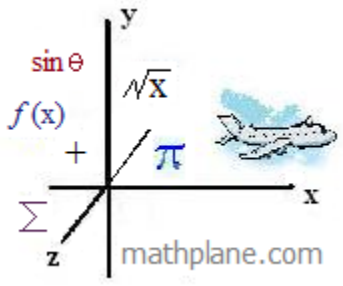
Greatest Common Factor &

Least Common Multiple

Notes, Examples, and Practice Quiz (w/Solutions)

Topics include factor trees, rainbow, GCF, LCM, and more…



Greatest Common Factor

Factors:

What are they? Numbers that are multiplied to get a number.

1, 2, 7, 14 are factors of 14
1, 2, 4, 5, 10, 20 are factors of 20
x, y, 1, 3 are factors of 3xy
1, 41 are the only factors of 41

Note: it is open to debate whether negative numbers can be considered factors; after all, -2 x -6 = 12.. So, are -2 and -6 factors of 12?

Note: Prime numbers have only 2 factors: 1 and the number itself.
1 is a factor of any number.

Common Factors:

What are they? Factors that are the same for 2 (or more) numbers.

1, 2, 11, 22 are factors of 22
1, 3, 11, 33 are factors of 33

1 & 11 are common factors of 22 and 33

1, 2, 4, 5, 8, 10, 20, 40 are factors of 40
1, 2, 4, 8, 16, x, y are factors of 16xy

1, 2, 4, & 8 are common factors of 40 and 16xy

Greatest Common Factor:

What is it? The largest number among common factors.

GCF of 22 and 33 is 11
GCF of 40 and 16xy is 8

When do you use it? To reduce fractions.

$$\frac{24}{60} = \frac{2}{5}$$

Factors of 24: 1, 2, 3, 4, 6, 12, 24
Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(1, 2, 3, 4, 6, and 12 are common factors.
But, 12 is the greatest common factor!)

Divide numerator and denominator by 12 to reduce fraction to lowest terms.

Least Common Multiple

Multiples:

What are they? Numbers added to themselves.

2, 4, 6, 8, 10... (multiples of 2)

5, 10, 15, 20... (multiples of 5)

3xy, 6xy, 9xy, 12xy... (multiples of 3xy)

Note: Multiples include zero and negative numbers.

Example: -44, -33, -22, -11, 0, 11, 22, 33, 44...
are multiples of 11

** However, when finding the "least common multiple", search for the smallest positive multiple.

Common Multiples:

What are they? Multiples that are the same for 2 (or more numbers)

3 6 9 12 15 18 21 24 27 30 33 36 39 ... (multiples of 3)

4 8 12 16 20 24 28 32 36 40 44 ... (multiples of 4)

12 24 36 ... (common multiples of 3 and 4)

observation: the common multiples of 3 and 4 are multiples of 12

2 4 6 8 10 12 ... (multiples of 2)
5 10 15 20 25 ... (multiples of 5)
10 20 30 40 50 ... (multiples of 10)

10 20 30 ... (common multiples of 2, 5, and 10)

observation: 2 and 5 are also factors of 10

Least Common Multiple:

What is it? The lowest (positive) number among common multiples.

6 12 18 24 30 36 42 48 54 60 ... (multiples of 6)
2 4 6 8 10 12 ... 24 26 28 30 ... (multiples of 2)
5 10 15 20 25 30 35 40 45 ... (multiples of 5)

Least Common Multiple of 2 and 5? 10

LCM of 2 and 6? 6

LCM of 5 and 6? 30

LCM of 2, 5, and 6? 30

Note: 30 60 90 120 ... are common multiples of 5 and 6;
But, the least common multiple is only 30

Least Common Multiple (continued)

Definition: "Smallest positive number that is divisible into all of the sets' members, leaving no remainder."

Example: Set A = multiples of 7 (all numbers 7n, where n is an integer)

Set B = multiples of 5 (all numbers 5n, where n is an integer)

Set A = {... -21, -14, -7, 0, 7, 14, 21, 28, 35, 42... }

Set B = {... -10, -5, 0, 5, 10, 15, 20, 25, 30, 35, 40... }

eliminate 0 and negative numbers...

find first common multiple...

