Logic and Reasoning

Notes and Exercises (with Solutions)

What is the number?
If the number is divisible by 2, then it is between 50 and 59...
If the number is NOT divisible by 3, then it is between 60 and 69...
If the number is NOT divisible by 4, then it is between 70 and 79...

(ANSWER at the end!)

Topics include truth tables, conditional statements, Venn and Euler Diagrams, and more.

Mathplane.com

Conditional Statements:
Quick Summary

Writing a conditional statement   If ________ , then ________ .   If: the hypothesis
then: the conclusion

"Fill in the blanks!"

Example: 20°F is a cold day.

If - then conditional true statement.

If it is 20 degrees Fahrenheit, then it is a cold day.

Converse: If it is a cold day, then it is 20 degrees Fahrenheit.

("Reverse the order") FALSE (It might be true, but there are exceptions.)

Counterexample -- It is a cold day, but it is 25 degrees (instead of 20)

Inverse: If it is not 20 degrees Fahrenheit, then it is not a cold day.

("Add 'not' ... 'not' to the statement") FALSE (The hypothesis is not valid, but the conclusion may still be OK)

Counterexample -- It is not 20 degrees, but it still might be a cold day!

Contrapositive: If it is not a cold day, then it is not 20 degrees Fahrenheit.

("Reverse the 'not' ... 'not' statement") TRUE (If the original statement is true, then the contrapositive is true!)

If P , then Q...

$P \Rightarrow Q$	Conditional Statement
$Q \Rightarrow P$	Converse
$\neg P \Rightarrow \neg Q$	Inverse
$\neg Q \Rightarrow \neg P$	Contrapositive

$P \Leftrightarrow Q$ Equivalent
Biconditional Statement

"Biconditional" Statements are "if and only if"
"one-to-one" statements. "Equivalent Statements"... So, the converse is TRUE.

Example: All 90 degree angles are right angles.

Conditional statement:	If an angle is 90 degrees, then it is a right angle. (hypothesis / conclusion)	TRUE
Converse:	If an angle is right, then it is 90 degrees.	TRUE
Inverse:	If an angle is NOT 90 degrees, then it is NOT a right angle.	TRUE
Contrapositive:	If an angle is not right, then it is not 90 degrees...	TRUE

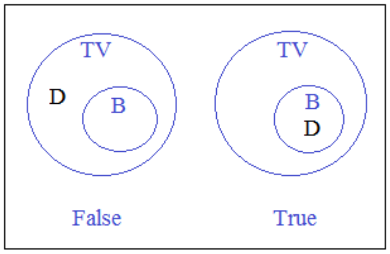
An angle is right *if and only if* it is 90 degrees.
(1-to-1 statement)

Valid or Invalid? Using truth statements and Euler diagrams.

Example: If you love baseball, then you watch baseball on TV.
Dean watches baseball on TV.

Conclusion: Dean loves baseball.

D: Dean
B: Love baseball
TV: Watch baseball



"if you love baseball, then you watch TV"

B ⇨ TV

"Dean watches baseball on TV"
(if you're Dean, then you watch baseball on TV)

D ⇨ TV

INVALID

Conclusion: "Dean loves baseball."
(If you're Dean, then you love baseball)

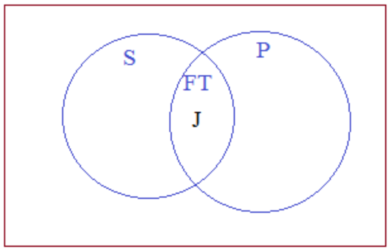
D ⇨ B

There is no way to link the statements to get the conclusion!

Example: Students who turn in permission slips can go on the field trip.
Jimmy turned in a permission slip.
Jimmy is a student in the class.

Conclusion: Jimmy can go on the field trip.

S: Students
FT: Field Trip
J: Jimmy
P: Permission Slip



"Students who turn in permission slips can go on field trip."
(if you're a student AND you turn in slip, then you go on trip)

$(S \wedge P)$ ⇨ FT

"Jimmy turned in permission slip."

J ⇨ P

VALID

"Jimmy is a student."

J ⇨ S

Yes, Jimmy is a student and turned in a permission slip.. Therefore, he can go on the field trip.

J ⇨ $(S \wedge P)$ ⇨ FT

Example:

A ⇨ B

~D ⇨ C

D ⇨ ~B

C ⇨ E

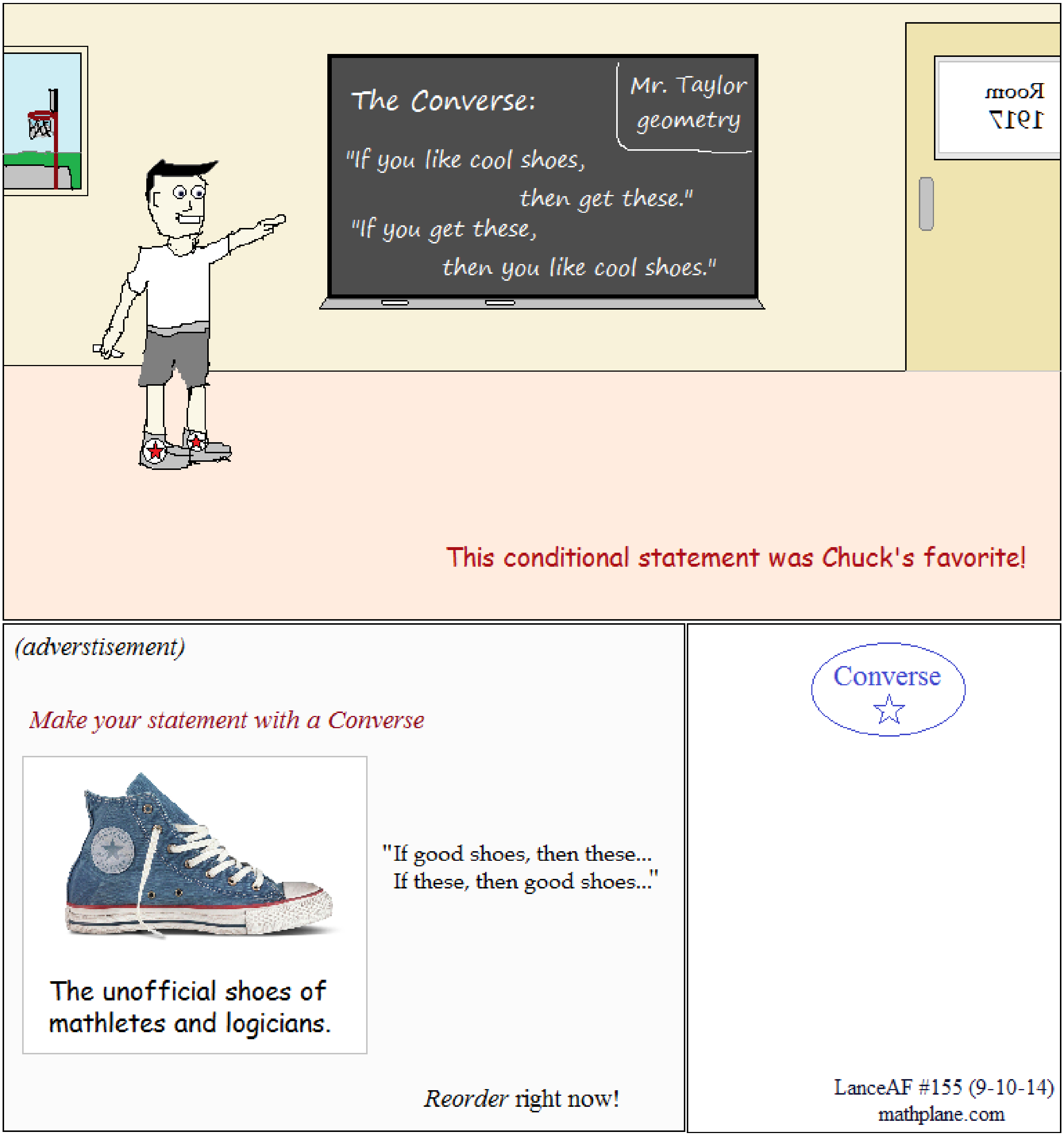
Conclusion: A ⇨ E

B ⇨ ~D (contrapositive has same truth value as conditional)

D ⇨ ~B

A ⇨ B ⇨ ~D ⇨ C ⇨ E

VALID


The Converse:
Mr. Taylor geometry
"If you like cool shoes, then get these."
"If you get these, then you like cool shoes."
Room 1917
This conditional statement was Chuck's favorite!
(adverstisement)
Make your statement with a Converse
The unofficial shoes of mathletes and logicians.
"If good shoes, then these...
If these, then good shoes..."
Reorder right now!
Converse
LanceAF #155 (9-10-14)
mathplane.com

Exercises ->

Write the following, and determine the truth value.

Conditional Statements

1) If it is 20°F, then it is cold outside.

Converse:

Inverse:

Contrapositive:

2) You're in California when you visit Los Angeles.

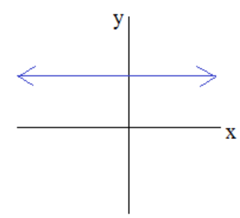
Conditional Statement:

Converse:

Inverse:

Contrapositive:

3) In a coordinate plane, the slope of a horizontal line is always zero.



Conditional Statement:

Biconditional Statement:

Converse:

Inverse:

Contrapositive:

4) A quarter is a coin.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

5) The girl is happy if she gets dessert.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

6) The boy is happy if and only if he gets dessert.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

Truth Tables

A conditional statement is an if-then statement in which P is the hypothesis and Q is the conclusion.

$P \Rightarrow Q$

The conditional statement is defined as 'true', UNLESS a true hypothesis leads to a false conclusion.

