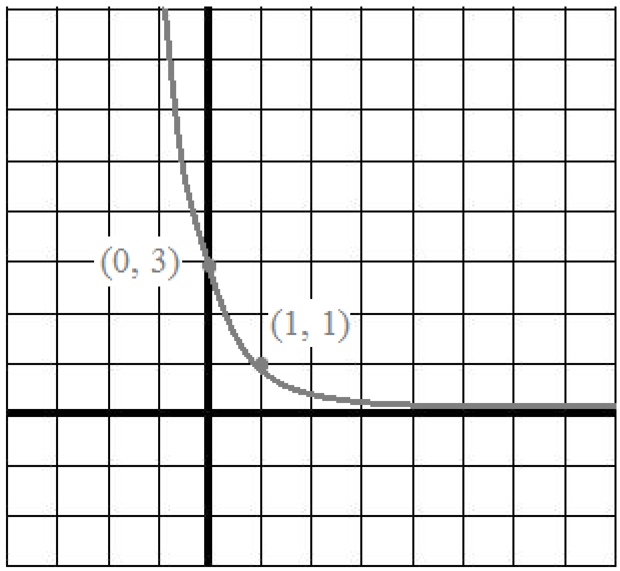
Exponents and Exponential Equations (Honors)

Practice Exercises (with Solutions)



Topics include exponential models, factoring, exponent rules, solving exponential equations, graphs, and more.

Example: What is the equation of the exponential curve?

since the end behavior approaches y = -5,

The exponential equation has a vertical shift down 5

$y = ab^x - 5$

(0, 3) is a point in the curve...

substitute the point into the equation...

$-3 = ab^0 - 5$

$a = 2$

so far, the model is:

$y = 2(b)^x - 5$

then, substitute a second point into the equation...

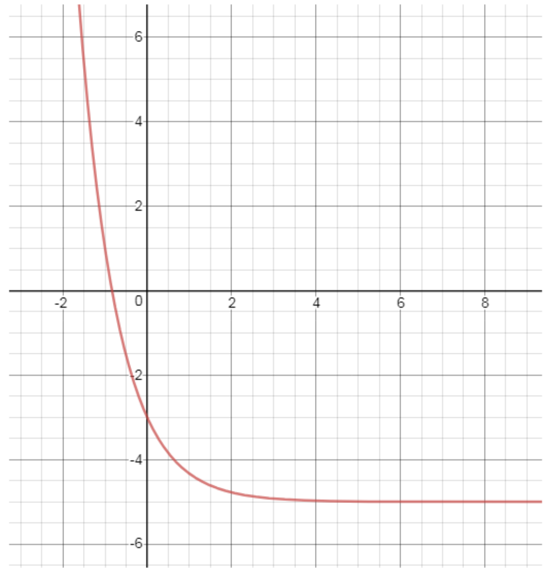
(-1, 1) is a point in the curve...

$1 = 2(b)^{-1} - 5$

$3 = b^{-1}$

$b = 1/3$

$y = 2(1/3)^x - 5$



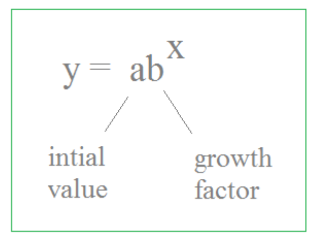
Example: What is the exponential model?

x	f(x)
3	15
4	45
5	135

$y = ab^x$

the b value is 3, because the function's growth factor is 3...

x	f(x)	
3	15	
		x3
4	45	
		x3
5	135	



To find the 'a' value,

$y = a(3)^x$

pick a point, such as (3, 15)

$15 = a(3)^3$

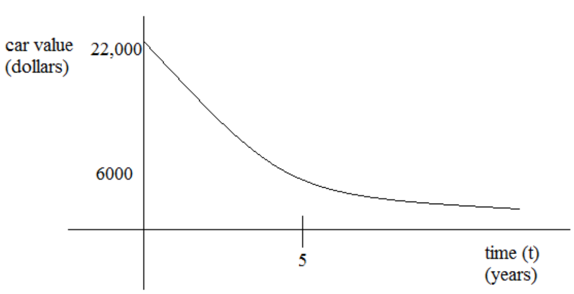
$a = 15/27$

$y = (5/9)(3)^x$

Example: The following exponential graph shows the value of a car as it relates to time.

a) What is the price of a new car?

b) What is the depreciation rate?



