Number Classifications

NATURAL NUMBERS

1, 2, 3, ...
"counting numbers"

WHOLE NUMBERS

0, 1, 2, 3, ...
"Natural Numbers + 0"

INTEGERS

... -2, -1, 0, 1, 2, ...
"Whole Numbers and their opposites"

RATIONALS

Numbers of the form $\frac{a}{b}$ where a & b are integers ($b \neq 0$)

"Any number that is a fraction"

e.g. 7, $1.6473\overline{73}$, $\frac{22}{5}$, -3.6×10^{8}

IRRATIONALS

Numbers that <u>cannot</u> be expressed as fractions.

These numbers do not have terminating or repeating decimals

e.g. $\sqrt{2}$, e , π

REAL NUMBERS

NUMBER GROUPS

COMPLEX NUMBERS

Numbers expressed as a + bi
where a is the 'real' part and
bi is the 'imaginary' part.
a & b are real numbers
i is an imaginary number: $i^2 = -1$

e.g. $\sqrt{-16}$, 3 + 7i

(Set Notations)

$\mathbb{R}$	Real Numbers
$\mathbb{Q}$	Rational Numbers
$\mathbb{Z}$	Integers
$\mathbb{N}$	Natural Numbers
$\mathbb{C}$	Complex Numbers

NUMBERS DEFINITIONS & EXAMPLES

PRIME NUMBER

A natural number ONLY divisible by itself or 1.
1 is not prime. 2 is the only even prime number.
e.g. 7, 23, 37

COMPOSITE NUMBER

A natural number that has a positive divisor other than 1 and itself.
e.g. 6, 9, 38

MIXED NUMBER

A mixed number is the sum of a fraction and a whole number.

e.g. $3\frac{4}{5}$ $2\frac{3}{7}$

RECIPROCAL

"A fraction with the numerator and denominator reversed"

