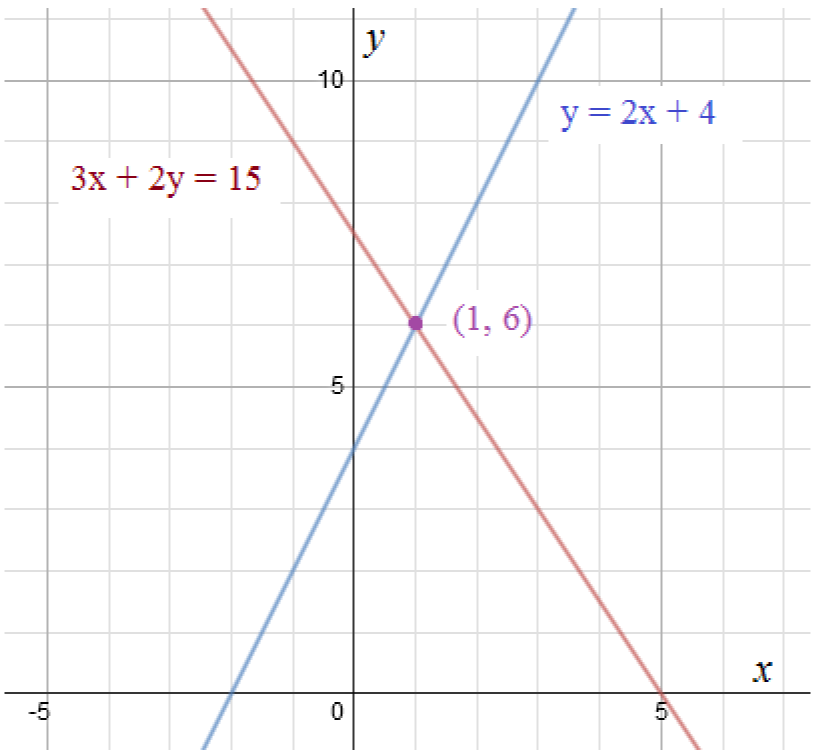
Linear Systems

Examples and Practice Tests (and, solutions)

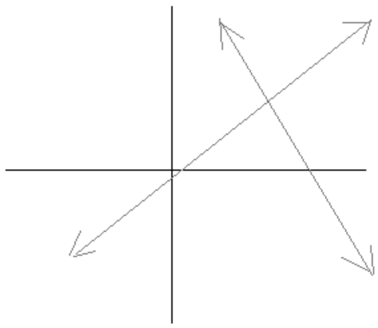


Topics include solving, graphing, elimination and substitution methods, word problems, classifying systems, 3 variables, and more.

Linear systems: Consistent or Inconsistent?

Classify the following linear systems:
(consistent/inconsistent - independent/dependent)

a)



Consistent and independent

one solution

b) $2x + 3y = -10$
$-4x - 6y = 20$

Consistent and dependent
(same lines)

infinite number of solutions

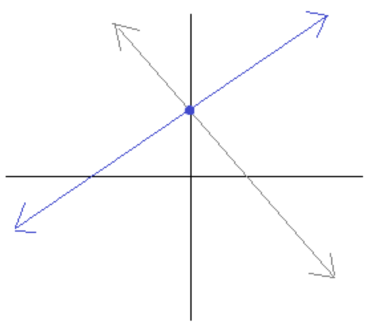
c) $y = 3x + 7$
$y = 3x - 7$

Inconsistent
(parallel lines)

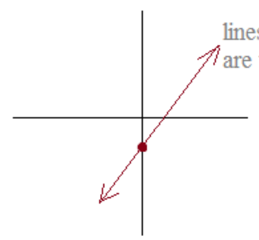
no solutions

Can you draw consistent, independent linear system where the lines have the same y-intercept?

Yes!



Note: a consistent, dependent linear system would obviously have a common y-intercept...



Inconsistent linear system: parallel lines

Consistent linear system: intersecting lines

Dependent ---> all points

Independent ---> one point of intersection

Six Ways to solve a linear system: $\begin{aligned} 2x + 3y &= 12 \\ y &= x - 11 \end{aligned}$

1) Elimination/Combination Method

(Write equations in standard form)

$$\begin{aligned} 2x + 3y &= 12 \\ x - y &= 11 \end{aligned}$$

(Choose variable to eliminate, and if necessary, change equation(s))

$$\begin{aligned} 2x + 3y &= 12 \\ 3x - 3y &= 33 \end{aligned}$$

(Combine equations and eliminate variable)

$$\begin{aligned} 5x + 0y &= 45 \\ x &= 9 \end{aligned}$$

Since $x = 9$, then

$y = (9) - 11$, so

$y = -2$

2) Substitution Method

$$\begin{aligned} 2x + 3y &= 12 \\ y &= x - 11 \end{aligned}$$

Substitute second equation into the first....

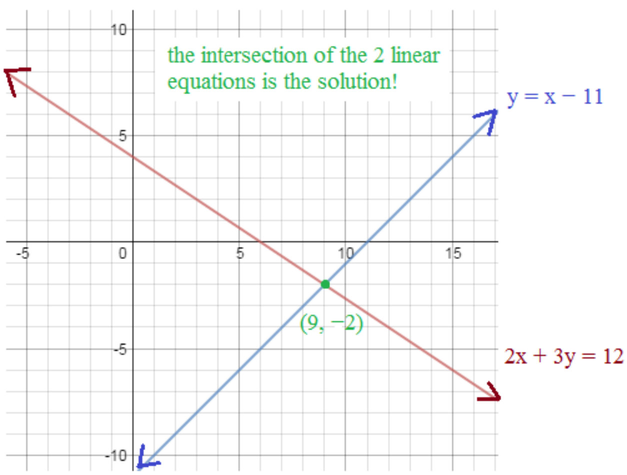
$$\begin{aligned} 2x + 3(x - 11) &= 12 \\ 2x + 3x - 33 &= 12 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

Since $x = 9$,

$2(9) + 3y = 12$

so, $y = -2$

3) Graphing



4) Matrix $AX = B$ then, $X = A^{-1}B$

$$\begin{aligned} 2x + 3y &= 12 \\ x - y &= 11 \end{aligned}$$

Place coefficients and solutions into matrices

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 12 \\ 11 \end{bmatrix}}_{B}$$

Use the inverse of A...

$$\underbrace{\begin{bmatrix} 1/5 & 3/5 \\ 1/5 & -2/5 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1/5 & 3/5 \\ 1/5 & -2/5 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 12 \\ 11 \end{bmatrix}}_{B}$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Identity Matrix I}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \begin{bmatrix} 45/5 \\ -10/5 \end{bmatrix}$$

5) Augmented Matrix

Place coefficients and solutions into 2x3 augmented matrix...

$$\left[\begin{array}{cc:c} 2 & 3 & 12 \\ 1 & -1 & 11 \end{array}\right]$$ switch rows..

$$\left[\begin{array}{cc:c} 1 & -1 & 11 \\ 2 & 3 & 12 \end{array}\right]$$ R1 x (-2), then add to R2

$$\left[\begin{array}{cc:c} 1 & -1 & 11 \\ 0 & 5 & -10 \end{array}\right]$$ R2 x (1/5)

$$\left[\begin{array}{cc:c} 1 & -1 & 11 \\ 0 & 1 & -2 \end{array}\right]$$ add R2 to R1 $\Rightarrow$ $$\left[\begin{array}{cc:c} 1 & 0 & 9 \\ 0 & 1 & -2 \end{array}\right]$$

Reduced Row Echelon Form (RREF) displays the solutions for x and y

6) Cramer's Rule (Using Determinants)

$$\begin{aligned} 2x + 3y &= 12 \\ 1x - 1y &= 11 \end{aligned}$$

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} \qquad D_x = \begin{vmatrix} 12 & 3 \\ 11 & -1 \end{vmatrix} \qquad D_y = \begin{vmatrix} 2 & 12 \\ 1 & 11 \end{vmatrix}$$

$-2 - 3 = -5$ $\qquad$ $-12 - 33 = -45$ $\qquad$ $22 - 12 = 10$

$$x = \frac{D_x}{D} = 9 \qquad y = \frac{D_y}{D} = -2$$

Linear Systems: Word Problem Applications

Example: 34 kids and 6 adult chaperones are going to the amusement park.
They can take cars and/or vans.
Each van seats 7, and each car seats 5.
If all 6 adults will drive, how many will go in each vehicle?

Step 1: Set Variables

V = # of vans
C = # of cars

Step 2: Set up equations/constraints

(Number of riders) $\underbrace{7V}_{\text{7 in each van}} + \underbrace{5C}_{\text{5 in each car}} = \underbrace{(34 + 6)}_{\text{40 riders}}$

(Van and car drivers) $V + C = 6$

Step 3: Solve (we have 2 equations and 2 unknowns)

use combination method:

$$\begin{aligned} 7V + 5C &= 40 \\ V + C &= 6 \end{aligned} \Rightarrow \begin{aligned} 7V + 5C &= 40 \\ -5V - 5C &= -30 \end{aligned} \Rightarrow \begin{aligned} 2V &= 10 \\ V = 5 \quad & C = 1 \end{aligned}$$

The group will take 5 vans and 1 car...

Example: An orchestra has a string to wind ratio of 9:4...
If there are 91 total instruments, how many of each are there?

Step 1: Set up variables

Let S = # of string instruments
W = # of wind instruments

Step 2: Set up equations

"91 total instruments" $S + W = 91$

"string to wind ratio of 9:4" $9W = 4S$ or $S = \frac{9}{4}W$ (ex: if there are 40 W, then there are 90 S

Step 3: Solve $S + W = 91$ $S = \frac{9}{4}W$

(since we have 2 equations and 2 unknowns, we have a system....)

Use substitution method:

$$\left(\frac{9}{4}W\right) + W = 91$$

$$\frac{13}{4}W = 91$$

$$\frac{4}{13} \cdot \frac{13}{4}W = 91 \cdot \frac{4}{13} \qquad W = 28 \quad \text{so,} \quad S = 63$$

The orchestra has 28 wind instruments and 63 string instruments.

Example: The math guy spends an afternoon rowing up and down a river.
In the morning, when he rowed with the current, he traveled 24 miles in 3 hours.
In the afternoon, when he rowed against the current, he went 16 miles in 4 hours.
What is the speed of the current?

Step 1: Figure out the variables

Let C = speed of the current Let R = speed of rower)

Step 2: Set equations

distance = rate x time

(with the current) 24 miles = (R + C)(3 hours)

(against the current) 16 miles = (R − C)(4 hours)

Step 3: Solve

$$8\,\frac{\text{miles}}{\text{hour}} = R + C$$

$$4\,\frac{\text{miles}}{\text{hour}} = R - C$$

$\xrightarrow{\text{combine equations}}$ $12\,\frac{\text{miles}}{\text{hour}} = 2R + 0C$

R = 6 miles/hour

The math guy is rowing at a speed of 6 miles per hour... So, the speed of the current is 2 miles per hour

Linear Systems

Example: Solve the system:

$4x + 9y = 8$
$8x + 6z = -1$
$6y + 6z = -1$

Rewrite the equations:

$4x + 9y + 0z = 8$
$8x + 0y + 6z = -1$
$0x + 6y + 6z = -1$

Combine 2nd and 3rd equations:

(-1)
$8x + 0y + 6z = -1$
$0x + 6y + 6z = -1$

$-8x + 0y - 6z = 1$
$0x + 6y + 6z = -1$ $\longrightarrow$ $-8x + 6y = 0$

then, combine the outcome with the 1st equation:

$-8x + 6y = 0$
(2) $4x + 9y + 0z = 8$
$8x + 18y = 16$ $\longrightarrow$ $24y = 16$
$y = 2/3$

Use substitution to get remaining terms:

$-8x + 6y = 0$
$-8x + 6(2/3) = 0$
$4 = 8x$
$x = 1/2$

$8x + 6z = -1$
$8(1/2) + 6z = -1$
$6z = -5$
$z = -5/6$

Example: An automobile gets 36 miles per gallon in the city, and 46 miles per gallon on the highway. With a 13-gallon gas tank, this automobile travelled 526 miles. How many gallons were used driving in the city?