$y = ab^x$

$22000 = ab^0$

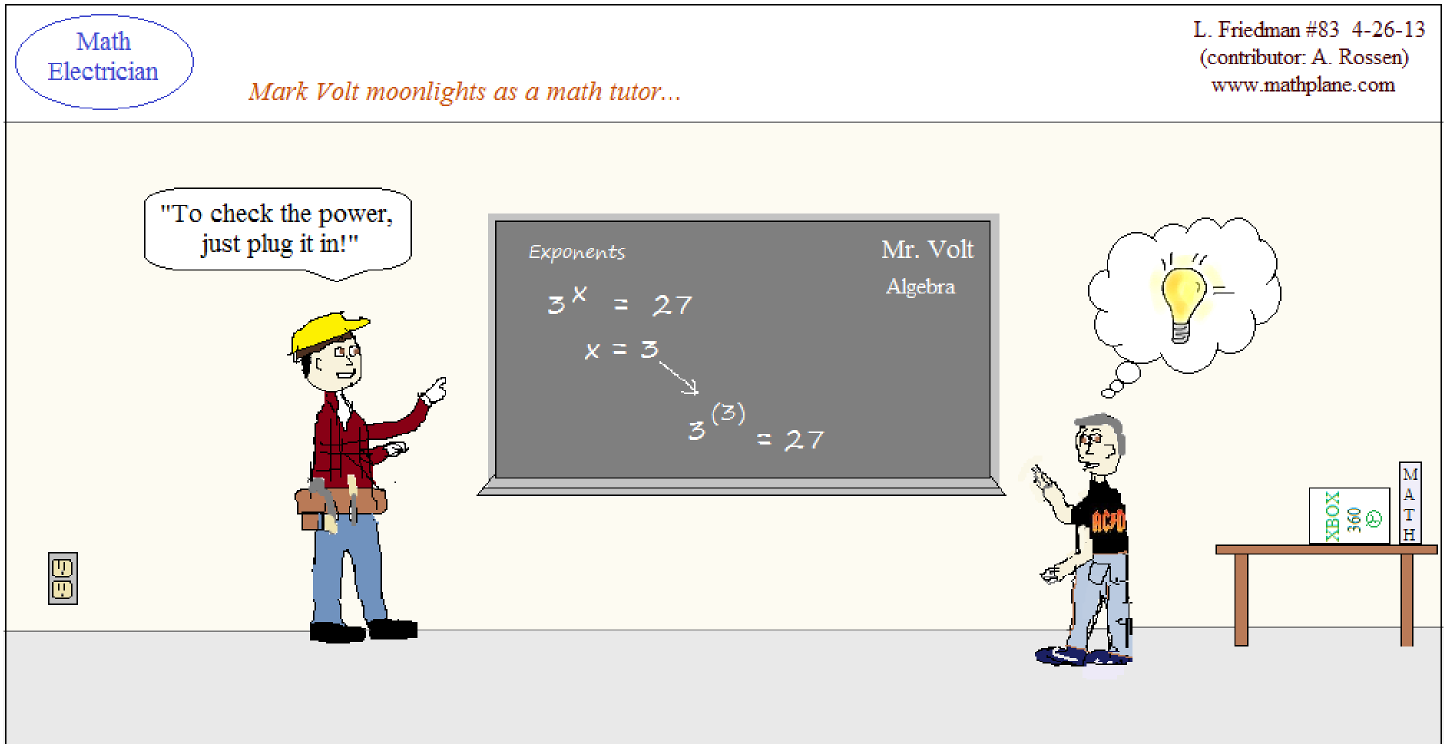
$a = 22000$ ⟸ price of new car

$y = 22000b^x$

$6000 = 22000b^5$

$b = .77$ ⟸ depreciation rate is 23%

$y = 22{,}000(.77)^x$



Alex gets the connection!

