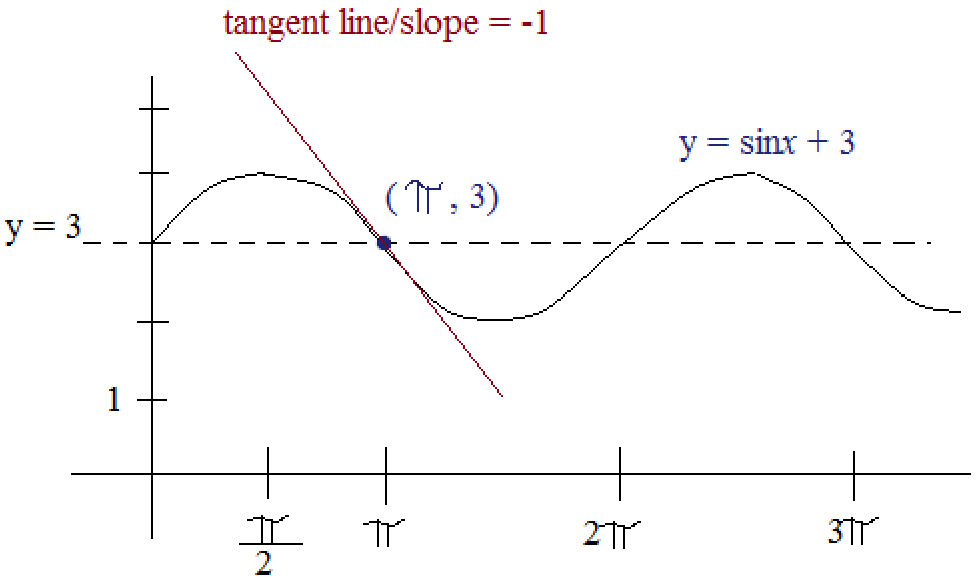
Calculus: Brief Intro to Trig Derivatives



Includes notes, examples, formulas, and practice worksheet (with solutions)

mathplane.com

Derivatives of Trigonometry Functions

Finding the Derivatives of the 6 basic Trig Functions

Using Instantaneous Rate of Change and trig & limit properties

$$\lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x}$$

$$\sin(A + B) = \sin A\cos B + \cos B\sin A$$

$$\lim_{\triangle x \to 0} \frac{\sin \triangle x}{\triangle x} = 1$$

derivative of sine:

$$\lim_{\triangle x \to 0} \frac{\sin(x + \triangle x) - \sin(x)}{\triangle x}$$

$$\lim_{\triangle x \to 0} \frac{\sin x\cos \triangle x + \cos x\sin \triangle x - \sin x}{\triangle x}$$

$$\lim_{\triangle x \to 0} \frac{\sin x\cos \triangle x - \sin x}{\triangle x} + \lim_{\triangle x \to 0} \frac{\cos x\sin \triangle x}{\triangle x}$$

$$\lim_{\triangle x \to 0} \sin x \frac{\cos \triangle x - 1}{\triangle x} + \lim_{\triangle x \to 0} \cos x \frac{\sin \triangle x}{\triangle x}$$

$$\sin x\,(0) \quad + \quad \cos x\,(1) \quad = \quad \cos x$$

derivative of cosine: Using the same limit techniques,
we can confirm for $y = \cos x$, the derivative $y' = -\sin x$

Using Quotient Rule (differentiation) & Trig Identities

$$\tan x = \frac{\sin x}{\cos x}$$

derivative of tangent:

$$\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} = \frac{\cos x\cos x - (-\sin x\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\cot x = \frac{\cos x}{\sin x}$$

derivative of cotangent:

$$\frac{d}{dx} \frac{\cos x}{\sin x} = \frac{-\sin x\sin x - (\cos x\cos x)}{(\sin x)^2} = \frac{(-1)(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

(quotient rule)

$$= \frac{(-1)(1)}{\sin^2 x} = -\csc^2 x$$

$$\sec x = \frac{1}{\cos x}$$

derivative of secant:

$$\frac{0(\cos x) - (-\sin x)(1)}{(\cos x)^2} = \frac{\sin x}{\cos x\,(\cos x)} = \frac{1}{\cos x}\,\frac{\sin x}{\cos x} = \sec x\tan x$$

$$\csc x = \frac{1}{\sin x}$$

derivative of cosecant:

$$\frac{(0)(\sin x) - (\cos x)(1)}{(\sin x)^2} = \frac{(1)\,(-\cos x)}{\sin x\ \ \sin x} = -\csc x\cot x$$

