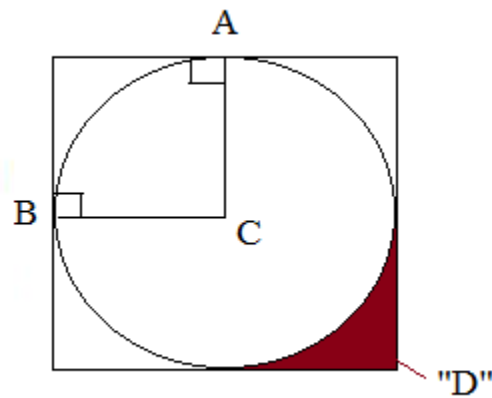# Geometry Review 001 Questions

(With Solutions)

Topics include area, angle measure, proofs, triangles, polygons, ratios, coordinate geometry, 'walk-about problem', and more.

Example: Find the point (6, y) equidistant from (2, 5) and (12, -1)

Method 1: Apply the distance formula

$$\text{distance} = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$$

distance to first point: $\sqrt{(6-2)^2 + (y-5)^2}$ distance to second point: $\sqrt{(6-12)^2 + (y--1)^2}$

$$\sqrt{16 + y^2 - 10y + 25}$$ $$\sqrt{36 + y^2 + 2y + 1}$$

Then, since they are equidistant, we can set them equal to each other...

$$\sqrt{16 + y^2 - 10y + 25} = \sqrt{36 + y^2 + 2y + 1}$$

$$y^2 - 10y + 41 = y^2 + 2y + 37$$

$$4 = 12y$$

$$y = 1/3 \checkmark$$

Method 2: Utilizing the Equidistance Theorem

Let's find the perpendicular bisector of the segment with endpoints (2, 5) and (12, -1)

Midpoint: (7, 2)

Slope of segment: $\dfrac{-6}{10}$

therefore, equation of perpendicular bisector....

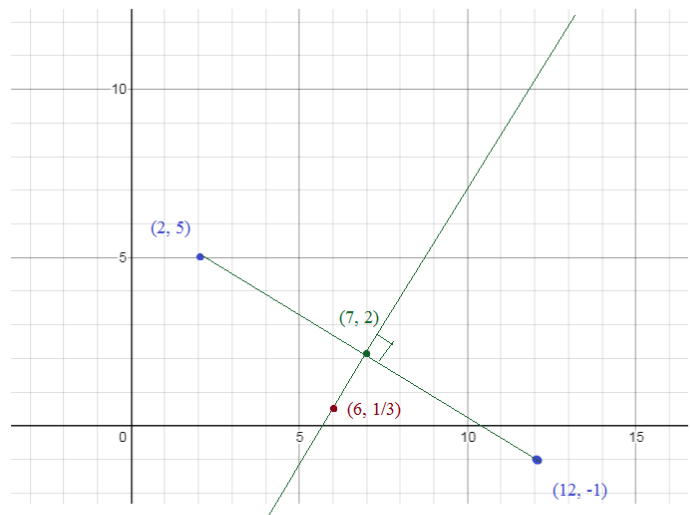(7, 2) with slope 10/6 = 5/3

$$y - 2 = \frac{5}{3}(x - 7)$$

→ According to the equidistance theorem, any point on the perpendicular bisector will be equidistant to each endpoint!

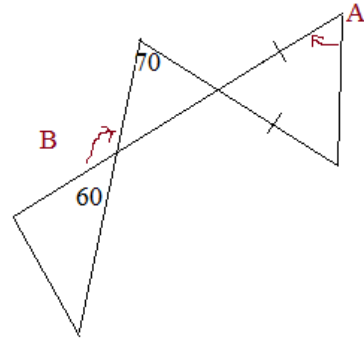substitute (6, y) into the equation....

$$y - 2 = \frac{5}{3}(6 - 7)$$

$$y - 2 = \frac{-5}{3}$$

$$y = 1/3 \checkmark$$

1) In the figure, what is the measure of $\angle A$? $\angle B$?

2) Write the equation of a line <u>perpendicular</u> to $4x + 2y = 6$
and that passes through $(4, 4)$.

3) Prove the diagonals of a parallelogram bisect each other.
(include a sketch of the parallelogram as well as a 2-column proof)

4) What is the measure of each *interior* angle of a *regular* hexagon?
And, the measure of each *exterior* angle?

5) What is an orthocenter of a triangle? Where is it located?
   Draw obtuse, right, and acute triangles. Then, identify the orthocenters in each.

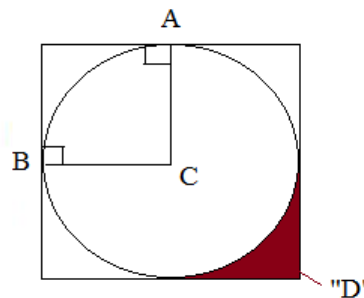6) Radius of circle P = 6 feet
   Measure of central angle QPR = 120°

   a) What is the area of section "A"?

   b) What percentage of the circle does triangle PQR occupy?
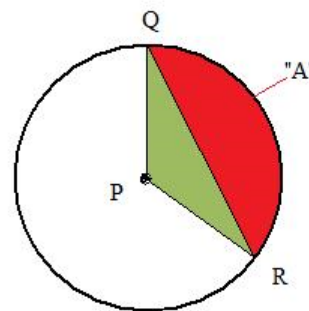
7) Circle C is inscribed in the square.
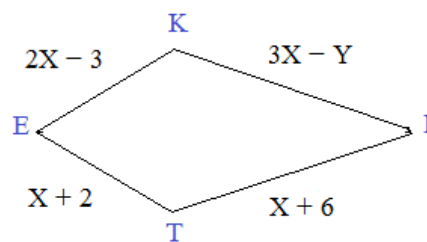
   Arc length $\overarc{AB}$ = 6π

   a) What is the measure of $\angle ACB$?

   b) What is the area of the region "D"?

8) In the following kite, $\overline{KE} = \overline{ET}$

    a) Find X and Y

$$K$$
$$2X - 3 \qquad 3X - Y$$
$$E \qquad\qquad\qquad I$$
$$X + 2 \qquad X + 6$$
$$T$$

    b) What is the perimeter of KITE?

   c) Is $\overline{IE}$ 21 units?   Why or why not?

9) A building is 1600 feet tall. The model of the building is 24 inches tall.
    What is the ratio of the model to the actual building?

    a) 200:3

    b) 3:200
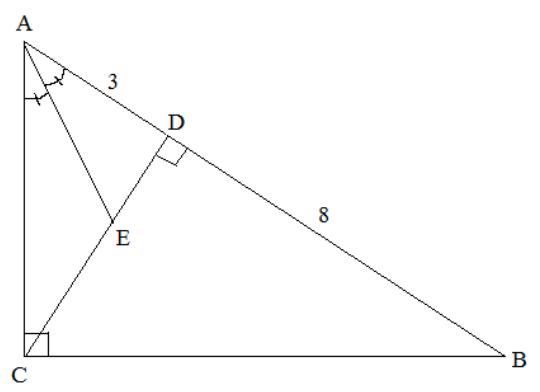
    c) 800:1

    d) 1:800

    e) none of the above

10) Two concentric circles have radii 3 and 7.
Find the length of the chord in the larger circle that
is tangent to the smaller circle.