Practice Exercises-→

Simplifying expressions: exponentials and roots

1) $9^5 \times 4^{10}$

2) $3^{14} \cdot 5^7$

3) $\dfrac{e^{4x}}{e^{x-5}}$

4) $\dfrac{3^{3t-1}}{3^{3t+2}}$

5) $\dfrac{8^{3\sqrt{5}}}{2^{2\sqrt{5}}} =$

6) $\dfrac{3^{7x+2}}{3^{7x-1}} =$

7) $\dfrac{64^{3x+1}}{4^{2x}} =$

8) $(5x)^{\frac{1}{2}}(3x^{\frac{1}{2}})$

9) $\dfrac{\sqrt[3]{9}\cdot\sqrt[3]{6}}{\sqrt[6]{2}\cdot\sqrt[6]{2}}$

10) $90^{\frac{1}{2}} - 10^{\frac{1}{2}}$

11) $\sqrt[3]{18}\cdot\sqrt[3]{15}$

12) $(8r)^{\frac{1}{3}}(2r^{\frac{1}{2}})$

Factoring exponentials

1) $x^{\frac{-1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}}$

2) $x^{\frac{-3}{2}}(x+3)^3 + x^{\frac{-1}{2}}(x+3)^2$

3) $x^{-3} - x^{-1} + x^{\cdot} - x^3$

Solving Exponential Equations

1) $\sqrt{\dfrac{16^{x+3}}{64^{x}}} = 256$

2) $2^{2x} - 3 \cdot 2^{x+1} + 8 = 0$

3) $\dfrac{3^{x^2}}{(3^x)^2} = 27$

4) $2^x + 8 \cdot 2^{-x} = 9$

5) $2^t + 2^{-t} - \dfrac{5}{2} = 0$

6) $2^{2x} - 3 \cdot 2^{x+2} + 32 = 0$

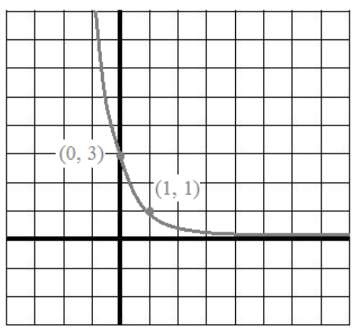
Exponential Function Models

1) What is the equation of a line that passes through (1, 20) and (2, 4)?

What exponential function passes through (1, 20) and (2, 4)?

Optional: Sketch both graphs.

2) Write an exponential equation describing the graph:



3) An exponential equation $f(x)$ includes the coordinates (1, 4) and (6, 7).

What is $f(10)$?

4) Find the exponential equation that goes through (0, 24) and (3, 8/9)

Exponential Equations

5) Find linear and exponential equations with graphs that pass through (1, 50) and (2, 25).

6) Find the exponential equation(s) which include (4, 8) and (6, 32).


Brewmasters Andy and Walt introduce a product for math geeks...
Math Marketing
advertisement
Since 2015
A&W
√BEER
16oz
"A radical beverage that uses base ingredients!"
LanceAF #187 (4/24/15)
mathplane.com
A&W Est. 2015
BREWERY
"Andy, sales are slow.. We need to expand our target audience..."
"I think packaging is the root of our problem. Our logo delivers a mixed message to consumers..."
*Also, mathematicians want nothing less than a full beer!
Unfortunately, due to poor branding and
an unsustainable niche market, this venture didn't last...*

SOLUTIONS-→

Simplifying expressions: exponentials and roots

SOLUTIONS

1) $9^5 \text{ x } 4^{10}$

$\left(3^2\right)^5 \text{ x } 4^{10} =$

$3^{10} \text{ x } 4^{10} =$

$\boxed{12^{10}}$

2) $3^{14} \cdot 5^7$

$(3^2)^7 \cdot 5^7$

$\boxed{45^7}$

3) $\dfrac{e^{4x}}{e^{x-5}}$

$\boxed{e^{3x+5}}$

4) $\dfrac{3^{3t-1}}{3^{3t+2}}$

$3^{-3} = \boxed{\dfrac{1}{27}}$

5) $\dfrac{8^{3\sqrt{5}}}{2^{2\sqrt{5}}} =$

$\dfrac{2^{9\sqrt{5}}}{2^{2\sqrt{5}}} = \boxed{2^{7\sqrt{5}}}$

6) $\dfrac{3^{7x+2}}{3^{7x-1}} =$

$3^3 = \boxed{27}$

7) $\dfrac{64^{3x+1}}{4^{2x}} =$

$\dfrac{(4^3)^{3x+1}}{4^{2x}} = \dfrac{4^{9x+3}}{4^{2x}}$

$= \boxed{4^{7x+3}}$

8) $(5x)^{\frac{1}{2}}(3x^{\frac{1}{2}})$

$5^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot 3 \cdot x^{\frac{1}{2}}$