e.g. 2/3 ---> 3/2
7 ---> 1/7

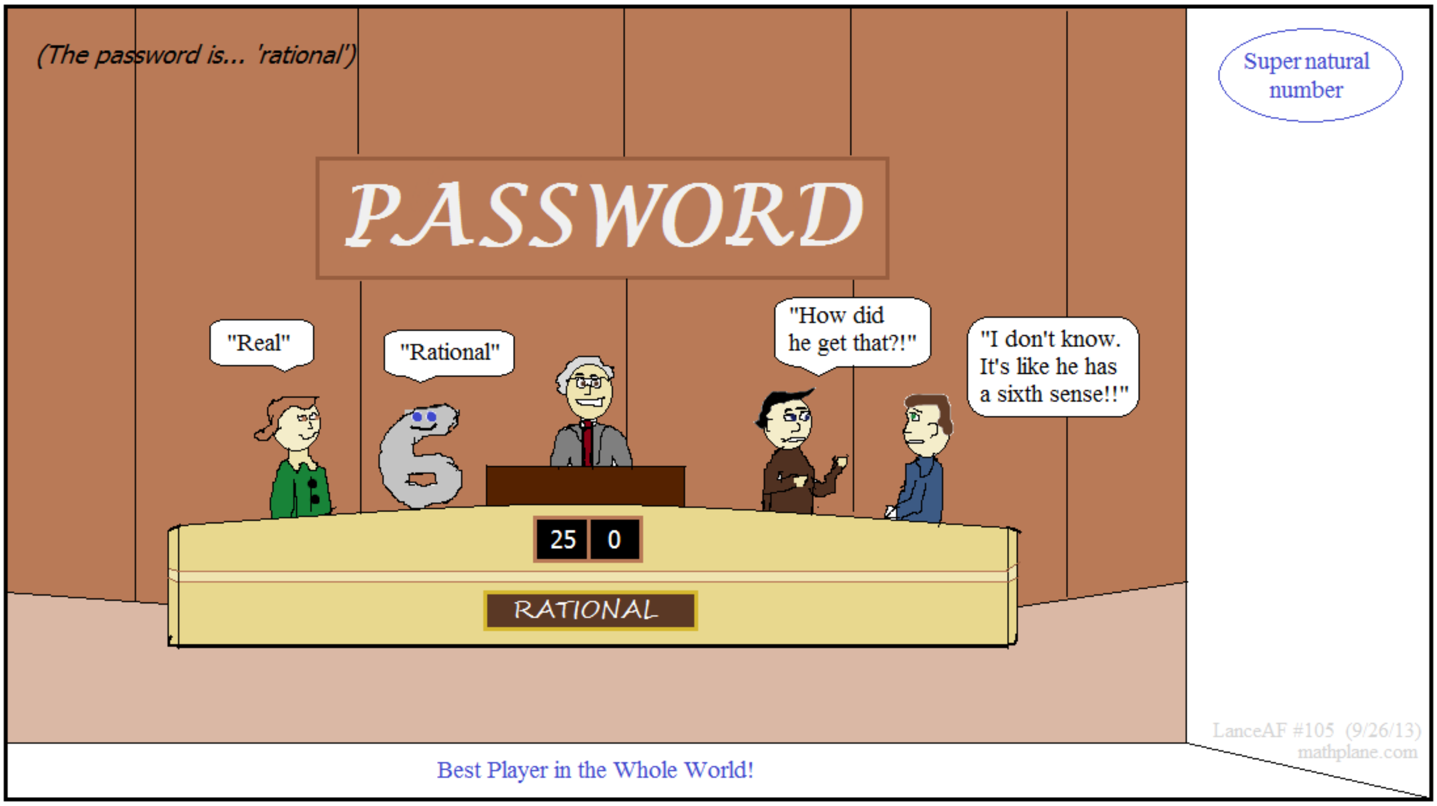
IDENTITY ELEMENTS

Additive Identity: 0 "Any number + 0 = the same number"
Multiplicative Identity: 1 "Any number x 1 = the same number"

PERFECT SQUARES

Any number that is the square of a rational number. (i.e. a "rational number times itself creates a perfect square")

e.g. 1, 4, 9, $\frac{9}{16}$, $\frac{100}{49}$



Practice Classifying Numbers.....

Place a check in any box that defines the number:

	natural	whole	integer	rational	irrational	real	imaginary
11							
$\sqrt{3}$							
-7							
$\frac{5}{2}$							
0							
$\sqrt{-9}$							
π							
$\frac{-8}{4}$							

What points (x, y) satisfy the equation $x + y = 10$, where x and y are *natural numbers*.

Answer always, sometimes, or never:

1) An integer is rational.

2) A whole number is natural.

3) A rational number is an integer.

4) An irrational number is a real number.

5) n and $-n$ are both natural numbers.

6) $\sqrt{x}$ is a real number.

7) a repeating decimal is a rational number.