Derivatives of Trig Functions

	Derivative		Derivative
$\text{Sin}x$	$\text{Cos}x$	$\text{Cos}x$	$-\text{Sin}x$
$\text{Tan}x$	$\text{Sec}^2 x$	$\text{Cot}x$	$-\text{Csc}^2 x$
$\text{Sec}x$	$\text{Sec}x\text{Tan}x$	$\text{Csc}x$	$-\text{Csc}x\text{Cot}x$

Examples:

$f(x) = \sin 3x$

$f'(x) = 3\cos 3x$

$g(x) = \frac{\tan x}{2} + 5x$

$g'(x) = \frac{1}{2}\sec^2 x + 5$

$h(x) = \cos(3x + 2)$

$h'(x) = -\sin(3x + 2)\cdot(3)$

$= -3\sin(3x + 2)$

Note: derivative of $\cos(u)$ is $(-\sin u)(u')$

derivative of $\sin(u)$ is $(\cos u)(u')$

(the chain rule applies to trig functions)

Example: $y = x^3 \sin x$

Using product rule, power rule, and derivative of trig function:

$$y' = 3x^2(\sin x) + (\cos x)x^3$$

$$= x^2(3\sin x + x\cos x)$$

Example:

$y = \sin^2 x + \cos^2 x$

Find y' (or dy/dx)

Using power rule, chain rule, and derivatives of trig functions:

$$y' = 2(\sin x)^1(\cos x) + 2(\cos x)^1(-\sin x)$$

$$y' = 2\sin x\cos x + (-2\cos x\sin x) = 0$$

Using trig identity:

$y = \sin^2 x + \cos^2 x$

$y = 1$

$y' = 0$

Derivatives of Trigonometry Functions

Example: $y = \frac{\cos x}{1 + \sin x}$

Using quotient rule and derivatives of trig functions:

$$y' = \frac{(-\sin x)(1 + \sin x) - (0 + \cos x)(\cos x)}{(1 + \sin x)^2} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$ factor -1 in the numerator

$$= \frac{(-1)[\sin x + \sin^2 x + \cos^2 x]}{(1 + \sin x)^2}$$ Use trig identity

$$= \frac{(-1)[\sin x + 1]}{(1 + \sin x)^2} = \frac{(-1)}{1 + \sin x}$$ Reduce and simplify

Example: Find the equation of the line tangent to the graph

$y = \sin x + 3$ at the point $x = \pi$

To find the equation of a line, you need a point and the slope.

Point: $x = \pi$

$y = \sin(\pi) + 3$ Point: $(\pi, 3)$

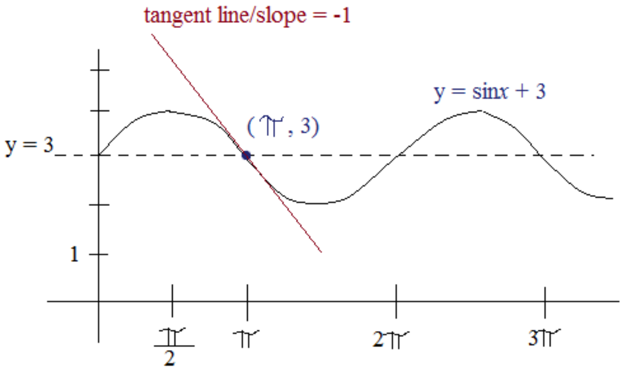
$= 0 + 3 = 3$

Slope: Use derivative to find instantaneous rate of change.

$y' = \cos x + 0$

So, the instantaneous rate of change (slope) at π is

$\cos(\pi) + 0 = -1$



Equation of the tangent line:

$\boxed{y - 3 = -1(x - \pi)}$ (Point slope form)

$y = -x + 6.14$ (slope intercept form/approximation)

Derivatives of Trig Functions

Examples:

$g\,(x) = \sin\sqrt{2x}$ function: $\sqrt{2x}$

derivative of the function: $\frac{1}{2}\,(2x)^{\frac{-1}{2}}\,(2) = \frac{1}{\sqrt{2x}}$

$$g'(x) = \cos\sqrt{2x}\left(\frac{1}{\sqrt{2x}}\right) = \frac{\cos\sqrt{2x}}{\sqrt{2x}}$$