Fill in the truth table:

Conditional Statements

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T		
T	F		
F	T		
F	F		

Conjuction: A compound statement formed by joining two statements with a connector ("AND") The symbol for 'and' is $\wedge$
(similar to the 'intersection' or overlap of sets)

Disjunction: A compound statement formed by joining two statements with a connector ("OR") The symbol for 'or' is $\vee$
(similar to the 'union' of sets)

Fill in the truth table:

Conjunctions & Disjunctions

P	Q	$P \vee Q$	$P \wedge Q$
T	T		
T	F		
F	T		
F	F		

In a 'conjunction', BOTH statements must be true for the compound to be true
(otherwise, the conjunction is false)

In a 'disjunction', BOTH statements must be false for the compound to be false
(otherwise, the disjunction is false)

The negation of a statement is the 'not'

$\sim P$ $\neg P$

Fill in the truth table:

Negations & Compound Statements

P	Q	$\neg P$	$\neg P \vee Q$	$\neg P \wedge Q$
T	T			
T	F			
F	T			
F	F			

Venn and Euler Diagrams

Match the diagram that best fits the statement:

1) If A, then B $\quad A \Rightarrow B$

2) If B, then A $\quad B \Rightarrow A$

3) A and B $\quad A \wedge B$

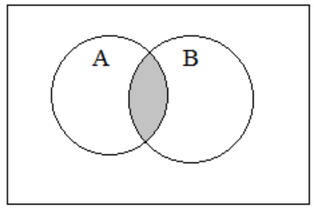
4) A or B $\quad A \vee B$

5) If ~A, then ~B $\quad \sim A \Rightarrow \sim B$

6) If A, then ~B $\quad A \Rightarrow \sim B$

7) B and ~A $\quad B \wedge \sim A$

I



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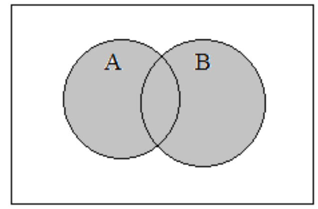
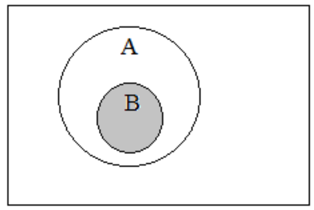
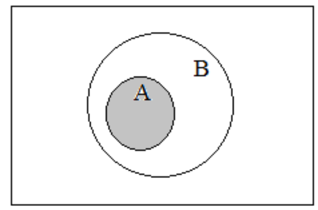


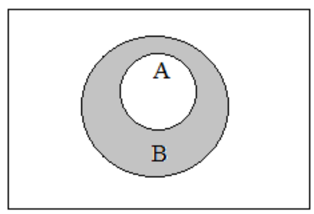
III



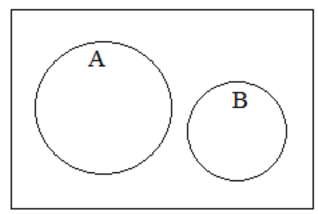
IV



V



VI



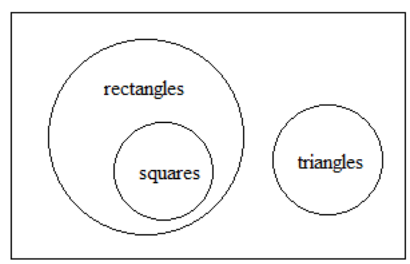
Conditional "if-then" statement: If a polygon is a ___square___ , then it is a ____________ .

Converse:

Inverse:

Contrapositive:

Polygons (sides and lengths)



True or false?

All squares are rectangles.

All rectangles are squares.

Triangles or Rectangles have 3 sides.

Triangles and Squares have 3 sides.

Provide a logical conclusion with a single 'if, then' statement....

Chain of Logic

1) $\sim Q \Rightarrow S \quad Q \Rightarrow N$

2) $P \Rightarrow B \qquad T \Rightarrow A \qquad \sim T \Rightarrow P$

3) If weebles wobble, then canaries call...
If minnows don't meander, then weebles wobble...
If doggies dig, then minnows don't meander...

4) If the food is not meat, then it is a vegetarian diet.

Food cooked by Wolfgang has lots of fat.

If the food is meat, then it is not recommended by my doctor.

Foods in a vegetarian diet do not have lots of fat..

5) If you eat too many blueberry pies, you will gain 5 pounds.

If you buy too many blueberries, you start making blueberry pies.

If you don't eat too many blueberry pies, you did not start making blueberry pies.

If you don't buy too many blueberries, you did not go blueberry picking.

Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

Euler Diagrams and Truth Statements

1) All golfers have rain gear.
JC has rain gear.
∴ JC is a golfer.

2) Some people love math.
All people who love math love physics.
∴ Some people love physics.

3) No apples are citrus.
All Fujis are apples.
∴ No fujis are citrus.

4) All rainy days are cloudy.
Today is cloudy.
∴ Today is a rainy day.

