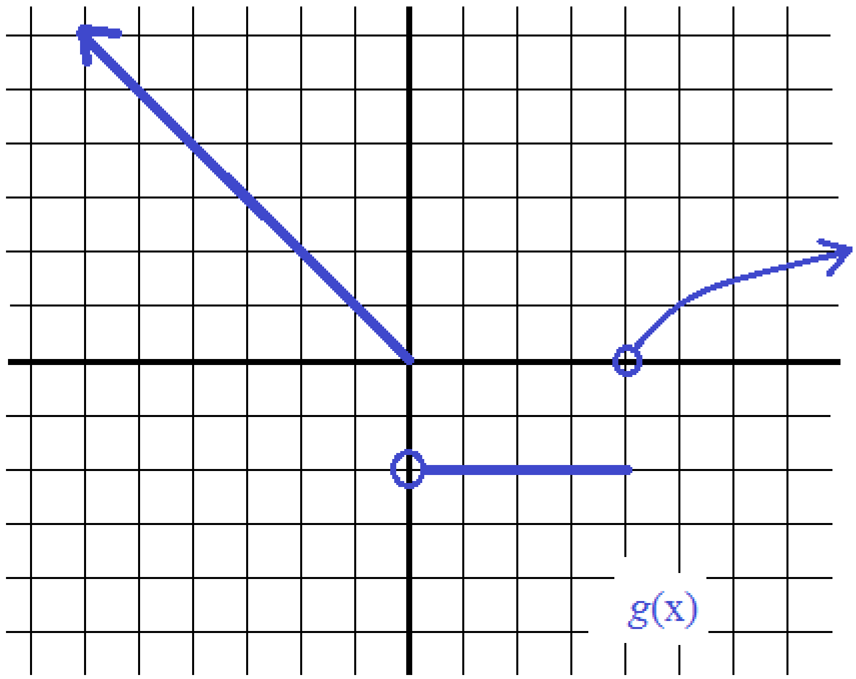
Piecewise Functions and f(x) Notation



Includes notes, examples, graphs, strategies, and practice questions (with solutions)

Functional Notation and Piecewise Functions

I. Functional Notation

What is it? A way to express a function.

Examples:

$f(x) = 3x + 8$

f identifies the function

x (inside the parentheses) is the "argument"

$f(2) = 3(2) + 8 = 14$

(substitute the x for 2)

$f(a) = 3(a) + 8 =$

$3a + 8$

$g(x) = 6x^2 - 3x + 7$

g identifies the function

x (inside the parentheses) is the "argument"

$g(4) = 6(4)^2 - 3(4) + 7$

(substitute each x with a 4)

$= 96 - 12 + 7 = 91$

$g(-1) = 6(-1)^2 + 3(-1) + 7$

$= 6 \cdot 1 + (-3) + 7 = 10$

(replaced each x with -1)

$h(t) = 3x + 4t - 5$

h identifies the function

t (inside the parentheses) is the "argument"

$h(3) = 3x + 4(3) - 5$

(substitute the t with a 3)

$= 3x + 7$

$h(x + 5) = 3x + 4(x + 5) - 5$

$= 3x + 4x + 20 - 5 = 7x + 15$

(replaced the t with (x + 5))

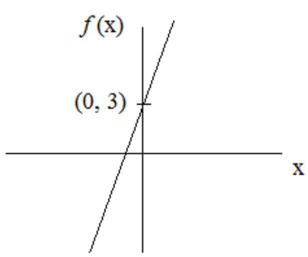
II. $f(x)$ vs. y

What is the difference between $f(x) = 4x + 3$ and $y = 4x + 3$?

The notation is different; everything else is the same... Every input x will have the same output in either expression.

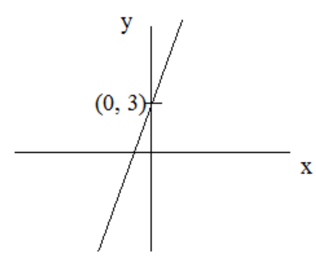
$f(0) = 3$

$f(3) = 15$



$y = 4(0) + 3 = 3$

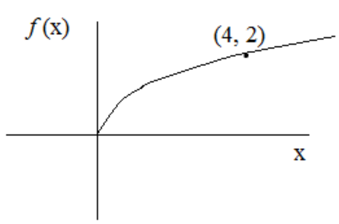
$y = 4(3) + 3 = 15$



What is the difference between $f(x) = \sqrt{x}$ and $y = \sqrt{x}$?

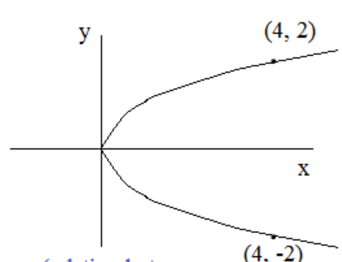
In this case, there is a subtle difference: $f(x)$ is a function but, y could be a relation (or function).

$f(4) = 2$



(function: only one output for each input)

$y = \sqrt{4} = \pm 2$



(relation between y and x)

"Piecewise defined functions" might be called *split functions*

III. Piecewise Function

What is it? A function that uses different calculations in different parts of its domain. (the formula will depend on the input!)

Examples:

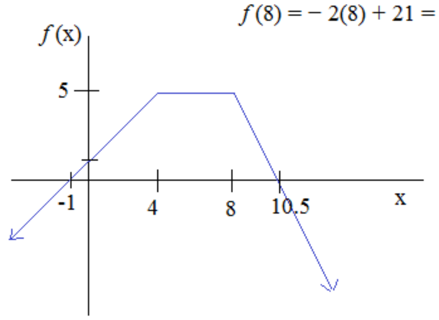
(formula) (domain)

$$f(x) = \begin{cases} x+1 & x \le 4 \\ 5 & 4 < x < 8 \\ -2x+21 & 8 \le x \end{cases}$$