$h(x) = \sec^3(2x)$ Let $u = 2x$

$u' = 2$

$h(x) = \sec(u)^3$

Note: according to the power rule, derivative of $[\sec(u)]^3$ is $3[\sec(u)]^2\cdot$ ("derivative of sec(u)")

$$h'(x) = 3\sec(u)^2\cdot \sec u(\tan u)u'$$

$$= 3\sec(2x)^2\cdot\sec(2x)\tan(2x)(2)$$

$$= 6\tan(2x)\sec^3(2x)$$

Find the relative extrema of $f\,(x) = \sin 2x + 4$ on the interval $[0, 2\pi)$

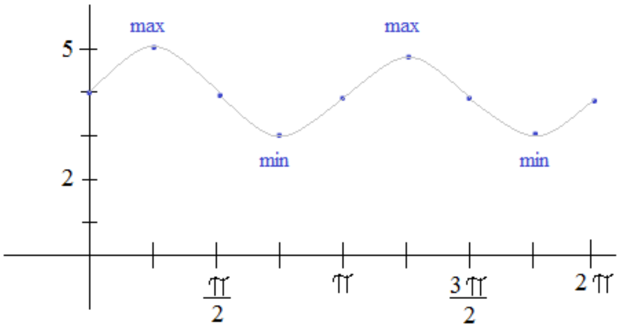
Find $f\,'(x)$: $\cos(2x)(2) + 0 = 2\cos(2x)$

To find the max/min, set $f\,'(x) = 0$

$2\cos(2x) = 0$

$\cos(2x) = 0$

$x = \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4}$



Also, $f\,''(x) = 0$ will help identify concavity and points of inflection.

Find derivative of $2\cos(2x)$: $2\cdot[-\sin(2x)(2)] = -4\sin(2x)$

To find points of inflection, set second derivative equal to zero.

$-4\sin(2x) = 0$

$\sin(2x) = 0$ points of inflection at $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Derivatives of Trig Functions

Some trigonometry examples with comments...

Example: $y = \sin^3(4x)$

Note: there are 3 "shells" or "layers"

$$y = (\sin(4x))^3$$

$$y' = \underbrace{3(\sin(4x))^2}_{1} \cdot \underbrace{\cos(4x)}_{2} \cdot \underbrace{4}_{3}$$

Example: $y = \dfrac{1}{\sqrt{\sin(3x^2)}}$

Utilizes the power rule, trig derivatives, and chain rule....

$$y = (\sin(3x^2))^{\frac{1}{2}}$$

$$y' = \underbrace{\frac{1}{2}(\sin(3x^2))^{-\frac{1}{2}}}_{\text{power}} \cdot \underbrace{\cos(3x^2)}_{\text{trig}} \cdot \underbrace{6x}_{\text{chain}} \Longrightarrow \frac{3x \cdot \cos(3x^2)}{\sqrt{\sin(3x^2)}}$$

Example: $g(x) = \sin(x)\sec(x)$

Utilizing product rule

$$g'(x) = \overset{u}{\sin(x)} \cdot \overset{v'}{\sec(x)\tan(x)} + \overset{u'}{\cos(x)} \cdot \overset{v}{\sec(x)}$$

$$= \sin(x) \cdot \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} + \cos(x) \cdot \frac{1}{\cos(x)}$$

$$= \frac{\sin^2 x}{\cos^2 x} + 1$$

$$= \tan^2 x + 1 = \sec^2 x$$

$\Longleftarrow$

SHORTCUT: use trig identities

$$\sec(x) = \frac{1}{\cos(x)} \qquad \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$g(x) = \sin(x)\sec(x) = \sin(x)\frac{1}{\cos(x)}$$

$$= \tan(x)$$

$$g'(x) = \sec^2(x)$$

Example: $y = \sec(5x^2)$

$$y' = \sec(5x^2)\tan(5x^2) \cdot 10x$$

Note: the angle measure is $5x^2$

the derivative is NOT $\text{sectan}(5x^2)$

Example: $h(x) = 2\sin(x)\cos(x)$

Using product rule

$$h'(x) = 2\,[\,\cos(x) \cdot \cos(x) + -\sin(x) \cdot \sin(x)\,]$$

$$2[\cos^2 x - \sin^2 x]$$

$$\downarrow$$

$$2\cos(2x)$$

$\Longleftarrow$

Using trig identiies and chain rule

$$\sin(2x) = 2\sin x\cos x$$

$$h(x) = 2\sin(x)\cos(x) = \sin(2x)$$

$$h'(x) = \cos(2x) \cdot 2$$

Example: $y = \sin(x) + \sin^2 x + \tan^2 x$

$$\frac{dy}{dx} = \cos(x) \cdot 1 + 2(\sin x) \cdot (\cos x) + 2(\tan x) \cdot (\sec^2 x)$$