5) Carl is 16 years old.
Carl drives a car.

Conclusion: If you're 16 years old, then you drive a car.

Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

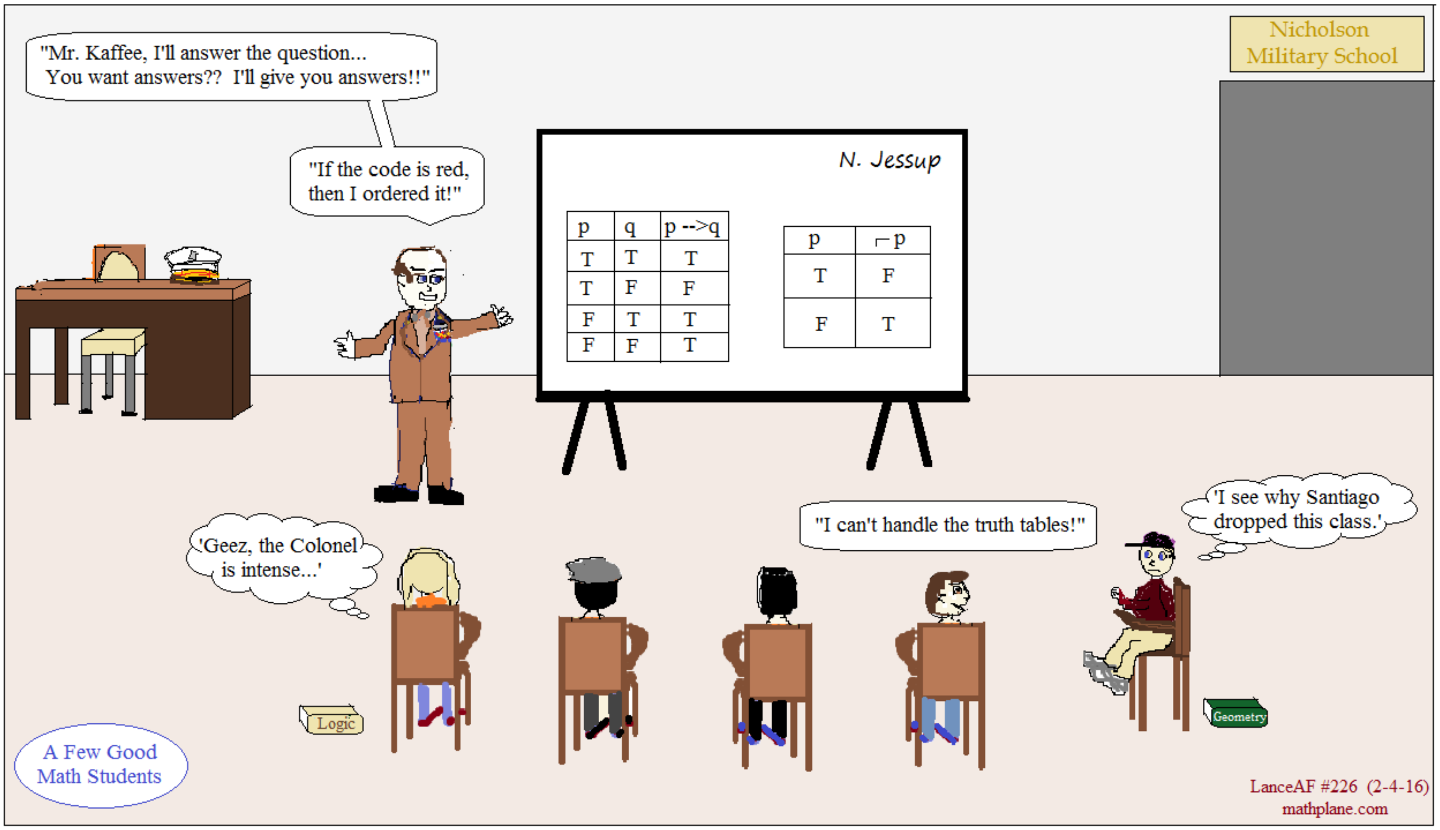
6) Some plants are poisonous.
Ivy is a plant.
∴ Ivy is poisonous.

7) All cordless phones have antennas.
All things with antennas are insects.
∴ All cordless phones are insects.

8) All cordless phones have antennas.
All insects have antennas.
∴ All cordless phones are insects.

9) If a lawyer passes the bar, then he/she can practice law.
Cindy passed the bar.

Conclusion: Cindy can practice law.


"Mr. Kaffee, I'll answer the question...
You want answers?? I'll give you answers!!"
"If the code is red, then I ordered it!"
Nicholson Military School
N. Jessup
p | q | p -->q
T | T | T
T | F | F
F | T | T
F | F | T
p | ⌐p
T | F
F | T
'Geez, the Colonel is intense...'
"I can't handle the truth tables!"
'I see why Santiago dropped this class.'
Logic
Geometry
A Few Good Math Students
LanceAF #226 (2-4-16)
mathplane.com

SOLUTIONS-→

Write the following, and determine the truth value.