11) The distance of the chord to the center of the circle is 6.
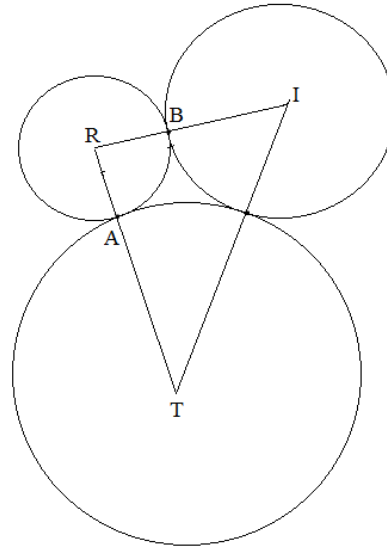The radius of the circle is 12.

What is the angle measure of arc ST?
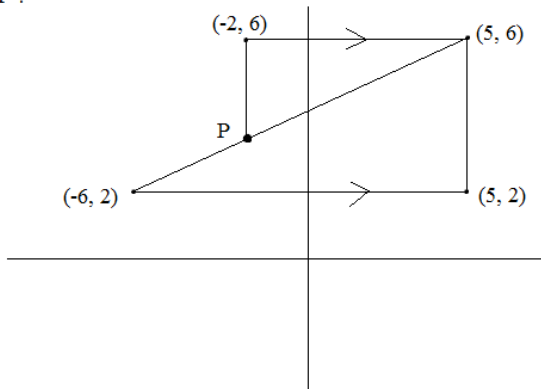
S   12

6

T

12) Find the length of $\overline{DE}$

A

3

D

8

E

C   B
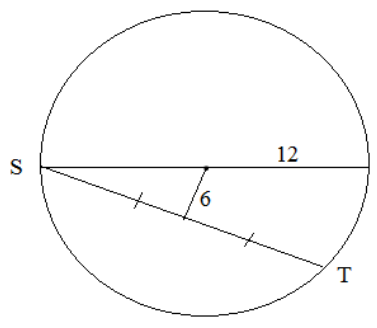
13)   Circles T, R, and I are tangent.

△ TRI joins the centers of the circles.

$\overline{RI}$ = 9 $\overline{TI}$ = 15 $\overline{RT}$ = 12

What is the arc length of arc $\overset{\frown}{AB}$?

14) What is coordinate P ?

(-2, 6)   (5, 6)

P

(-6, 2)   (5, 2)

15) (-4, -7)   (a, 5)   distance between points: 13

Find the possible value(s) of a.

How many edges in a square pyramid?   Edges in a triangular pyramid?   Sides in a triangular prism?

What is the surface area of a triangular pyramid with all edges of 10 feet?

Find the *lateral* area of a rectangular pyramid with base 4 x 5 feet and *slant height* of 8 feet.
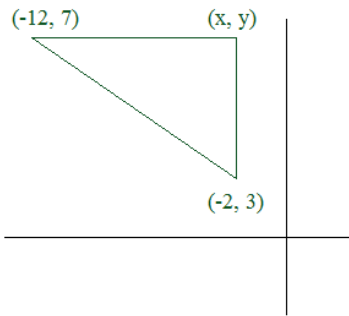

Volume Riddle: How much dirt is in a square hole that is 3 meters wide, 3 meters long,
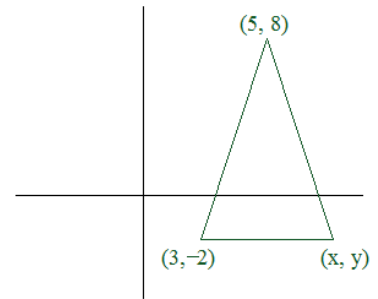   and 3 meters deep?   (Hint: It is not 27 cubic meters.)
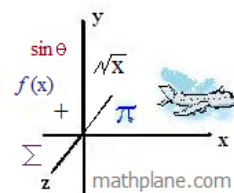
Identify the missing coordinates. Then, find the area of each figure.

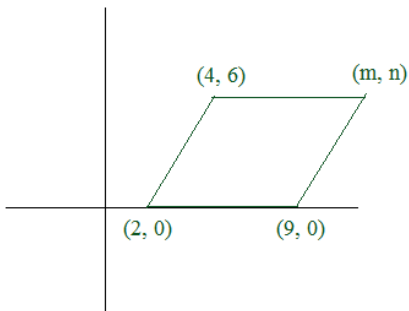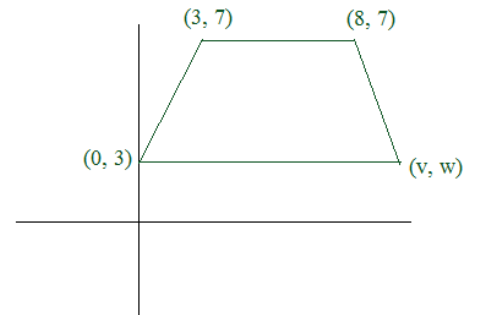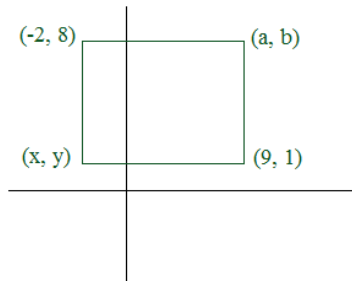Quadrilaterals, Triangles, and Coordinates

1) Right Triangle

(-12, 7)   (x, y)

(-2, 3)

2) Isosceles Triangle

(5, 8)

(3, -2)   (x, y)

3) Parallelogram

(4, 6)   (m, n)

(2, 0)   (9, 0)

4) Isosceles Trapezoid

(3, 7)   (8, 7)

(0, 3)   (v, w)

5) Rectangle

(-2, 8)   (a, b)

(x, y)   (9, 1)

6) Square

(a, b)

(c, d)   (5, 0)

(f, g)

7) Rhombus

(-1, 5)   (4, 5)

(x, y)   (1, 1)

8) Parallelogram

(2, 6)

(5, 4)

(2, -1)   (w, r)

mathplane.com

Math in Hell

"The final exam is 97% of your grade. It is one question: find the surface area of the following pyramid...."