Step 1: Establish variables (and make a grid)

x = number of gallons
y = number of miles

	City	Highway
Fuel	x	(13 − x)
Rate	36 m/g	46 m/g
Distance	y	(526 − y)

Step 2: Construct system

$$y = 36\,\frac{\text{miles}}{\text{gallon}}\,(x \text{ gallons})$$

$$(526 - y) = 46\,\frac{\text{miles}}{\text{gallon}}\,(13 - x)(\text{gallons})$$

(2 equations, 2 unknowns)

Step 3: Solve (using substitution)

$(526 - (36x)) = 46(13 - x)$

$526 - 598 = 36x - 46x$

$-72 = -10x$

$\boxed{x = 7.2 \text{ gallons}}$

Step 4: Check answer

city: 7.2 gallons x 36m/g = 259.2 miles

highway: 5.8 gallons x 46m/g = 266.8 miles

total miles = 526 miles

Example: Solve the system of linear equations

$$2x + 3y - z = 12$$
$$3x - 4y + z = -9$$
$$x + 5y + z = 7$$

Step 1: Recognize the efficient approach...

In this case, it seems the elimination method is easiest.. (get rid of the z's first)

Step 2: Solve

Combine 1st and 2nd equations:

$$\begin{array}{r} 2x + 3y - z = 12 \\ 3x - 4y + z = -9 \\ \hline 5x - y \quad = 3 \end{array}$$

Combine 1st and 3rd equations:

$$\begin{array}{r} 2x + 3y - z = 12 \\ x + 5y + z = 7 \\ \hline 3x + 8y \quad = 19 \end{array}$$

Then, solve the 2 x 2 linear system....

$$5x - y = 3 \Rightarrow 40x - 8y = 24$$
$$3x + 8y = 19 \Rightarrow \begin{array}{r} 3x + 8y = 19 \\ \hline 43x \quad = 43 \end{array}$$

$x = 1$

so, $y = 2$

If $x = 1$ and $y = 2$, then $z = -4$

Step 3: Check solutions..

(1, 2, -4) Plug into ALL 3 EQUATIONS!

$2x + 3y - z = 12 \Rightarrow 2 + 6 - (-4) = 12$ ✓

$3x - 4y + z = -9 \Rightarrow 3 - 8 + (-4) = -9$ ✓

$x + 5y + z = 7 \Rightarrow 1 + 10 + (-4) = 7$ ✓

Example: Solve the system of linear equations

$$2x + 4y - 7z = 15$$
$$3y + z = 10$$
$$-6x + 2z = -28$$

Step 1: Recognize the efficient approach...

In this case, the substitution method seems most efficient...

Step 2: Using the middle equation, we can solve for z....

$$3y + z = 10 \Rightarrow z = 10 - 3y$$

Then, substitute into the 3rd equation...

$$-6x + 2z = -28$$
$$-6x + 2(10 - 3y) = -28$$
$$-6x + 20 - 6y = -28$$
$$-6x - 6y = -48$$

and, into the 1st equation...

$$2x + 4y - 7z = 15$$
$$2x + 4y - 7(10 - 3y) = 15$$
$$2x + 4y - 70 + 21y = 15$$
$$2x + 25y = 85$$

Now combine the results:

$$-6x - 6y = -48 \Rightarrow -2x - 2y = -16$$
$$2x + 25y = 85 \Rightarrow \begin{array}{r} 2x + 25y = 85 \\ \hline 23y = 69 \end{array}$$

$y = 3$.... then, $x = 5$

and, $z = 1$

Step 3: Check the answer...

(5, 3, 1)

$2x + 4y - 7z = 15 \quad 10 + 12 - 7 = 15$ ✓

$3y + z = 10 \quad 9 + 1 = 10$ ✓

$-6x + 2z = -28 \quad -30 + 2 = -28$ ✓

Specifying infinite solutions

Example: Solve the following system

$$2x - 3y = 10$$
$$-4x + 6y = -20$$

Using elimination method:

$2(2x - 3y = 10) \Longrightarrow$ $4x - 6y = 20$

$$\begin{array}{r} 4x - 6y = 20 \\ -4x + 6y = -20 \\ \hline 0 = 0 \end{array}$$

$\Longrightarrow$ there are infinitely many solutions

(5, 0) (1/2, -3) are 2 examples....

To specify, isolate one of the variables...

$$2x - 3y = 10$$
$$2x - 10 = 3y$$
$$\frac{2x - 10}{3} = y$$

$\Longrightarrow$ solutions are $\left(T, \frac{2}{3}T - \frac{10}{3}\right)$

Example: The following is an augmented matrix. Find the solution.

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & -2 \\ 0 & 1 & -3 & 4 \\ 0 & -1 & 3 & -4 \end{array}\right]$$

$$x - 2y + 4 = -2$$
$$y - 3z = 4$$
$$-y + 3z = -4$$

$$x - 2y + 4z = -2$$
$$y - 3z = 4$$
$$-y + 3z = -4$$

add together and get $0 = 0$

$\Longrightarrow$ dependent...

So, we'll take one of the dependent equations and isolate a variable...

$$y = 3z + 4$$

Then, plug it into the first equation...

$$x - 2(3z + 4) + 4z = -2$$
$$x - 6z - 8 + 4z = -2$$
$$x - 2z = 6$$
$$2z = x - 6$$
$$\boxed{z = \frac{1}{2}x - 3}$$

$$y = 3z + 4$$
$$y = 3\left(\frac{1}{2}x - 3\right) + 4$$
$$\boxed{y = \frac{3}{2}x - 5}$$

$$\boxed{\begin{array}{l} x = x \\ y = \frac{3}{2}x - 5 \\ z = \frac{1}{2}x - 3 \end{array}}$$

Checking the answer:

$$x - 2y + 4z = -2$$
$$y - 3z = 4$$
$$-y + 3z = -4$$

Let's check some solutions...

$x = 1$, $y = -7/2$, $z = -5/2$ $\Longrightarrow$

$(1) - 2(-7/2) + 4(-5/2) = -2$

$(-7/2) - 3(-5/2) = 4$

$-(-7/2) + 3(-5/2) = -4$

$x = 6$, $y = 4$, $z = 0$ $\Longrightarrow$

$(6) - 2(4) + 4(0) = -2$

$(4) - 3(0) = 4$

$-(4) + 3(0) = -4$

Note: everything is solved for x....
(similar to working with parametric equations)

$$\left(x,\ \frac{3}{2}x - 5,\ \frac{1}{2}x - 3\right)$$

or, the (x, y, z) solution can be written with another variable

ex: (B, 3/2(B) - 5, 1/2(B) - 3)

Example: In the following system, solve:

a) in terms of x

b) in terms of y

c) in terms of z

$$2x - 3y + z = 11$$
$$5x + y - 2z = 8$$

Using "elimination" method, get rid of the y terms...

$$2x - 3y + z = 11$$
$$5x + y - 2z = 8$$
$$15x + 3y - 6z = 24$$

$$17x - 5z = 35$$

$$17x = 35 + 5z$$

$$x = \frac{35}{17} + \frac{5}{17}z \qquad \text{x in terms of z}$$

$$17x - 35 = 5z$$

z in terms of x $\quad \frac{17}{5}x - 7 = z$

Using "elimination" method, get rid of the x terms...

$$2x - 3y + z = 11$$
$$5x + y - 2z = 8$$
$$-10x + 15y - 5z = -55$$
$$10x + 2y - 4z = 16$$

$$17y - 9z = -39$$

$$17y = -39 + 9z$$

$$y = \frac{-39}{17} + \frac{9}{17}z \qquad \text{y in terms of z}$$

$$17y + 39 = 9z$$

z in terms of y $\quad \frac{17}{9}y + \frac{39}{9} = z$

Then, using "elimination", get rid of the z's ...
This will find x in terms of y and y in terms of x....

OR, use "substitution" with the above equations...

z in terms of y $\quad \boxed{\frac{17}{9}y + \frac{39}{9}} = z$

x in terms of z $\quad x = \frac{35}{17} + \frac{5}{17}z$

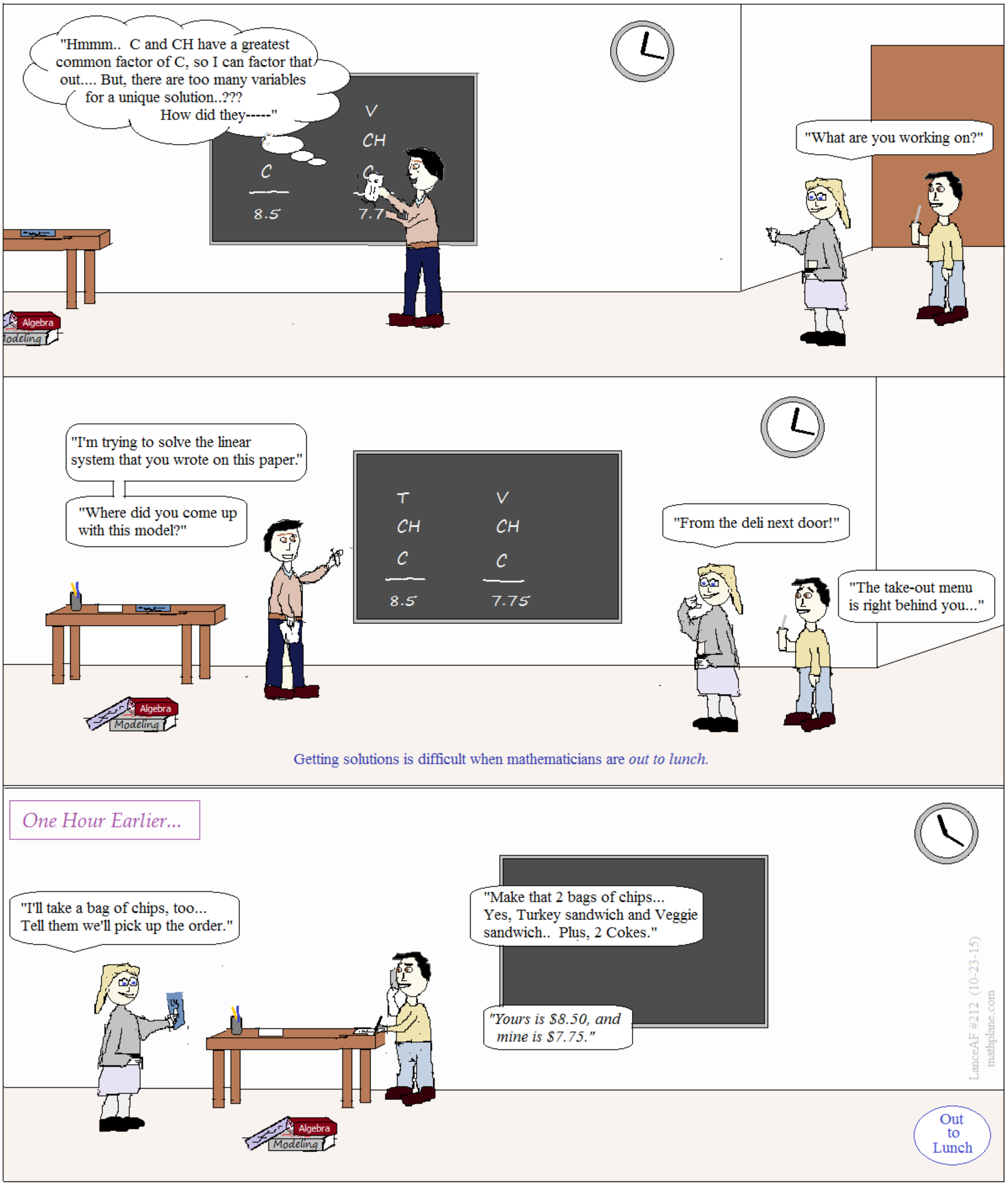
$$x = \frac{35}{17} + \frac{5}{17}\left(\boxed{\frac{17}{9}y + \frac{39}{9}}\right)$$

