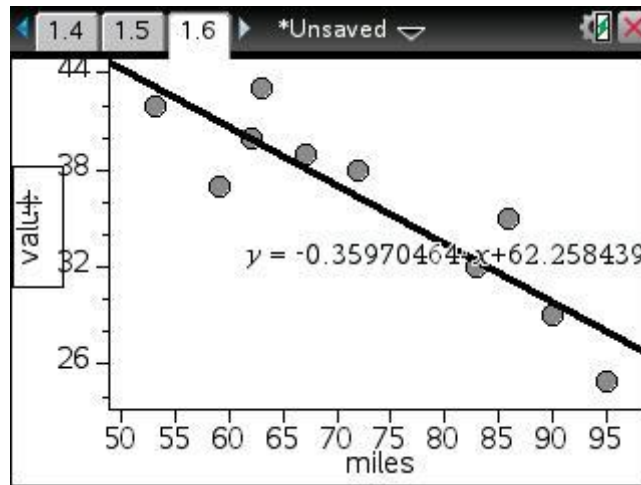# Linear Equations and Models

Notes, Examples, and Practice Quiz (with solutions)

Topics include Linear Forms, Best fit lines, slope, graphing, and more.
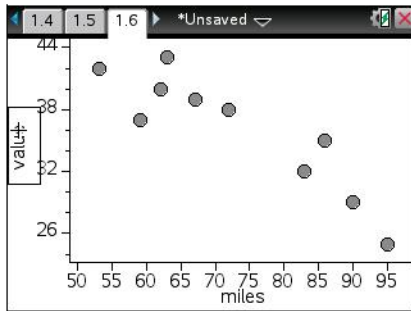
Linear Regression Models

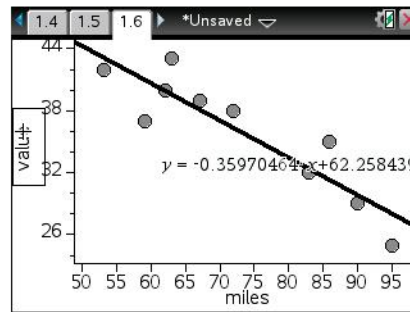*Example:* The following are 10 five-year old sports cars

odometer (1000s miles)	59	90	63	72	53	67	86	62	95	83
trade-in value (1000s $$)	37	29	43	38	42	39	35	40	25	32

Construct a linear model:

Scatterplot of the 10 data points:

Linear regression/'line of best fit'

$$y = -.3597x + 62.258$$

What is the approximate trade-in value of a car with 50,000 miles?

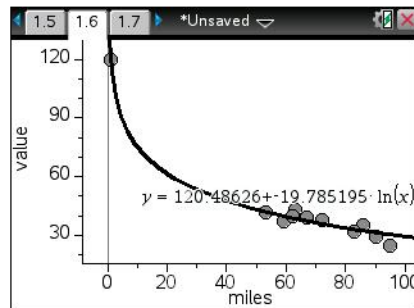trade-in (input) x = 50 $y = -.3597(50) + 62.258$

$y = 44.273$

Approx. $44,273

What does the y-intercept represent?   the theoretical price of the brand new sports car

When x = 0 (new car), $y = -.3597(0) + 62.258$ Approx. $62,258

What does the x-intercept represent?   the number of miles when the car is worthless

When y = 0 (car has no value), $0 = -.3597(x) + 62.258$ Approx. 173 years

Is a linear model best?   Not necessarily, because cars lose most of the value when driven off the lot

Logarithmic Model

What other factors are not considered?   During 5 years, an owner can influence the resale value beyond miles driven (interior, dents, maintenence, etc...)

Example: A math book salesman earns $200 per week plus $5 for each book he sells.

   a) Write an equation expressing the salesman's income per week as a function of books sold.

   b) What is the income if he sells 45 books?

   c) If his income were $755, how many books were sold?

   d) Graph the relationship.

a) The independent variable (input) is # of books   (B)

   The dependent variable (output) is the income   (I)

   Since we're looking for income (per week) as a function of books sold....

$$I \ = \ \$5B$$

Then, we add the fixed base which is $200...

$$I \ = \ \$200 + \$5B$$

   where $B \geq 0$

      and, B are whole numbers
      (assuming partial books aren't sold!)

b) Let B = 45

   $I = \$200 + \$5(45)$
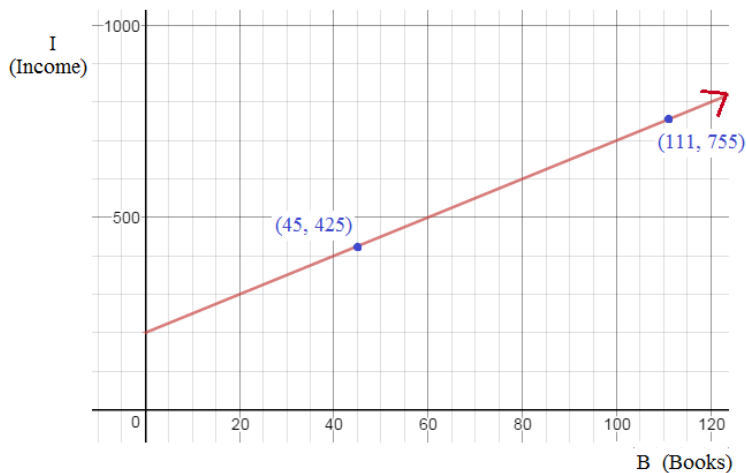
   $I = \$425$

c) Let I = $755

   $\$755 \ = \ \$200 + \$5B$

   $\$555 \ = \ \$5B$

   $B \ = \ 111$ books were sold
            in one week

d)   Note: the "y-intercept" represents income when 0
            books are sold.. i.e. the base salary...

For real world application, since
you cannot sell 'partial books' or
'negative books', the variable B
must be a whole number!

→   technically, the graph should
not be solid. Instead, it should
be a sequence of points where
B = 0, 1, 2, 3, ...

Example: Snoopy goes to the store to buy bones and treats.
            Each bone cost $1, and each treat costs 25 cents.
            If Snoopy has 12 dollars, create a linear model showing ways he can spend all of his money.
            Draw a line (segment), showing the number of bones as a function of the number of treats...
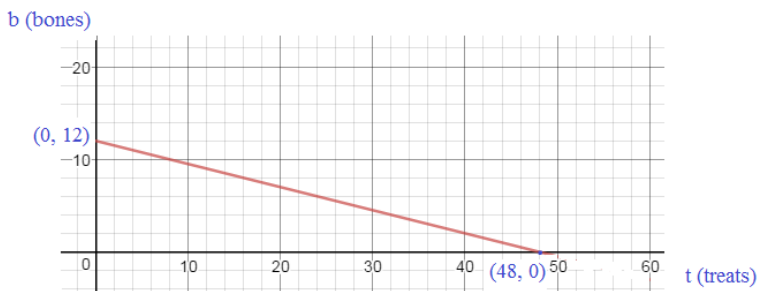   (Assume the store will sell partial treats and bones.)