"... There is no partial credit... Round your answer to the nearest centimeter..."
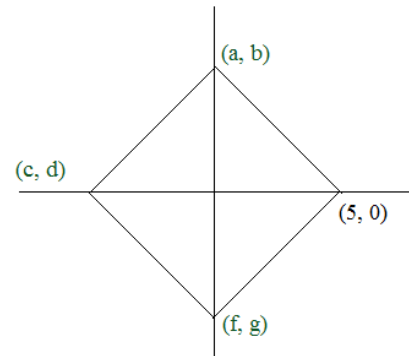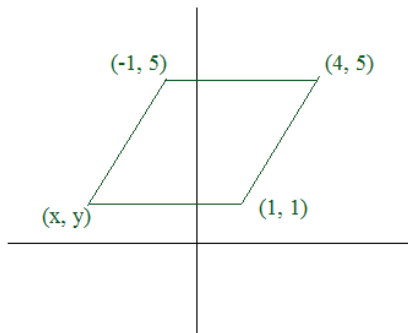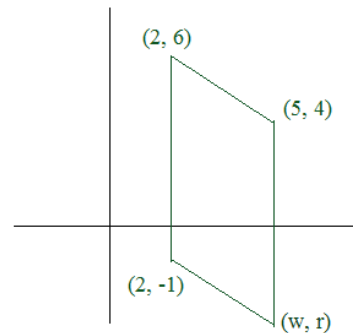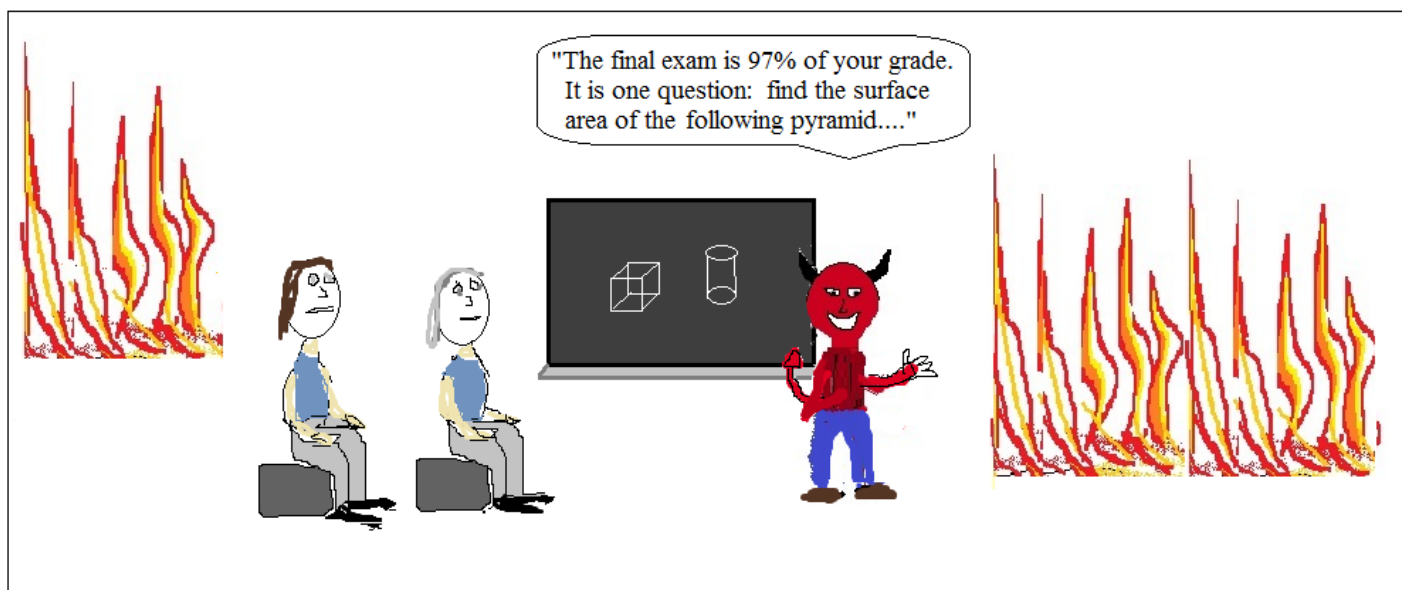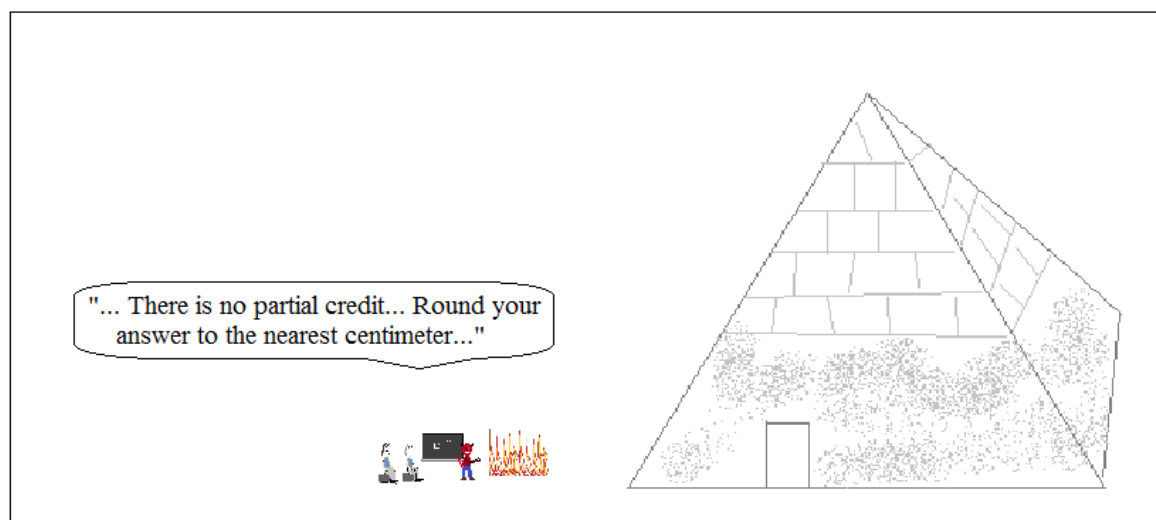
In its 1000 year history, no one ever passed Mr. Devlin's Geometry class.