ANSWERS

Conditional Statements

1) If it is 20°F, then it is cold outside.

Converse: If it is cold outside, then it is 20° F (False. Counterexample: 23 degrees)

Inverse: If it is NOT 20°F, then it is NOT cold outside. (False)

Contrapositive: If it is not cold outside, then it is not 20° F. (True)

2) You're in California when you visit Los Angeles.

Conditional Statement: If you visit Los Angeles, then you're in California. (True)

Converse: If you're in California, then you're visiting Los Angeles. (False. Counterexample: San Francisco)

Inverse: If you're not visiting Los Angeles, then you're not in California. (False)

Contrapositive: If you're NOT in California, then you're NOT visiting Los Angeles. (True)

3) In a coordinate plane, the slope of a horizontal line is always zero.

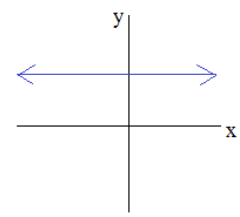
Conditional Statement: In a coordinate plane, if the line is horizontal, then its slope is zero. (True)

Biconditional Statement: In a coordinate plane, the slope of a line is zero IF AND ONLY IF it is horizontal. (True)

Converse: In a coordinate plane, if the slope is zero, then the line is horizontal. (True)

Inverse: (In a coordinate plane), if the line is NOT horizontal, then its slope is NOT zero. (True)

Contrapositive: (In a coordinate plane), if the slope is not zero, then the line is not horizontal. (True)



4) A quarter is a coin.

Conditional Statement: If it's a quarter, then it's a coin. (True)

Converse: If it's a coin, then it's a quarter. (False. Counterexamples: penny, dime, etc...)

Inverse: If it's not a quarter, then it's not a coin. (False)

Contrapositive: If it's not a coin, then it's not a quarter. (True)

5) The girl is happy if she gets dessert.

Conditional Statement: If the girl gets dessert, then she is happy. (true)

Converse: If the girl is happy, then she got/gets dessert. (false. she may or may not)

Inverse: If the girl doesn't get dessert, then she is not happy. (false)

Contrapositive: If she does not get dessert, then the girl is not happy. (true)

6) The boy is happy if and only if he gets dessert.

***"if and only if" indicates that this is an 'equivalence statement'... all of the statements are true!

Conditional Statement: If the boy gets dessert, then he is happy. (True)

Converse: If the boy is happy, he got dessert. (True)

Inverse: If the boy doesn't get dessert, then he is not happy. (True)

Contrapositive: If the boy is unhappy, then he did not get dessert. (True)

A conditional statement is an if-then statement in which P is the hypothesis and Q is the conclusion.

$$P \Rightarrow Q$$

The conditional statement is defined as 'true', UNLESS a true hypothesis leads to a false conclusion.

Fill in the truth table:

Conditional Statements

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	T

Conjuction: A compound statement formed by joining two statements with a connector ("AND") The symbol for 'and' is $\wedge$
(similar to the 'intersection' or overlap of sets)

Disjunction: A compound statement formed by joining two statements with a connector ("OR") The symbol for 'or' is $\vee$
(similar to the 'union' of sets)

Fill in the truth table:

Conjunctions & Disjunctions

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

In a 'conjunction', BOTH statements must be true for the compound to be true
(otherwise, the conjunction is false)

In a 'disjunction', BOTH statements must be false for the compound to be false
(otherwise, the disjunction is false)

The negation of a statement is the 'not'

$\sim P \quad \neg P$

Fill in the truth table:

Negations & Compound Statements

P	Q	$\neg P$	$\neg P \vee Q$	$\neg P \wedge Q$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

Venn and Euler Diagrams

SOLUTIONS

Match the diagram that best fits the statement:

1) If A, then B $\quad A \Rightarrow B \quad$ IV

2) If B, then A $\quad B \Rightarrow A \quad$ III

3) A and B $\quad A \wedge B \quad$ I

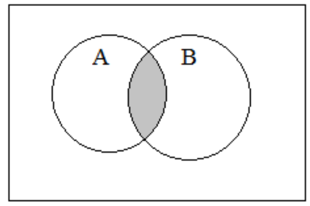
4) A or B $\quad A \vee B \quad$ II

5) If ~A, then ~B $\quad \sim A \Rightarrow \sim B \quad$ III (contrapositive of 'if B, then A')

6) If A, then ~B $\quad A \Rightarrow \sim B \quad$ VI

7) B and ~A $\quad B \wedge \sim A \quad$ V

I



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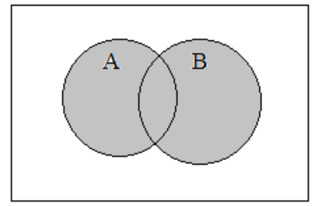

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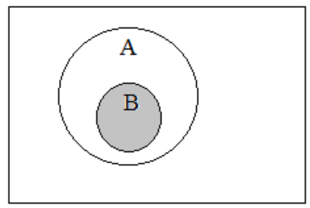