Standard Form

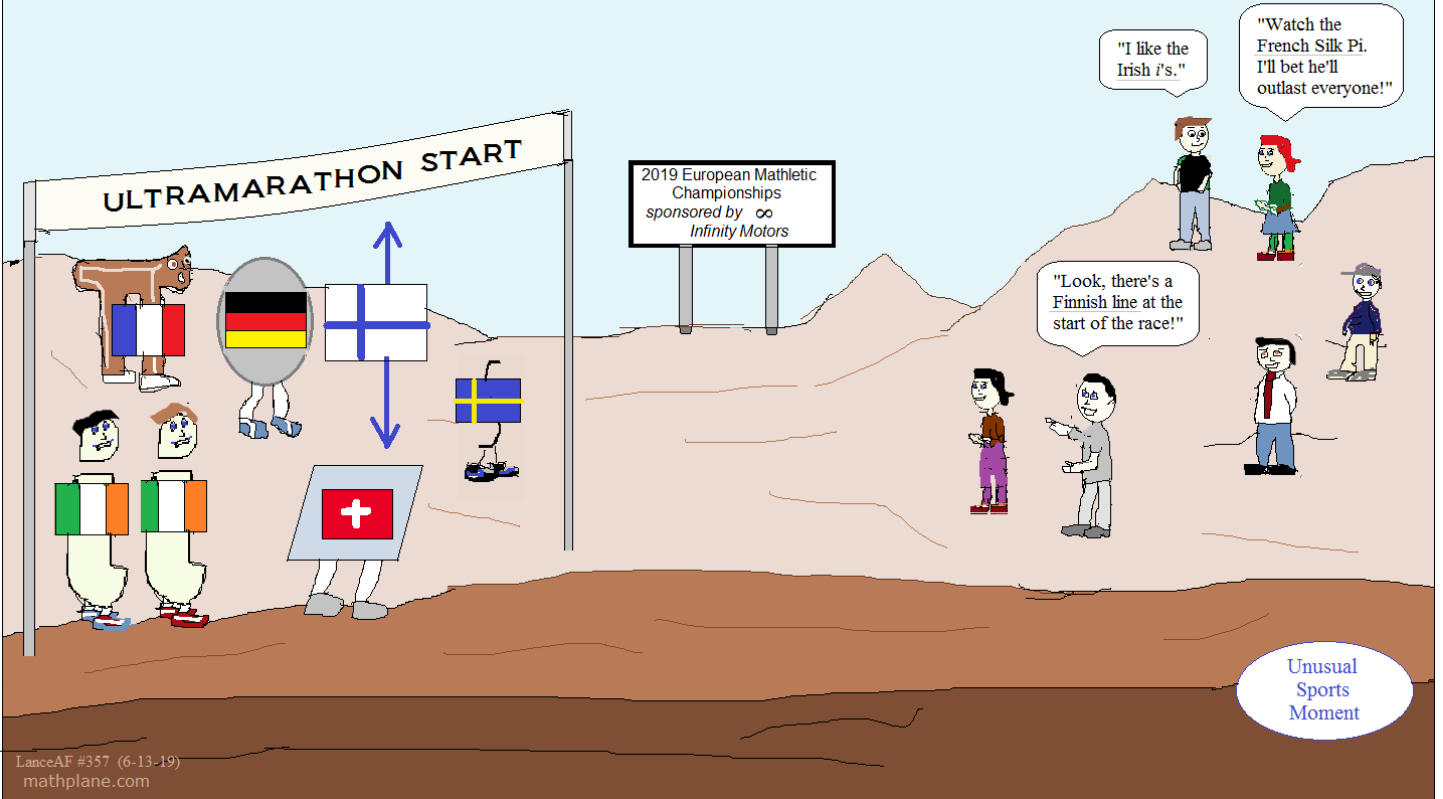Let B = # of bones
   T = # of treats

$.25T + 1B = 12$

   where T and B are
   positive real numbers...

Practice Exercises-→

A) Slope

1) (-2, 6)   (x, 10)   Slope = 2

   Find x

2) (1, y + 3)   (-3, 6)   Slope = 3

   Find y

3) If the lines are parallel,   line m   (1, 5) (2, 10)

                                     line p   (-3, -1) (k, 7)

   Find k

4) The following points are collinear:

   (3, -2) (8, 8) and (-7, m)

   What is m?

B) Given the line l: $3x + 6y = 18$

   1) Find the x-intercept:

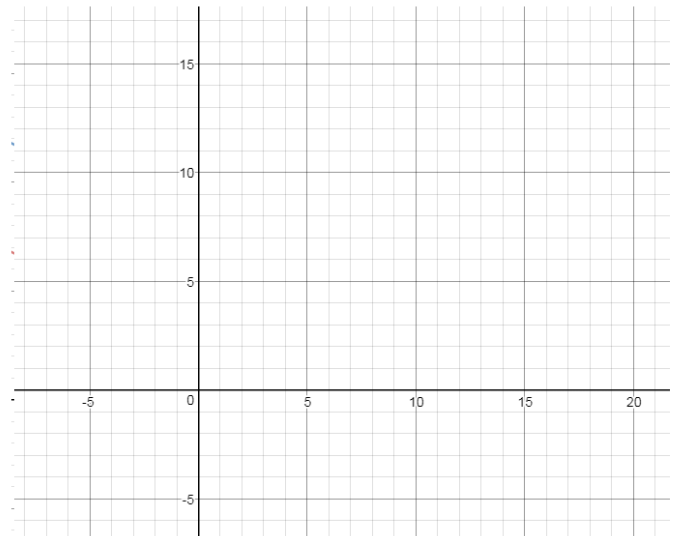   y-intercept:

            slope:

   2) Write an equation of the line parallel to l that passes through (2, 7).

   3) Write an equation of the line perpendicular to l that passes through (-1, 4).
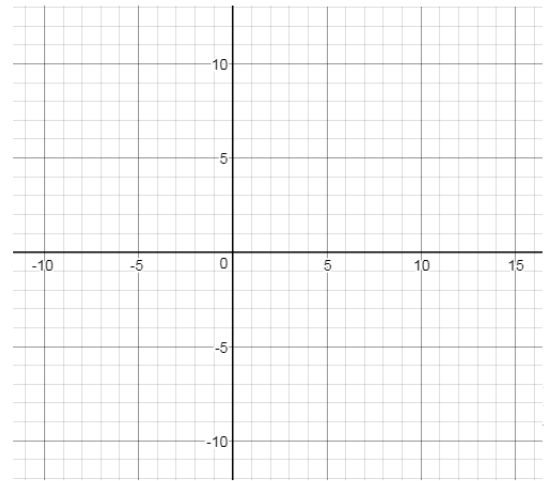
       Graph the 3 lines...