LanceAF #39 7-1-12
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SOLUTIONS→

1) $\angle B = 120°$

    $60 + \angle B = 180$ (supplementary angles)

$\angle A = 65°$

    middle triangle angles: 60 (vertical angles =)

                            70 (given)

                            50 (triangles --> 180)

    last triangle angles: 50 / X / X (isosceles)

                        $50 + 2X = 180 \longrightarrow X = 65$

2) $4x + 2y = 6$

       $2y = -4x + 6$ ---> changing to slope-intercept form

       $y = -2x + 3$ ---> slope of line is -2

       Therefore, slope of perpendicular line is 1/2

line with slope = 1/2 through (4, 4)
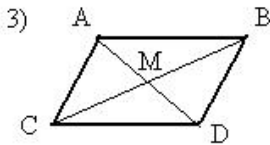
    $y = 1/2\ x + b$

    $4 = 1/2\ (4) + b$

    $4 = 2 + b$

    $b = 2$
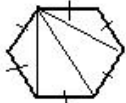
    Therefore, line is $y = 1/2\ x + 2$
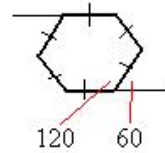
3)   A   B

Given: $\square$ABDC

Prove: AM = MD

       CM = MB

| Statement	Reason
$\overline{AB} = \overline{CD}$	Definition of $\square$
$\angle AMB = \angle DMC$	Vertical angles $\cong$
$\angle BAM = \angle CDM$	parallel lines cut by transversal, alternate interior angles $\cong$
$\triangle ABM = \triangle DCM$	Angle Angle Side
$\overline{AM} = \overline{MD}, \overline{CM} = \overline{MB}$	CPCTC
	(Corresponding Parts of Congruent Triangles are Congruent)

4)

$180(n - 2) = 180(6 - 2) = 720°$

total of interior angles

$\dfrac{720}{6} = 120°$

each interior angle

120   60

$\dfrac{360}{n} = \dfrac{360}{6}$

each exterior angle is $60°$

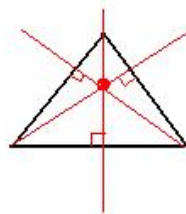5) Orthocenter: The point where the <u>three altitudes</u> of a triangle meet

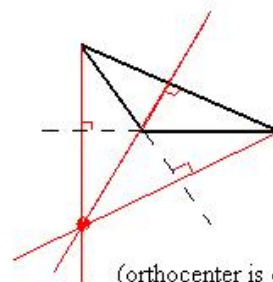   (are <u>concurrent</u>)

Right   Acute   Obtuse
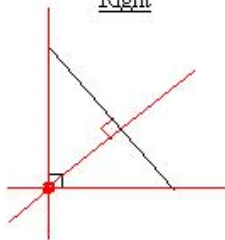
(orthocenter is on the right triangle)
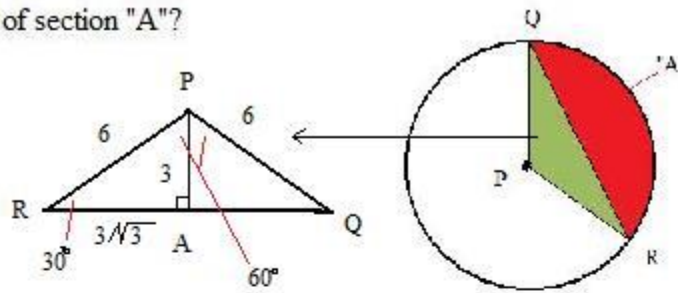
(orthocenter is inside an acute triangle)

(orthocenter is outside an obtuse triangle)

6) a) What is the area of section "A"?

30-60-90 triangle establishes lengths of sides.
then, area$\triangle PQR$
$= 1/2 bh$

$= 1/2 (6\sqrt{3})3$

$= 9\sqrt{3}$

P

6   6

3

R

$3\sqrt{3}$ A

$30°$

$60°$

Q

Area of entire circle $= \pi r^2$

$= 36\pi$ square feet

Area of portion $\overset{\frown}{QPR} = \dfrac{120}{360} \times 36\pi$ square feet

$= 12\pi$ square feet

(Now, subtract $\triangle PQR$)

Area of "A" $= \left(12\pi - 9\sqrt{3} \right)$ square feet

b) What percentage of the circle does triangle PQR occupy?

$\dfrac{\text{Area of triangle}}{\text{Area of Circle}} = \dfrac{9\sqrt{3}}{36\pi} = \dfrac{\sqrt{3}}{4\pi}$

approx. 13.8%

7) Circle C is inscribed in the square.

a) What is the measure of $\angle ACB$? 90 degrees

b) What is the area of region "D"?

Arc length $\overset{\frown}{AB} = 6\pi$ therefore, circumference of circle $= (4 \times \overset{\frown}{AB}) = 24\pi$

$\longrightarrow \overline{AC} = \overline{AB} = 12$

Area of square $= 24 \times 24 = 576$
Area of circle $= \pi r^2 = 144\pi$
4 corner pieces $= 576 - 144\pi$

corner "D" $= \dfrac{(576 - 144\pi)}{4}$

$144 - 36\pi$

A

B

C

"D"

SOLUTIONS

8) In the following kite, $\overline{KE} = \overline{ET}$

K

2X − 3   3X − Y
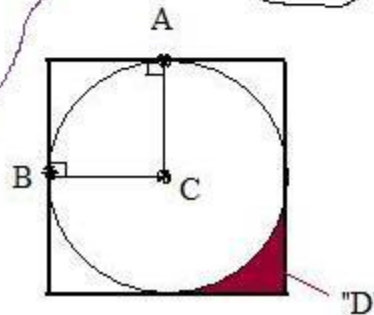
   a) Find X and Y

   Since KE = ET,   2X - 3 = X + 2

E   I
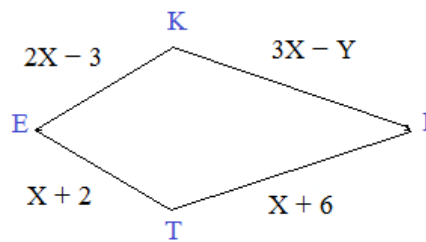
       X = 5

           If X = 5, then $\overline{IT}$ = (5) + 6 = 11

X + 2   X + 6

           Since it is a kite, $\overline{IT} = \overline{IK}$

T

           IK = 11: 3(5) − Y = 11   Y = 4

   b) What is the perimeter of KITE?

       $\overline{KE}$ = 7

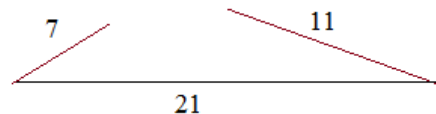 $\overline{ET}$ = 7

               perimeter is 36 units

       $\overline{IK}$ = 11

       $\overline{IT}$ = 11

   c) Is $\overline{IE}$ 21 units? Why or why not?

7   11

       It cannot be 21 units...

21

       Length of $\overline{KE}$ is 7... Length of $\overline{IK}$ is 11...
       $\overline{IE}$ must be less than 18 units!