Place a check in any box that defines the number: Answers

		natural	whole	integer	rational	irrational	real	imaginary
	11	X	X	X	X		X	
	$\sqrt{3}$					X	X	
	-7			X	X		X	
	$\frac{5}{2}$				X		X	
	0		X	X	X		X	
3i	$\sqrt{-9}$							X
	π					X	X	
-2	$\frac{-8}{4}$			X	X		X	

What points (x, y) satisfy the equation $x + y = 10$, where x and y are *natural numbers*.

natural numbers include: 1, 2, 3, 4, ...

(1, 9) (6, 4)
(2, 8) (7, 3)
(3, 7) (8, 2)
(4, 6) (9, 1)
(5, 5)

Answer always, sometimes, or never:

1) An integer is rational. Always.... (all integers are rational, because any integer can be expressed as n/1)

2) A whole number is natural. Sometimes.... 0 is whole, but NOT natural...

3) A rational number is an integer. Sometimes... 5 is rational integer... 1/2 is rational non-integer...

4) An irrational number is a real number. Always... (imaginary numbers are not rational nor irrational)

5) n and $-n$ are both natural numbers. Never... If n = 1, then -n = -1... natural numbers are positive..

6) $\sqrt{x}$ is a real number. Sometimes... If $x < 0$, then it is not...