C) Find all the linear equations....
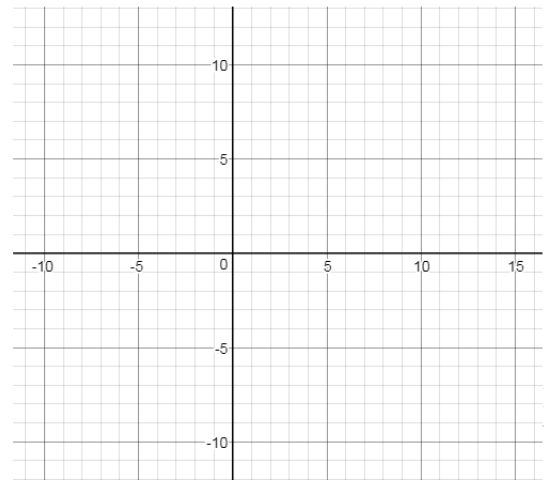   Identify the slopes and intercepts...
   Then, graph...

1) Line contains the points (2, 3) and (7, -7)
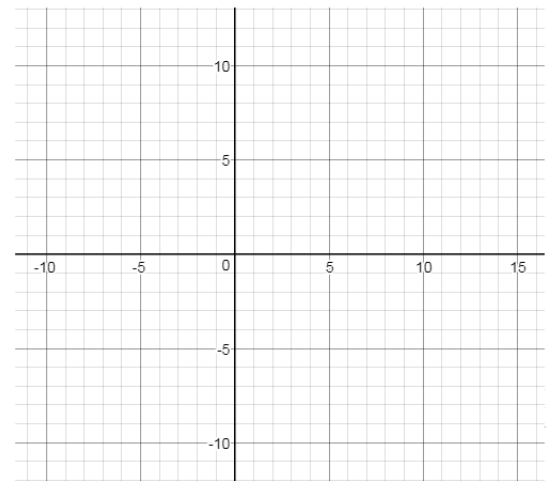
   And, a line parallel going through (0, 10)

2) Line passes through the origin and (2, 5)

   And, a line perpendicular going through (0, -4)

3) Horizontal line through (4, 7) and (-2, 7)

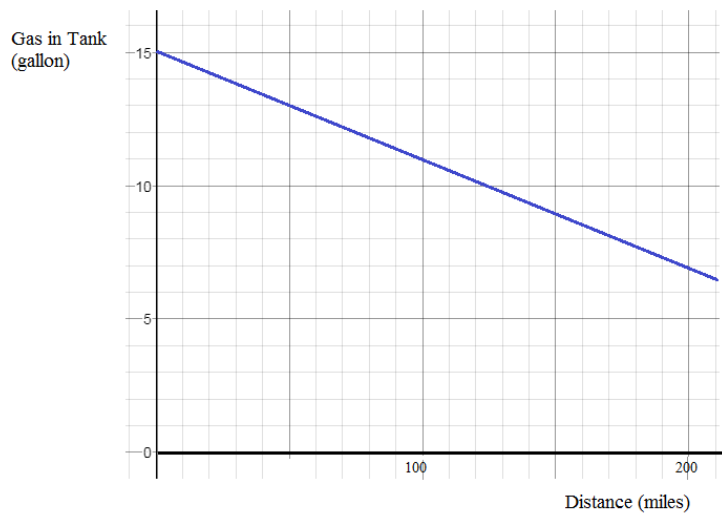   And, a line parallel going through (-3, -5)

D) Write equation of a line that includes (7, 13)
   and has the same y-intercept as $y = 2x + 1$

E) The following chart displays values from a linear model.
   Fill in the rest of the chart, AND write the equation of the model...

x	-3	-2	-1	0			3
y	17		7		-3		-13

F) The graph shows the relationship between the distance traveled
   and the gallons of gas in a blue sports car.

Gas in Tank (gallon)

Distance (miles)

a) What is the capacity of the car's gas tank?

b) How far does the car travel on 1 gallon of gas?

c) Write an equation that shows the amount of gas (g) that
   remains after the car travels (m) miles.

d) How far could the car travel before it runs out of gas?

G) Answer the following:

1) The cost of a taxicab ride is modeled by the linear function

$C(m) = 2.25 + .75(m)$ where m is the number of miles driven

        a) What is the minimum cost of a ride?

        b) How much does a 6 mile ride cost?

        c) If you pay 12 dollars for a ride, how far did you travel?

Graph the cost function, labeling the axes....

2) Calvin opens a lemonade stand, after buying 10 dollars of lemonade mix.
The cost of each cup is 5 cents, and he plans to charge 25 cents per cup of lemonade sold.
Write the cost function and revenue functions.

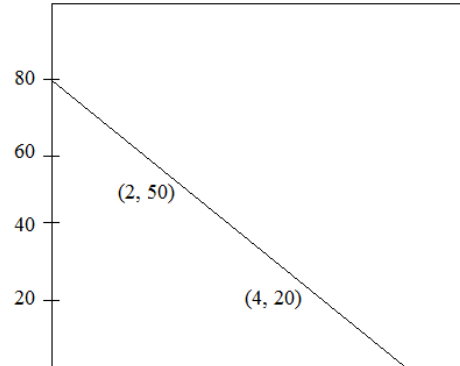How many cups must Calvin sell to earn a profit?

H) Three friends at camp receive money from their parents.
    Each spends their money at a constant rate, modeled by the following:

Camper A: $y = -22.5x + 90$

Camper B:

(end of) Week	$ remaining
1	100
2	75
3	50
4	25

Camper C:

a) Which camper received the most money from his parent?

b) Which camper spent his money the fastest?

c) Which camper ran out of money first?

I) A car has a tank that holds 18 gallons of gas.

If a car travels at 2 gallons/hour, write an function for gallons (g) as a function of time (t)

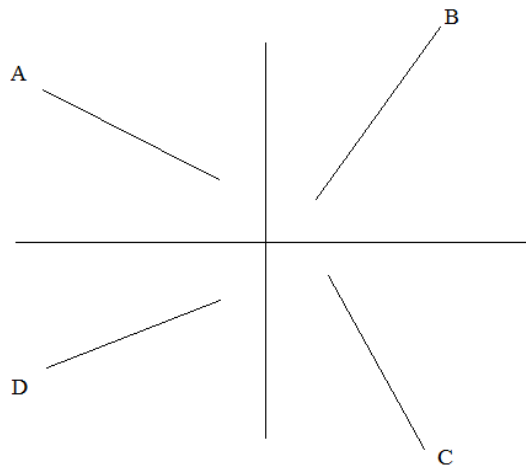Graph the equation.