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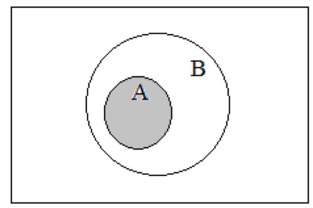
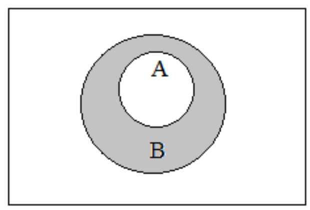
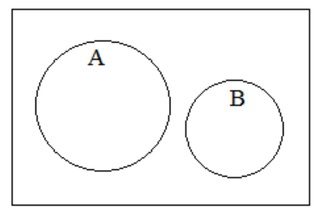


V



VI



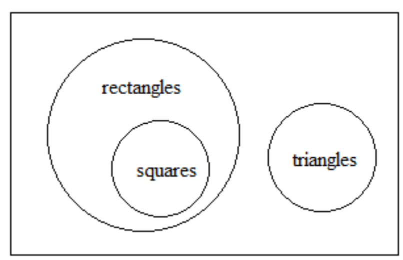
Conditional "if-then" statement: If a polygon is a square, then it is a rectangle.

Converse: If the polygon is a rectangle, then it is a square. (False... Only equilateral rectangles are squares)

Inverse: If a polygon is not a square, then it is not a rectangle. (False)

Contrapositive: If a polygon is not a rectangle, then it is not a square. (True)

Polygons (sides and lengths)



True or false?

All squares are rectangles. True

All rectangles are squares. False (some are.. some are not)

Triangles or Rectangles have 3 sides. True

Triangles and Squares have 3 sides. False

Provide a logical conclusion with a single 'if, then' statement....

SOLUTIONS

Chain of Logic

1) $\sim Q \Rightarrow S \quad Q \Rightarrow N$

To link the Q's, we'll use the contrapositive....

$\sim S \Rightarrow Q \qquad Q \Rightarrow N$

$\sim S \Rightarrow N$ (Or, $\sim N \Rightarrow S$)

2) $P \Rightarrow B \qquad T \Rightarrow A \qquad \sim T \Rightarrow P$

WE can link the P's together...

$\sim T \Rightarrow P \quad P \Rightarrow B$

$\sim T \Rightarrow P \Rightarrow B$

so, $\sim T \Rightarrow B$

$T \Rightarrow A$

using contrapositive

$\sim A \Rightarrow \sim T$

Now, we can link....

$\sim A \Rightarrow \sim T \quad \sim T \Rightarrow P \Rightarrow B$

$\sim A \Rightarrow B$ or, $\sim B \Rightarrow A$

3) If weebles wobble, then canaries call...
If minnows don't meander, then weebles wobble...
If doggies dig, then minnows don't meander...

Translate and list the 'if, then' statements...

ww ---> cc

not mm ---> ww

dd ---> not mm

Then, link the parts....

dd ---> not mm ---> ww ---> cc

If doggies dig, then canaries call...

4) If the food is not meat, then it is a vegetarian diet.

Food cooked by Wolfgang has lots of fat.

If the food is meat, then it is not recommended by my doctor.

Foods in a vegetarian diet do not have lots of fat..

Write symbols for logic chain links...

$\neg$ meat $\Rightarrow$ veg diet

Wolf $\Rightarrow$ fat

meat $\Rightarrow$ $\neg$ doctor recommended

veg diet $\Rightarrow$ $\neg$ fat

Pick out singular items... These must be ends of chain (if all links are used..)

Wolf $\Rightarrow$ fat

fat $\Rightarrow$ $\neg$ veg diet

$\neg$ veg diet $\Rightarrow$ meat

meat $\Rightarrow$ $\neg$ doctor recommended

Food cooked by Wolfgang is not recommended by my doctor...

Chain of Logic

SOLUTIONS

5) If you eat too many blueberry pies, you will gain 5 pounds.

If you buy too many blueberries, you start making blueberry pies.

If you don't eat too many blueberry pies, you did not start making blueberry pies.

If you don't buy too many blueberries, you did not go blueberry picking.

List the chain links:

Eat too many ---> gain 5

buy too many ---> start making

$\neg$ eat too many ---> $\neg$ start making

$\neg$ buy too many ---> $\neg$ picking

Start with 'picking' (or, 'gain 5'), because they are single terms..
(They are likely the endpoints.)

picking ---> buy too many

buy too many ---> start making

start making ---> eat too many

eat too many ---> gain 5

If you go blueberry picking, then you gain 5 pounds.

Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

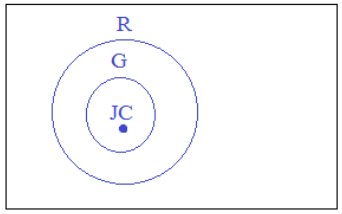
ANSWERS

Euler Diagrams and Truth Statements

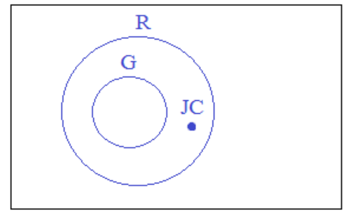
1) All golfers have rain gear.
JC has rain gear.
∴ JC is a golfer.

It could be true..
But, it could be false..

Therefore, "invalid"



JC is a golfer

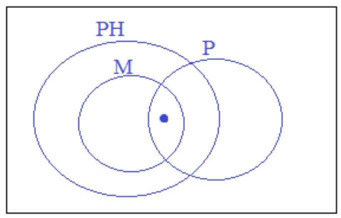


JC is not a golfer

2) Some people love math.
All people who love math love physics.
∴ Some people love physics.

Anyone who loves math will love physics... Since some people love math, there are some people who love physics..

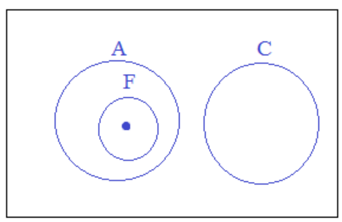
"Valid"



P: People
PH: Love Physics
M: Love Math

3) No apples are citrus.
All Fujis are apples.
∴ No fujis are citrus.

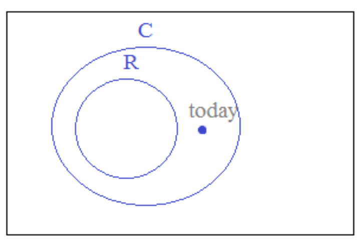
"Valid"



A: Apples
F: Fujis
C: Citrus

4) All rainy days are cloudy.
Today is cloudy.
∴ Today is a rainy day.

"Invalid"



Exception..

C: Cloudy days
R: Rainy days

5) Carl is 16 years old.
Carl drives a car.

Conclusion: If you're 16 years old, then you drive a car.

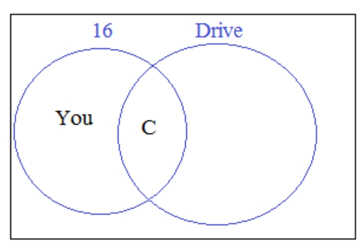
"Invalid"

C ⇨ 16

C ⇨ D

~D ⇨ ~C

(there's no way to make a deductive link!)



Counterexample

Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

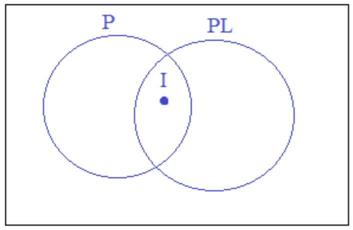
ANSWERS

6) Some plants are poisonous.
Ivy is a plant.
∴ Ivy is poisonous.

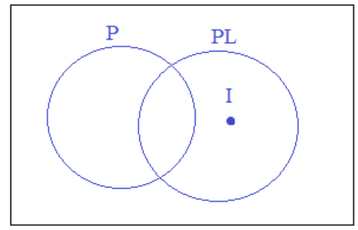
Ambiguous, so

"invalid"

P: Poisonous things
PL: Plants
I: Ivy



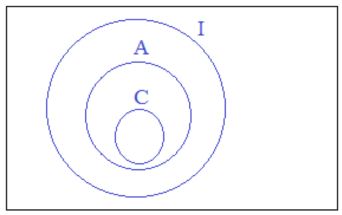
Yes



No

7) All cordless phones have antennas.
All things with antennas are insects.
∴ All cordless phones are insects.

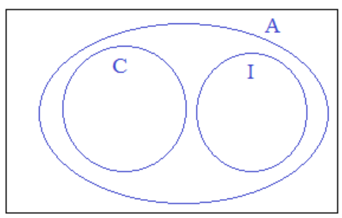
"Valid" (but, bizarre!)



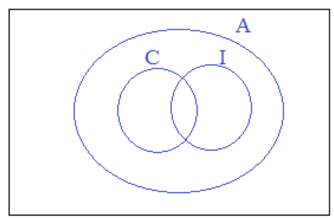
Maybe the cordless phones are shaped like insects? (cartoon characters)

8) All cordless phones have antennas.
All insects have antennas.
∴ All cordless phones are insects.

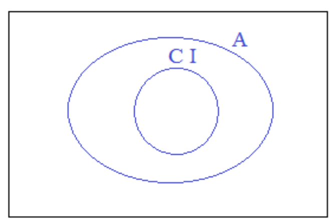
"Invalid"



False



False



true

9) If a lawyer passes the bar, then he/she can practice law.
Cindy passed the bar.

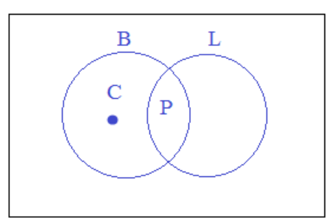
Conclusion: Cindy can practice law.