$f(-2) = (-2) + 1 = -1$

$f(4) = 5$

$f(8) = -2(8) + 21 = 5$



Note: this is a *continuous* function

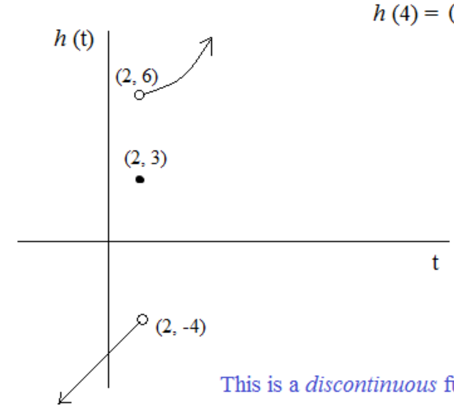
(formula) (domain)

$$h(t) = \begin{cases} t-6 & t < 2 \\ 3 & t = 2 \\ t^2+2 & t > 2 \end{cases}$$

$h(0) = -6$

$h(2) = 3$

$h(4) = (4)^2 + 2 = 18$



This is a *discontinuous* function

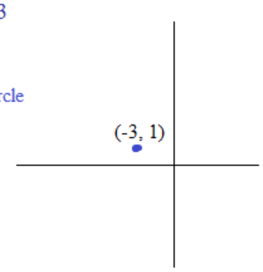
(although, it is a continuous piecewise function, because each domain piece has a continuous function)

Example: Graph

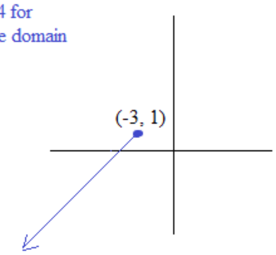
$$f(x) = \begin{cases} x+4 & x \le -3 \\ 3 & -3 < x \le 4 \\ x^2 & x > 4 \end{cases}$$

For $x \le -3$

Go to -3
$f(-3) = 1$
Closed circle

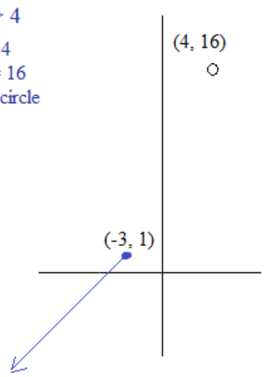


Extend $x + 4$ for the rest of the domain

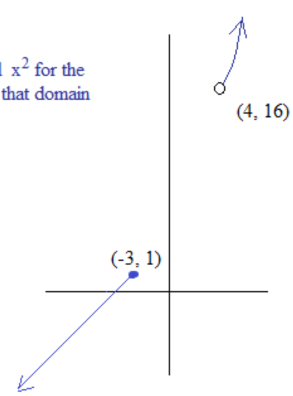


For $x > 4$

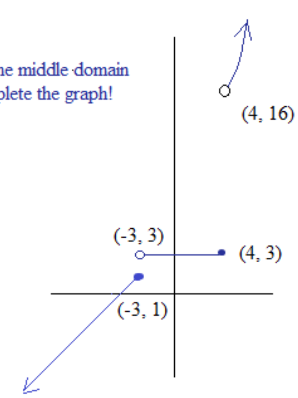
Go to 4
$f(4) = 16$
Open circle



Extend x^2 for the rest of that domain



Insert the middle domain to complete the graph!



How to graph a piecewise function

Here are 2 approaches to graphing a piecewise function:

Method 1: "Endpoint and Extend"

This is more effective for linear pieces.

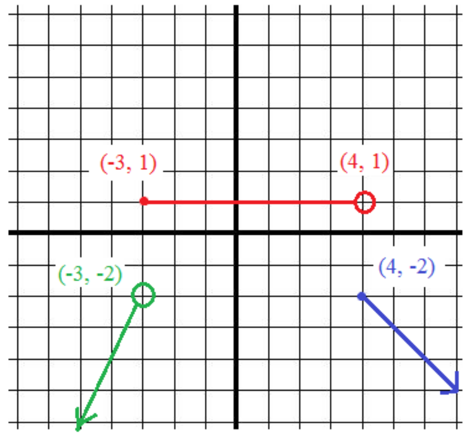
Example:

$$f(x) = \begin{cases} 2x + 4 & \text{if} \quad x < -3 \\ 1 & \text{if} \quad -3 \leq x < 4 \\ -x + 2 & \text{if} \quad x \geq 4 \end{cases}$$

Start at $x = -3$:
$f(-3) = 2(-3) + 4 = -2$
since $x < -3$, it's an open circle
$2x + 4$ is a line with slope 2, so extend a line to the left...

Start at $x = -3$:
$f(-3) = 1$
since $x \geq -3$, it's a closed circle
"$y = 1$" is a horizontal line that extends to $x = 4$ (open circle)

Start at $x = 4$:
$f(4) = -(4) + 2 = -2$
since $x \geq 4$, it's a closed circle
$-x + 2$ is a line with slope -1, so extend a line to the right



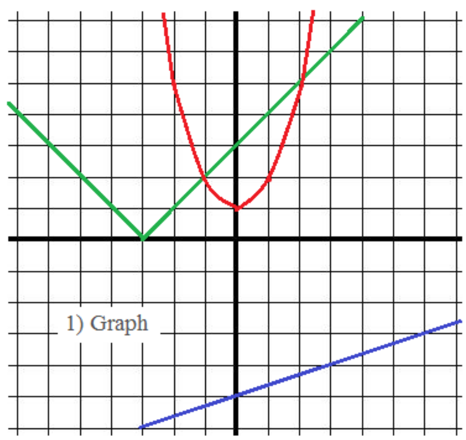
"Endpoint and Extend"
For each domain piece:
1) Find endpoint
2) Open/Close circle
3) Extend

Method 2: "Graph and Cut"

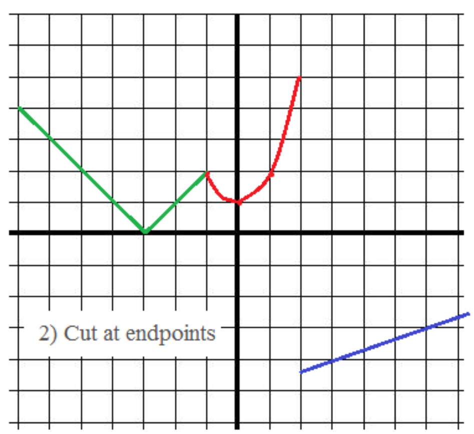
Example:

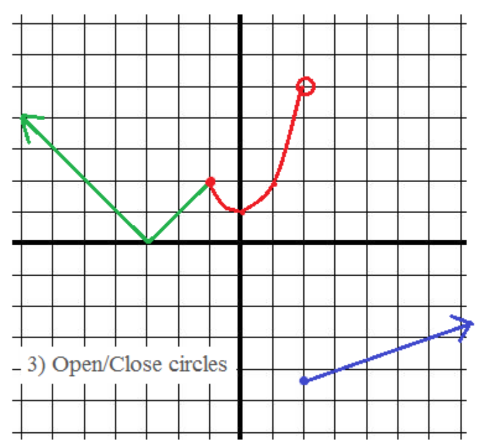
$$f(x) = \begin{cases} |x + 3| & \text{if} \quad x < -1 \\ x^2 + 1 & \text{if} \quad -1 \leq x < 2 \\ \frac{1}{3}x - 5 & \text{if} \quad x \geq 2 \end{cases}$$

absolute value function $|x + 3|$
parabola $x^2 + 1$
line $\frac{1}{3}x - 5$



"Graph and Cut"
For each domain piece:
1) Graph the function
2) Cut at the endpoints of the domain
3) Open/Close circle





Example: Find the value(s) of x, such that $f(x) = 2$.
Then, graph to confirm your answer.

$$f(x) = \begin{cases} 2x^2 - 6 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ -8 + x & \text{if } x > 1 \end{cases}$$

obviously, $f(1) = 2$

then, for the 3rd equation: $-8 + x$...

$f(10) = 2$

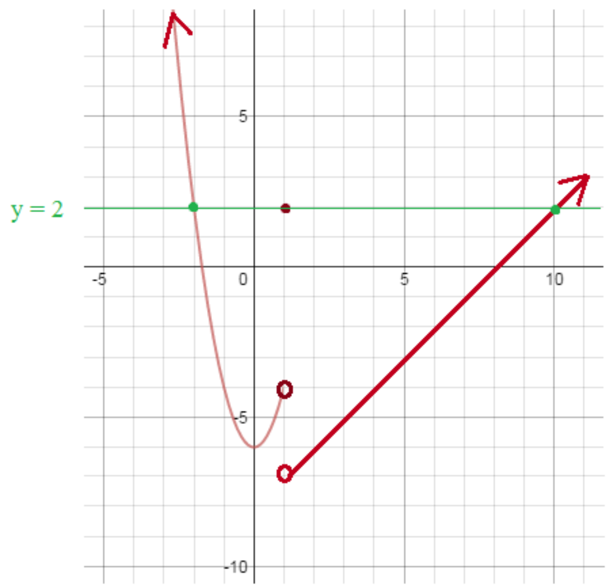
and, for the 1st equation: $2x^2 - 6 = 2$

$2x^2 = 8$

$f(-2) = 2$ $\qquad x = -2$ or 2...

Since this equation only applies if $x < 1$, we only consider -2

$x = -2, 1, 10$



Example: If g(x) is continuous, what are m and d?
Graph this continuous piecewise function to verify.

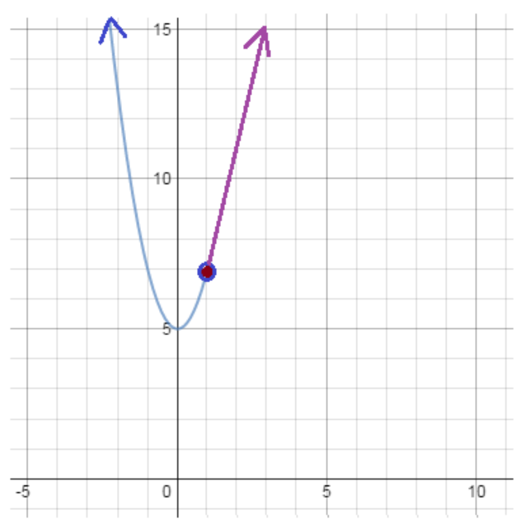
$$g(x) = \begin{cases} 2x^2 + 5 & \text{if } x < 1 \\ m & \text{if } x = 1 \\ 4x + d & \text{if } x > 1 \end{cases}$$

for $x < 1$, the equation $2x^2 + 5$ ends at $(1, 7)$

therefore, at $x = 1$, m must be 7

and, since m is 7, $4x + d = 7$.... d must be 3

(at $x = 1$, all 3 equations equal 7)



Piecewise Absolute Value Functions

Examples: Write each equation as a piecewise function (i.e. using linear equations)

1) $y = 3|x + 2| + 4$

slope will be -3 on the left and $+3$ on the right

going through the point (-2, 4), we can determine the lines

vertex

$$\begin{cases} -3x + (-2) & \text{if } x < -2 \\ 3x + 10 & \text{if } x \geq -2 \end{cases}$$

2) $y = 5 - |x + 7|$

$y = -1\,|x + 7| + 5$

vertex is at (-7, 5)

slope is -1 for $x \leq -7$

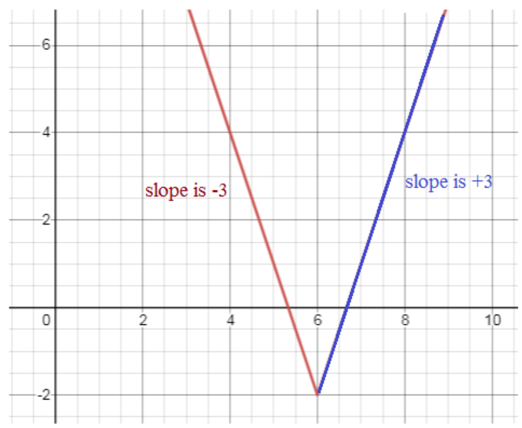
1 for $x > -7$

$$\begin{cases} -x - 2 & \text{if } x \leq -7 \\ x + 12 & \text{if } x > -7 \end{cases}$$

3) $y = 3|x + 6| - 2$

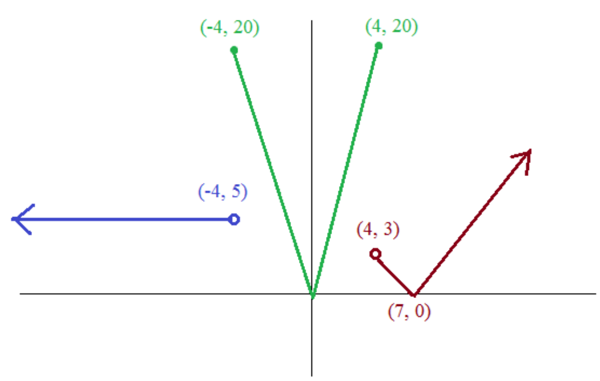
vertex where slopes change is (-6, -2)

$$f(x) = \begin{cases} 3x + 16 & \text{if } x > -6 \\ -3x - 20 & \text{if } x \leq -6 \end{cases}$$