$$= \cos(x) + \sin(2x) + 2\tan(x)\sec^2 x$$

Notice: the derivative of x^6 is $6x^5$

OR, showing the chain rule:

$$6(x)^5 \cdot 1$$

Inverse Trig Derivatives & Implicit Differentiation

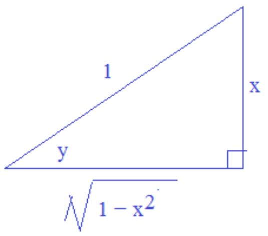
Example: $y = \sin^{-1} x$ What is $\frac{dy}{dx}$?

Step 1: Change the inverse trig term

$$\sin y = \sin\left(\sin^{-1} x\right)$$

$$\sin(y) = x$$

Step 2: "Draw the triangle"



Sine y = $\frac{\text{opposite}}{\text{hypotenuse}}$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Step 3: Use implicit diffentiation to find *dy*/*dx*

$$\cos(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} \quad \Longrightarrow \quad \text{using the triangle,} \quad \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

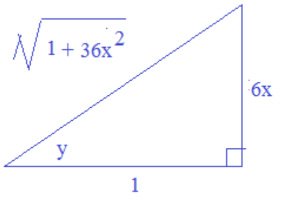
Example: $y = \tan^{-1}(6x)$ Find the derivative.

Step 1: Change the inverse trig term

$$\tan y = \tan\left(\tan^{-1}(6x)\right)$$

$$\tan(y) = 6x$$

Step 2: "Draw the triangle"



Tan y = $\frac{\text{opposite}}{\text{adjacent}}$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Step 3: Use implicit diffentiation to find *dy*/*dx*

$$\sec^2(y)\,\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{\sec^2(y)} \quad \Longrightarrow \quad \text{using the triangle,} \quad \frac{dy}{dx} = \frac{6}{\sqrt{1 + 36x^2}^{\,2}} = \frac{6}{1 + 36x^2}$$

Inverse Trig Derivatives & Implicit Differentiation

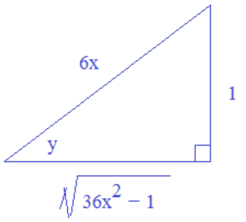
Example: $f(x) = \csc^{-1}(6x)$ Find $f'(x)$.

Step 1: Rewrite the inverse trig term

$$\csc\ y = \csc\left(\csc^{-1}(6x)\right)$$

$$\csc y = 6x$$

Step 2: "Draw the Triangle"



$\csc y = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{6x}{1}$

Pythagorean Theorem: $a^2 + b^2 = c^2$

$1 + b^2 = 36x^2$

Step 3: Use implicit differentiation to find derivative

$$-\csc(y)\cot(y)\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{-\csc(y)\cot(y)} \xrightarrow[\text{using the triangle}]{} \frac{6}{-6x\sqrt{36x^2-1}}$$

Important: x must be a positive value! (the length of the hypotenuse cannot be negative)

$$\Downarrow$$

$$f'(x) = \frac{-1}{|x|\sqrt{36x^2-1}}$$

Inverse Trig Derivative Formulas

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1}u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad \frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{d}{dx}\cot^{-1}u = \frac{-1}{1+u^2}\frac{du}{dx}$$

Example: $y = \sec^{-1}\left(\frac{x}{3}\right)$ Find y'.

$u = \frac{x}{3}$ $\frac{du}{dx} = \frac{1}{3}$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$y' = \frac{1}{\left|\frac{x}{3}\right|\sqrt{\frac{x^2}{9}-1}}\ \frac{1}{3}$$

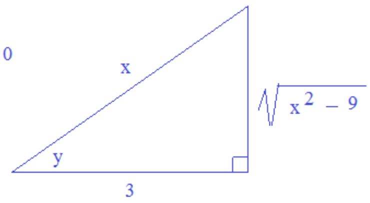
$$= \frac{1}{|x|\sqrt{\frac{x^2}{9}-1}}$$

$$= \frac{1}{|x|\sqrt{\frac{x^2-9}{9}}}$$

$$\boxed{= \frac{3}{|x|\sqrt{x^2-9}}}$$

(verify with triangles)

$x > 0$



$$\sec(y) = \frac{x}{3}$$

$$\sec(y)\tan(y)\frac{dy}{dx} = \frac{1}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}\cos(y)\cot(y)$$

$$\frac{dy}{dx} = \frac{1}{3}\ \frac{3}{x}\ \frac{3}{\sqrt{x^2-9}}$$

$$\boxed{\frac{dy}{dx} = \frac{3}{|x|\sqrt{x^2-9}}}$$

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

Preferable to ordinary computer cookies...