$$x = \frac{35}{17} + \frac{5}{9}y + \frac{65}{51}$$

x in terms of y $\quad x = \frac{5}{9}y + \frac{170}{51}$

$$x - \frac{170}{51} = \frac{5}{9}y$$

y in terms of x $\quad \frac{9}{5}x - 6 = y$



Practice Quizzes-→

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

1) $y = 3x + 8$

$y = \frac{-1}{2}x + 15$

2) $y = -2x - 7$

$y = 4x + 11$

3) $y = 5x + 12$

$(y - 4) = 6(x + 1)$

4) $y = -4x + 17$

$(y + 2) = \frac{1}{3}(x - 8)$

5) $2x + 7y = 25$

$3x - y = 3$

6) $2x - 3y = 9$

$y = \frac{2}{3}x - 3$

7) $.3x - .5y = 1$

$y = .2x + 6$

8) $2x + y = 7$

$x = 4$

9) $-3x + 6y = 12$

$\frac{1}{2}x - y = 8$

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

10) $y + 5 = -(6 - x)$
$x = -y + 19$

11) $\frac{1}{2}x - 5y = 9$
$\frac{1}{4}x + 6y = -4$

12) $x + y = 2$
$y = -2(x - 5) + 1$

13) $y - 6 = .2(x + 10)$
$y + 1 = .5(x + 15)$

14) $x = 2y - 12$
$(y - 6) = 4x - 7$

Convert each equation into *slope intercept form*

a) $2x + 6y = 12$

b) $y + 3 = 2(x + 8)$

c) $x + \frac{1}{2}y = 5$

Convert each equation into *standard form*

a) $y = 3x + 5$

b) $y + 1 = \frac{1}{2}(x - 6)$

c) $.2x + .7y = 11$

d) $y = -(x + 7) + 8$

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

SOLUTIONS

1) $y = 3x + 8$

$y = \frac{-1}{2}x + 15$

substitution: $3x + 8 = \frac{-1}{2}x + 15$

for ease, let's get rid of fractions by multiplying both sides by 2

$6x + 16 = -x + 30$

$7x = 14$ $\quad y = 3(2) + 8$

$x = 2$ $\quad y = 14$

$\boxed{(2, 14)}$

**to check: plug (2, 4) into the other equation...

$14 = \frac{-1}{2}(2) + 15$

$14 = -1 + 15$ ✓

2) $y = -2x - 7$

$y = 4x + 11$

substitution: $-2x - 7 = 4x + 11$

$-18 = 6x$

$x = -3$

$y = 4(-3) + 11 = -1$

$\boxed{(-3, -1)}$

elimination: $y = -2x - 7$

$-\ (y = 4x + 11)$

$0 = -6x - 18$

$18 = -6x$ $\quad y = -2(-3) - 7$

$x = -3$ $\quad y = -1$

3) $y = 5x + 12$

$(y - 4) = 6(x + 1)$

substitution: put "y" into 2nd equation

$((5x + 12) - 4) = 6(x + 1)$

$5x + 8 = 6x + 6$

$2 = x$

If $x = 2$, then $y = 5(2) + 12$

$y = 22$

$\boxed{(2, 22)}$

To check: plug solution into the other equation...

$(y - 4) = 6(x + 1)$

$(22 - 4) = 6(2 + 1)$

$18 = 18$ ✓

4) $y = -4x + 17$

$(y + 2) = \frac{1}{3}(x - 8)$

substitution: put the "y" into the 2nd equation

$(-4x + 17) + 2) = \frac{1}{3}(x - 8)$

$-4x + 19 = \frac{1}{3}x - \frac{8}{3}$

for ease, multiply by 3 to get rid of fractons

$-12x + 57 = x - 8$

$65 = 13x$ $\quad y = -4(5) + 17$

$x = 5$ $\quad y = -3$

$\boxed{(5, -3)}$

5) $2x + 7y = 25$

$3x - y = 3$

elimination: (easier because equations are in standard form)

$2x + 7y = 25$

$7\ (3x - y = 3)$

$2x + 7y = 25$

$+\ 21x - 7y = 21$

$23x = 46$

$x = 2$

then, $y = 3$

$\boxed{(2, 3)}$

6) $2x - 3y = 9$

$y = \frac{2}{3}x - 3$

substitution:

$2x - 3(\frac{2}{3}x - 3) = 9$

$2x - 2x + 9 = 9$

$0 = 0$

elimination:

$\boxed{\text{infinite solutions}}$

$2x - 3y = 9$

$y = \frac{2}{3}x - 3$

$3y = 2x - 9$ $\quad$ SAME LINES!

$9 = 2x - 3y$

7) $.3x - .5y = 1$

$y = .2x + 6$

For ease, let's multiply both equations by 10

$3x - 5y = 10$

$10y = 2x + 60$

elimination method:

$(3x - 5y = 10)(2)$

$-2x + 10y = 60$

$6x - 10y = 20$

$4x = 80$

$x = 20$ then, $y = 10$

$\boxed{(20, 10)}$

8) $2x + y = 7$

$x = 4$

substitution:

Just place second equation into first!

$2(4) + y = 7$

$y = -1$ and, of course $x = 4$

$\boxed{(4, -1)}$

9) $-3x + 6y = 12$

$\frac{1}{2}x - y = 8$

elimination:

first, multiply second equation by 6...

$-3x + 6y = 12$

$3x - 6y = 48$

$0 + 0 = 60$

PARALLEL LINES!!

$\boxed{\text{No real Solution}}$

Solve the following linear systems (using elimination/combination or substitution method)

SOLUTIONS

Systems of Linear Equations

10) $y + 5 = -(6 - x)$

$x = -y + 19$

substitution:

$y + 5 = -(6 - (-y + 19))$

$y + 5 = -(-13 + y)$

$2y = 8$

$y = 4$

If $y = 4$, then $x = -(4) + 19$

$x = 15$

$\boxed{(15, 4)}$

To check, plug solution into BOTH equations:

$(4) + 5 = -(6 - (15))$

$9 = 9$ ✓

11) $\frac{1}{2}x - 5y = 9$

$\frac{1}{4}x + 6y = -4$

elimination:

For ease, I'll get rid of the fractions..
(multiply 1st by 2; multiply 2nd by 4)

$x - 10y = 18$

$-\quad x + 24y = -16$

$-34y = 34$

$y = -1$

if $y = -1$, then using substitution, we can see $x = 8$

$\boxed{(8, -1)}$

12) $x + y = 2$

$y = -2(x - 5) + 1$

substitution:

since y is isolated in the second equation, we'll substitute it into the first...

$x + -2(x - 5) + 1 = 2$

$x - 2x + 10 + 1 = 2$

$9 = x$

so, $y = -7$

$\boxed{(9, -7)}$

To check: plug solution into 2nd equation

$(-7) = -2((9) - 5) + 1$

$-7 = -2(4) + 1$

$-7 = -7$ ✓

13) $y - 6 = .2(x + 10)$

$y + 1 = .5(x + 15)$

First, multiply both equations by 10

$10y - 60 = 2x + 20$

$10y + 10 = 5x + 75$

Since y coefficient are the same, we'll use elimination:

$10y - 60 = 2x + 20$

$-\ (10y + 10 = 5x + 75)$

$0y - 70 = -3x - 55$

$3x = 15$

$x = 5$

then, with substitution, $y = 9$

$\boxed{(5, 9)}$

14) $x = 2y - 12$

$(y - 6) = 4x - 7$

since x is by itself in the 1st equation...

substitution:

$(y - 6) = 4(2y - 12) - 7$

$y - 6 = 8y - 48 - 7$

$49 = 7y$

$y = 7$ then, $x = 2$

$\boxed{(2, 7)}$

Convert each equation into *slope intercept form*

$y = mx + b$

a) $2x + 6y = 12$

$6y = 12 - 2x$

$\boxed{y = -\frac{1}{3}x + 2}$

b) $y + 3 = 2(x + 8)$

$y + 3 = 2x + 16$

$\boxed{y = 2x + 13}$

c) $x + \frac{1}{2}y = 5$

$\frac{1}{2}y = -x + 5$

$\boxed{y = -2x + 10}$

Convert each equation into *standard form*

$Ax + By = C$ where A is positive integer (and B and C are integers)