9) A building is 1600 feet tall. The model of the building is 24 inches tall.
   What is the ratio of the model to the actual building?

       a) 200:3

       b) 3:200   The ratio is model:actual

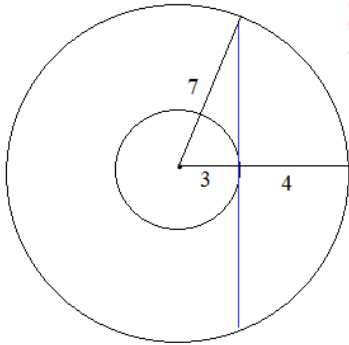   c) 800:1   24 inches:1600 feet

       d) 1:800   2 feet:1600 feet

       e) none of the above
                                                   1:800

10) Two concentric circles have radii 3 and 7.
    Find the length of the chord in the larger circle that
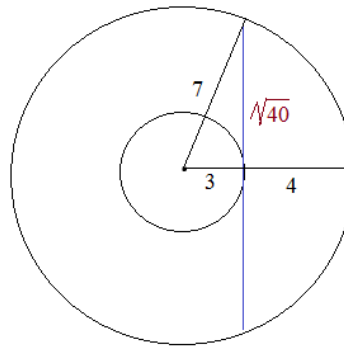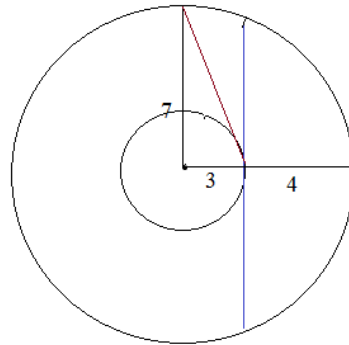   is tangent to the smaller circle.

Step 1: Draw a diagram

(Endpoints of chord are
on the larger circle;
The chord is tangent
to the smaller circle)

7

3   4

Step 2: Identify useful geometry property

7     $\sqrt{40}$

3   4

Since radius is perpendicular
bisector of chord, we create
a right triangle.
Using Pythagorean Theorem,
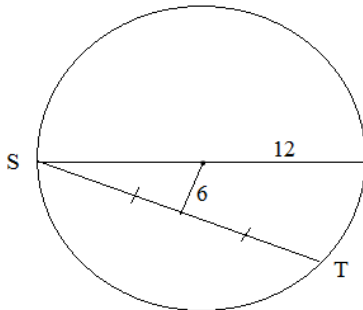1/2 the chord is $\sqrt{40}$

Therefore, entire chord is

$2 \times \sqrt{40} \longrightarrow 4\sqrt{10}$

7

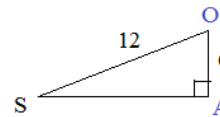3   4

Incorrect...
Wrong right triangle

11) The distance of the chord to the center of the circle is 6.
    The radius of the circle is 12.

    What is the angle measure of arc ST?

S   12

6

T

Since the chord segments $\overline{AS}$ and $\overline{AT}$ are congruent, $\overline{AO}$ must be
perpendicular to $\overline{ST}$.

Since the radius is 12, SO is 12 (because all radii are congruent)
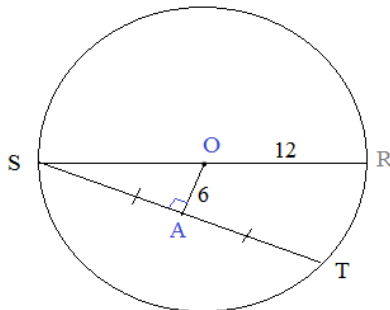
O

12   6

S   A

Since the hypotenuse is 2x the leg, it is a 30-60-90 right triangle.

---> Angle S is 30 degrees...

(inscribed)
And, because angle S is 30 degrees, the angle measure of arc RT is 60 degrees...

We know RTS is a semi-circle (180 degrees)...

S   O   12   R
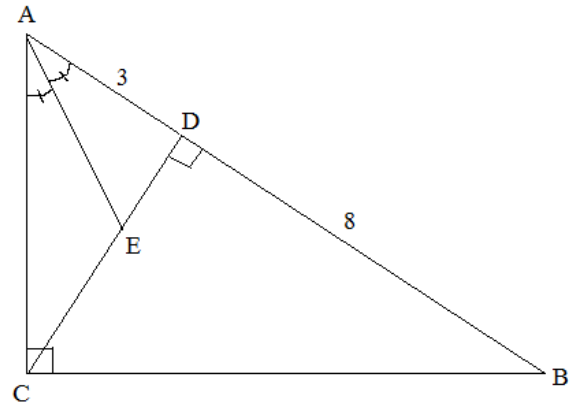
6

A

T

Therefore, ST is 120 degrees...

12) Find the length of $\overline{DE}$

Step 1: Utilize the "Geometric Mean of divided Hypotenuse"

$$\frac{AD}{DC} = \frac{DC}{DB}$$

$$DC^2 = AD \cdot DB$$

$$DC = \sqrt{24}$$

Step 2: Utilize the Pythagorean Theorem
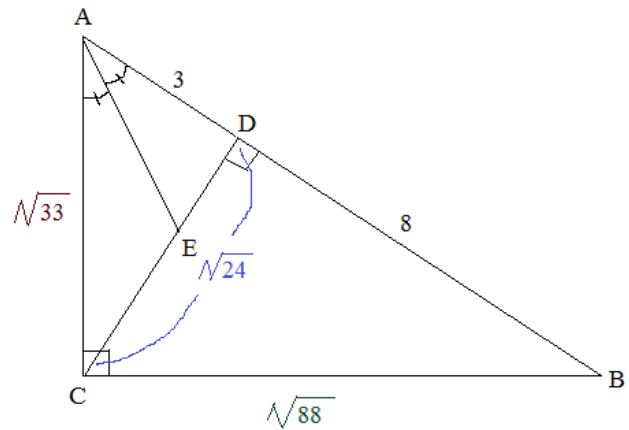
$$DB^2 + DC^2 = CB^2$$

$$64 + 24 = CB^2$$

$$CB = \sqrt{88}$$

$$CB^2 + AC^2 = AB^2$$

$$88 + AC^2 = 121$$

$$AC = \sqrt{33}$$

Step 3: Use the "Angle Bisector Theorem"

Since AE is an angle bisector in triangle CAD,

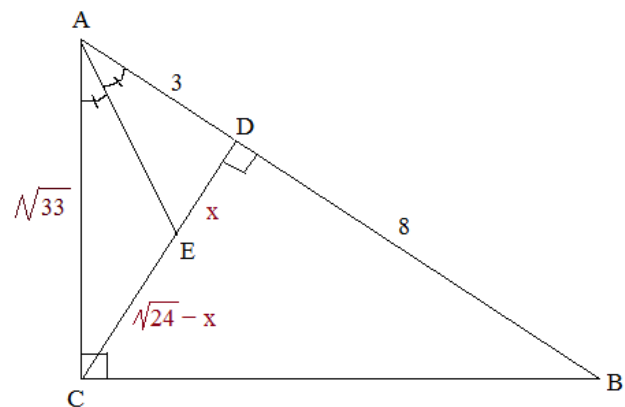$$\frac{AD}{AC} = \frac{DE}{CE}$$

$$\frac{3}{\sqrt{33}} = \frac{x}{\sqrt{24} - x}$$

$$3\sqrt{24} - 3x = \sqrt{33}\,x$$

$$3\sqrt{24} = \sqrt{33}\,x + 3x$$

$$14.697 = 8.745x$$

$$\boxed{x = 1.68}$$

13)   Circles T, R, and I are tangent.