Essential part of a well-rounded, *academic diet.*

Try with (*t*), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

Trigonometry Differentiation Worksheet

Section I:
Find *dy/dx*

1) $y = \sin(2x)$

2) $y = \cos x^4$

3) $y = \cos^2 x$

4) $y = \tan^3(3x)$

5) $y = \sec(3x)$

Section II:
Find y'

1) $y = x^2 \sin^2 x$

2) $y = 5\csc(3x)$

3) $y = \dfrac{3\sin x}{(2x + 5)}$

4) $y = \sin^2(4x) + \cos^2(4x)$

5) $y = 4\sin x\cos x$

Section III:
Find *dy/dx*

1) $\sin(xy) + 3x = 4$

2) $\sin y + \cos x = 1$

Trigonometry Derivatives Questions

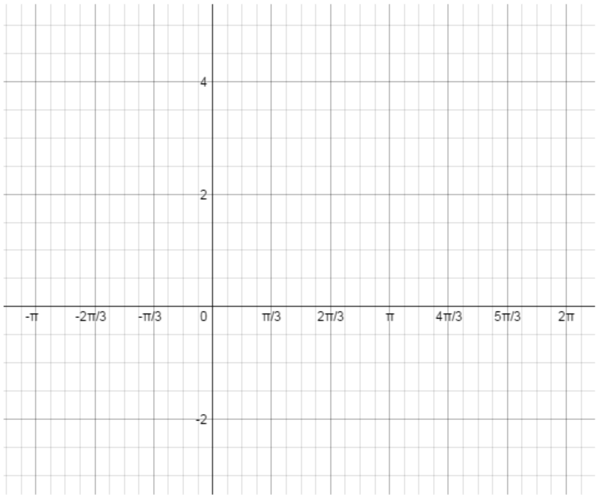
A) $\frac{d^{713}}{dx^{713}} \sin x$

- a) $\sin x$
- b) $\cos x$
- c) $-\cos x$
- d) $-\sin x$
- e) 0

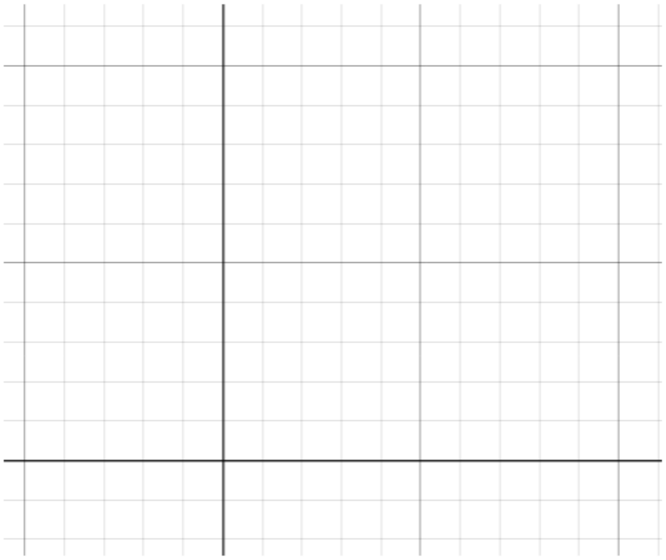
B) Use the definition of derivative to prove $\frac{d}{dx}\cos x = -\sin x$

C) Write the equation of a line tangent to $y = \csc x$ @ $x = \frac{\pi}{3}$

Graph the trig equation and the tangent line.