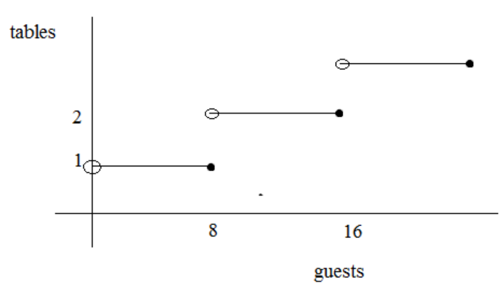
Example: Graph the following

$$f(x) = \begin{cases} 5 & \text{if } x < -4 \\ 3x + 8 & \text{if } |x| \leq 4 \\ |x - 7| & \text{if } x > 4 \end{cases}$$



Example: Application of Step Function

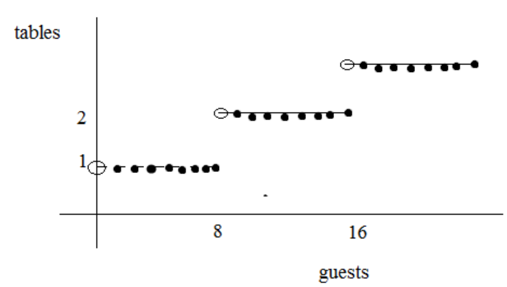
A company's banquet will be held at a club where the tables seat 8 people.
Show a graph representing the number of tables needed as a function of guests..
Then, write the equation...



$$f(x) = [[\, \frac{x}{8} \,]] + 1$$

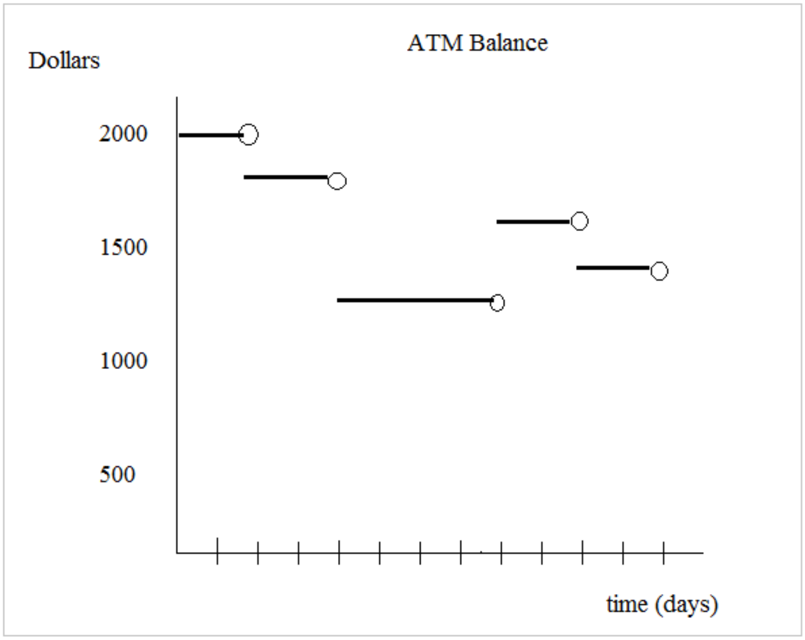
where x is number of guests..

Since you can't have "partial people",



Models: Using Piecewise functions

Example:



What is the initial balance?

Initial balance occurs when t = 0...
Therefore, initial balance is 2000 dollars

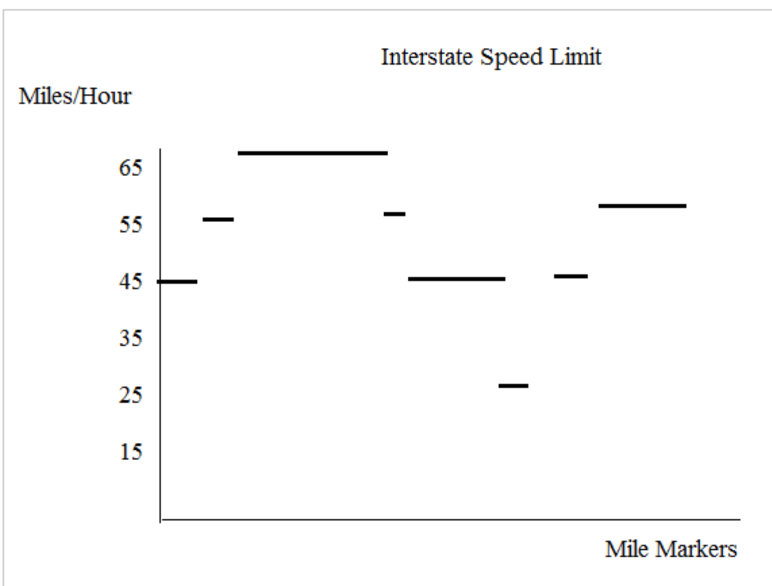
How many withdrawals were made?

If a withdrawal occurs, the amount will "gap lower"...
This occurs 3 times...

How many deposits?

If a deposit occurs, the amount will "gap higher"..
This occurs once..

Example:



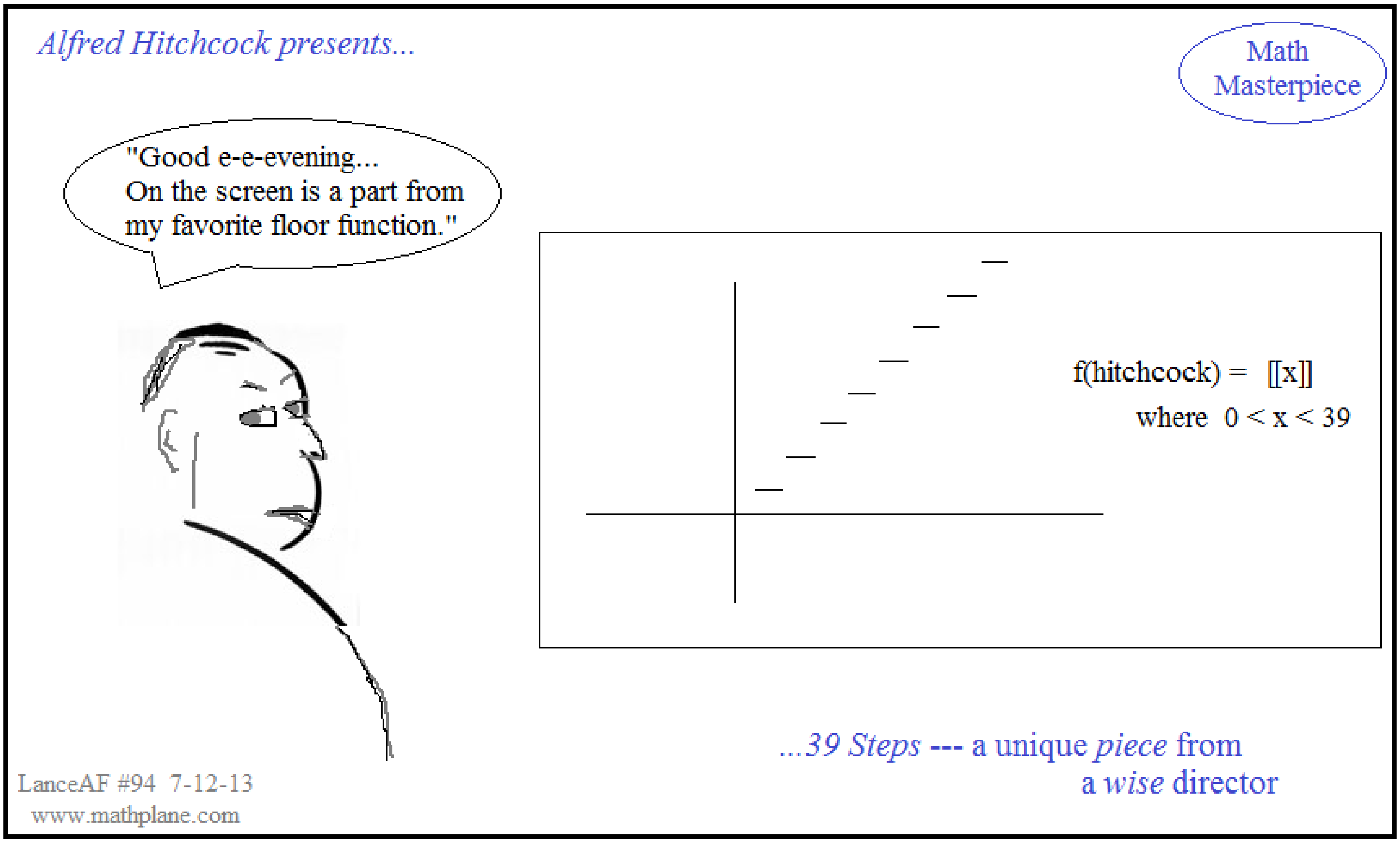
Explain a possible representation of the graph.

Each discontinuity represents a speed limit sign.

The 65 mph would occur on an interstate highway.

The drop to 25 mph would occur when the road passes through a town.

(Note: this is just a model. It's unlikely a car would instantaneously change speeds. Instead the graph would be continuous.)



Practice Exercises -→

Solving and Graphing f(x) Functions

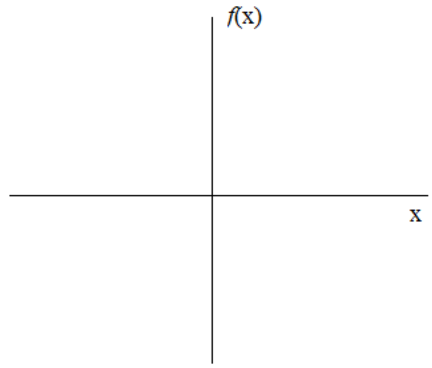
Find the solutions AND graph each function.

1) $f(x) = 3x + 2$

a) $f(2) =$

b) $f(0) =$

c) $f(-6) =$

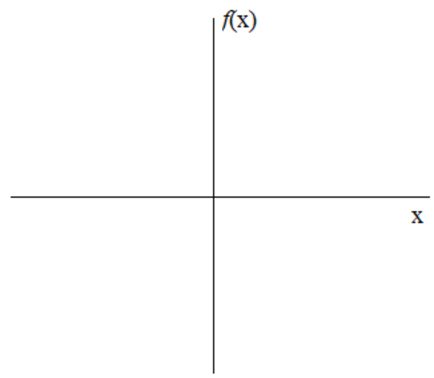


2) $f(x) = |x - 4| + 1$

a) $f(5) =$

b) $f(-5) =$

c) $f(2) =$

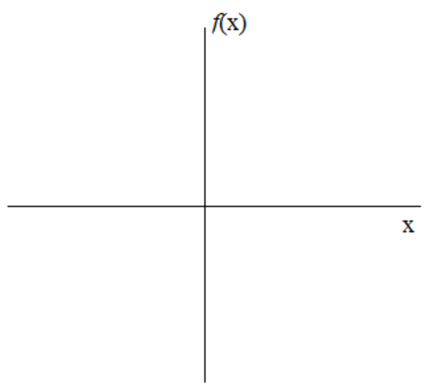


3) $f(x) = \begin{cases} x + 3 & \text{if } x < 4 \\ x - 3 & \text{if } x \geq 4 \end{cases}$

a) $f(0) =$

b) $f(7) =$

c) $f(4) =$

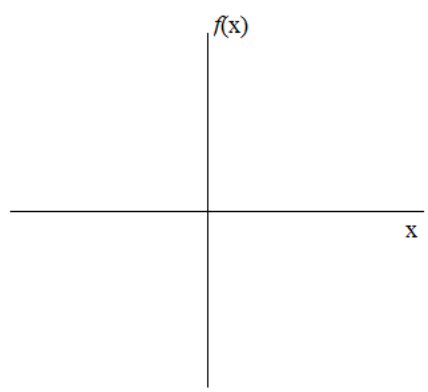


4) $f(x) = \begin{cases} 2x - 7 & \text{if } x < -7 \\ 3 & \text{if } -7 \leq x < 5 \\ x^2 - 12 & \text{if } x \geq 5 \end{cases}$

a) $f(-8) =$

b) $f(0) =$

c) $f(7) =$



Piecewise Functions Quiz

I. Function notation - answer the following:

a)

$$f(x) = \begin{cases} x + 2, & \text{if } x < 3 \\ x + 7, & \text{if } x \geq 3 \end{cases}$$