a) $y = 3x + 5$

$-3x + y = 5$

$\boxed{3x - y = -5}$

b) $y + 1 = \frac{1}{2}(x - 6)$

$2y + 2 = (x - 6)$

$-x + 2y = -8$

$\boxed{x - 2y = 8}$

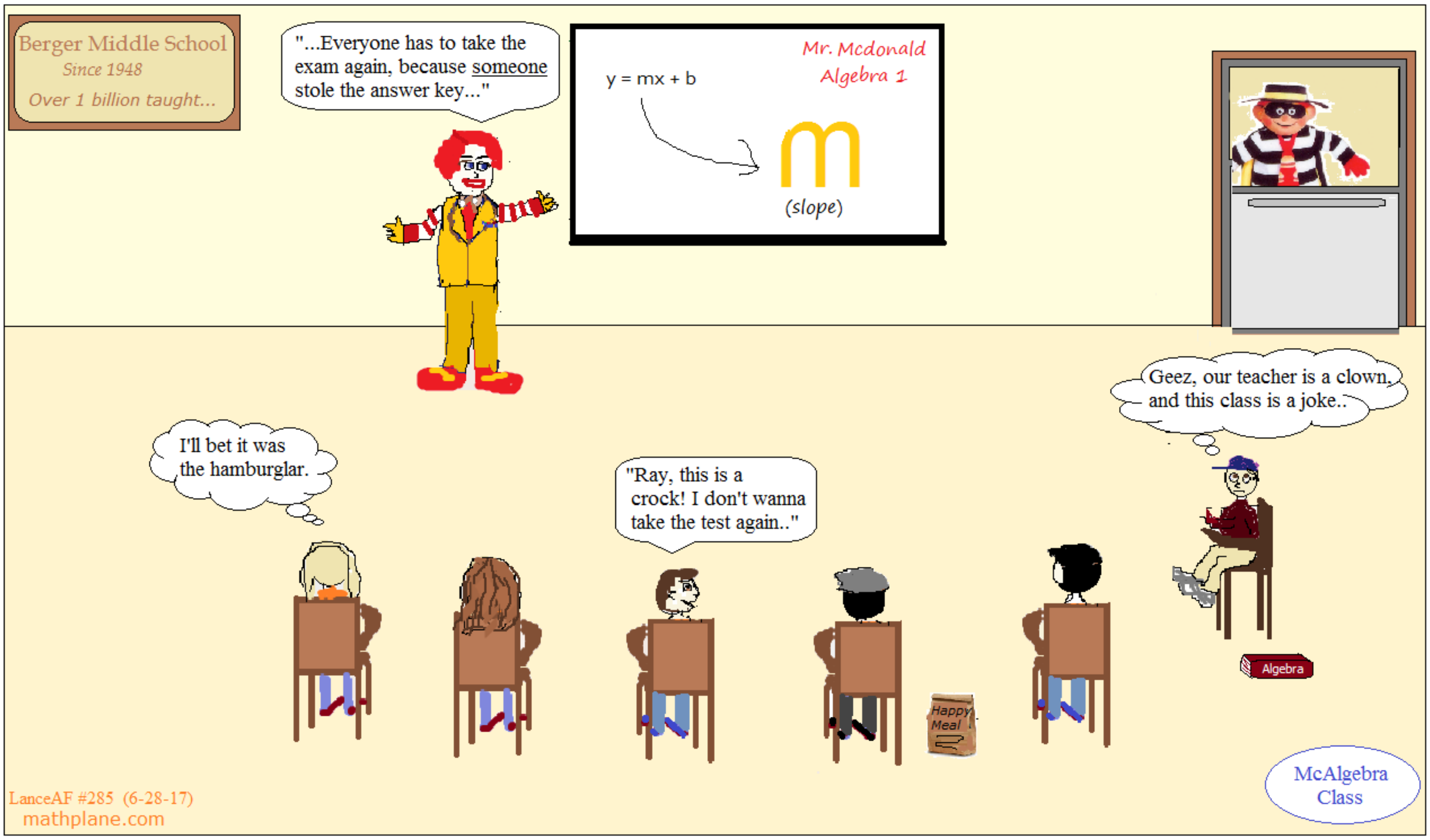
c) $.2x + .7y = 11$

$\boxed{2x + 7y = 110}$

d) $y = -(x + 7) + 8$

$y = -x - 7 + 8$

$\boxed{x + y = 1}$

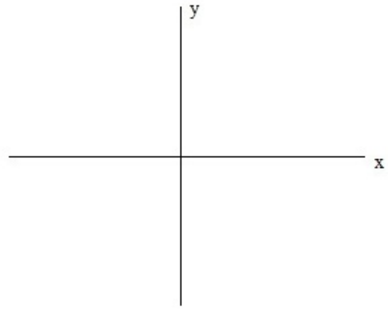


More Practice questions→

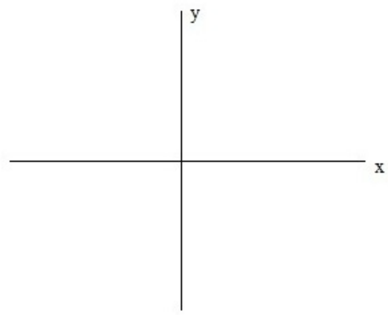
Linear Systems Quick Quiz

Solve and Graph the following Systems:

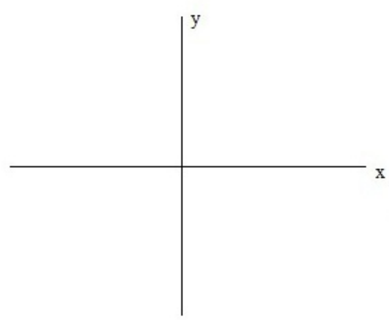
1) $3x + y = 9$
$y = -2x + 4$



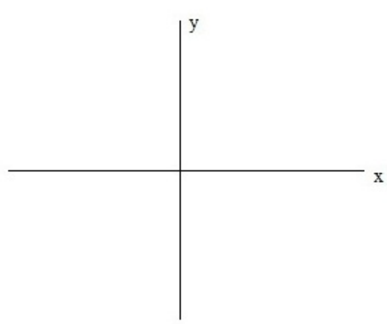
2) $y = 6$
$2x - 3y = 4$



3) $y = \frac{1}{2}x - \frac{1}{4}$
$2x + 4y = 1$



4) $x = 5$
$y = 6$



Linear Systems Quick Quiz

Answer the following:

1) Jim bought 65 cupcakes and cookies for his birthday party. Each cupcake cost $1 and each cookies cost 75 cents.
   If he paid $57.50 for the treats, how many of each did he buy?

2) A high school play has 2 freshmen, 5 sophomores, and 11 juniors.
   If 1/3 of the cast is composed of seniors, how many seniors are in the play?

3) A movie theater charges 9 dollars for adults, 5 dollars for kids, and 3 dollars for seniors.
   Last month, the theater sold 9,500 tickets and generated $57,920.
   If the theater sold twice as many tickets to kids as seniors,
   how many of each ticket did the theater sell?

4) A bartender wants to make 81 ounces of a 20% cranberry drink mix.
How much pure cranberry juice should he mix with a 10% cranberry blend?

Linear Systems Quick Quiz

5) Solve the following linear system:

$$9x + 9y + 4z = -56$$
$$-4x - 4y + z = 11$$
$$x + y + z = -9$$

Linear Systems Quick Quiz

SOLUTIONS

Solve and Graph the following Systems:

1) $3x + y = 9$
$y = -2x + 4$

$\boxed{\text{solution is } (5, -6)}$

(substitution method)

$3x + (-2x + 4) = 9$
$x = 5$

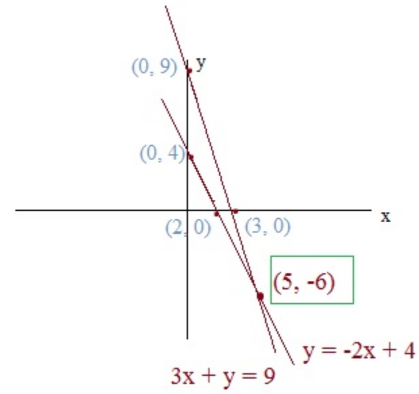
substitute 2nd equation into 1st. solve.

$3(5) + y = 9$
$y = -6$

place x value into one of the equations to get y.

check: $(-6) = -2(5) + 4$
$-6 = -6$

check solution in other equation.



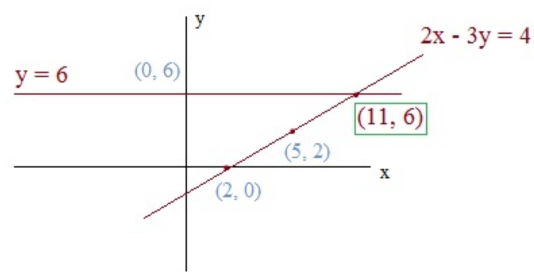
2) $y = 6$
$2x - 3y = 4$

(substitution/combine the equations)

$2x - 3(6) = 4$
$2x = 22$
$x = 11$

$\boxed{\text{solution is } (11, 6)}$

and, $y = 6$



3) $y = \frac{1}{2}x - \frac{1}{4}$ equation 1

$2x + 4y = 1$ equation 2

(combination/elimination method)

$4y = 2x - 1$ 1

$-2x + 4y = -1$ 1

Re-write equation 2.
Then, combine with equation 1.

$2x + 4y = 1$ 2

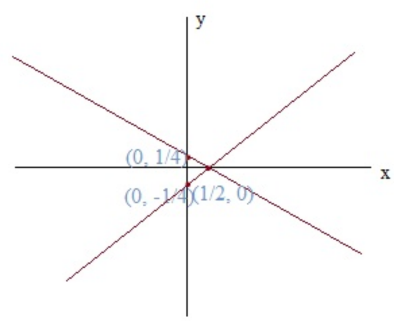
$8y = 0$ 1 + 2

$\boxed{\text{intersection at } (1/2, 0)}$

$y = 0$ solution

$2x + 4(0) = 1$
$2x = 1$
$x = 1/2$

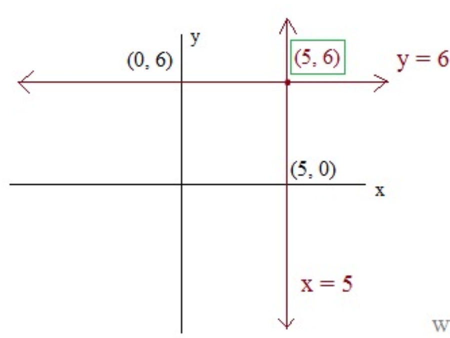
place y = 0 into the second equation to get x



4) $x = 5$
$y = 6$

Graph it first, and you'll see the solution!

The lines intersect at (5, 6)



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Answer the following: SOLUTIONS

Linear Systems Quick Quiz

1) Jim bought 65 cupcakes and cookies for his birthday party. Each cupcake cost $1 and each cookies cost 75 cents. If he paid $57.50 for the treats, how many of each did he buy?

Let CP = number of cupcakes
CK = number of cookies

$CP + CK = 65$

$\$1(CP) + \$.75(CK) = \$57.50$

Using substitution method:

$CP = 65 - CK$

then,

$\$1(65 - CK) + \$.75(CK) = \$57.50$

$65 - 1CK + .75CK = 57.50$

$-.25CK = -7.50$

$CK = 30$

and, $CP = 35$

$\boxed{\text{30 cookies, 35 cupcakes}}$

quick check: 30 cookies will cost $22.50; and, 35 cupcakes will cost $35.00; total: $57.50 ✓

2) A high school play has 2 freshmen, 5 sophomores, and 11 juniors. If 1/3 of the cast is composed of seniors, how many seniors are in the play?

2 + 5 + 11 = 18 non seniors

S = # of seniors
C = # of cast members

seniors + non seniors = total cast

S + 18 = C

1/3(C) = S

1/3(C) + 18 = C
2/3C = 18
2C = 54
C = 27

$\boxed{S = 9}$

3) A movie theater charges 9 dollars for adults, 5 dollars for kids, and 3 dollars for seniors. Last month, the theater sold 9,500 tickets and generated $57,920. If the theater sold twice as many tickets to kids as seniors, how many of each ticket did the theater sell?

A = # of adult tickets
K = # of kid tickets
S = # of senior tickets

1) A + K + S = 9500

2) $9(A) + $5(K) + $3(S) = $57,920

3) K = 2S

3 equations, 3 unknowns...