"Invalid"

Is Cindy a lawyer?
Or, not?
Maybe she was disbarred?

$(L \wedge B) \Rightarrow PL$

$C \Rightarrow B$

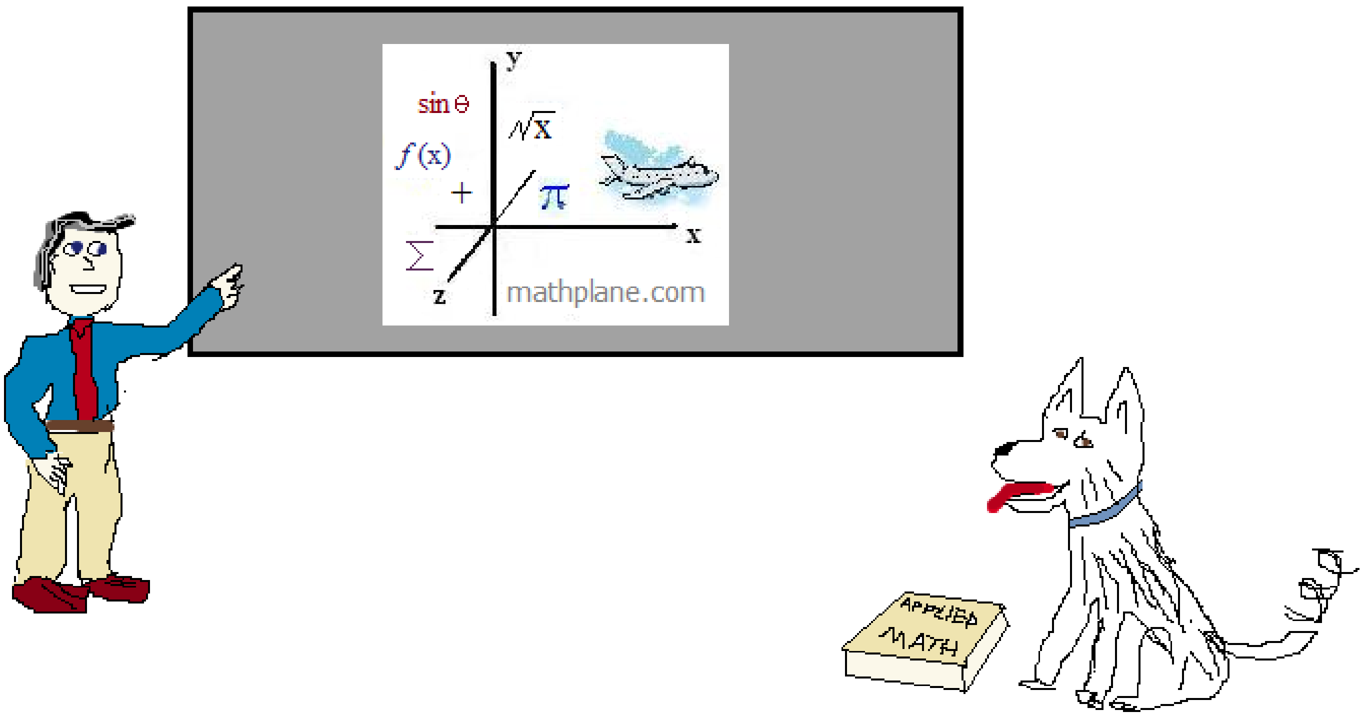


P: Practice Law
B: Pass Bar
C: Cindy
L: Lawyers

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, then let us know.

Cheers



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What is the number?

If the number is divisible by 2, then it is between 50 and 59...

If the number is NOT divisible by 3, then it is between 60 and 69...

If the number is NOT divisible by 4, then it is between 70 and 79...

SOLUTION

First, a quick look...

If the number is divisible by 2, then it is between 50 and 59... Therefore, the number is ODD, unless it is between 50 and 59...

If the number is NOT divisible by 3, then it is between 60 and 69... this eliminates 63, 66, 69...

If the number is NOT divisible by 4, then it is between 70 and 79... this eliminates 72, 76...

Then, recognize that this question is implementing logic:

Recall: conditional statement, inverse, converse, and contrapositive!

***Here's the key...

Contrapositive of 1st statement: "If NOT between 50 and 59, then NOT divisible by 2"..

math property of factors: If NOT divisible by 2, then it can't be divisible by 4!!
(any number divisible by 4 must be divisible by 2...)

3rd statement: If NOT divisible by 4, then it is between 70 and 79...

Conclusion: If NOT between 50 and 59, then it is between 70 and 79...

NOT 50-59 ⇒ Not divisible by 2 ⇒ Not divisible by 4 ⇒ between 70-79

So, numbers can be 50 52 54 56 58 71 73 75 77 79

Contrapositive of 2nd statement: "If NOT between 60 and 69, then number IS divisible by 3..."

So, the numbers can be 54 or 75

But, 54 violates the 3rd statement!

So, the number must be 75....