$f(-5) =$

$f(3) =$

$f(5) =$

b)

$$g(x) = \begin{cases} 3x + 2, & \text{if } x < -6 \\ 5, & \text{if } -6 \leq x < 10 \\ x^2, & \text{if } x \geq 10 \end{cases}$$

$g(0) =$

$g(-6) =$

$g(10) =$

c)

$$j(x) = \begin{cases} -10, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 10, & \text{if } x > 0 \end{cases}$$

$j(-25) =$

$j(1/2) =$

$j(0) =$

d)

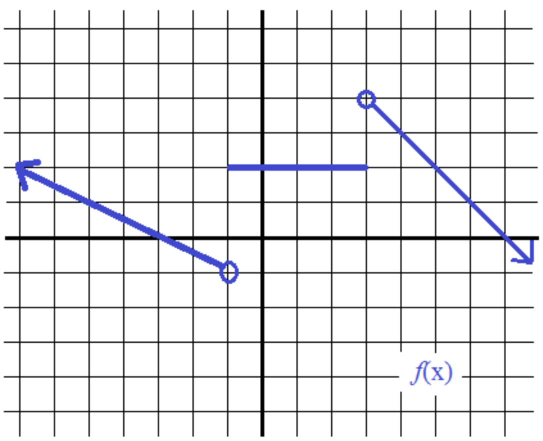
$$h(t) = \begin{cases} \sqrt{(-t)}, & \text{if } t < 0 \\ 5, & \text{if } 0 \leq t < 5 \\ -2x, & \text{if } 5 \leq t \end{cases}$$

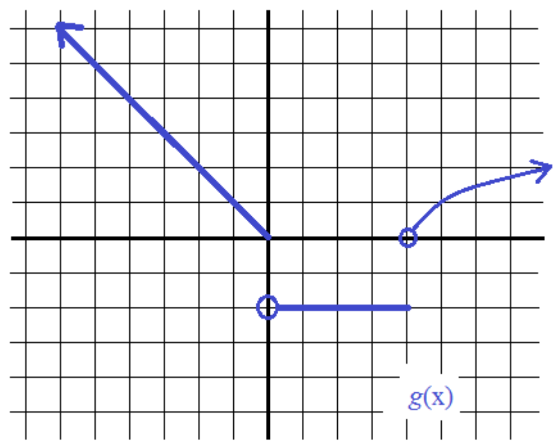
$h(-4) =$

$h(5) =$

$h(10) =$

II. Using a graph -- answer the following





$f(-5) =$

$f(-1) =$

$f(1) =$

$f(7) =$

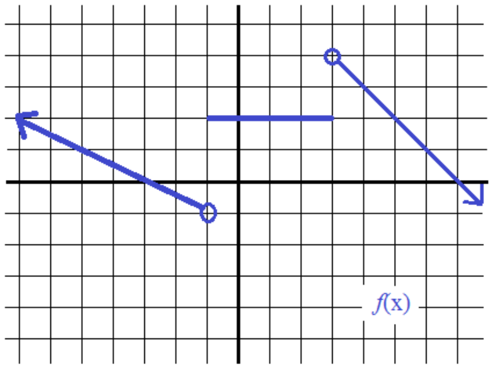
$g(-3) =$

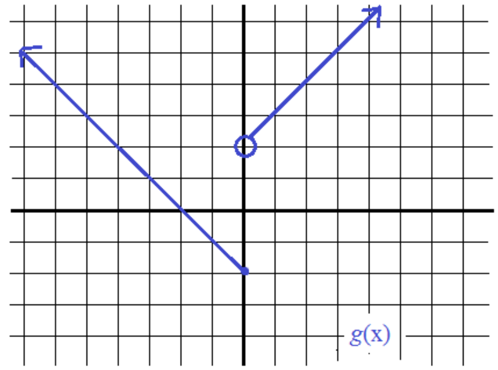
$g(4) =$

$g(5) =$

$g(-20) =$

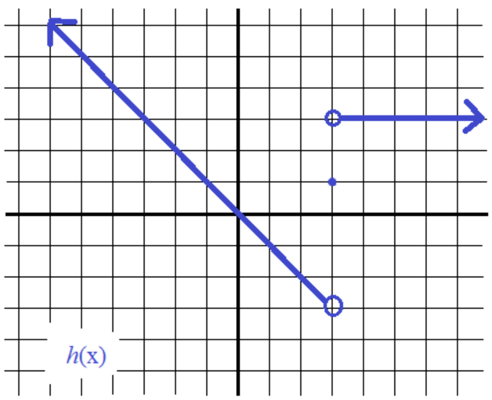
III. Identifying the Piecewise function -- write an expression to describe the graph

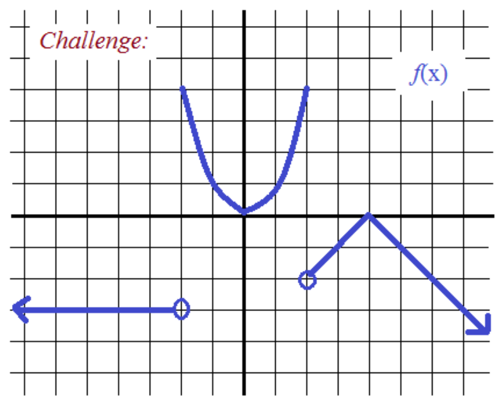




$f(x) = \{$

$g(x) = \{$





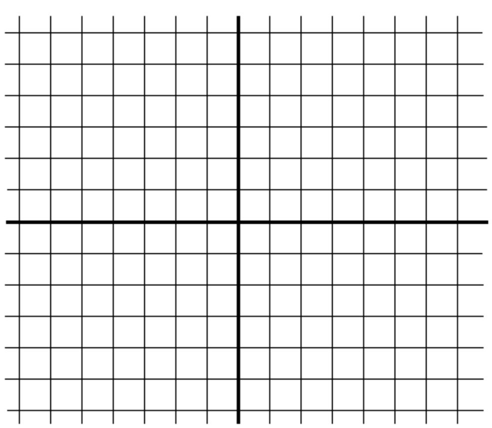
$h(x) = \{$

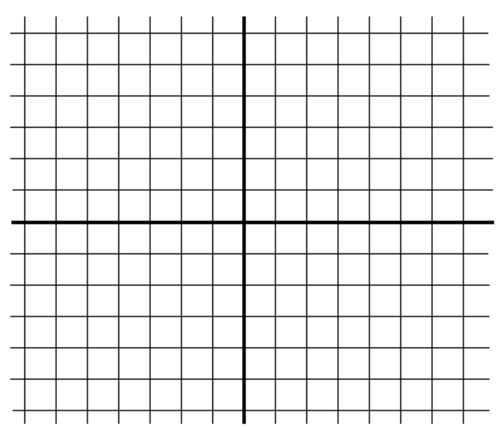
$f(x) = \{$

IV: Graphing Piecewise functions

$$f(x) = \begin{cases} 4, & \text{if } x < 3 \\ -x + 3, & \text{if } x \geq 3 \end{cases}$$