State the domain and range.

After the trip, there is 3 gallons of gas left in the tank.

How long was the trip?

I. Which line segment matches
   each slope?

   1) -2/3

   2) 2/3

   3) 3

   4) -3

II. Write the equation of a line

   1) parallel to y = 3 and passing through (-5, 6)

   2) perpendicular to the x-axis and passing through (1, 12)
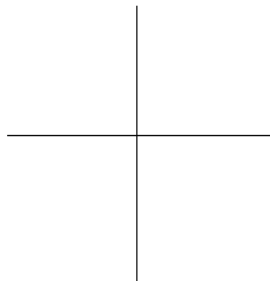
   3) perpendicular to x = 5 and passing through (-2, 7)

III. Graph the following. Then, identify the properties..

   1) x = -2   2) y = 8

      x-intercept: x-intercept:

      y-intercept: y-intercept:
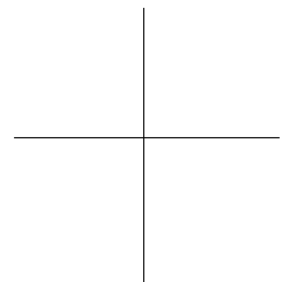
      slope: slope:

Hidden Message

Clue: "first step to finding the slope."

Solve the following. Then, convert the numbers into letters to find the answer.

Letter Key:

1	2	3	4	5	6	7	8	9	0
B	E	G	H	L	N	O	S	T	W

1) Slope of $y = 3x + 22/53$

2) Given: line A $\Rightarrow$ x + 7y = 10

line A $\perp$ line B

What is the slope of line B?

3) Find the y-intercept of the line

$(y - 3) = 3(x + 2)$

4) Slope between the origin and (2, 14)

5) C = (1, 5)   D = (-7, 9)   E = (w, 1)

If C, D, and E are collinear points, what is w?

6) Slope of a line having points

(3, 15) & (-2, -5)
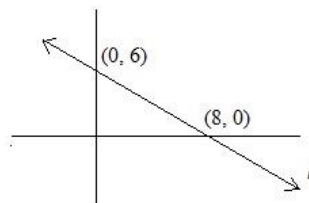
7) Slope of the line $4x - 2y = 7$

8) What is the x-intercept of line l ?

(0, 6)

(8, 0)

l

9) Lines C and D are parallel. If the equation for C is
$y = 6x + 4$, what is the slope of D?

10) What is the x-intercept of the line $2x + 3y = 14$?

11) Slope of a horizontal line

SOLUTIONS-→

A) Slope   SOLUTIONS

1) $(-2, 6)$ $(x, 10)$ Slope = 2

   Find x

   slope m = $\dfrac{y_1 - y_2}{x_1 - x_2}$ $2 = \dfrac{6 - 10}{-2 - x}$ $2 = \dfrac{-4}{(-2 - x)}$ $\boxed{x = 0}$

2) $(1, y + 3)$ $(-3, 6)$ Slope = 3

   Find y

   slope m = $\dfrac{\triangle y}{\triangle x}$ $3 = \dfrac{y + 3 - 6}{1 - (-3)}$ $3 = \dfrac{y - 3}{4}$ $\boxed{y = 15}$

3) If the lines are parallel,   line m $(1, 5)$ $(2, 10)$ the slope of line m is... $\dfrac{10 - 5}{2 - 1} = 5$

   line p $(-3, -1)$ $(k, 7)$

   Find k   sincle m and p are parallel, the slope of p is 5 $5 = \dfrac{-1 - 7}{-3 - k}$ $-15 - 5k = -8$

   $-5k = 7$ $\boxed{k = -7/5}$

4) The following points are collinear:

   $(3, -2)$ $(8, 8)$ and $(-7, m)$ collinear ---> all on the same line ---> all must have same slope...

   What is m?   slope between $(3, -2)$ and $(8, 8)$ $\dfrac{-2 - 8}{3 - 8} = 2$ slope between $(8, 8)$ and $(-7, m)$ must be equal to 2 $2 = \dfrac{8 - m}{8 - (-7)}$ $2 = \dfrac{8 - m}{15}$ $\boxed{m = -22}$
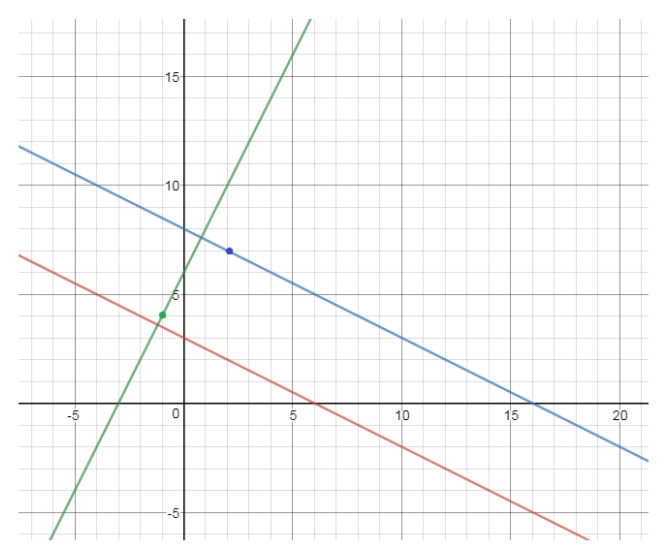
B) Given the line l : $3x + 6y = 18$

1) Find the x-intercept: x-intercept is where line crosses the x-axis ---> (something, 0) $\boxed{(6, 0)}$

   y-intercept: y-intercept is where line crosses the y-axis ---> (0, something) $\boxed{(0, 3)}$

   slope: rewrite in y = mx + b form ---> $3x + 6y = 18$

   $6y = 18 - 3x$ $\boxed{y = \dfrac{-1}{2}x + 3}$

   $y = 3 - \dfrac{3}{6}x$

2) Write an equation of the line parallel to l that passes through $(2, 7)$.

   parallel lines have same slope ---> slope m = -1/2

   must go through (2, 7)

   $y = mx + b$ $b = 8$ $\boxed{y = -1/2 + 8}$

   $7 = (-1/2)(2) + b$

3) Write an equation of the line perpendicular to l that passes through $(-1, 4)$.

   perpendicular lines have "opposite reciprocal" slope ---> slope m = 2

   must go through (-1, 4)

   $y = mx + b$ $b = 6$ $\boxed{y = 2x + 6}$

   $4 = (2)(-1) + b$

   Graph the 3 lines...