When would we use 'least common multiple'?

-----> To find the common denominator of 2 or more fractions.

$$\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40}$$

(40 is the least common multiple of 5 and 8)

multiples of 5: 5 10 15 20 25 30 35 40 45 50 ...

multiples of 8: 8 16 24 32 40 48 56 ...

Finding factors: "The rainbow"

Start with the obvious factors: 1 and the number itself...

Then, work your way to the inside by finding factor pairs...

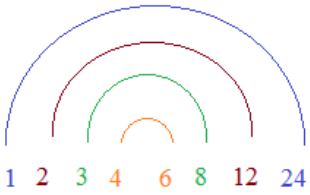
Example: Find the factors of 24

1 24

1 2 12 24

1 2 3 8 12 24

1 2 3 4 6 8 12 24



Example: Find the factors of 100

1 100

1 2 50 100

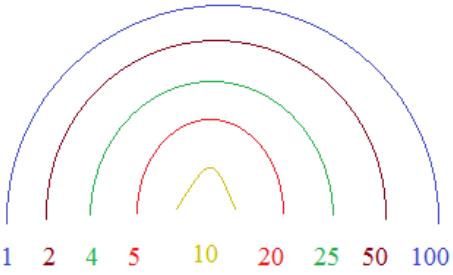
(skip 3, because it's not a factor)

1 2 4 25 50 100

1 2 4 5 20 25 50 100

(skip 6, 7, 8, and 9... Not factors)

1 2 4 5 10 20 25 50 100

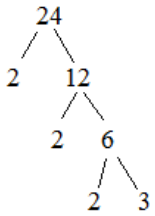


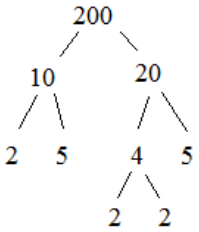
Factor Tree Applications

What is a factor tree?

A branching diagram showing the factors of number.

Examples:

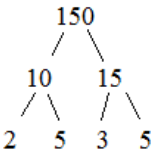




Note: the end of each branch is a prime number.

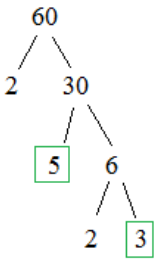
Applications:

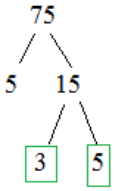
1) Displaying all prime factors



2, 3, and 5 are all prime factors

2) Finding greatest common factor





Since 3 and 5 are common factors, the greatest common factor is 3 x 5 = 15


7
1
3
5
The *Nature* of *Mathematics*
Sitting under a factor tree may inspire *numerous* ideas.
L Friedman #25 3-25-12
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QUICK QUIZ-→

GCF, LCM, and Factor Tree Quick Quiz

I. Find the GCF (Greatest Common Factor)

A) 8 and 20 B) 42 and 91 C) 5 and 25

II. Find the LCM (Least Common Multiple)

A) 6 and 11 B) 3 and 4 C) 5 and 25

III. Using a factor tree, find all the prime factors.

A) 42 B) 78 C) 350

IV. Miscellaneous

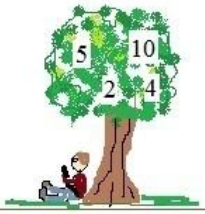
A) What is the greatest common factor of 2 prime numbers?

B) What is the least common multiple of 3, 5, and 7?

C) List all factors of 120.

Hidden Message

Find the solutions below. Then, convert the numbers to letters to reveal the answer!



Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	C	E	F	H	L	O	R	T	W

Clue: "A tree that doesn't need water?"