Using substitution:

A + (2S) + S = 9500 — 3rd equation into 1st equation

9A + 5(2S) + 3S = 57920 — 3rd equation into 2nd equation

Then, combine these two equations:

A + 3S = 9500

9A + 13S = 57,920

-9A - 27S = -85,500

-14S = -27,580

$\boxed{S = 1970}$

Since K = 2S,

K = 2(1970)

$\boxed{K = 3940}$

And, since

A + K + S = 9500

A + 3940 + 1970 = 9500

$\boxed{A = 3590}$

Quick Check:

3590 + 3940 + 1970 = 9500 ✓

Seniors: 1970
Kids : 3940

twice as many kids ✓

$9 x 3590 tix = $32,310

$5 x 3940 tix = $19,700

$3 x 1970 tix = $5,910

total sales: $57,920 ✓

Linear Systems Quick Quiz

4) A bartender wants to make 81 ounces of a 20% cranberry drink mix.
How much pure cranberry juice should he mix with a 10% cranberry blend?

After reading the quesiton, we can see it is a 'mixture' problem.

Method 1: Using a chart

Step 1: Identify the "core item" -- in this case, it is "cranberry"

Step 2: Fill in the chart

	quantity	%	amount of cranberry
10% blend	X	.10	.10X
pure juice	Y or, 81 - X	1.00	1.00(81 - X)
20% desired mix	81	.20	.20(81) or, 16.2

Step 3: Using column 3, solve the equation....

10% blend   pure   20% amount

.10X + (81 - X) = 16.2

-.9X = -64.8

X = 72

9 ounces of pure cranberry

72 ounces of 10% mix

Method 2: System of linear equations

Let X = amount of 10% blend
Y = amount of pure cranberry

first equation: amounts   X + Y = 81

second equation: concentration of cranberry

.10X + 1.00(Y) = .20(81)

Then, solve...

X + Y = 81

.10X + Y = 16.2

Elimination method

X + Y = 81

— .10X + Y = 16.2

.90X = 64.8

X = 72   then, Y = 9

5) Solve the following linear system:

1   9x + 9y + 4z = -56
2   - 4x - 4y + z = 11
3   x + y + z = -9

Using elimination method:

multiply row 2 by -1 and add to row 3

- 4x - 4y + z = 11
x + y + z = -9

multiply row 3 by -4 and add to row 1

9x + 9y + 4z = -56
x + y + z = -9

combined 2nd and 3rd rows:

5x + 5y = -20

combined 1st and 3rd rows:

5x + 5y = -20

The result is 2 identical lines...
----> dependent system....

Using the combined equation 5x + 5y = -20

x + y = -4

y = -x - 4

Then, using the 3rd equation x + y + z = -9

z = -9 − x − y

z = -9 − x − (-x - 4)

z = -5

Therefore, the solution (in terms of x) is (x, -x - 4 , -5)

To check our answer, pick out an x value:

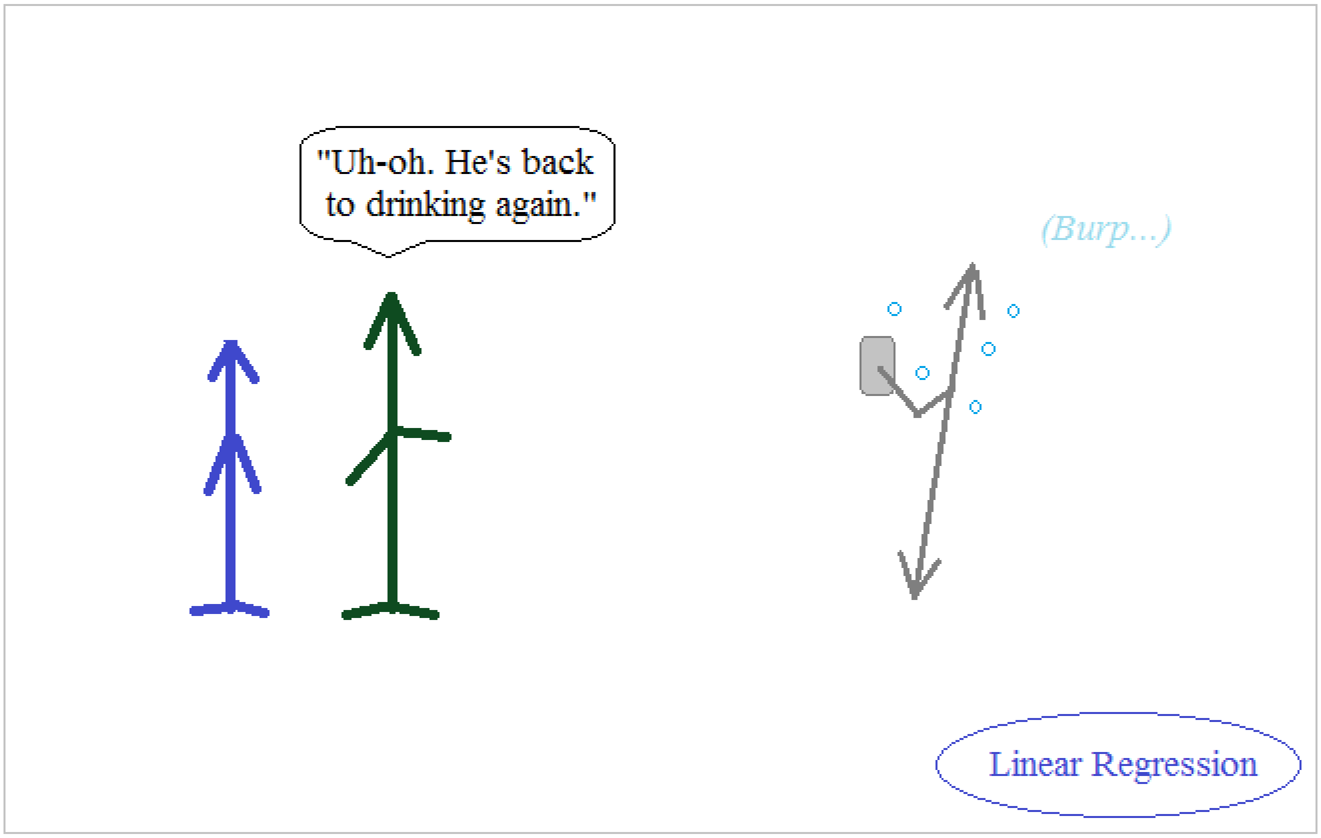
if x = 3, then, solution is (3, -7, -5)

9(3) + 9(-7) + 4(-5) = 27 - 63 - 20 = -56

-4(3) - 4(-7) + (-5) = -12 + 28 - 5 = 11

(3) + (-7) + (-5) = -9

No longer on the straight and narrow?



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Another Practice Test→

Linear Systems Test 2

Part I: Solving Systems

Use Substitution (and show your work)

1) $y = 3x + 10$
$2x + 3y = -3$

2) $y = 2x - 4$
$3x - y = 9$

Use Elimination (Combination) Method (and show your work)

3) $3x + 7y = 1$
$6x - 5y = -17$

4) $x + 3y = 6$
$3x - y = -12$

Use Any Method

5) $y = 4$
$3x + 5y = 8$

6) $\frac{2}{3}x - y = 4$
$y = 2x - 12$

7) $y = -3x + 10$
$3x + y = 15$

Part II: Graphing

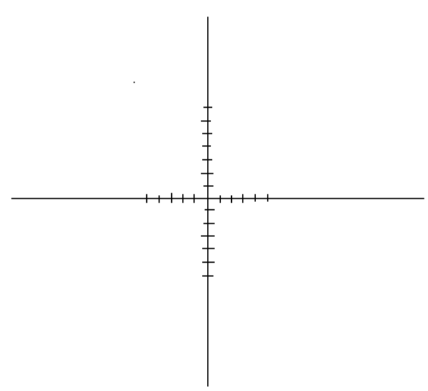
Graph the following: $3x + 5y = 15$

What is the x-intercept?

y-intercept?

slope?

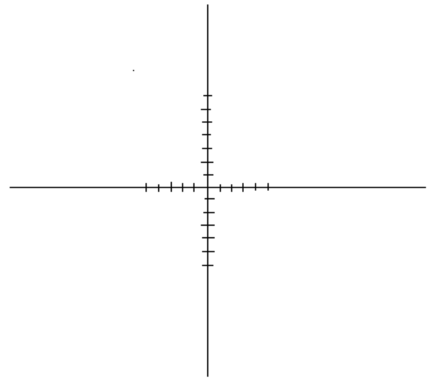
Is (20, -8) a point on this line?



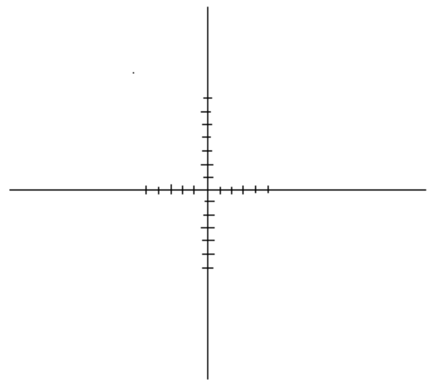
Part III: Graph and Solve

Graph each system. Then, identify the solutions on the graphs.

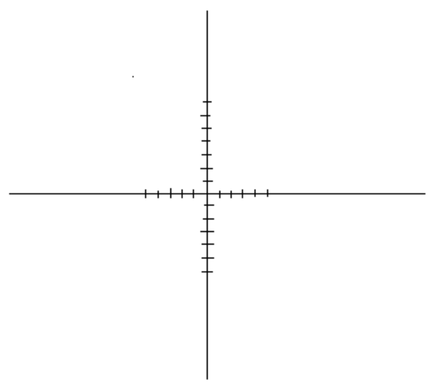
$3x + 2y = 15$
$y = 2x + 4$



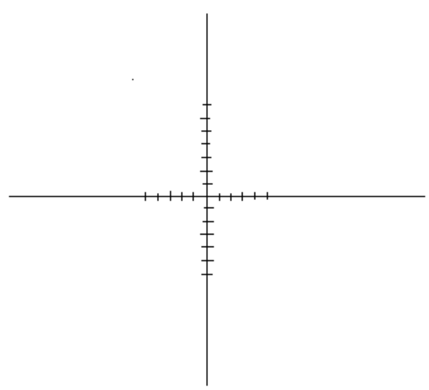
$y < 2x + 4$
$2x - y \leq 4$



$y \leq 3x + 5$
$6x + 2y > -6$



$y = -5$
$x - 6y = 13$



Part IV: Word Problems

Solve the Linear Systems. (Label the variables and show your work.)

1) A movie theater charges \$2.50 for kids and \$4.00 for adults. Last Friday, 260 people attended the show. If the theater collected \$782, how many of the viewers were adults?

2) At the movie, Lance wants to buy popcorn and candy for himself and four friends. Popcorn cost \$2 and Candy cost \$1.
If he wants to spend less than \$20 and needs to get at least one treat per person, graph a system that describes all the possible combinations of popcorn and candy he can buy.

3) There is a cafe next to the movie theater. The daily costs for the cafe are \$200 plus \$2 per order. If each customer pays \$5 per order, how many daily customers does the cafe need to make a profit?
(Show your solutions algebraically AND graphically)

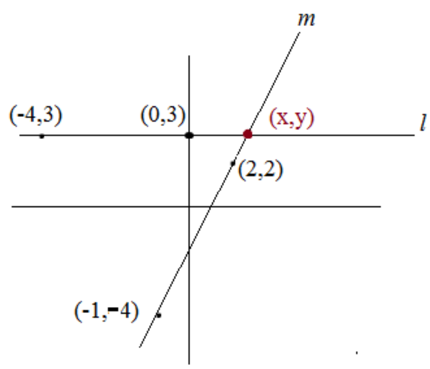
Part V: Miscellaneous Concepts

1) Describe the linear system and solve.

