1$ is 5 units

Step 3: Duplicate the distance from line of reflection to mirror point B'

Since distance from B (-8, 4) to $y = -1$ is 5 units,

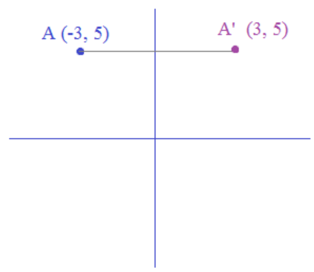
we'll continue down another 5 units from (-8, -1) to

$\boxed{(-8, -6)}$

4) ***Challenge

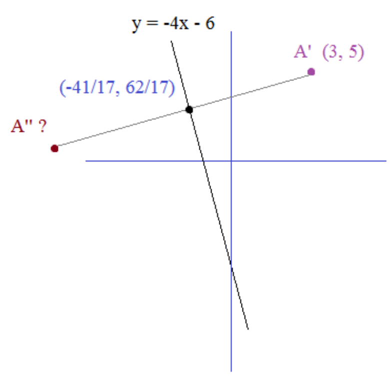
If point A (-3, 5) is reflected over the y-axis and THEN reflected over the line $y = -4x - 6$, where does the point land?

Step 1: Sketch and identify the first reflection



The length from A to the y-axis is 3 units, so, the coordinate of A' is (3, 5)

Step 2: Sketch the second reflection and identify the line segment perpendicular to the line of reflection



Since the slope of $y = -4x - 6$ is -4,

the slope of the segment A'A" is 1/4

then, using the point (3, 5), the equation of the segment is

$$y - 5 = \frac{1}{4}(x - 3)$$

Step 3: find the intersection of segment and line of reflection..

$$\boxed{\begin{gathered} y - 5 = \frac{1}{4}(x - 3) \\ y = -4x - 6 \end{gathered}}$$

Using substitution, $(-4x - 6) - 5 = \frac{1}{4}(x - 3)$

$$-4x - 11 = \frac{1}{4}x - \frac{3}{4}$$

multiply by 4 to get rid of fractions

$$-16x - 44 = x - 3$$

$$-41 = 17x$$

$$x = -41/17$$

$$y = -4(-41/17) - 6$$

$$y = \frac{164}{17} - \frac{102}{17} = 62/17$$

$\boxed{A'' \ (-133/17, 39/17)}$

Step 4: Using midpoint formula, find the reflection point..

Looking at y values: distance from 5 to 62/17 is 23/17

therefore, 62/17 to A" must be 23/17 ----> 39/17

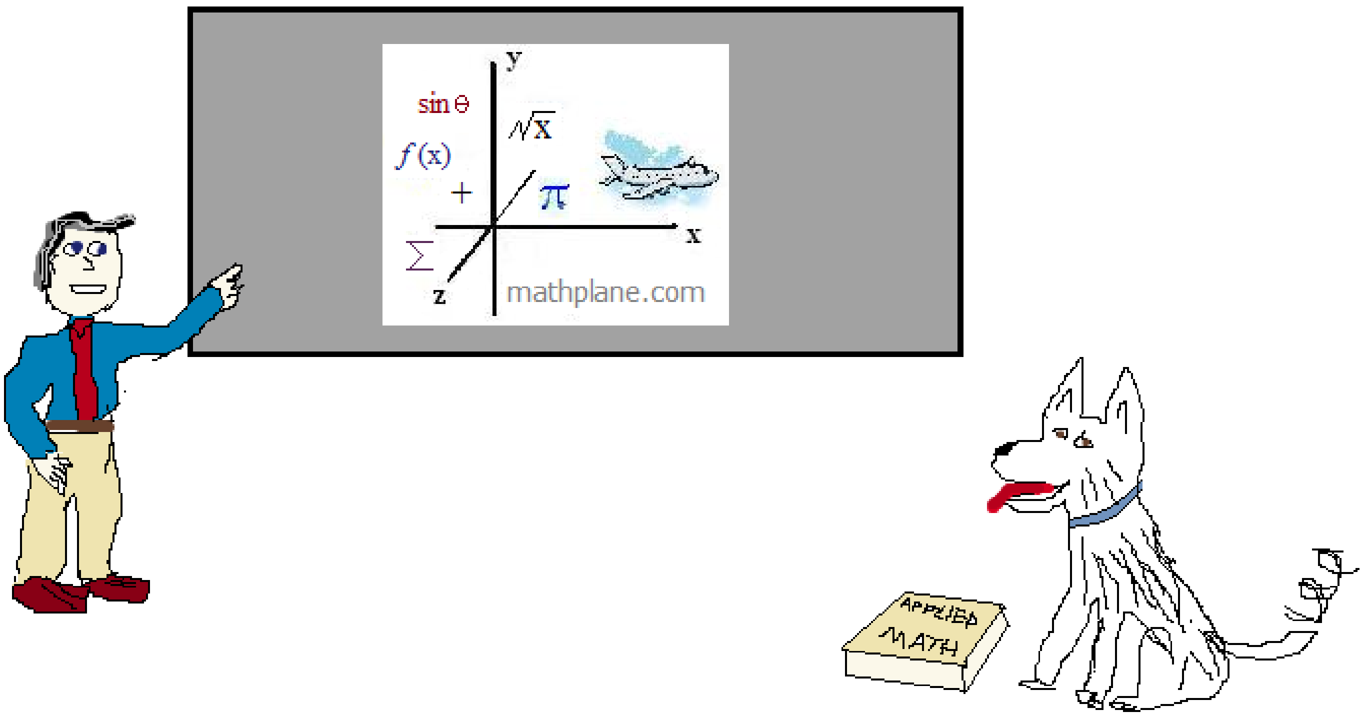
Looking at x values: distance from 3 to -41/17 is 92/17