1) Least Common Multiple of 2, 4, and 8. → ☐ ____

2) The numerator after $\frac{32}{56}$ is reduced to lowest terms. → ☐ ____

3) Greatest Common Factor of 2, 4, and 8. → ☐ ____

4) $\frac{1}{3} + \frac{1}{4} + \frac{1}{9} = \frac{25}{\square 6}$ → ☐ ____

5) It's never a factor of another number. → ☐ ____

6) It's a factor of every number. → ☐ ____

7) Which of the following is not a factor of 210: 5, 6, 7, or 8? → ☐ ____

8) LCM of 3, 5, and 12. → ☐0 ____

9) GCF of 28, 49, 70, and 2100. → ☐ ____

10) The number of different factors of 24. → ☐ ____

11) The denominator of $\frac{36}{63}$ after reducing to lowest terms. → ☐ ____

12) The LCM of the numbers in the illustration above. → ☐0 ____

13) A factor of all even numbers. → ☐ ____

GCF, LCM, and Factor Tree Quick Quiz

SOLUTIONS

I. Find the GCF (Greatest Common Factor)

A) 8 and 20

factors of 8: 1, 2, 4, 8
factors of 20: 1, 2, 4, 5, 10, 20

4

B) 42 and 91

factors of 42: 1, 2, 3, 6, 7, 14, 21, 42
factors of 91: 1, 7, 13, 91

7

C) 5 and 25

factors of 5: 1, 5
factors of 25: 1, 5, 25

5

II. Find the LCM (Least Common Multiple)

A) 6 and 11

multiples of 6: 6, 12, 18, 24, ... 60, 66
multiples of 11: 11, 22, 33, ... 55, 66

66

B) 3 and 4

multiples of 3: 3, 6, 9, 12, 15
multiples of 4: 4, 8, 12, 16, ...

12

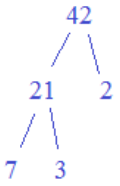
C) 5 and 25

multiples of 5: 5, 10, 15, 20, 25, 30
multiples of 25: 25, 50, ..

25

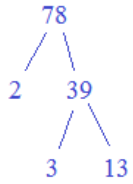
III. Using a factor tree, find all the prime factors.

A) 42



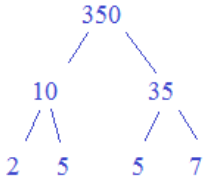
2, 3, and 7

B) 78



2, 3, and 13

C) 350



2, 5, and 7

IV. Miscellaneous

A) What is the greatest common factor of 2 prime numbers?

Since prime numbers have only 2 factors -- 1 and themselves -- the GCF between two primes is 1.

B) What is the least common multiple of 3, 5, and 7?

3, 5, and 7 are prime.. Therefore, the first common multiple will be 3 x 5 x 7 = 105

C) List all factors of 120.

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

1 x 120
2 x 60
3 x 40
4 x 30
5 x 24
6 x 20
8 x 15
10 x 12

Hidden Message

Find the solutions below. Then, convert the numbers to letters to reveal the answer!

Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	C	E	F	H	L	O	R	T	W

Clue: "A tree that doesn't need water?"

SOLUTIONS

1) Least Common Multiple of 2, 4, and 8.

multiples of 2: 2, 4, 6, **8**, 10....
4: 4, **8**, 12, 16, 20...
8: **8**, 16, 24, 32...

2) The numerator after $\frac{32}{56}$ is reduced to lowest terms. $\frac{32}{56} = \frac{4}{7}$ (greatest common factor of 32 & 56 is 8)

3) Greatest Common Factor of 2, 4, and 8.

factors of 2: 1, **2**
4: 1, **2**, 4
8: 1, **2**, 4, 8

4) $\frac{1}{3} + \frac{1}{4} + \frac{1}{9} = \frac{25}{36}$ $\frac{12}{36} + \frac{9}{36} + \frac{4}{36} = \frac{25}{36}$

5) It's never a factor of another number. 0 can never be a factor of another number. (0 is only a factor of itself)

6) It's a factor of every number. 1 is a factor of every number.

7) Which of the following is not a factor of 210: 5, 6, 7, or 8?

210/5 = 42
210/6 = 35
210/7 = 30
210/8 = 26.25

8) LCM of 3, 5, and 12. Looking for common multiples, focus on the largest number: 12...

12, 24, 36, 48, **60**, 72.... Then, 60 is the first number that is a multiple of 5... And, it also is a multiple of 3!

9) GCF of 28, 49, 70, and 2100.

Focus on the smallest number: 28.. It's factors are
1, 2, 4, 7, 14, 28 ⟹ 7 is the only one that goes into 49, 70, and 2100 — 7

10) The number of different factors of 24.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24... 8 total..