7) a repeating decimal is a rational number. Always... Any repeating decimal can be expressed as a fraction -- ratio of 2 integers..

Vinculum (Bar) and "Repeating Decimals"

Vinculum: A horizontal bar drawn over multiple quantities to indicate they are grouped together.

Examples include:

radicals	$\sqrt{9X^3}$
line segments	$\overline{AB}$ (joining points A & B)
repeating decimals	$0.776\overline{76}$

Repeating Decimals: A decimal number that eventually becomes periodic (i.e. "the end repeats indefinitely")

Examples:

$$\frac{1}{3} = 0.333333... = 0.\overline{3}$$

$$\frac{22}{7} = 3.142857142857... = 3.\overline{142857}$$

$$12.0340353535... = 12.0340\overline{35}$$

Converting Fractions to Decimals: Divide the numerator by the denominator

Examples:

$\frac{42}{9}$

$$\begin{array}{r} 4.6\overline{66} \\ 9\,\overline{)\,42} \\ -36 \\ \hline 60 \\ -54 \\ \hline 60 \\ -54 \\ \hline \end{array}$$

etc....

$4.\overline{6}$

$\frac{3}{700}$

$$\begin{array}{r} 0.00428571 \\ 700\,\overline{)\,3.0000000} \\ -2.800 \\ \hline .2000 \\ -.1400 \\ \hline 6000 \\ -5600 \\ \hline 4000 \\ -3500 \\ \hline 5000 \\ -4900 \\ \hline 1000 \\ -700 \\ \hline 3000 \end{array}$$

etc...
(repeats indefinitely)

$0.00\overline{428571}$

Converting 'Repeating Decimals' to Fractions: Using algebra

Examples: $.\overline{7}$ let $n = .7\overline{7}$ then, $10n = 7.7\overline{7}$

$$\begin{array}{r} 10n \\ -\;\;n \\ \hline 9n \end{array} \qquad \begin{array}{r} 7.7\overline{7} \\ -.7\overline{7} \\ \hline 7.0 \end{array}$$

substitution reveals that $9n = 7$ $\quad n = \frac{7}{9}$

$11.\overline{18}$ let $m = .\overline{18}$ then, $100m = 18.\overline{18}$

$$\begin{array}{r} 100m \\ -\;\;m \\ \hline 99m \end{array} \qquad \begin{array}{r} 18.\overline{18} \\ -\;.\overline{18} \\ \hline 18 \end{array}$$

$11.\overline{18} = 11 + m$

$m = \frac{18}{99}$

$11.\overline{18} = 11\frac{18}{99}$

$234.00176\overline{76}$

Separate the number into parts: $234 + .001 + .000\overline{76}$

Convert the parts to fractions: $234 + .001 = \frac{1}{1000} = \frac{99}{99000}$ $+$ let $p = .00076\overline{76}$

$1000p = .76\overline{76}$

$100000p = 76.\overline{76}$

$$\begin{array}{r} 100000p \\ -\;1000p \\ \hline 99000p \end{array} \qquad \begin{array}{r} 76.\overline{76} \\ -\;.\overline{76} \\ \hline 76 \end{array}$$

$p = \frac{76}{99000}$

Combine the Fractions: $234 + \frac{99}{99000} + \frac{76}{99000} = 234\frac{175}{99000}$

$.000\overline{76} = \frac{76}{99000}$

Specifying number sets

Write the solution set:

1) $5x + 3 = 14$ $\quad x \in$ {real numbers}

2) $3x + 7 = 20$ $\quad x \in$ {integers}

3) $|3x + 4| > 10$ $\quad x \in$ {natural numbers}

4) $|4x + 3| = 9$ $\quad x \in$ {positive numbers}

5) $-5 < x + 2 \leq 7$ $\quad x \in$ {whole numbers}

6) $x^2 = 64$ $\quad x \in$ {real numbers}

SOLUTIONS

1) $5x + 3 = 14$ $\quad x \in$ {real numbers}

$5x = 11$

$x = 11/5$

{11/5}

2) $3x + 7 = 20$ $\quad x \in$ {integers}

$3x = 13$

$x = 13/3$

But, 13/3 is not an integer! $\quad \emptyset$

3) $|3x + 4| > 10$ $\quad x \in$ {natural numbers}

$3x + 4 > 10$ or $3x + 4 < -10$

$3x > 6$ $\qquad 3x < -14$

$x > 2$ $\qquad x < -14/3$

{3, 4, 5, ...}

Since natural numbers are positive integers, the solution is 3, 4, 5, 6, ...