△ TRI joins the centers of the circles.

$\overline{RI} = 9$ $\overline{TI} = 15$ $\overline{RT} = 12$

What is the measure of arc $\overset{\frown}{AB}$?

Part 1: Find the measures of the radii
("Walk-about question")

Using "all radii in a circle are congruent",

we can identify the lengths of the radii
then, since $\overline{RT} = 12$,

$$2x + 6 = 12 \qquad x = 3$$
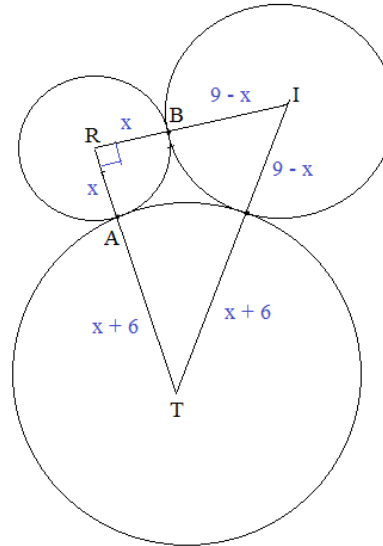
so, the radii measures are

R = 3
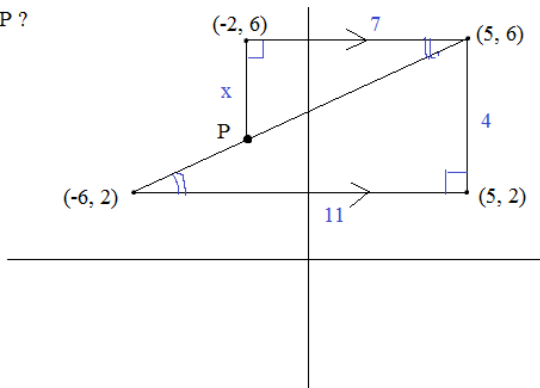I = 6
T = 9

Part 2: Find the arc length of $\overset{\frown}{AB}$

Since the lengths of the triangle are 9, 12, and 15,
we can see it's a right triangle!

Therefore, the measure of cental angle ARB is 90 degrees

$$\text{Arc } \overset{\frown}{AB} \ = \ \frac{90}{360} \ 6\pi \ = \ \boxed{\frac{3\pi}{2}}$$

Diagram labels: 9 - x, I, B, R, x, x, 9 - x, A, x + 6, x + 6, T

14) What is coordinate P ?

Diagram labels: (-2, 6), 7, (5, 6), x, P, 4, (-6, 2), 11, (5, 2)

The two triangles are proportional...

$$\frac{4}{x} \ = \ \frac{11}{7}$$

$$x = 28/11$$

so, P is (-2, 6 − 28/11)

$$\boxed{(-2, 38/11)}$$

15) (-4, -7)   (a, 5)   distance between points: 13

Find the possible value(s) of a.

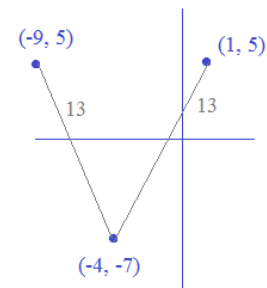$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$13 = \sqrt{(-4 - a)^2 + (-7 - 5)^2}$$

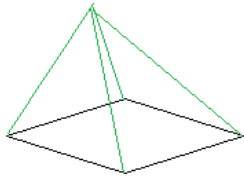$$169 = (-4 - a)^2 + 144$$

$$25 = (-4 - a)^2$$

$$-4 - a = 5 \qquad -4 - a = -5$$

$$\boxed{a = -9 \qquad a = 1}$$

Diagram labels: (-9, 5), (1, 5), 13, 13, (-4, -7)
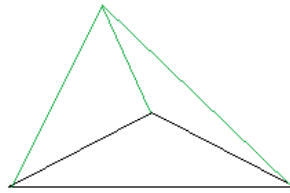
How many edges in a square pyramid?

Square Pyramid

4 "vertical" edges
4 "base" edges
8 total edges

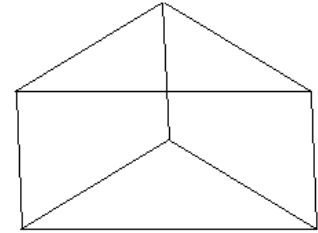**Edges in a triangular pyramid?**

Triangular Pyramid

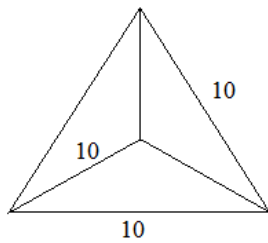3 "vertical" edges
3 "base" edges
6 total edges

Sides in a triangular prism?
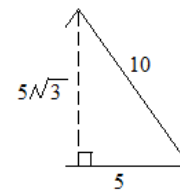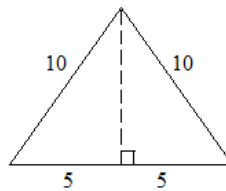
Triangular Prism

2 triangle bases
3 rectangle (lateral) sides
5 total sides (or faces)

What is the surface area of a triangular pyramid with all edges of 10 feet?

Since all edges are 10 feet, each of the 4 (triangle) sides of the pyramid is the same area.
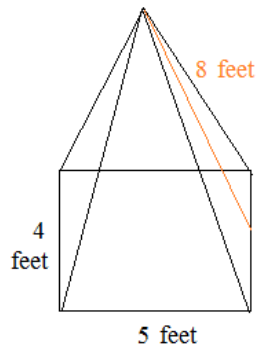
So, find the area of one side:

10   10
10

10   10
5   5

$5\sqrt{3}$ 10
5

(using pythagorean theorem or recognizing 30-60-90 triangle, we find the height of the triangle is $5\sqrt{3}$)

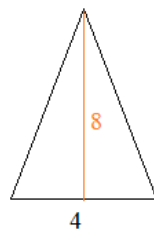Area of one side: $1/2(b)(h) = 1/2 (10)(5\sqrt{3}) = 25\sqrt{3}$

Surface area of pyramid $= 100\sqrt{3}$ square feet

Find the _lateral_ area of a rectangular pyramid with base 4 x 5 feet and _slant height_ of 8 feet.
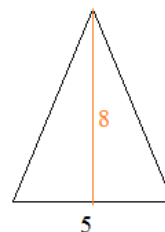
Rectangular Pyramid

8 feet

4 feet

5 feet

The lateral area is the area of the lateral sides (so, it excludes the base!)