$\boxed{3x\sqrt{5}}$

9) $\dfrac{\sqrt[3]{9} \cdot \sqrt[3]{6}}{\sqrt[6]{2} \cdot \sqrt[6]{2}}$

$\dfrac{\sqrt[3]{54}}{\sqrt[6]{4}} = \dfrac{\sqrt[3]{27 \cdot 2}}{\sqrt[6]{4}}$

$\dfrac{3 \cdot 2^{\frac{1}{3}}}{(2^2)^{\frac{1}{6}}} = \dfrac{3 \cdot 2^{\frac{1}{3}}}{2^{\frac{1}{3}}} = \boxed{3}$

10) $90^{\frac{1}{2}} - 10^{\frac{1}{2}}$

$\sqrt{90} - \sqrt{10} =$

$3\sqrt{10} - \sqrt{10} =$

$\boxed{2\sqrt{10}}$

11) $\sqrt[3]{18} \cdot \sqrt[3]{15}$

$\sqrt[3]{18 \cdot 15}$

$\sqrt[3]{3 \cdot 3 \cdot 2 \cdot 3 \cdot 5}$

$3\sqrt[3]{2 \cdot 5}$

$\boxed{3\sqrt[3]{10}}$

12) $(8r)^{\frac{1}{3}}(2r^{\frac{1}{2}})$

$8^{\frac{1}{3}} \cdot r^{\frac{1}{3}} \cdot 2 \cdot r^{\frac{1}{2}}$

$2 \cdot r^{\frac{1}{3}} \cdot 2 \cdot r^{\frac{1}{2}}$

$4 \cdot r^{\frac{1}{3}+\frac{1}{2}}$

$\boxed{4r^{\frac{5}{6}}}$

Factoring exponentials

1) $x^{\frac{-1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}}$

$x^{\frac{-1}{2}} \cdot \left(1 + 2x + x^2\right)$

$x^{\frac{-1}{2}} \cdot (x+1)(x+1)$

$\boxed{x^{\frac{-1}{2}} \cdot (x+1)^2}$

2) $x^{\frac{-3}{2}}(x+3)^3 + x^{\frac{-1}{2}}(x+3)^2$

take out "greatest common factor"

GCF of (x + 3) terms: $(x+3)^2$

CF of x terms: $x^{\frac{-3}{2}}$

$\left(x^{\frac{-3}{2}}(x+3)^2\right)\ [(1)(x+3)] + [(x)(1)]$

$\boxed{\left(x^{\frac{-3}{2}}(x+3)^2\right)\ [2x+3]}$

3) $x^{-3} - x^{-1} + x - x^3$

factor by regrouping

$x^{-3}(1 - x^2) + x(1 - x^2)$

$(1 - x^2) \cdot (x^{-3} + x)$

difference of squares

$(1+x)(1-x) \cdot x^{-3}(1 + x^4)$

$\boxed{\dfrac{1}{x^3}(1+x)(1-x)(1+x^4)}$

Solving Exponential Equations

SOLUTIONS

1) $\sqrt{\dfrac{16^{x+3}}{64^{x}}} = 256$ Recognize the 'common base' is 4...

$\sqrt{\dfrac{(4^2)^{x+3}}{(4^3)^{x}}} = 4^4$

$\sqrt{\dfrac{4^{2x+6}}{4^{3x}}} = 4^4$

$\sqrt{4^{-x+6}} = 4^4$

$4^{-x+6} = 4^8$

$x = -2$

2) $2^{2x} - 3\cdot 2^{x+1} + 8 = 0$ Recognize that $2^{x+1} = 2^x\cdot 2^1$

$2^{2x} - 3\cdot 2^x\cdot 2^1 + 8 = 0$

$2^{2x} - 6\cdot 2^x + 8 = 0$ Notice the exponents 2x and x....