C) Find all the linear equations....
   Identify the slopes and intercepts...
   Then, graph...

SOLUTIONS

Linear Equations and Models

1) Line contains the points (2, 3) and (7, -7)

$$\text{slope} = \frac{'\text{rise}'}{'\text{run}'} = \frac{3 - (-7)}{2 - 7} = \boxed{-2}$$

$y = mx + b$

(using the first coordinate)

$(3) = -2(2) + b$

$b = 7$ $\boxed{y = -2x + 7}$

x-intercept: (7/2, 0)

y-intercept: (0, 7)

   And, a line parallel going through (0, 10)

parallel line has slope -2

since $\boxed{\text{y-intercept is (0, 10),}}$

$\boxed{y = -2x + 10}$ $\boxed{\text{x-intercept is (5, 0)}}$

2) Line passes through the origin and (2, 5)

   we need the slope: (0, 0) and (2, 5)

   "rise" over "run" is 5/2

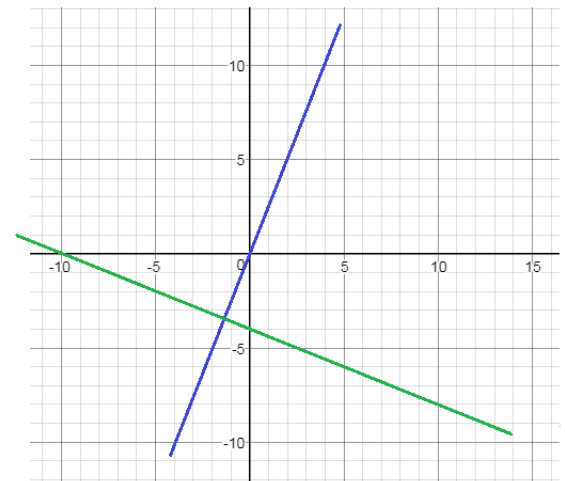$\boxed{y = \frac{5}{2}x + 0}$ $\boxed{\text{x-intercept AND y-intercept is (0, 0)}}$

   And, a line perpendicular going through (0, -4)

since slope is 5/2, a perpendicular line will have slope of -2/5

$\boxed{y = \frac{-2}{5}x - 4}$ $\boxed{\begin{array}{l}\text{y-intercept is (0, -4)} \\ \text{x-intercept is (-10, 0)}\end{array}}$

3) Horizontal line through (4, 7) and (-2, 7)

   horizontal line has a $\boxed{\text{slope of 0}}$

   since all y-values will be 7, the equation of the line is $\boxed{y = 7}$

   $\boxed{\begin{array}{l}\text{y-intercept is (0, 7)} \\ \text{There is NO x-intercept}\end{array}}$

   And, a line parallel going through (-3, -5)

   (a perpendicular line would have a slope of 'undefined')
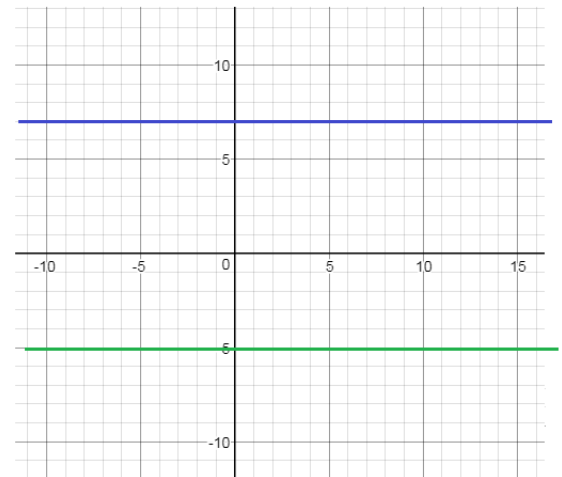
   A parallel line will be horizontal and have $\boxed{\text{a slope of 0}}$

   since it goes through (-3, -5), all the y-values will be -5....

$\boxed{\begin{array}{l} y = -5 \\ \text{y-intercept is (0, -5)} \\ \text{NO x-intercept} \end{array}}$

D) Write equation of a line that includes (7, 13)
and has the same y-intercept as $y = 2x + 1$

SOLUTIONS

$y = mx + b$

$13 = m(7) + 1$

$m = 12/7$ $y = 12/7 (x) + 1$

E) The following chart displays values from a linear model.
Fill in the rest of the chart, AND write the equation of the model...

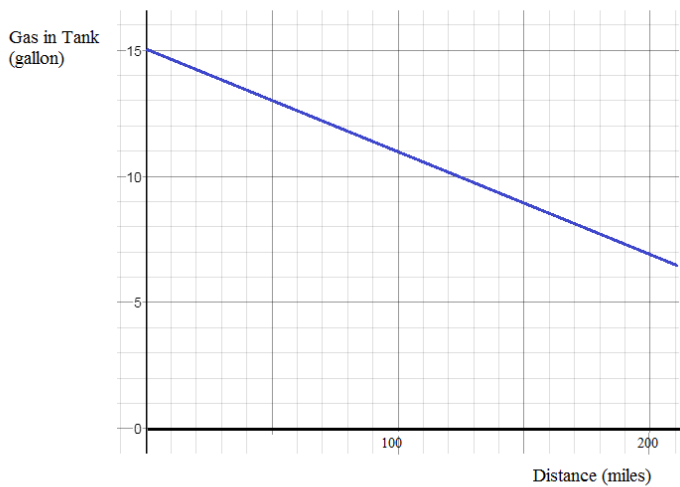| x	-3	-2	-1	0	1	2	3
y	17	12	7	2	-3	-8	-13

First set of differences are constant...
Therefore, it is *linear*.

Slope is -5
y-intercept is (0, 2) $y = -5x + 2$

the sketch is *decreasing*

F) The graph shows the relationship between the distance traveled
and the gallons of gas in a blue sports car.

Gas in Tank
(gallon)

Distance (miles)

a) What is the capacity of the car's gas tank?

The capacity occurs when the car starts: 15 gallons

b) How far does the car travel on 1 gallon of gas?

Since the car uses 4 gallons to travel 100 miles, it's rate is $\dfrac{4 \text{ gallons}}{100 \text{ miles}} = \dfrac{1 \text{ gallon}}{25 \text{ miles}}$

c) Write an equation that shows the amount of gas (g) that
remains after the car travels (m) miles.

We know the g-intercept is 15
and the slope (rate of change) is -1/25, the equation is

$g = \dfrac{-1}{25}m + 15$

d) How far could the car travel before it runs out of gas?