8
4
("left and right sides")

8
5
("front and back sides")

area of the front and back:

$2 \times [1/2 (b)(h)] = 2 \times [1/2 (5)(8)] = 40$

area of left and right:

$2 \times [1/2 (b)(h)] = 2 \times [1/2 (4)(8)] = 32$

Total lateral area: 72 square feet

Volume Riddle: How much dirt is in a square hole that is 3 meters wide, 3 meters long, and 3 meters deep? (Hint: It is not 27 cubic meters.)

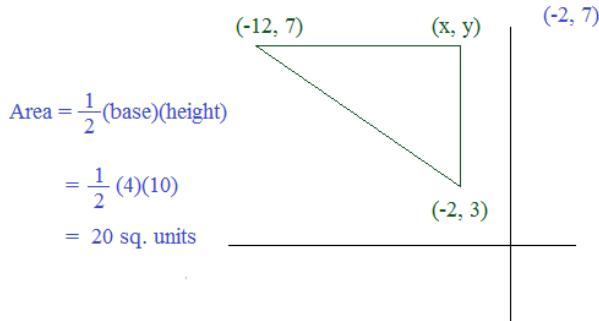Answer: Zero. There is no dirt in a hole. It's empty!

sin θ
$f(x)$
$\sqrt{x}$
π
$\sum$
x
y
z

mathplane.com
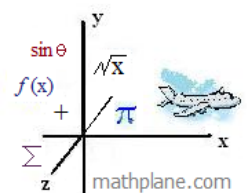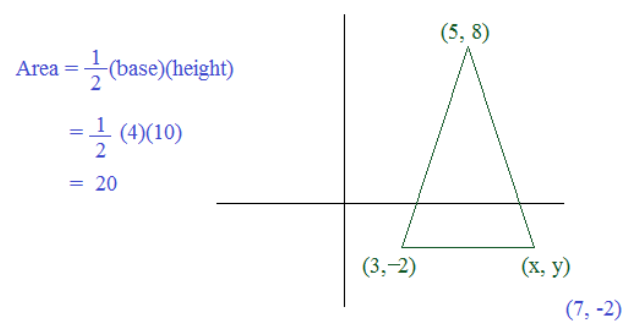
Identify the missing coordinates. Then, find the area of each figure.   SOLUTIONS   Quadrilaterals, Triangles, and Coordinates

1) Right Triangle
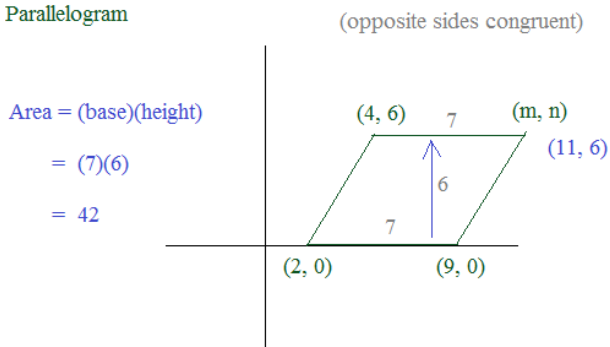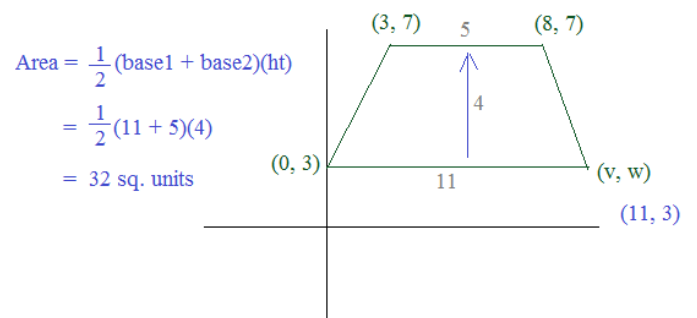
(-12, 7)   (x, y)   (-2, 7)

Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

= 20 sq. units

(-2, 3)

2) Isosceles Triangle

(5, 8)

Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

= 20

(3, -2)   (x, y)

(7, -2)

3) Parallelogram   (opposite sides congruent)

Area = (base)(height)

= (7)(6)

= 42

(4, 6)   7   (m, n)

(11, 6)

6

7

(2, 0)   (9, 0)

4) Isosceles Trapezoid

Area = $\frac{1}{2}$(base1 + base2)(ht)

= $\frac{1}{2}$(11 + 5)(4)

= 32 sq. units

(3, 7)   5   (8, 7)

4

(0, 3)   11   (v, w)

(11, 3)

5) Rectangle

(-2, 8)   (a, b)   (9, 8)

(-2, 1)   (x, y)   (9, 1)

Area = (length)(width)

= (11)(7)

= 77

6) Square

Area = (side)^2

= $(5\sqrt{2})^2$

= 50

Or,

Area = $\frac{1}{2}$ (diagonal 1)(diagonal 2)

= $\frac{1}{2}$ (10)(10)   = 50

(a, b)   (0, 5)

$5\sqrt{2}$

(-5, 0)

(c, d)   5

5   (5, 0)

(f, g)   (0, -5)

7) Rhombus

Area = (base)(height)

= (4)(5)

= 20

Or,

Area = $\frac{1}{2}$ (d1)(d2)

= $\frac{1}{2}$ ($\sqrt{80}$)($\sqrt{20}$)

= 20

(-1, 5)   (4, 5)

4

(x, y)   (1, 1)

(-4, 1)

8) Parallelogram

Area = (base)(height)

= (7)(3)

= 21 sq. units

(2, 6)

(5, 4)

7   3

(2, -1)