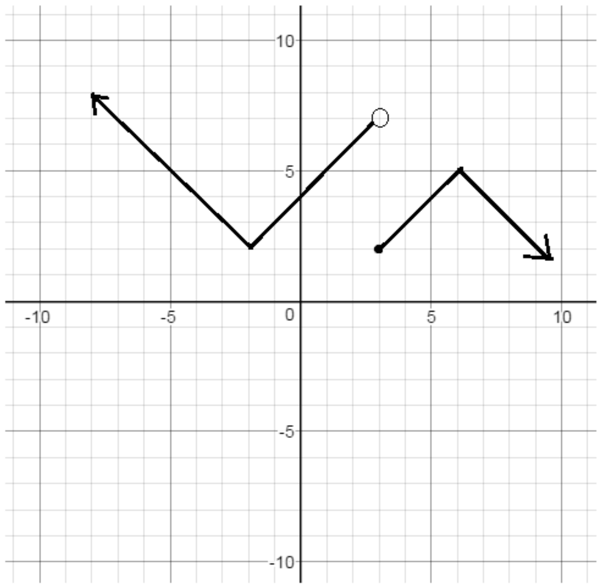
$$g(x) = \begin{cases} 2x, & \text{if } x < -3 \\ |x|, & \text{if } -3 \leq x < 3 \\ 5, & \text{if } x \geq 3 \end{cases}$$





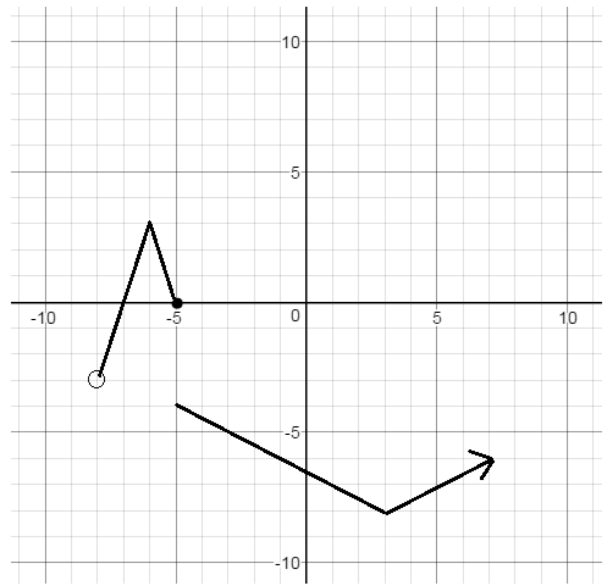
V. Use a minimal number of "pieces" to describe the graphs...

1)



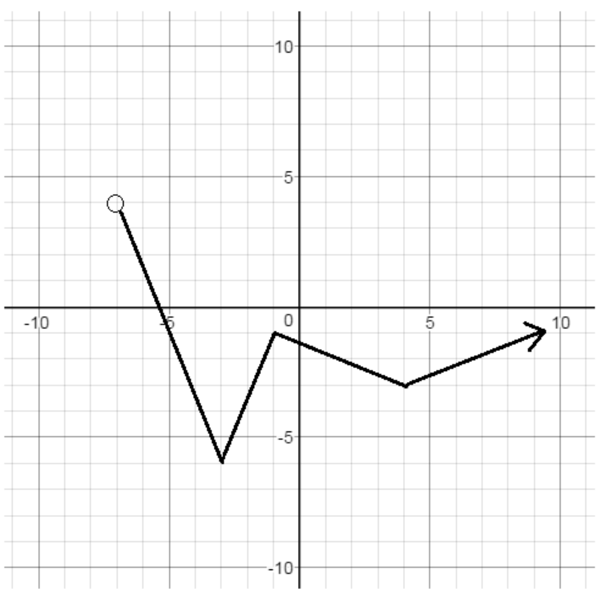
$f(x) = \Bigg\{$

2)



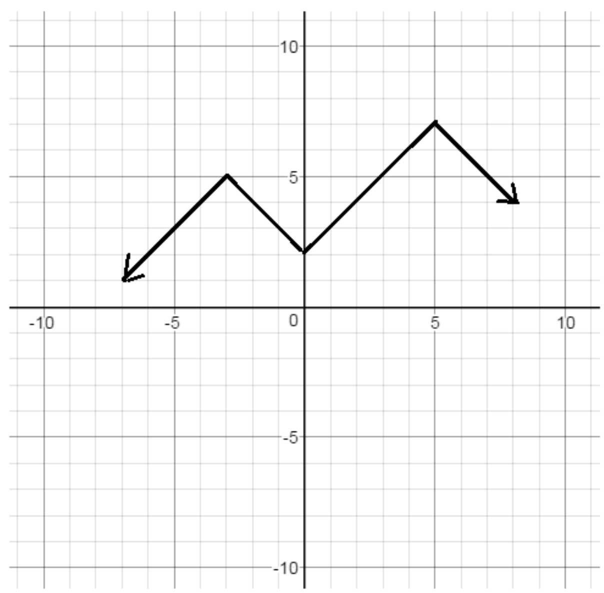
$g(x) = \Bigg\{$

3)



$h(x) = \Bigg\{$

4)



$p(x) = \Bigg\{$

Piecewise Functions: Linear pieces

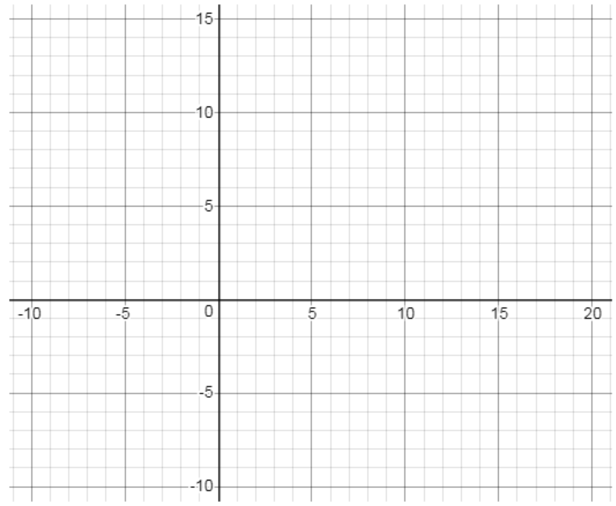
Graph the following piecewise functions. Then, identify the domain and range.