Using substitution, let $A = 2^x$

$2^x\cdot 2^x = 2^{x+x} = 2^{2x}$

$A^2 - 6A + 8 = 0$

$(A-2)(A-4) = 0$

$A = 2, 4$

$2^x = 2$

$2^x = 4$

$x = 1, 2$

3) $\dfrac{3^{x^2}}{(3^x)^2} = 27$ Recognize the common base is 3...

Then, simplify the parenthesis...

$\dfrac{3^{x^2}}{3^{2x}} = 3^3$

$3^{(x^2-2x)} = 3^3$ Drop the base and solve..

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$

Check solutions:

if x = -1, then $\dfrac{3^{(-1)^2}}{(3^{(-1)})^2} = \dfrac{3}{(1/9)} = 27$ ✓

If x = 3, then $\dfrac{3^{(3)^2}}{(3^{(3)})^2} = \dfrac{3^9}{3^6} = 27$ ✓

4) $2^x + 8\cdot 2^{-x} = 9$ multiply by 2^x to get rid of negative exponent

$2^x(2^x + 8\cdot 2^{-x} - 9 = 0)$

$2^{2x} + 8\cdot 2^0 - 9\cdot 2^x = 0$

$2^{2x} - 9\cdot 2^x + 8 = 0$ Using substitution, let $2^x = A$

$A^2 - 9A + 8 = 0$

$(A-8)(A-1) = 0$

$A = 8, 1$

$2^x = 8$

$2^x = 1$

x = 3

x = 0

5) $2^t + 2^{-t} - \dfrac{5}{2} = 0$ Factor out least exponent..

$2^{-t}\left(2^{2t} + 1 - \dfrac{5}{2}\cdot 2^t\right) = 0$

$2^{-t} = 0$ no solution

$2^{2t} - \dfrac{5}{2}\cdot 2^t + 1 = 0$ Using Substitution, let $B = 2^t$

$B^2 - \dfrac{5}{2}B + 1 = 0$

Note: the discriminant of this quadratic is

$b^2 - 4ac = \dfrac{9}{4}$

since this is a perfect square, we can factor it...

$\dfrac{1}{2}\left(2B^2 - 5B + 2\right) = 0$

$\dfrac{1}{2}(2B-1)(B-2) = 0$

$B = 1/2$ $2^t = 1/2$

$B = 2$ $2^t = 2$

t = -1

t = 1

6) $2^{2x} - 3\cdot 2^{x+2} + 32 = 0$

$2^{2x} - 3\cdot 2^x\cdot 2^2 + 32 = 0$

$2^{2x} - 12\cdot 2^{2x} + 32 = 0$

$A^2 - 12A + 32 = 0$

$(A-4)(A-8) = 0$

$A = 4, 8$

$2^x = 4$

$2^x = 8$

x = 2

x = 3

Exponential Function Models

SOLUTIONS

1) What is the equation of a line that passes through (1, 20) and (2, 4)?

What exponential function passes through (1, 20) and (2, 4)?

Optional: Sketch both graphs.

slope: $\frac{20-4}{1-2} = -16$

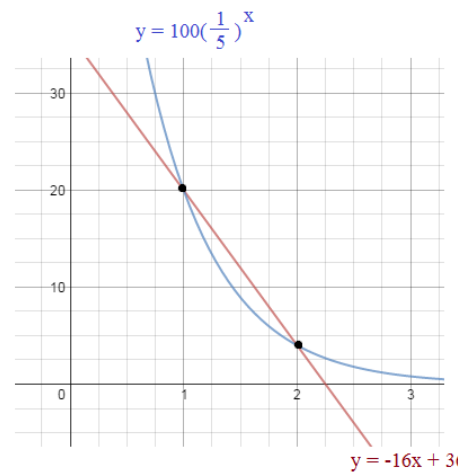
point: (1, 20)

$(y - 20) = -16(x - 1)$

or

$y = -16x + 36$

Note: To check the equations, test the points (1, 20) and (2, 4)