4) $|4x + 3| = 9$ $\quad x \in$ {positive numbers}

$4x + 3 = 9$ or $4x + 3 = -9$

$4x = 6$ $\qquad 4x = -12$

$x = 3/2$ $\qquad$ ~~$x = -3$~~

{3/2}

5) $-5 < x + 2 \leq 7$ $\quad x \in$ {whole numbers}

$-5 < x + 2$ and $x + 2 < 7$

$-7 < x$ and $x \leq 5$

and, x must be a whole number!

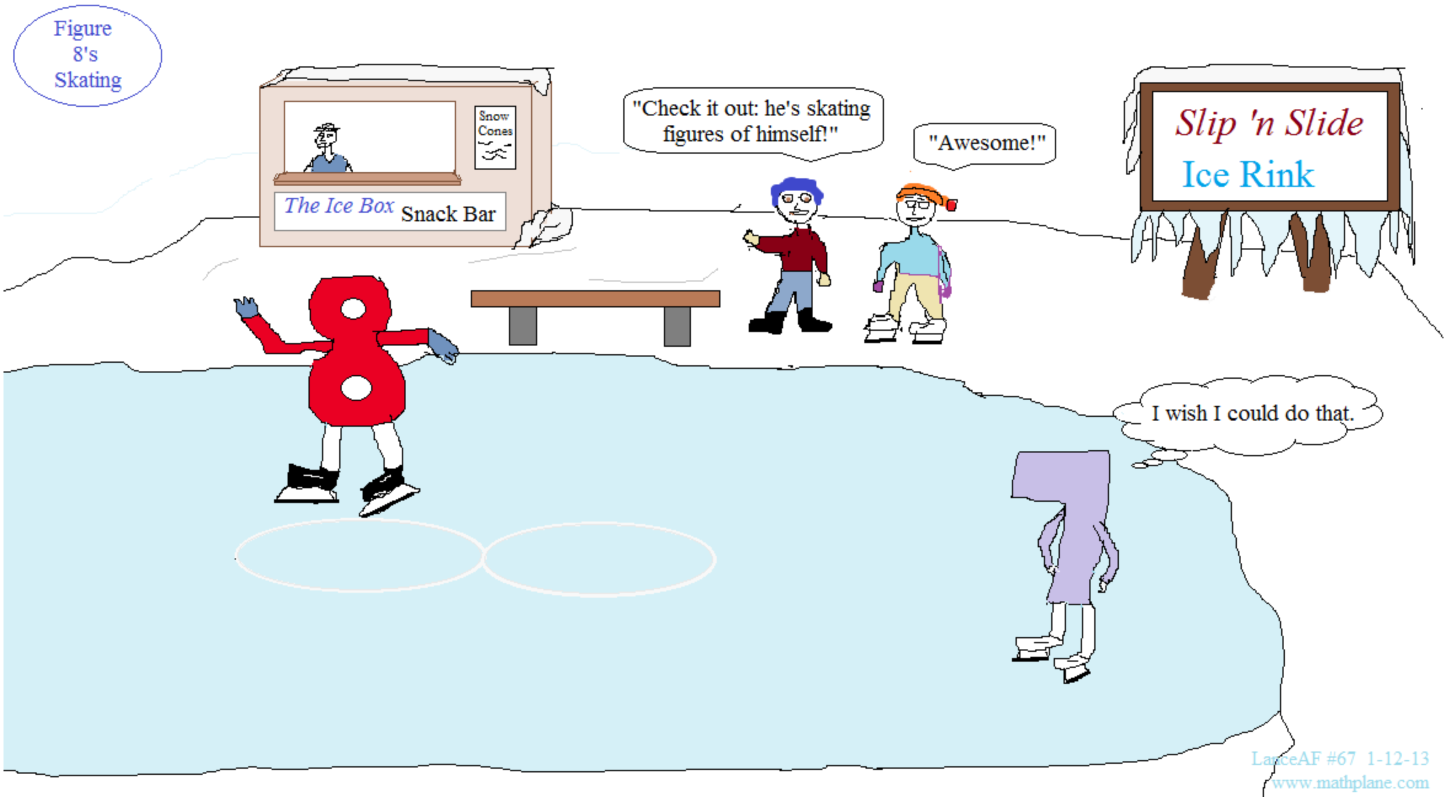
{0, 1, 2, 3, 4, 5}

6) $x^2 = 64$ $\quad x \in$ {real numbers}

(square root both sides)

$x = 8$ or -8

{-8, 8}



Quick Quiz (with solutions)

Nine Number Questions

1) List the 5 smallest prime numbers.

2) Is the product of two prime numbers ever prime?

3) {perfect squares < 100} ∩ {odd numbers}

4) How many rational numbers are between 2 and 9?

5) List the 4 integers closest to 1 on the number line.

6) Are natural numbers a subset of integers?

7) What is the product of a number and its reciprocal?

8) Give 3 examples of irrational numbers.

9) Express $.29\overline{29}$ as a fraction.

Nine Number Questions

ANSWERS

1) List the 5 smallest prime numbers.

2, 3, 5, 7, 11

2) Is the product of two prime numbers ever prime?

No.. The product of 2 primes will always produce a number which has a factor other than itself and one.

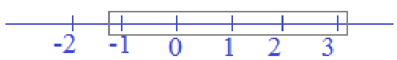
3) {perfect squares < 100} ∩ {odd numbers}

{1, 4, 9, 16, 25, 36, 49, 64, 81} ∩ {1, 3, 5, 7, 9, ... } = {1, 9, 25, 49, 81}

4) How many rational numbers are between 2 and 9?

There are an infinite number of rational numbers! notice: 2.001 2.00001 2.1001 are examples.. There are countless others..

5) List the 4 integers closest to 1 on the number line.



-1, 0, 2, and 3

6) Are natural numbers a subset of integers?

yes, every natural number is an integer..

7) What is the product of a number and its reciprocal?

one $a \cdot \frac{1}{a} = 1$

8) Give 3 examples of irrational numbers.

$\sqrt{2}$ e pi

9) Express $.29\overline{29}$ as a fraction.