(w, r)   (5, -3)
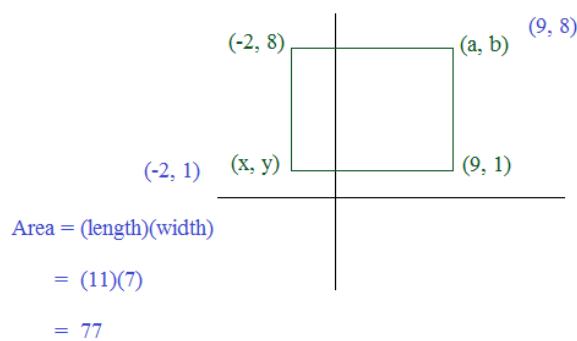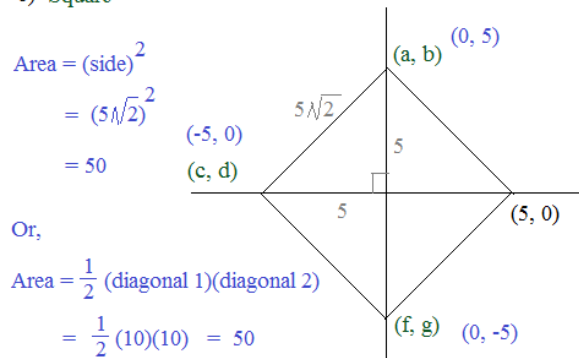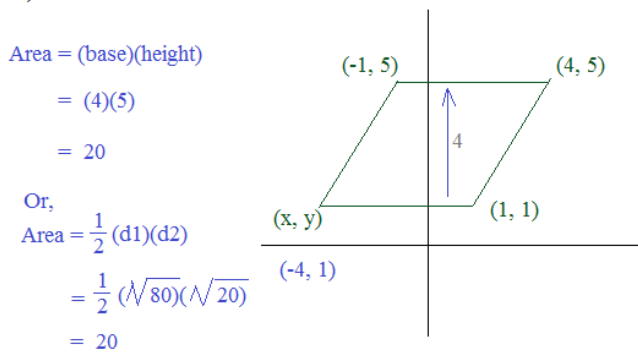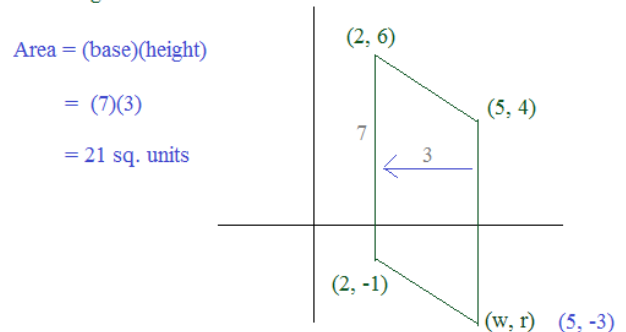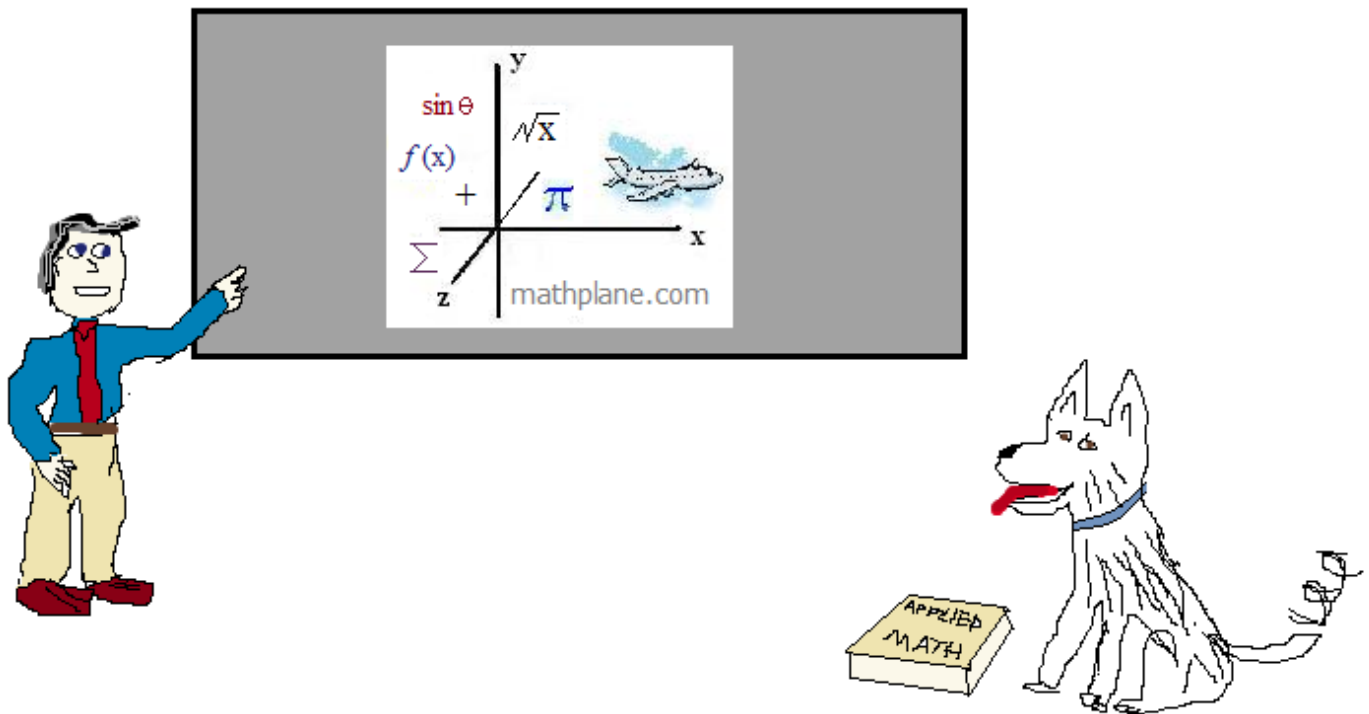
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Thanks for visiting. (Hope it helped!)

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If you have questions, suggestions, or requests, let us know.
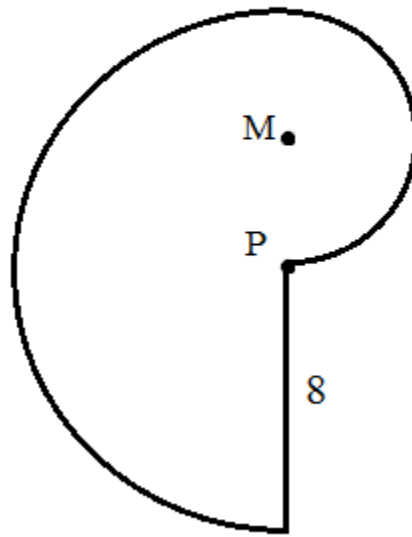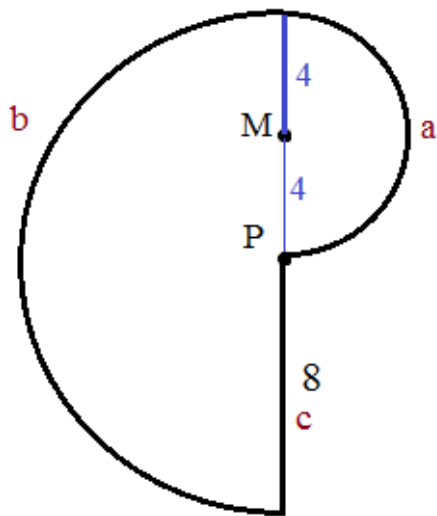
Enjoy

Also, at TES and TeachersPayTeachers

Plus, Mathplane.ORG for mobile and tablets

One more question:

Points M and P are the *centers* of the semi-circles.
What is the perimeter of the figure?

Points M and P are the *centers* of the semi-circles.
What is the perimeter of the figure?

a   arc length of small semi-circle:

$$\frac{180}{360} \cdot 2\pi(4) = 4\pi$$

b   arc length of large semi-circle:

$$\frac{180}{360} \cdot 2\pi(8) = 8\pi$$

c   length of segment $= 8$

$$12\pi + 8$$

or

$$45.7$$