$y = ab^x$

Using substitution: $20 = ab^1$ $\quad 4 = ab^2$

$a = \frac{20}{b}$

$4 = \frac{20}{b}b^2$

$4 = 20b$

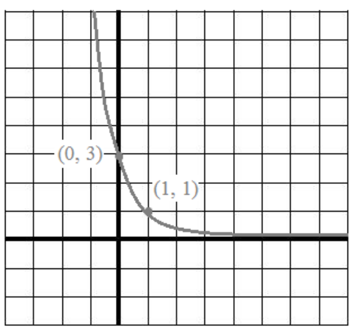
$b = \frac{1}{5}$

$4 = a\left(\frac{1}{5}\right)^2$

$a = 100$

$y = 100(\frac{1}{5})^x$

2) Write an exponential equation describing the graph:



$y = ab^x$

substitute first point:

$3 = ab^0$

$3 = a(1)$

$a = 3$

$y = 3b^x$ substitute second point:

$1 = 3b^1$

$b = \frac{1}{3}$

$y = 3(\frac{1}{3})^x$

quick check:

(0, 3): $3 = 3(\frac{1}{3})^0$

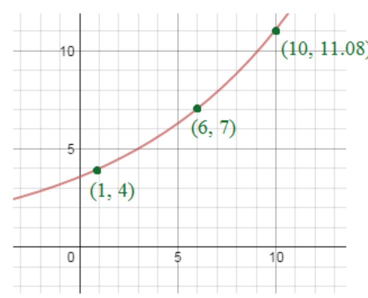
$3 = 3$ ✓

(1, 1): $1 = 3(\frac{1}{3})^1$

$1 = 1$ ✓

3) An exponential equation $f(x)$ includes the coordinates (1, 4) and (6, 7).

What is $f(10)$?



$y = ab^x$

substitute (1, 4): $4 = ab^1$

$a = \frac{4}{b}$

then, substitute a $\quad y = \frac{4}{b}b^x$

$y = 4b^{-1}b^x$

$y = 4b^{x-1}$

then, substitute (6, 7)

$7 = 4b^{(6-1)}$

$1.75 = b^5$

$b = \sqrt[5]{1.75}$

$b = 1.12$

since $b = 1.12$

and $a = \frac{4}{b}$

$a = 3.57$

$f(x) = (3.57)(1.12)^x$

$f(10) = (3.57)(1.12)^{10}$ $= 11.08$

4) Find the exponential equation that goes through (0, 24) and (3, 8/9)

Using the short-cut, we know that the inital value 'a' = 24 (because the y-intercept is (0, 24))

$y = 24b^{x}$

Then, $b = \left(\frac{8/9}{24}\right)^{\frac{1}{3}}$

"b" is the growth factor

therefore, $y = 24\left(\frac{8/9}{24}\right)^{\frac{x}{3}} = 24\left(\frac{1}{27}\right)^{\frac{x}{3}}$

or, $24\left(\frac{1}{3}\right)^{x}$

quick check:

(0, 24) $24 = 24\left(\frac{1}{27}\right)^{\frac{0}{3}}$ ✓

(3, 8/9) $8/9 = 24\left(\frac{1}{27}\right)^{\frac{3}{3}}$ ✓

5) Find linear and exponential equations with graphs that pass through (1, 50) and (2, 25).

Using the exponential function,

$y = ab^{x}$

$50 = ab^{1}$

$25 = ab^{2}$

then, solve the system with substitution:

$a = \frac{50}{b}$ (first equation)

so, $25 = \frac{50}{b}b^{2}$ (substitute into second equation)

$25 = 50b$

$b = \frac{1}{2}$

and, then a = 100

$y = 100(.5)^{x}$

Using the linear equation $y = mx + b$,

we need the slope: $m = \frac{25 - 50}{2 - 1} = -25$

and, a point: (1, 50)

(substitute to find b)

$50 = (-25)(1) + b$

$b = 75$

$y = -25x + 75$

6) Find the exponential equation(s) which include (4, 8) and (6, 32).