Let $n = .29\overline{29}$

Then, let $100n = 29.29\overline{29}$

$100n - n = 99n$

Substitution: $29.29\overline{29} - .29\overline{29} = 29$

So, $99n = 29$

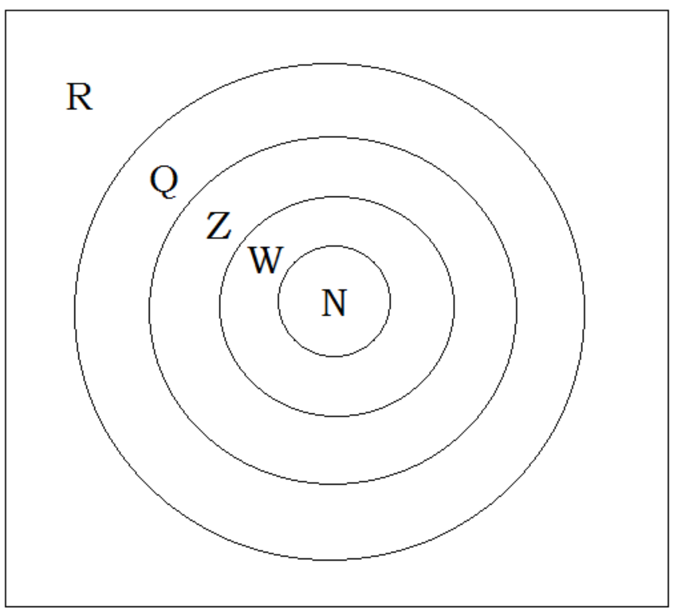
$n = \frac{29}{99}$

Quick review of Number Classes

Natural Numbers	"Count to 10 and Beyond"	1, 2, 3, 4, 5, ...	$\mathbb{N}$
Whole Numbers	"Add a zero to the Naturals"	0, 1, 2, 3, 4, 5, ...	$\mathbb{W}$
Integers	"Add the opposites to the Wholes"	.... -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...	$\mathbb{Z}$
Rational Numbers	"Use the Integers to make Fractions"	-.65,... -1/3, ..., 0, ... , 1/2, 3, 5.6, ...	$\mathbb{Q}$
Real Numbers	"Add the rest of the numbers on the number line"	$-\sqrt{3}$, ... -1, ... , 2, *e*, ... 8/3, ...	$\mathbb{R}$
Complex Numbers	"Add a perpendicular line to the number line"	a + b*i* where a is any real number, and, b is any real number except 0 and, *i* is $\sqrt{-1}$	$\mathbb{C}$

$$\mathbb{R} - \mathbb{Q} = \mathbb{P}$$

Reals + Rationals = Irrationals



NUMBER PUZZLES

I. Each capital letter represents a unique digit.
Using logic, can you match each letter with the number it represents?

Here are the properties:

a) A x D = A

b) B, C, and G are prime numbers

c) B, F, and D are even numbers

d) G < E

e) H has 3 factors

f) F = 3B

g) G x D = 20

h) C + A = E

A	B	C	D	E	F	G	H
___	___	___	___	___	___	___	___

II. If A, B, and C are unique digits,

and

```
   AA
   BB
+  CC
  ABC
```

what are A, B, and C?

III. Place the following categories inside the rectangles...

Then, arrange the numbers inside the squares....

Nine Categories:

Odd
Even
Factors of 60
Greater than 20
Less than 20

Perfect Squares
Prime
Multiples of 3
Multiples of 5

Twenty Numbers:

2, 4, 5, 7, 9,   11, 12, 15, 16, 18,

20, 21, 23, 24, 25,   30, 35, 36, 45, 60

Categories

Numbers

Categories

IV. What are the 3 prime factors of 2294 ?

NUMBER PUZZLES

SOLUTIONS

I. Each capital letter represents a unique digit.
Using logic, can you match each letter with the number it represents?

Here are the properties:

a) A x D = A

b) B, C, and G are prime numbers

c) B, F, and D are even numbers

d) G < E

e) H has 3 factors

f) F = 3B

g) G x D = 20

h) C + A = E

A	B	C	D	E	F	G	H
1	2	7	4	8	6	5	9

a) if A x D = A, then A is either 0 or 1

b) B, C, and G are 2, 3, 5, 7

c) since B is even, B must be 2...

f) F = 3B, so F = 6

g) since G x D = 20, G and D must be 4 and 5...

since D is even ---> D is 4 and G is 5

h) since C + A = E, A cannot be 0...

So, A = 1

C + 1 = E C cannot be 3 (because 4 is already equal to D)
C must be 7...
and, E is 8...

e) H has 3 factors
(possible answers: 4 or 9)
H is 9
it's factors are 1, 3, 9

II. If A, B, and C are unique digits,

and

$$\begin{array}{r} AA \\ BB \\ +\ CC \\ \hline ABC \end{array}$$

what are A, B, and C?

A must be 1 or 2...

Then, B must be 8 or 9

(because A + B must equal 10)

Then, B is 9, A is 1, and C is 8

$$\begin{array}{r} 11 \\ 88 \\ 99 \\ \hline 189 \end{array}$$

III. Place the following categories inside the rectangles...

Then, arrange the numbers inside the squares....