$0 = \dfrac{-1}{25}m + 15$ $-15 = \dfrac{-1}{25}m$ $m = 375 \text{ miles}$
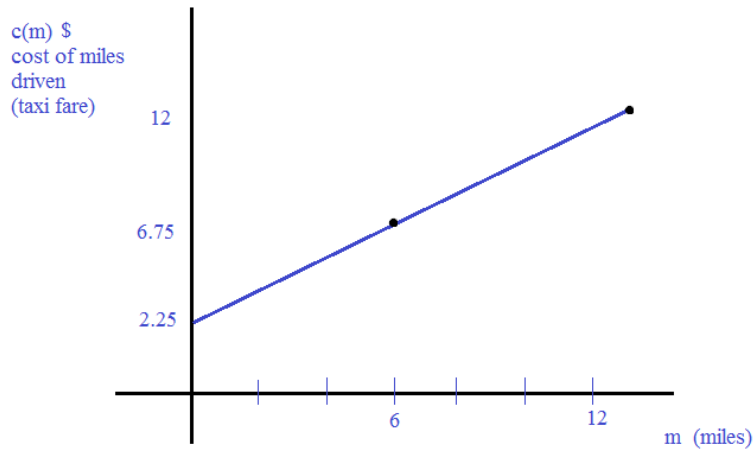
G) Answer the following:

1) The cost of a taxicab ride is modeled by the linear function

$C(m) = 2.25 + .75(m)$ where m is the number of miles driven

   a) What is the minimum cost of a ride?   The minimum cost of the ride occurs when m = 0   **$2.25**

   b) How much does a 6 mile ride cost? $c(6) = 2.25 + .75(6) = $ **$6.75**

   c) If you pay 12 dollars for a ride, how far did you travel? $12 = 2.25 + .75(m)$ $9.75 = .75(m)$ **m = 13 miles**

Graph the cost function, labeling the axes....

c(m) $
cost of miles
driven
(taxi fare)

2) Calvin opens a lemonade stand, after buying 10 dollars of lemonade mix.
The cost of each cup is 5 cents, and he plans to charge 25 cents per cup of lemonade sold.
Write the cost function and revenue functions.

$C(x) = 10 + .05(x)$ where x is the number of cups sold

$R(x) = .25(x)$ where x is the number of cups sold

How many cups must Calvin sell to earn a profit?

Profit occurs when $R(x) > C(x)$

So, when are they equal? (when does Calvin break even?)

$C(x) = R(x)$ $10 + .05(x) = .25(x)$

$10 = .20(x)$

$x = 50$

Calvin must sell 51 cups of lemonade to earn a profit!

H) Three friends at camp receive money from their parents.
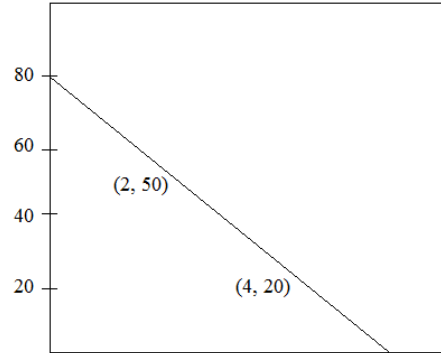Each spends their money at a constant rate, modeled by the following:

SOLUTIONS

Camper A: $y = -22.5x + 90$

Camper C:

Camper B:

(end of) Week	$ remaining
1	100
2	75
3	50
4	25

$y = -25x + 125$

80

60

(2, 50)

40

20   (4, 20)

$y = -15x + 80$

a) Which camper received the most money from his parent?   y-intercept

Camper A: 90
Camper B: 125 ⟵
Camper C: 80

b) Which camper spent his money the fastest?   slope

Camper A: $22.50/week
Camper B: $25/week ⟵
Camper C: $15/week

c) Which camper ran out of money first?   x-intercept

Camper A: 4 weeks ⟵
Camper B: 5 weeks
Camper C: 5.33 weeks

I) A car has a tank that holds 18 gallons of gas.

If a car travels at 2 gallons/hour, write an function for gallons (g) as a function of time (t)

$g = 18 - 2t$

Graph the equation.

State the domain and range.   domain: $0 \le t \le 9$ range: $0 \le g \le 18$
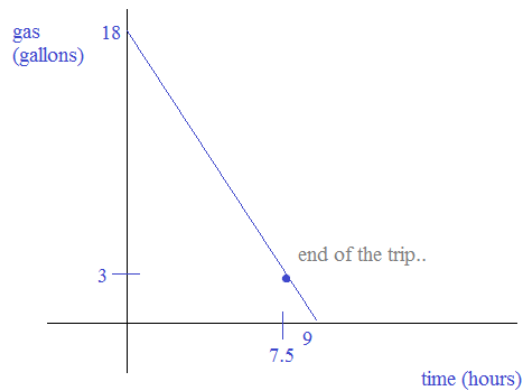
After the trip, there is 3 gallons of gas left in the tank.
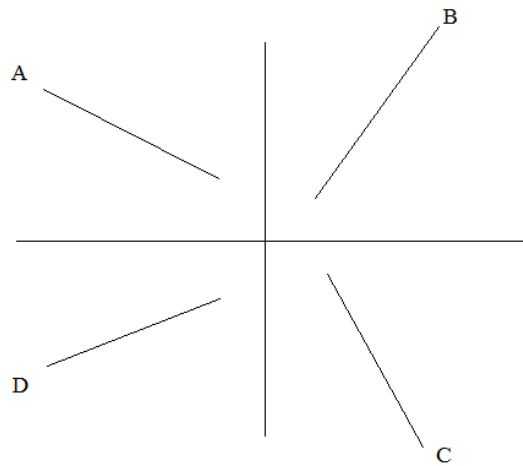
How long was the trip?

$g = 18 - 2t$

$3 = 18 - 2t$

$t = 7 \ 1/2$ hours

gas   18
(gallons)

3

end of the trip..

9
7.5

time (hours)

I. Which line segment matches each slope?

1) -2/3   A

2) 2/3   D

3) 3   B

4) -3   C

To help determine:

positive slopes go up (B and D)
negative slopes go down (A and C)

the "flatter" lines are less than 1
the "steeper" lines are greater than 1

B

A

D

C

II. Write the equation of a line

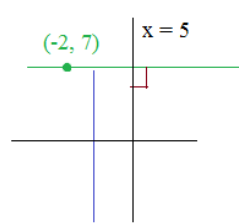1) parallel to $y = 3$ and passing through (-5, 6)

$y = 6$
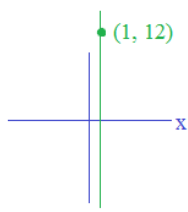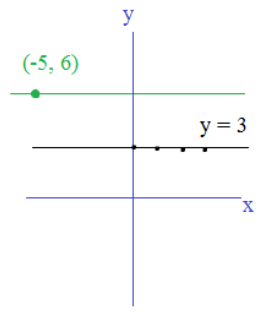
2) perpendicular to the x-axis and passing through (1, 12)

$x = 1$

3) perpendicular to $x = 5$ and passing through (-2, 7)

$y = 7$

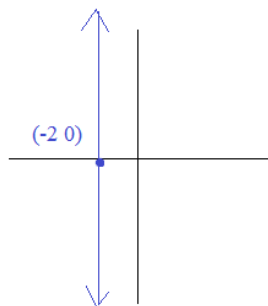steps a) graph $y = 3$
           (0, 3), (1, 3), (2, 3) etc..

       b) plot (-5, 6)

       c) draw parallel line

       d) describe the line...