$y = ab^{x}$

$32 = ab^{6}$

$8 = ab^{4}$

solve two equations with two unknowns

$a = \frac{8}{b^{4}}$

substitution

$32 = \frac{8}{b^{4}} \cdot b^{6}$

$32 = 8b^{2}$

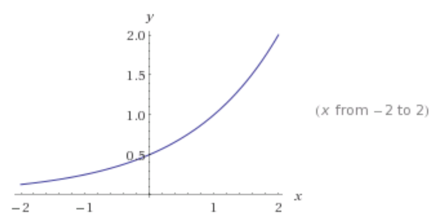
$4 = b^{2}$

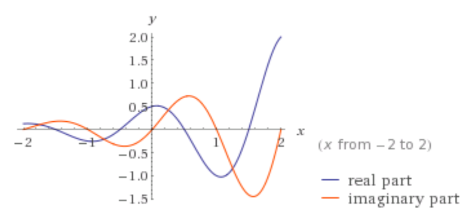
$b = -2, 2$

then, $a = \frac{1}{2}$

$y = \frac{1}{2}(2)^{x}$

$y = \frac{1}{2}(-2)^{x}$

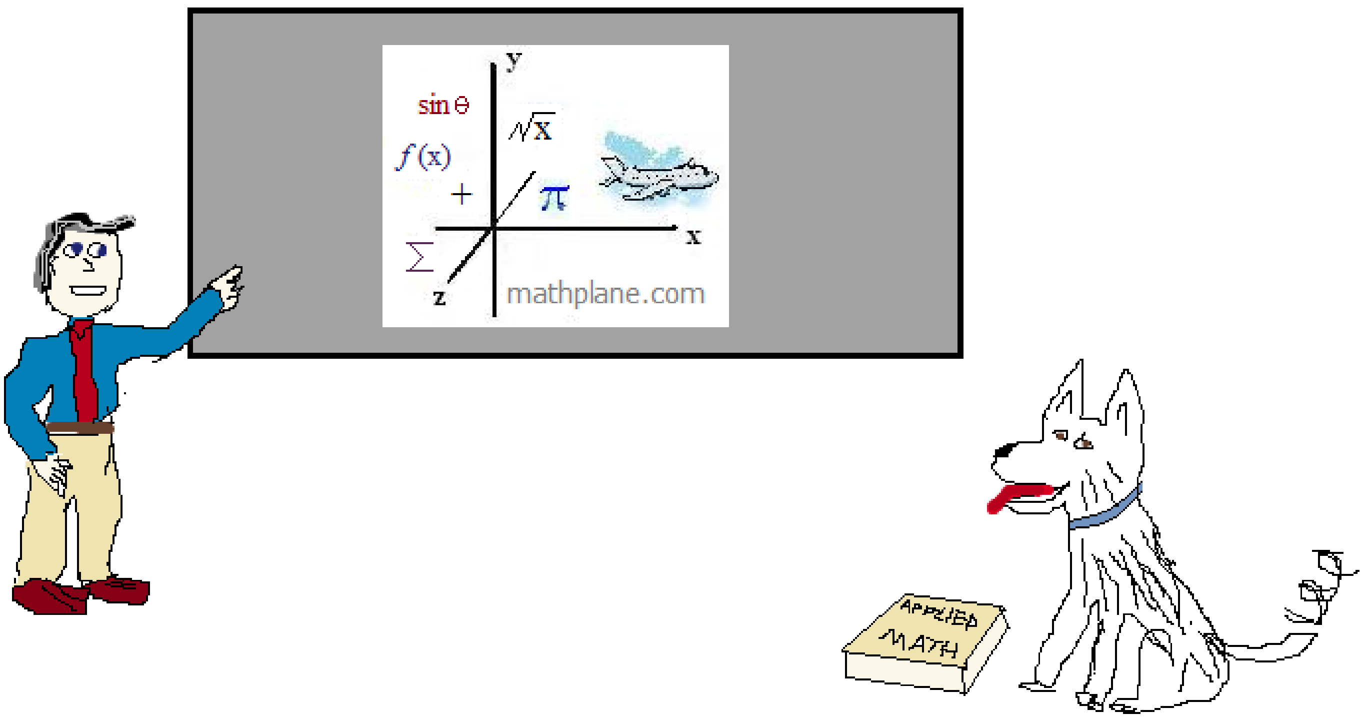




Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



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