D) Find and graph the tangent line for the function $f(x) = \frac{1 + \cos x}{1 - \cos x}$ @ $\frac{\pi}{2}$



E) Find the second derivative of $y = \sin^2 x$

Trigonometry Differentiation Worksheet

SOLUTIONS

Section I:

Find *dy/dx*

1) $y = \sin(2x)$

$$\frac{dy}{dx} = \cos(2x) \cdot 2 = \boxed{2\cos(2x)}$$

$u = 2x$
$y = \sin(u)$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

2) $y = \cos x^4 \longrightarrow \cos(x^4)$

$$\frac{dy}{dx} = \underset{\text{"derivative trig part"}}{-\sin x^4} \cdot \underset{\text{"derivative of the ( ) term"}}{4x^3} = \boxed{-4x^3\sin(x^4)}$$

3) $y = \cos^2 x = (\cos x)^2$

(use power rule)

$dy/dx = 2\cos x^1 \cdot (-\sin x) = \boxed{-2\sin x\cos x}$

or $-\sin(2x)$

4) $y = \tan^3(3x) = [\tan(3x)]^3$

power/chain rules:

$dy/dx = 3\tan(3x)^2 \cdot \sec^2(3x) \cdot (3)$

$\boxed{9\tan^2(3x)\sec^2(3x)}$

5) $y = \sec(3x)$

derivative of sec: $\sec \cdot \tan$

derivative of 3x: 3

$dy/dx = \sec(3x)\tan(3x) \cdot 3$

$= \boxed{3\sec(3x)\tan(3x)}$

Section II:

Find y'

1) $y = x^2\sin^2 x$

product rule: $2x(\sin^2 x) + (x^2)2\sin x^1 \cdot \cos x$

$2x(\sin^2 x) + 2x^2 \sin x\cos x$

or, factor out the 2xsinx:

$\boxed{2x\sin x(\sin x + x\cos x)}$

2) $y = 5\csc(3x)$

find derivative of csc(3x):

$-\csc(3x)\cot(3x) \cdot 3$

then, multiply by 5:

$\boxed{-15\csc(3x)\cot(3x)}$

3) $y = \dfrac{3\sin x}{(2x + 5)}$

quotient rule:

$$dy/dx = \frac{3\cos x(2x + 5) - 2(3\sin x)}{(2x + 5)^2}$$

$$= \boxed{\frac{(6x + 15)\cos x - 6\sin x}{(2x + 5)^2}}$$

4) $y = \sin^2(4x) + \cos^2(4x)$

shortcut: $\sin^2 + \cos^2 = 1$

(trigonometry identity)

so, $y = 1$ $\boxed{dy/dx = 0}$

long way: $dy/dx =$

$2\sin(4x)^1 \cdot \cos(4x) \cdot 4 + 2\cos(4x)^1 \cdot -\sin(4x) \cdot 4$

$8\sin(4x)\cos(4x) - 8\sin(4x)\cos(4x) = 0$

5) $y = 4\sin x\cos x$

Using trig identity: $\sin 2x = 2\sin x\cos x$

$y = 2\sin 2x$

$dy/dx = 2 \cdot [\cos 2x \cdot 2] = \boxed{4\cos(2x)}$

Using product rule:

$4[\cos x \cdot (\cos x) + (\sin x) \cdot -\sin x]$

$4[\cos^2 x - \sin^2 x]$ (trig identity)

$= 4\cos(2x)$

Section III:

Find *dy/dx*

Implicit differentiation:

note: to find derivative of ( ), use product rule for xy

1) $\sin(xy) + 3x = 4$

$\cos(xy) \cdot [(1)(y) + (x)(dy/dx)] + 3 = 0$

$$[(1)(y) + (x)(dy/dx)] = \frac{-3}{\cos(xy)}$$

$$(x)(dy/dx) = \frac{-3}{\cos(xy)} - y$$

$$dy/dx = \frac{\frac{-3}{\cos(xy)} - y}{x} = \boxed{\frac{-(3\sec(xy) + y)}{x}}$$

2) $\sin y + \cos x = 1$

$\cos(y)\, dy/dx + (-\sin(x)) = 0$

$\cos(y)\, dy/dx = \sin(x)$

$$\boxed{dy/dx = \frac{\sin(x)}{\cos(y)}}$$

or $\sin(x)\sec(y)$

Trigonometry Derivatives Questions

SOLUTIONS

A) $\dfrac{d^{713}}{dx^{713}}\ \sin x$

a) $\sin x$

$\boxed{\text{b) } \cos x}$

c) $-\cos x$

d) $-\sin x$

e) 0

$\sin x$

1st derivative $\dfrac{d}{dx}\sin x = \cos x$

2nd derivative $\dfrac{d^2}{dx^2}\sin x = -\sin x$

3rd derivative $\dfrac{d^3}{dx^3}\sin x = -\cos x$

4th derivative $\dfrac{d^4}{dx^4}\sin x = \sin x$

Since every 4th derivative returns to $\sin x$,

$$\frac{d^{712}}{dx^{712}}\sin x = \sin x$$

therefore, the 713th derivative is $\cos x$

Note the similarity to i where

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

B) Use the definition of derivative to prove $\dfrac{d}{dx}\cos x = -\sin x$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