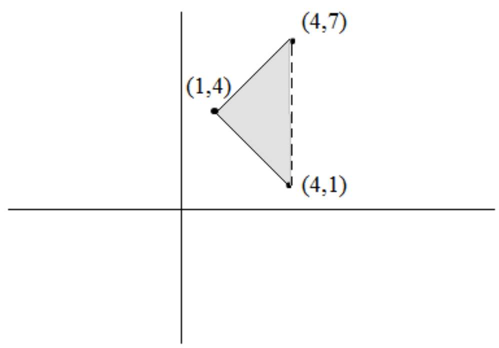
l:

m:

(x, y) = ?



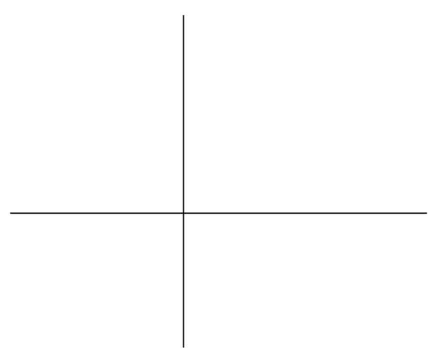
2) Describe the system:

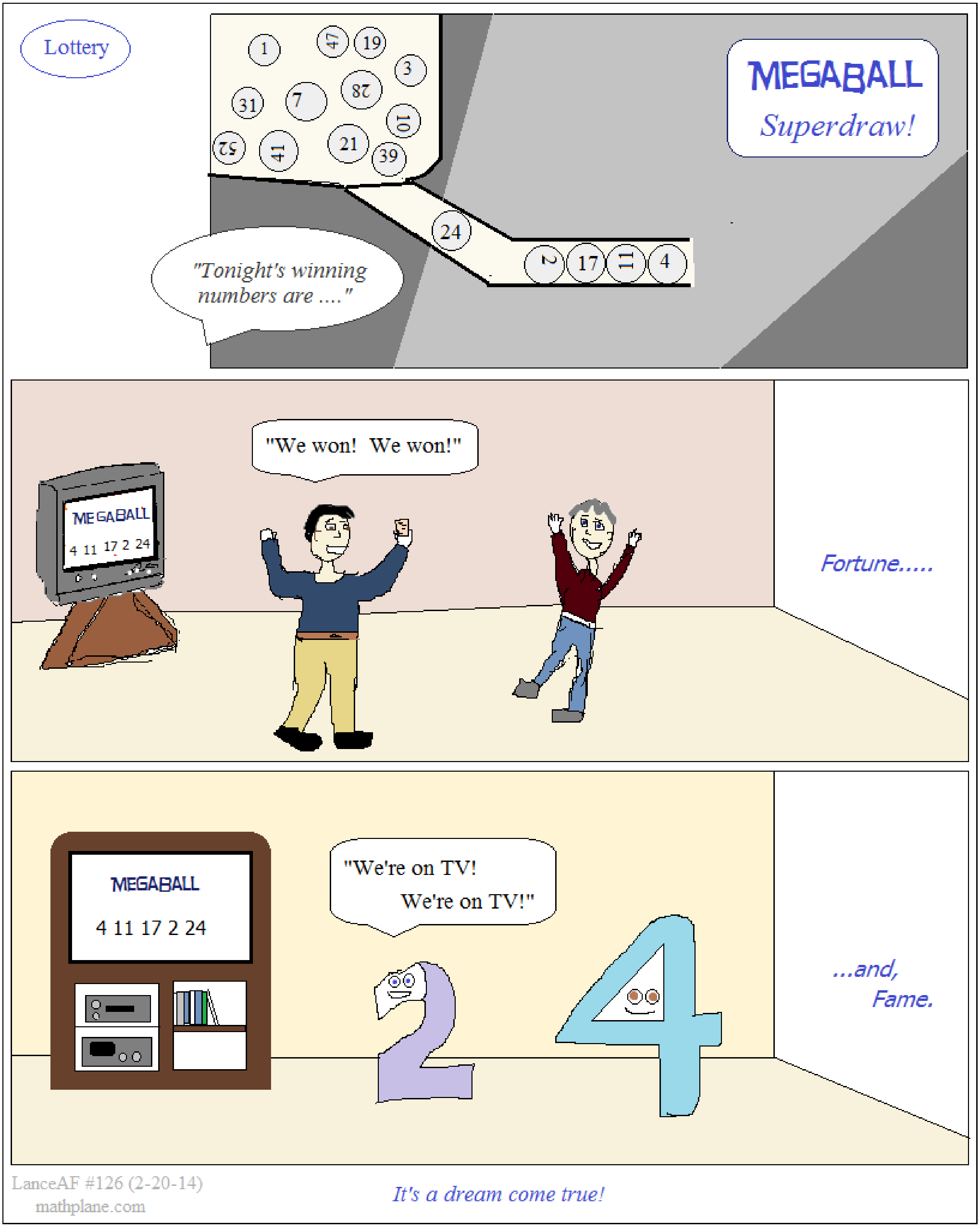


3) Graph and write the linear equation (in standard form):

The x-intercept is (8, 0)

y-intercept is (0, 5)




Lottery
MEGABALL
Superdraw!
"Tonight's winning numbers are ...."
24
4 11 17 2
"We won! We won!"
MEGABALL
4 11 17 2 24
Fortune.....
"We're on TV!
We're on TV!"
MEGABALL
4 11 17 2 24
...and,
Fame.
It's a dream come true!
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Linear Systems Test 2

Solutions

Part I: Solving Systems

Use Substitution (and show your work)

1) $y = 3x + 10$
$2x + 3y = -3$

(substitute y into 2nd equation)
$2x + 3(3x + 10) = -3$
$2x + 9x + 30 = -3$
$11x = -33$
$x = -3$

$\boxed{(-3, 1)}$

(put x into 1st equation)
$y = 3(-3) + 10$
$y = 1$

(check solution!)
$2(-3) + 3(1) = -3$
$-6 + 3 = -3$
$-3 = -3$ ✓

2) $y = 2x - 4$
$3x - y = 9$

$\boxed{(5, 6)}$

(substitute y into 2nd equation)
$3x - (2x - 4) = 9$
$3x - 2x + 4 = 9$
$x = 5$

(put x into 2nd equation)
$3(5) - y = 9$
$15 - y = 9$
$y = 6$

(check solutions)
$(6) = 2(5) - 4$
$6 = 10 - 4$
$6 = 6$ ✓

$3(5) - (6) = 9$
$15 - 6 = 9$
$9 = 9$ ✓

Use Elimination (Combination) Method (and show your work)

3) $3x + 7y = 1$
$6x - 5y = -17$

(multiply top by -2) $-6x - 14y = -2$

(combine equations)
$-6x - 14y = -2$
$6x - 5y = -17$
$-19y = -19$
$y = 1$

$\boxed{(-2, 1)}$

(plug y into top equation)
$3x + 7(1) = 1$
$3x = -6$
$x = -2$

(Check bottom equation)
$6(-2) - 5(1) = -17$
$-12 - 5 = -17$
$-17 = -17$ ✓

4) $x + 3y = 6$
$3x - y = -12$

(multiply bottom by 3) $9x - 3y = -36$

(combine equations)
$x + 3y = 6$
$9x - 3y = -36$
$10x = -30$
$x = -3$

$\boxed{(-3, 3)}$

(plug x into top equation)
$(-3) + 3y = 6$
$3y = 9$
$y = 3$

(check bottom equation)
$3(-3) - (3) = -12$
$-9 - 3 = -12$
$-12 = -12$ ✓

Use Any Method

5) $y = 4$
$3x + 5y = 8$

(easily plug top equation into bottom)

$3x + 5(4) = 8$
$3x + 20 = 8$
$x = -4$

$\boxed{(-4, 4)}$

and, obviously $y = 4$

6) $\frac{2}{3}x - y = 4$
$y = 2x - 12$

(rewrite top equation) $y = \frac{2}{3}x - 4$

(set equations equal to each other/substituting y) $\frac{2}{3}x - 4 = 2x - 12$
$8 = \frac{4}{3}x$
$x = 6$

(plug x into top) $\frac{2}{3}(6) - y = 4$
$4 - y = 4$
$y = 0$

$\boxed{(6, 0)}$

7) $y = -3x + 10$
$3x + y = 15$

(rewrite bottom equation)
$y = -3x + 15$

(compare equations!)
$y = -3x + 15$
$y = -3x + 10$

Same slope, different intercepts!
Parallel lines

$\boxed{\text{NO SOLUTION}}$

Part II: Graphing

Graph the following: $3x + 5y = 15$

What is the x-intercept? $\boxed{(5, 0)}$

$3x + 5(0) = 15$
$x = 5$

y-intercept? $\boxed{(0, 3)}$

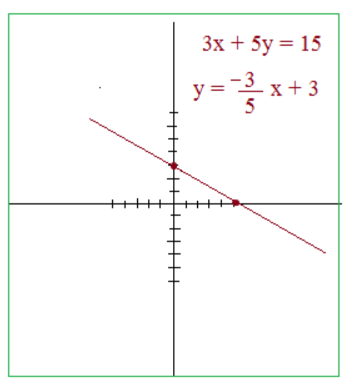
$3(0) + 5y = 15$
$y = 3$

slope? $\boxed{\frac{-3}{5}}$

$m = \frac{0 - 3}{5 - 0} = \frac{-3}{5}$

Is (20, -8) a point on this line? $\boxed{\text{NO}}$

(plug in the point!)
$3(20) + 5(-8) = 15$
$60 - 40 = 15$
$20 \neq 15$



Part III: Graph and Solve

Solutions

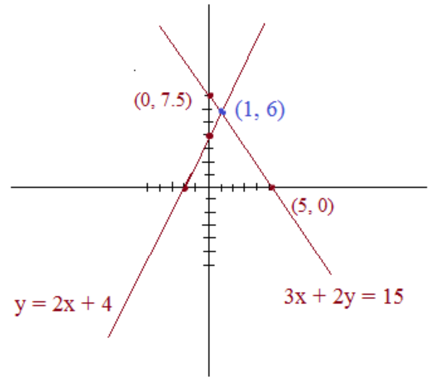
Graph each system. Then, identify the solutions on the graphs.

$3x + 2y = 15$
$y = 2x + 4$

Use substitution method to verify solution:

$3x + 2(2x + 4) = 15$
$3x + 4x + 8 = 15$
$7x = 7$
$x = 1$
$3(1) + 2y = 15$
$3 + 2y = 15$
$2y = 12$
$y = 6$

$\boxed{(1, 6)}$

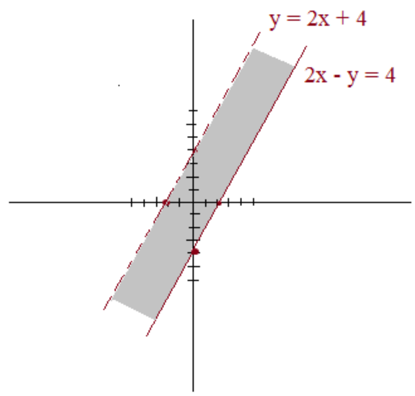


$y < 2x + 4$
$2x - y \le 4$

Notice, these are parallel lines!

draw the line $y = 2x + 4$
since it is $<$, it is a slashed line..
then, test (0, 0)
$(0) < 2(0) + 4$
$0 < 4$ yes.
Region below the line that includes (0, 0) is shaded!