11) The denominator of $\frac{36}{63}$ after reducing to lowest terms. 9 is the greatest common factor..

$\frac{36}{63} = \frac{4}{7}$

12) The LCM of the numbers in the illustration above.

The numbers are 2, 4, 5, and 10.. The LCM is 20..

13) A factor of all even numbers.

All even numbers are divisible by 2.
So, 2 is a factor of all even numbers.

"A tree that doesn't need water?"

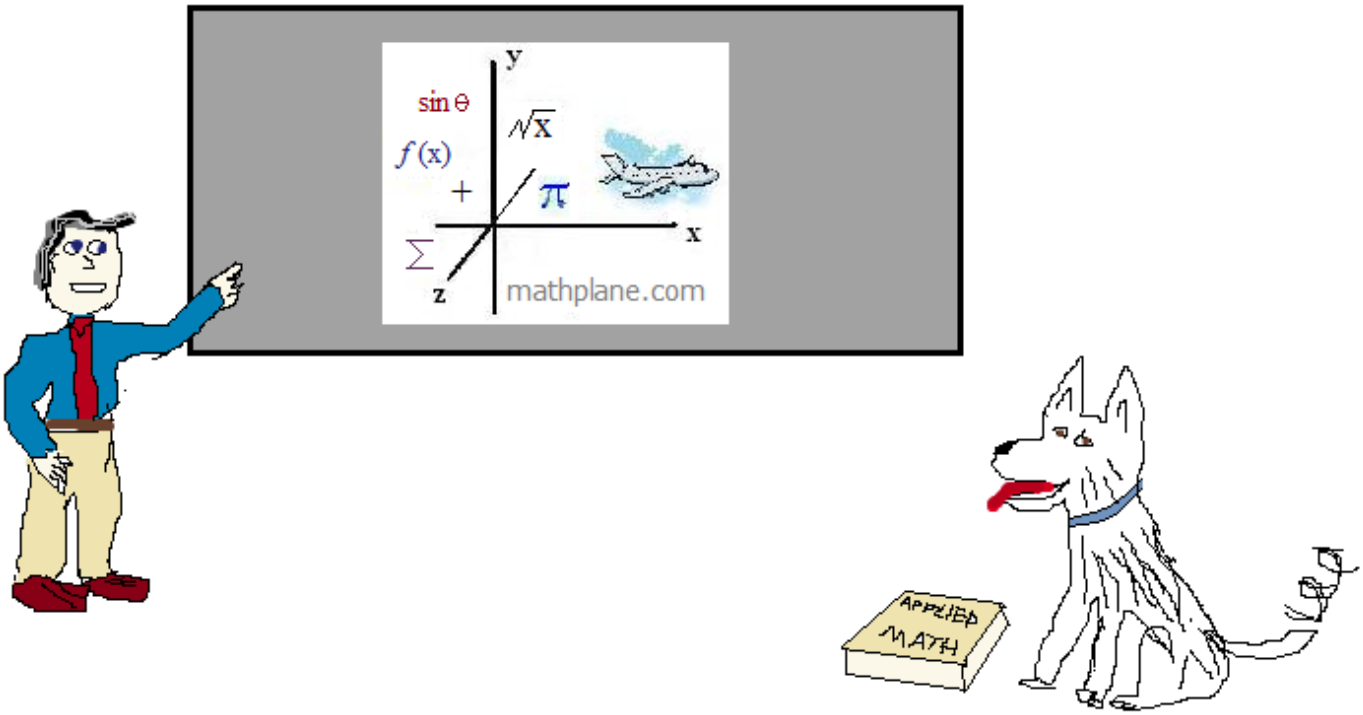
THE 'FACTOR TREE'

8	→	T
4	→	H
2	→	E
3	→	F
0	→	A
1	→	C
8	→	T
6 0	→	O
7	→	R
8	→	T
7	→	R
2 0	→	E
2	→	E

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know

Cheers



ONE MORE MATH QUESTION:

How many numbers are both multiples of 3 AND factors of 60?

(Answer on next page)

ANSWER:

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Multiples of 3: 3, 6, 9, 12 … 54, 57, 60…

BOTH: 3, 6, 12, 15, 30, 60

SIX TOTAL!!