Using direct substitution, we get

$$\frac{\cos(x+0)-\cos(x)}{0} = \frac{0}{0}$$

Indeterminate...

$$\lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$ (Sum Identity)

$$\lim_{h\to 0}\frac{\cos(x)\cos(h)-\cos(x)}{h} - \frac{\sin(x)\sin(h)}{h}$$ (regroup and separate)

$$\cos(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} - \sin(x)\lim_{h\to 0}\frac{\sin(h)}{h}$$

$$\cos(x)\cdot 0 \quad - \sin(x)\cdot(1) \quad = -\sin(x)$$

$$\lim_{h\to 0}\frac{\cos(h)-1}{h}$$

Using direct substitution, we get 0/0..
So, multiply by the conjugate..

$$\lim_{h\to 0}\frac{\cos(h)-1}{h}\cdot\frac{\cos(h)+1}{\cos(h)+1}$$

$$\lim_{h\to 0}\frac{\cos^2(h)-1}{\cos^2(h)+h}$$

$$\lim_{h\to 0}\frac{\sin^2(h)}{\cos^2(h)+h}$$

$$\frac{0}{1+0} = 0$$

C) Write the equation of a line tangent to $y = \csc x$ @ $x = \dfrac{\pi}{3}$

Graph the trig equation and the tangent line.

To find the equation of a line, we need the slope and a point...

a) point: If $x = \dfrac{\pi}{3}$, then $y = \dfrac{2}{\sqrt{3}}$

$$\left(\frac{\pi}{3}, \frac{2}{\sqrt{3}}\right)$$

b) slope: find the derivative (IROC)

$y' = -\csc x \cot x$

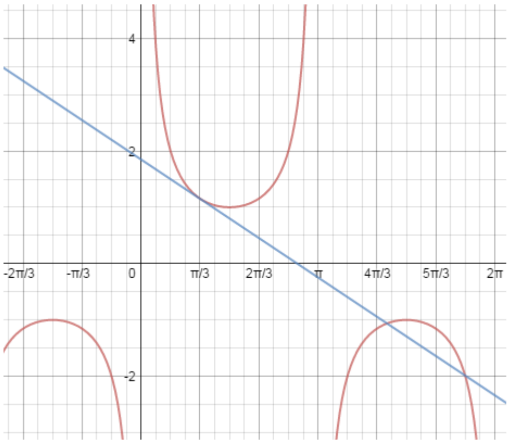
then, at $x = \dfrac{\pi}{3}$ the slope is $-\csc\left(\dfrac{\pi}{3}\right)\cot\left(\dfrac{\pi}{3}\right) =$

$$\frac{-2}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} = \frac{-2}{3}$$

So, the equation of the tangent line (in point slope form): $\boxed{y - \dfrac{2}{\sqrt{3}} = \dfrac{-2}{3}\left(x - \dfrac{\pi}{3}\right)}$

$$y = \frac{-2}{3}x + \frac{2\pi}{9} + \frac{2}{\sqrt{3}}$$

$$\boxed{y = \frac{-2}{3}x + \frac{2\pi + 6\sqrt{3}}{9}}$$



$y = \dfrac{-2}{3}x + 1.85$

D) Find and graph the tangent line for the function $f(x) = \frac{1 + \cos x}{1 - \cos x}$ @ $\frac{\pi}{2}$

To find the equation of a line, we need a point and the slope....

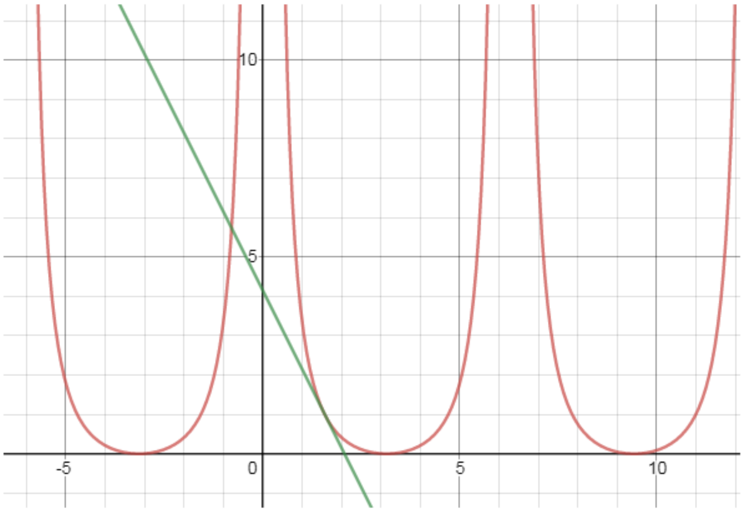
To find the point.... $f(\frac{\pi}{2}) = \frac{1 + \cos\frac{\pi}{2}}{1 - \cos\frac{\pi}{2}} = 1$ the point is $\boxed{(\frac{\pi}{2}, 1)}$