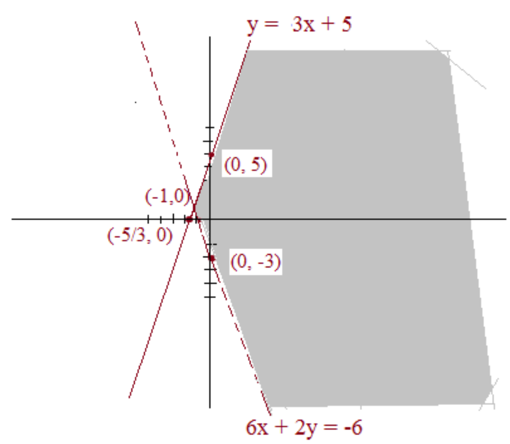
Then,
draw line $2x - y = 4$
or $y = 2x - 4$
since it is $\le$, it is a solid line.
then, test (0, 0)
$2(0) - (0) \le 4$ yes.
Region above the line that includes (0, 0) is shaded..



$y \le 3x + 5$
$6x + 2y > -6$

First, graph the top equation by identifying the y-intercept and x-intercept. Then, draw a line that goes through both. (since it is $\le$, the line is solid)
then, test (0, 0)
$0 \le 0 + 5$ yes! The area under the line may be shaded.

Then, graph the second equation by drawing line through intercepts. Then, the line is dashed (because it is $>$)
Test (0, 0):
$6(0) + 2(0) > -6$
$0 > -6$ yes... Area above the dashed line may be shaded.

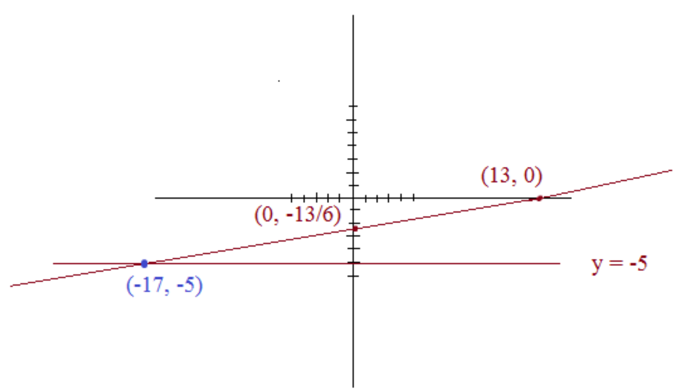


$y = -5$ (horizontal line)
$x - 6y = 13$

$x - 6(-5) = 13$

$x + 30 = 13$

$x = -17$



Part IV: Word Problems

Solutions

Solve the Linear Systems. (Label the variables and show your work.)

1) A movie theater charges \$2.50 for kids and \$4.00 for adults. Last Friday, 260 people attended the show. If the theater collected \$782, how many of the viewers were adults?

Let K = # of kids $2.5 per kid
A = # of adults $4.0 per adult

$2.5K + $4.0A = $782
or
$2.5K + 4A = 782$

$A + K = 260$

Use elimination method to find A and K:

$$\begin{array}{rl} & 2.5K + 4A = 782 \\ - & 4K + 4A = 1040 \\ \hline & -1.5K = -258 \\ & K = 172 \end{array}$$

$A + 172 = 260$
$A = 88$

(Check Solution)

\$2.50 x 172 = \$430

\$4.00 x 88 = \$352

\$782 total!

** Now, answer the question: How many viewers were adults? 88

2) At the movie, Lance wants to buy popcorn and candy <u>for himself and four friends</u>.
Popcorn cost \$2 and Candy cost \$1.
If he wants to spend less than \$20 and needs to get at least one treat per person, graph a system that describes all the possible combinations of popcorn and candy he can buy.

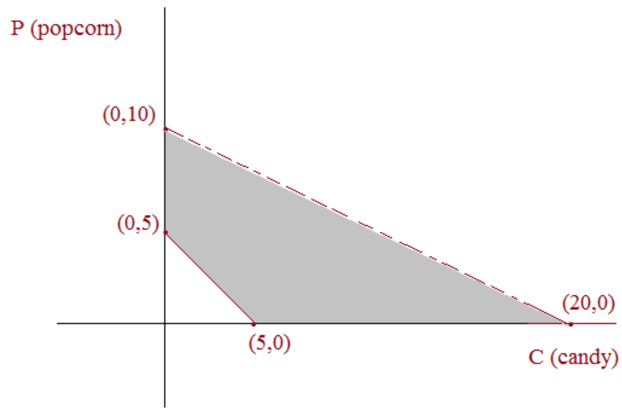
Let P = # of Popcorn
C = # of Candy

Price of candy = \$1
Price of popcorn = \$2

$\$1C + \$2P < \$20$ (cost constraint)

$P + C \geq 5$ (quantity constraint)

Any combination of popcorn and candy in the gray region would satisfy the cost constraint (< \$20) and satisfy the quantity constraint (everyone gets at least one treat).



3) There is a cafe next to the movie theater. The daily costs for the cafe are \$200 plus \$2 per order. If each customer pays \$5 per order, how many daily customers does the cafe need to make a profit?
(Show your solutions algebraically AND graphically)

Let X = # of customers

Cafe Costs = \$200 + \$2X

Cafe Revenues = \$5X

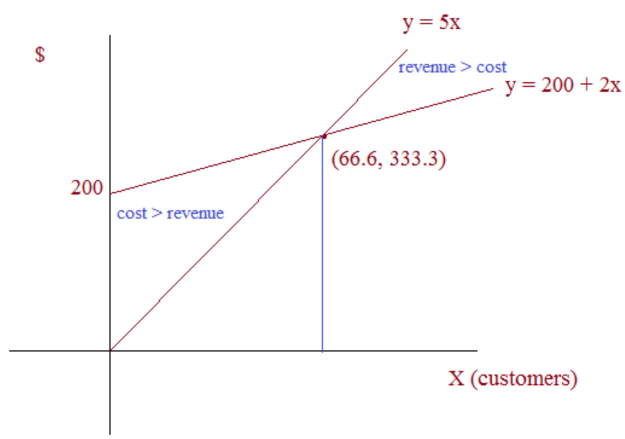
Let's find where revenue > cost...

$C(x) = 200 + 2x$
$R(x) = 5x$

Where does revenue = cost?

$5x = 200 + 2x$
$3x = 200$
$x = 66.6$

Since you can't have "fractional customers", the cafe must have 67 or more customers to make a profit.



Part V: Miscellaneous Concepts

Solutions

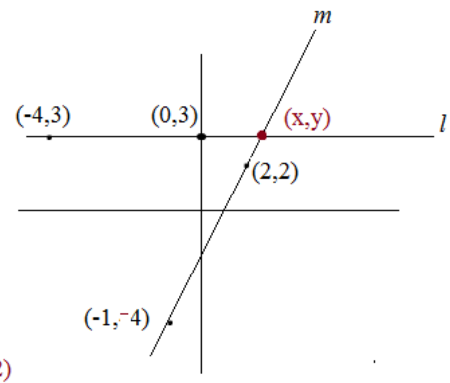
1) Describe the linear system and solve.

l: (horizontal line) $\boxed{y = 3}$

m: slope is 6/3 = 2

so line in point slope form is:

$\boxed{(y - 2) = 2(x - 2)}$



(x, y) = ?

$\boxed{\left(\frac{5}{2}, 3\right)}$

l and m intersect at $(3 - 2) = 2(x - 2)$

$1 = 2x - 4$

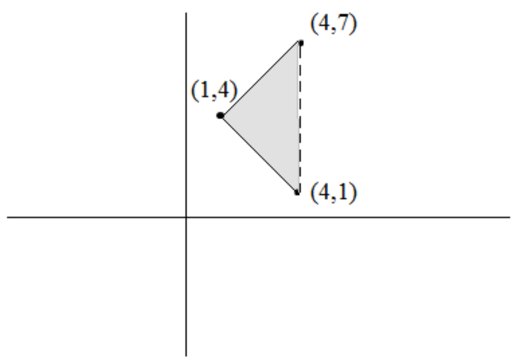
$x = 5/2$

2) Describe the system:

$x < 4$

$y \leq x + 3$

$y \geq -x + 5$



3) Graph and write the linear equation (in standard form):

The x-intercept is (8, 0)

y-intercept is (0, 5)

find slope:

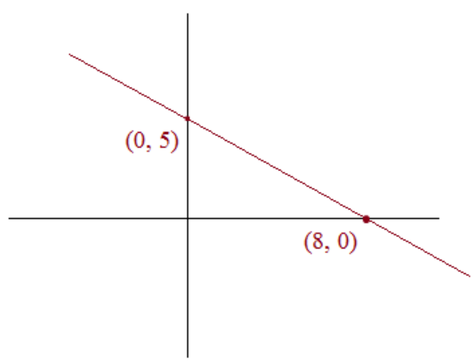
$$m = \frac{5 - 0}{0 - 8} = \frac{-5}{8}$$

$$y - 0 = \frac{-5}{8}(x - 8)$$

$$y = \frac{-5}{8}x + 5$$

$$8y = -5x + 40$$

$$\boxed{5x + 8y = 40}$$

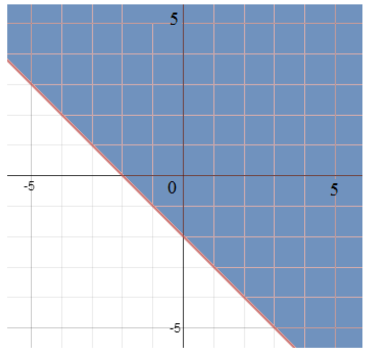


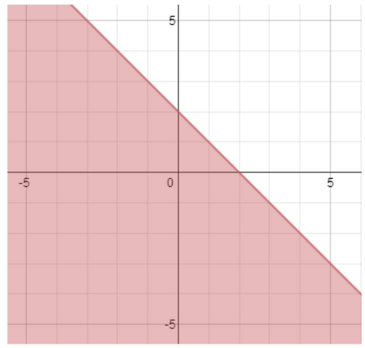
Example: Graph $|x + y| \leq 2$

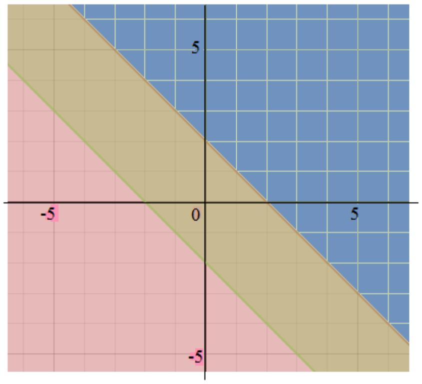
Graph $x + y \leq 2$

Graph $x + y \geq -2$

The intersection is the solution..