Nine Categories:

Odd
Even
Factors of 60
Greater than 20
Less than 20

Perfect Squares
Prime
Multiples of 3
Multiples of 5

Twenty Numbers:

2, 4, 5, 7, 9, 11, 12, 15, 16, 18,

20, 21, 23, 24, 25, 30, 35, 36, 45, 60

SOLUTIONS

Categories	Numbers			
odd	9	7	21	45
factors of 60	4	5	60	20
even	36	2	12	30
> 20	25	23	24	35
< 20	16	11	18	15
	perfect square	prime	mult 3	mult 5

Categories

IV. What are the 3 prime factors of 2294 ?

Since 2294 is even, we know 2 is one of the prime factors...

2294 divided by 2 equals 1147...

A quick check can eliminate 3, because it leaves a remainder...
Since 1147 ends in 7, we can eliminate 5...
And, dividing by 7, 11, or 13 leaves a remainder...

So, the remaining possibities are between 17 and 67...

Try combinations that multiply to 1147 --- i.e. end in 7....

2

31

37

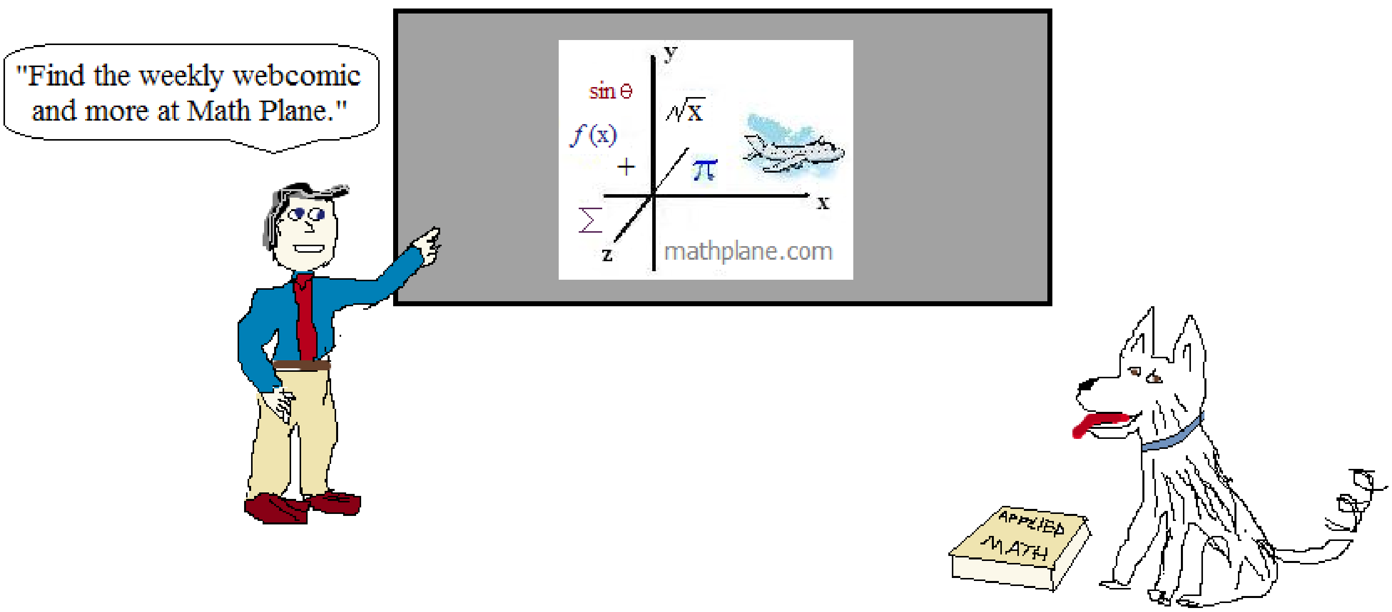
31, 41, 61 with 17, 37, 47, 67....

Trial and error leads to 31 x 37

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, stores, comics and content at Pinterest, TES, and TeachersPayTeachers

And, Mathplane *Express* for mobile at Mathplane.ORG