To find the slope.... $f'(x) = \frac{(0 - \sin x)(1 - \cos x) - (0 + \sin x)(1 + \cos x)}{(1 - \cos x)^2}$

quotient rule

Then, @ $(\frac{\pi}{2}, 1)$ $f'(\frac{\pi}{2}) = \frac{(-1)(1) - (1)(1)}{(1)^2} = \boxed{-2}$

$\boxed{\text{equation of line: } y - 1 = -2(x - \frac{\pi}{2})}$



E) Find the second derivative of $y = \sin^2 x$

$y' = 2\sin x^1 \cos x = \sin 2x$ (using trig double angle identity)

$\boxed{y'' = \cos 2x \cdot 2}$

long way (w/o trig identity)...

$y' = 2\sin x\cos x$

$y'' = 2\cos x\cos x - 2\sin x\sin x$ (product rule)

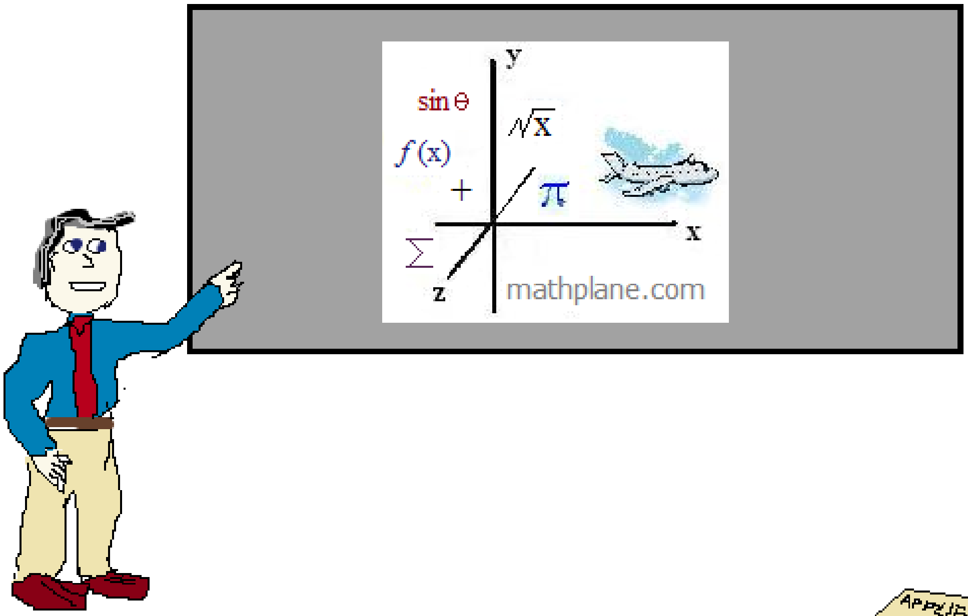
$2(\cos^2 x - \sin^2 x) \Longrightarrow$ equals $\boxed{2\cos 2x}$

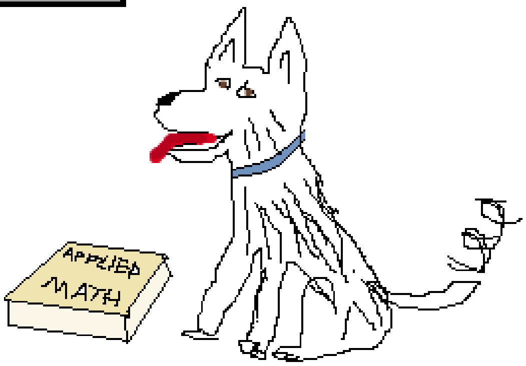
(Applying cosine double angle identity)

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.





Also, at teacherspayteachers and TES

And, Mathplane.ORG for mobile and tablets