(-5, 6)

y = 3

x

y

• (1, 12)

x

(-2, 7)   x = 5

III. Graph the following. Then, identify the properties..
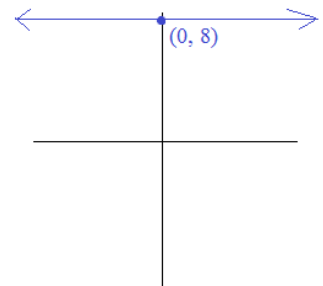
1) x = -2

x-intercept: (-2, 0)

y-intercept: none

slope: undefined
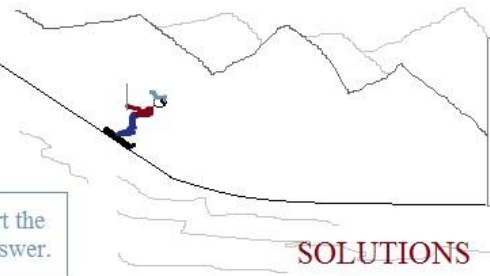
(-2 0)

2) y = 8

x-intercept: none

y-intercept: (0, 8)

slope: 0

(0, 8)

Hidden Message

Clue: "first step to finding the slope."

Solve the following. Then, convert the numbers into letters to find the answer.

Letter Key:

1	2	3	4	5	6	7	8	9	0
B	E	G	H	L	N	O	S	T	W

SOLUTIONS

1) Slope of $y = 3x + 22/53$

(slope intercept form $y = mx + b$)

m = slope

$3 \rightarrow G$

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2) Given: line A ⟹ $x + 7y = 10$

line A ⊥ line B

What is the slope of line B?

Line A $7y = -x + 10$

$y = -x/7 - 10/7$

slope of line A = $-1/7$

Slope of perpendicular line B is

"opposite reciprocal"   7

$7 \rightarrow O$

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3) Find the y-intercept of the line

$(y - 3) = 3(x + 2)$
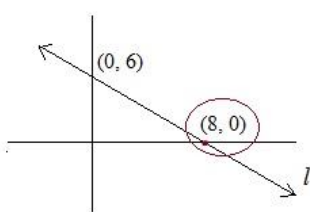
$y - 3 = 3x + 6$

$y = 3x + 9$

y-intercept is 9

$9 \rightarrow T$

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4) Slope between the origin and (2, 14)

(0, 0)

slope m $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{14 - 0}{2 - 0} = 7$

$7 \rightarrow O$

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5) C = (1, 5) D = (-7, 9) E = (w, 1)

If C, D, and E are collinear points, what is w?

If collinear, same slopes....

slope CD $= \dfrac{9 - 5}{-7 - 1} = \dfrac{-1}{2}$

slope DE $= \dfrac{-1}{2} = \dfrac{1 - 9}{w - (-7)} = \dfrac{-8}{w + 7}$

$w = 9$

$9 \rightarrow T$

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6) Slope of a line having points

(3, 15) & (-2, -5)

$\dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-5 - 15}{-2 - 3} = \dfrac{-20}{-5} = 4$

$4 \rightarrow H$

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7) Slope of the line $4x - 2y = 7$ ⟹ $-2y = -4x + 7$

$y = 2x - 7/2$ (slope is 2)

$2 \rightarrow E$

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8) What is the x-intercept of line l ?

x-intercept is the point where the line intercepts the x-axis.

$x = 8$ or, (8, 0)

(0, 6)

(8, 0)

First step to finding the (ski) slope?

"Go to the Snow"

$8 \rightarrow S$

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9) Lines C and D are parallel. If the equation for C is $y = 6x + 4$, what is the slope of D?

parallel lines have identical slopes

slope of C is 6.. therefore, slope of D is 6..

$6 \rightarrow N$

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10) What is the x-intercept of the line $2x + 3y = 14$?

The x-intercept will be (x, 0).. so, plug in the point:

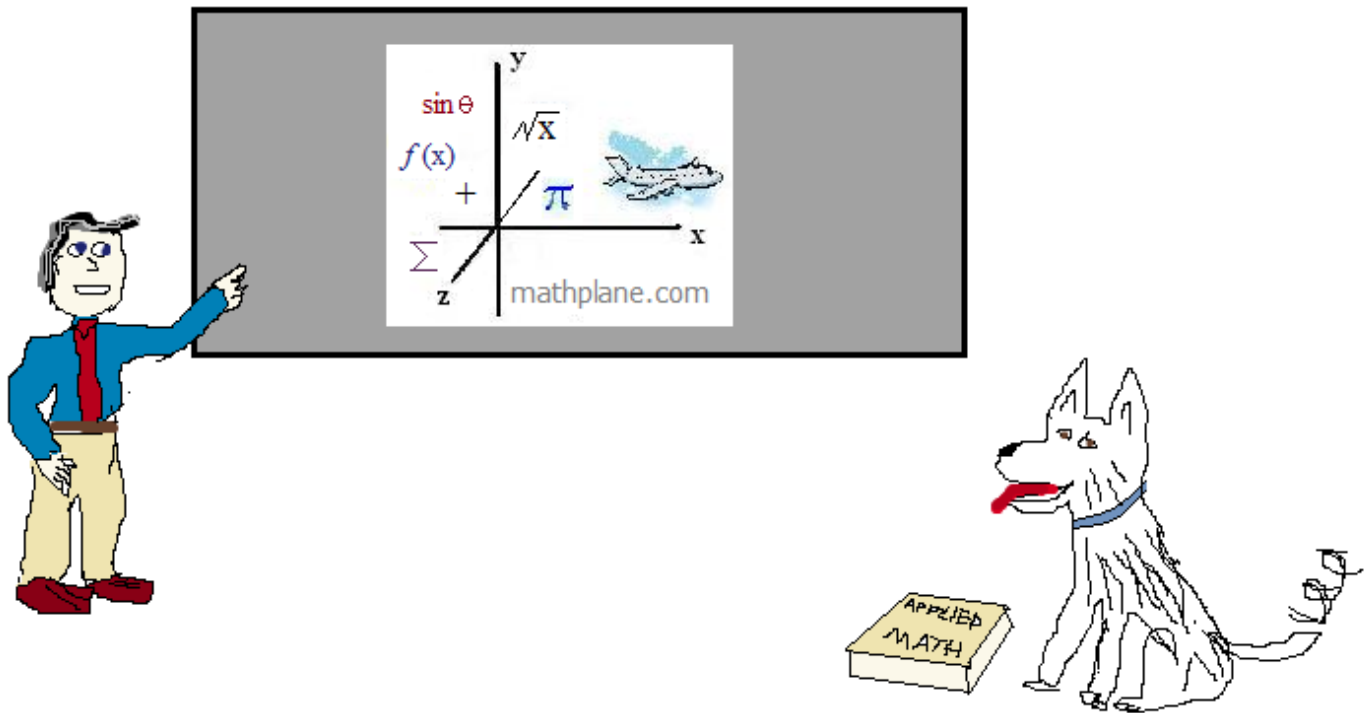$2x + 3(0) = 14$

$2x = 14$

$x = 7$

(7, 0)

$7 \rightarrow O$

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11) Slope of a horizontal line

the slope of a horizontal line is 0..

$0 \rightarrow W$
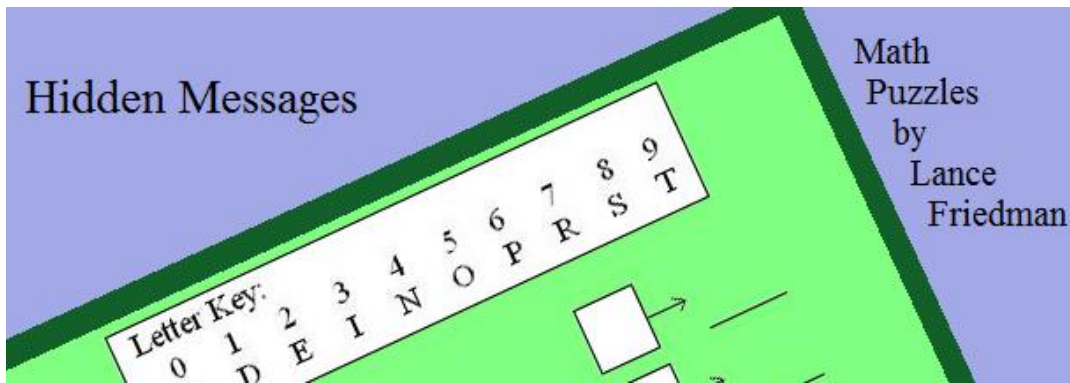
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Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

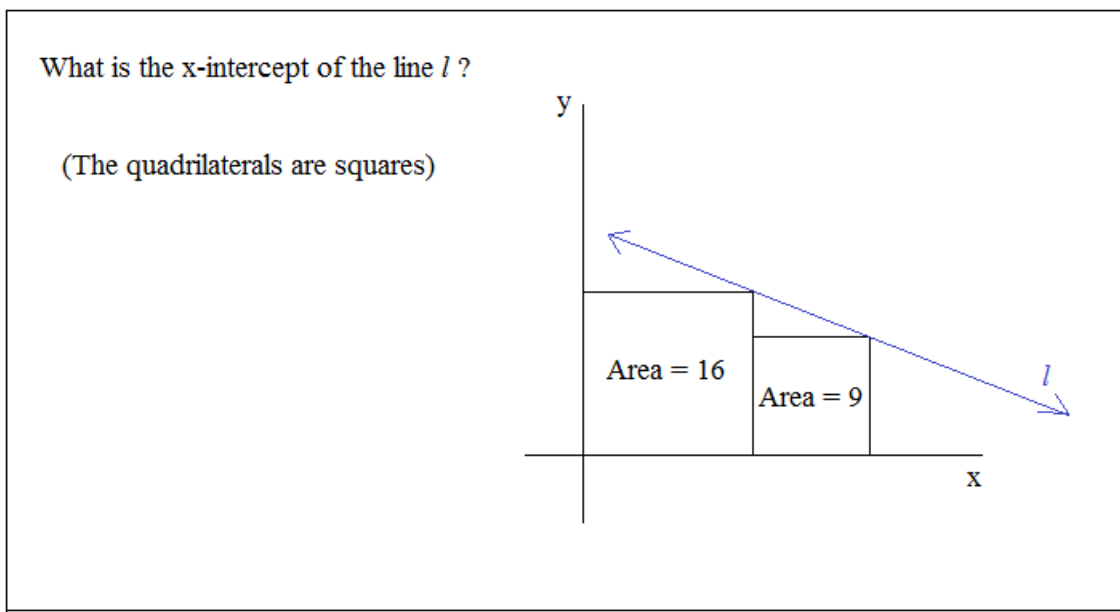Cheers

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One More Question....

What is the x-intercept of the line *l* ?
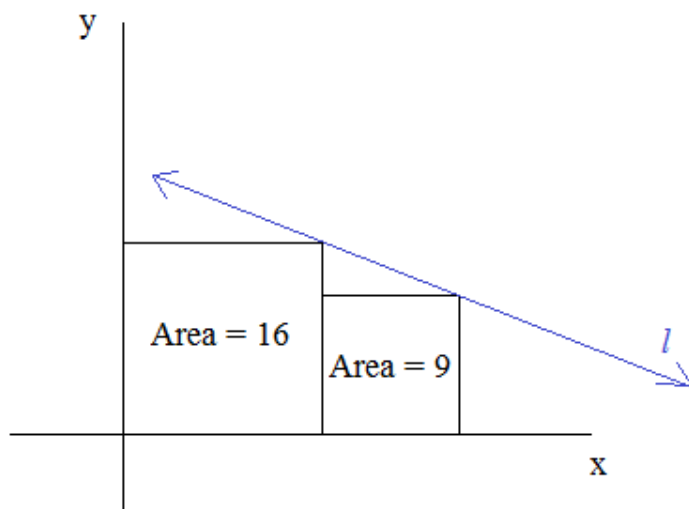
(The quadrilaterals are squares)

Area = 16

Area = 9

l

Answer-→

What is the x-intercept of the line *l* ?

(The quadrilaterals are squares)

2 of the points on line *l* are (4, 4) and (7, 3)...

Slope: $\dfrac{3-4}{7-4} = \dfrac{-1}{3}$

therefore, equation of the line is $y - 4 = \dfrac{-1}{3}(x-4)$

$$y = \dfrac{-1}{3}x + \dfrac{4}{3} + 4$$

$$y = \dfrac{-1}{3}x + \dfrac{16}{3}$$

ANSWER

The x-intercept is (16, 0)