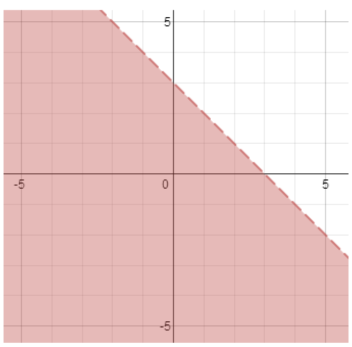




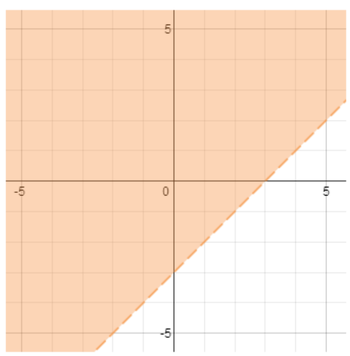
Example: Graph $|x| + |y| < 3$

Graph all 4 possibilities...

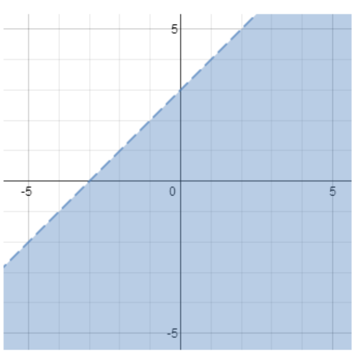
$x + y < 3$



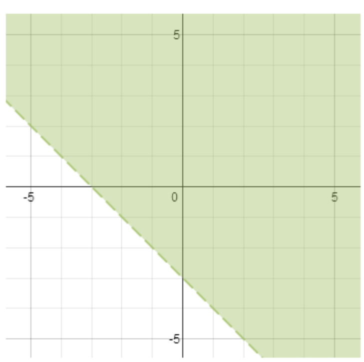
$x + (-y) < 3$

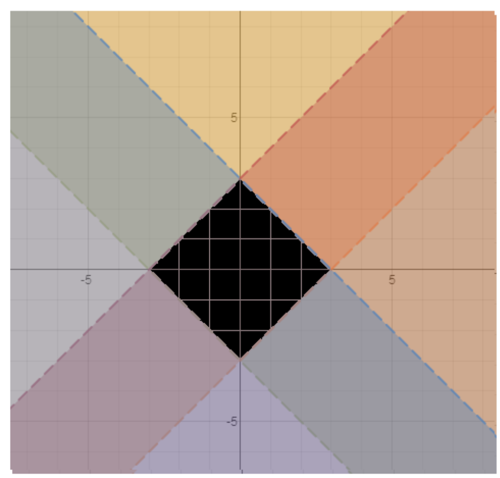


$(-x) + y < 3$



$(-x) + (-y) < 3$





NOTE: To check answer, test points in each region..

(0, 0): $|0| + |0| < 3$ ✓

(5, 0): $|5| + |0| < 3$ ✗

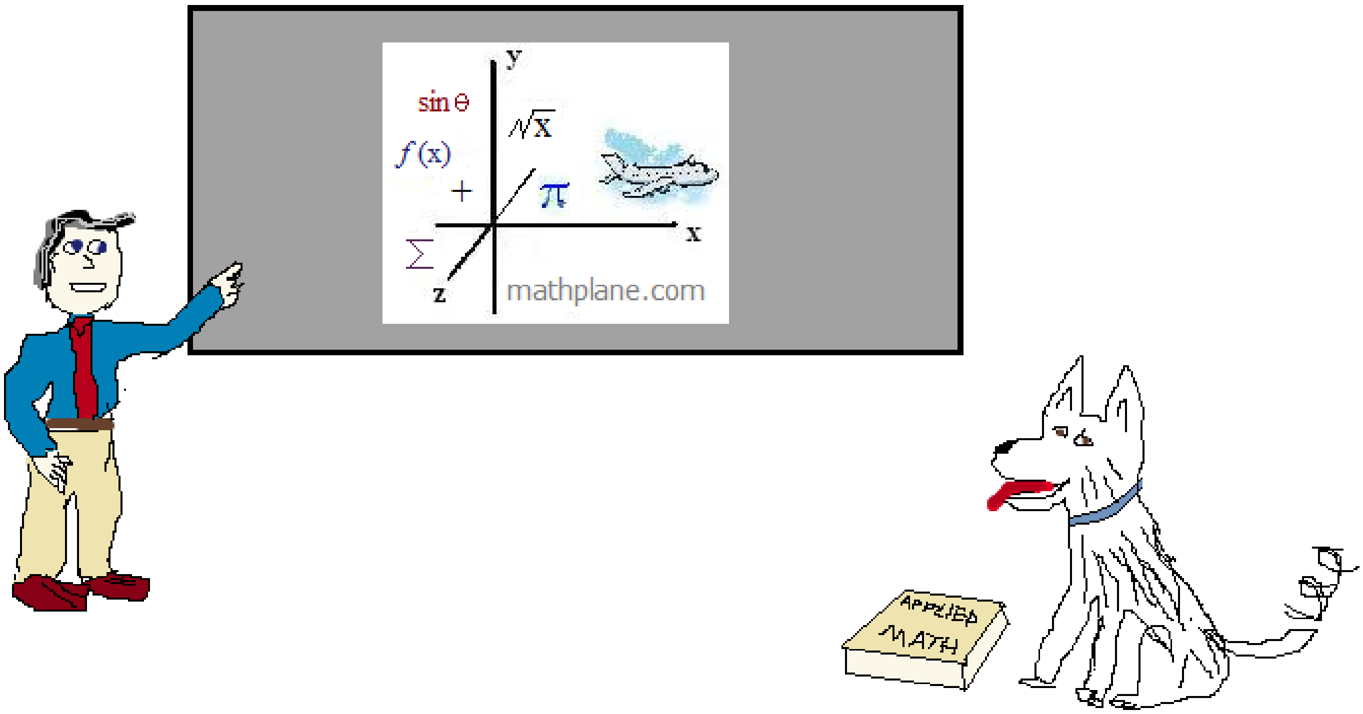
(-1, -1): $|-1| + |-1| < 3$ ✓

(-3, 4): $|-3| + |4| < 3$ ✗

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Good luck!



Also, at TeachersPayTeachers

And, Mathplane *Express* for mobile at Mathplane.ORG

One more question:

Can you graph this linear system?

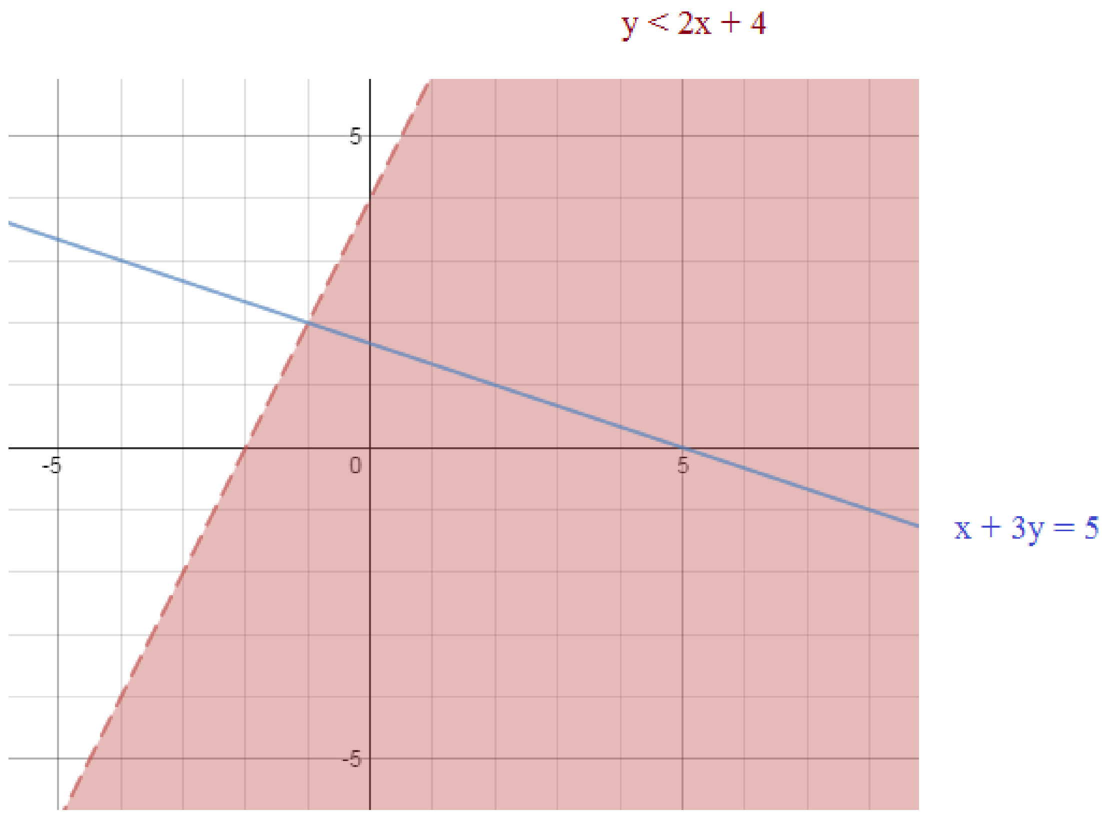
$y < 2x + 4$

$x + 3y = 5$

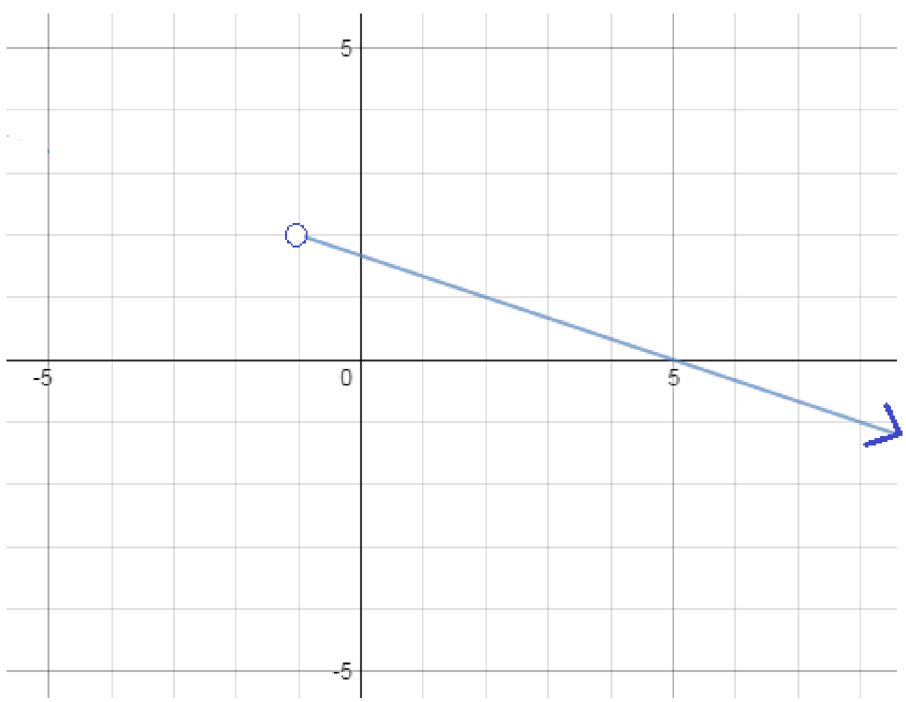
Solution on next page-→

Graph the following system: $y < 2x + 4$

$x + 3y = 5$



The solution set must satisfy both equations!



The solution set is $x + 3y = 5$ on the interval $(-1, \infty)$