therefore, -41/17 to A" must be 92/17 -----> -133/17

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

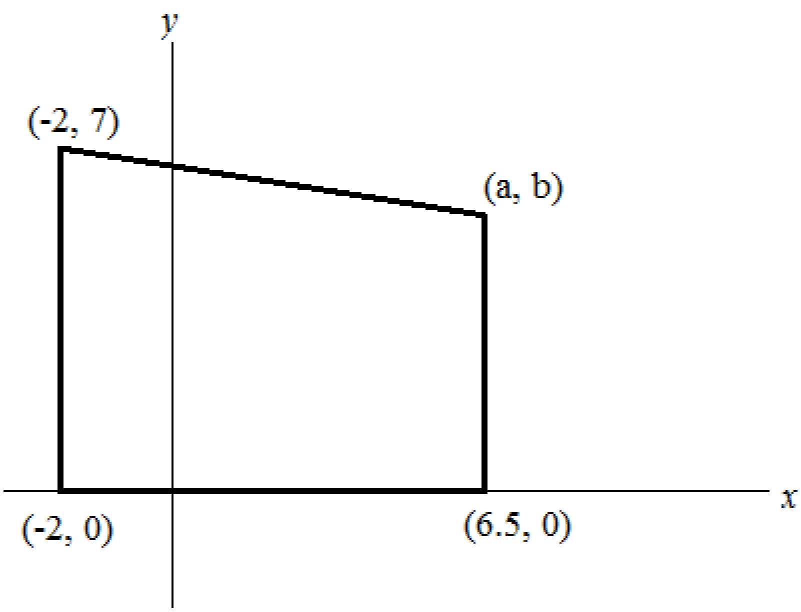


Check out Mathplane *Express* for mobile and tablets at Mathplane.org

Also, at TES and TeachersPayTeachers

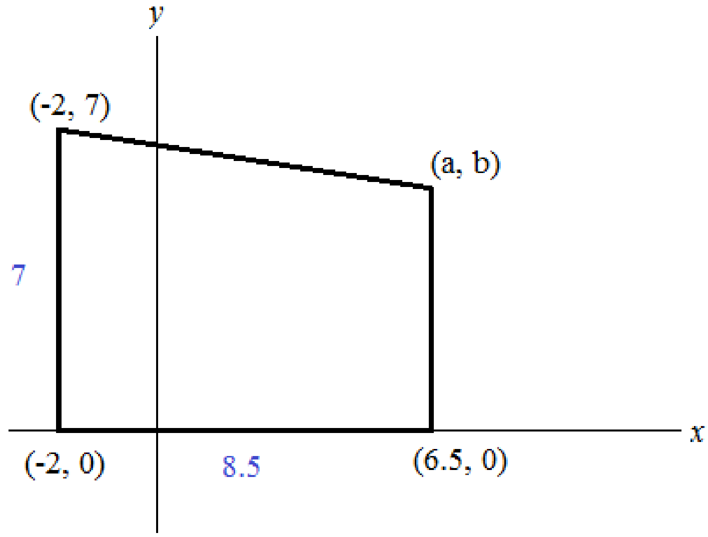
If the area of the trapezoid is 53,

Then what is the coordinate of (a, b)?



Answer-→

If the area of the trapezoid is 53, what is the coordinate (a, b)?



Area of trapezoid:

$\frac{1}{2}$ (base1 + base2)(height)

$\frac{1}{2}$ (7 + base2)(6.5 − (-2))

$\frac{1}{2}$ (7 + base2)(8.5) = 53

(7 + base2)(8.5) = 106

(7 + base2) = 12.47

base2 = 5.47

$\boxed{(6.5, 5.47)}$

Also, suppose the diagram isn't drawn to scale!

In other words, what if the horizontal lines are parallel, and the vertical lines (which appear parallel) are not?!?!

$\frac{1}{2}$(8.5 + base 2')(7) = 53

(8.5 + base 2')(7) = 106

(8.5 + base 2') = 15.14

base 2' = 6.64

$\boxed{(4.64, 7)}$