1)

$$f(x) = \begin{cases} x + 3 & \text{if} \quad x < 2 \\ -1 & \text{if} \quad 2 \le x < 6 \\ -x + 10 & \text{of} \quad x \ge 6 \end{cases}$$

domain:

range:

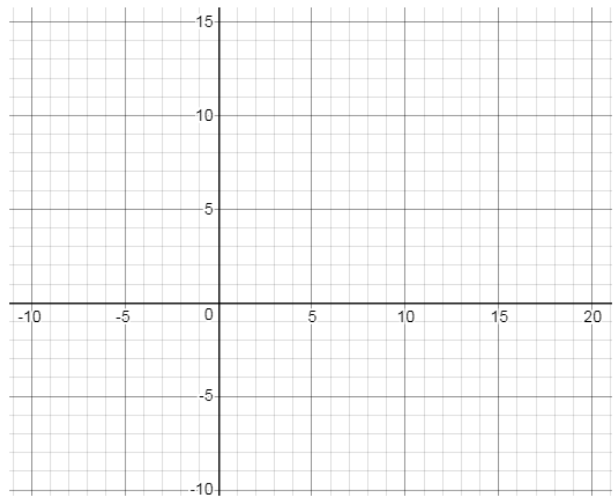


2)

$$g(x) = \begin{cases} 3 - 4x & \text{if} \quad x < 0 \\ 5 & \text{if} \quad 3 \le x \le 7 \\ -x + 10 & \text{if} \quad x \ge 12 \end{cases}$$

domain:

range:

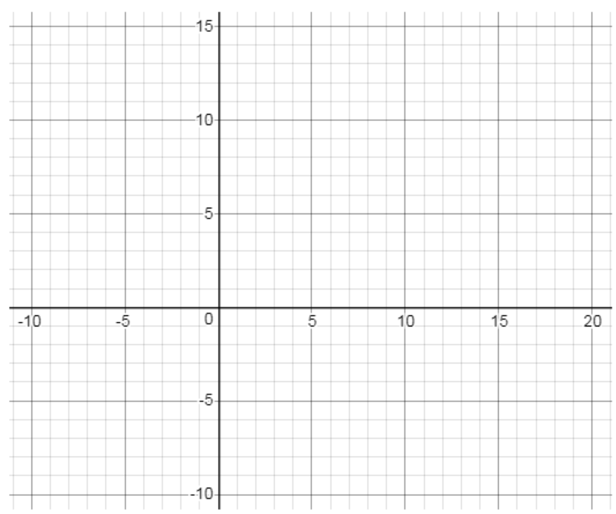


3)

$$h(t) = \begin{cases} 2t + 2 & \text{if} \quad 0 < t \le 4 \\ t + 6 & \text{if} \quad 4 < t \le 8 \\ 14 & \text{if} \quad t > 8 \end{cases}$$

domain:

range:



Describe the following piecewise functions. Determine the domain and range.

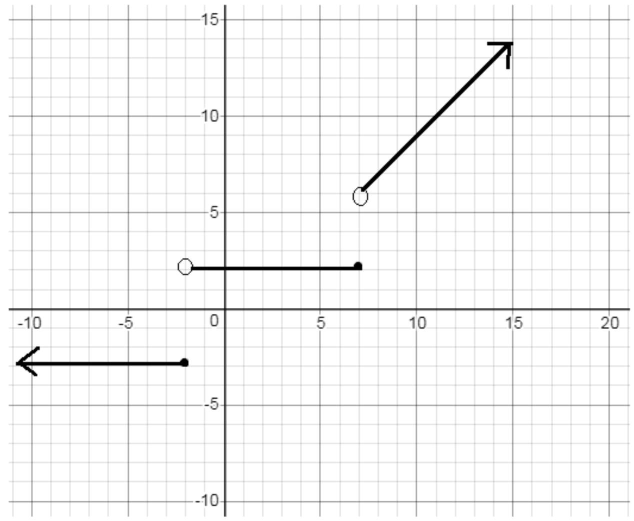
Piecewise Functions: Linear pieces

4)

$f(x) = \begin{cases} & \text{if} \\ & \text{if} \\ & \text{if} \end{cases}$

domain:

range:

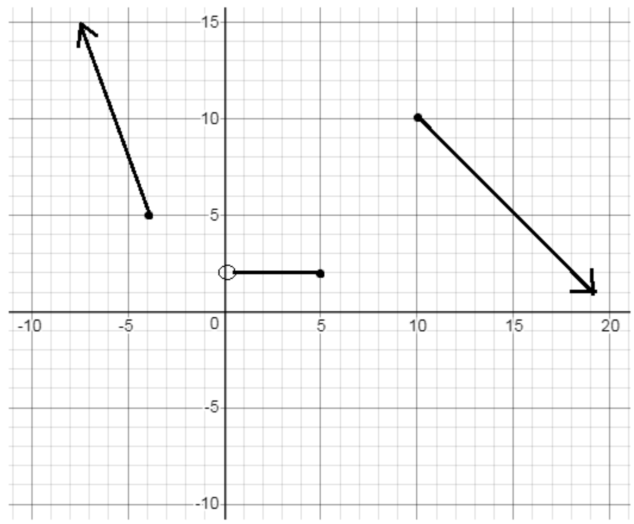


5)

$g(x) = \begin{cases} & \text{if} \\ & \text{if} \\ & \text{if} \end{cases}$

domain:

range:

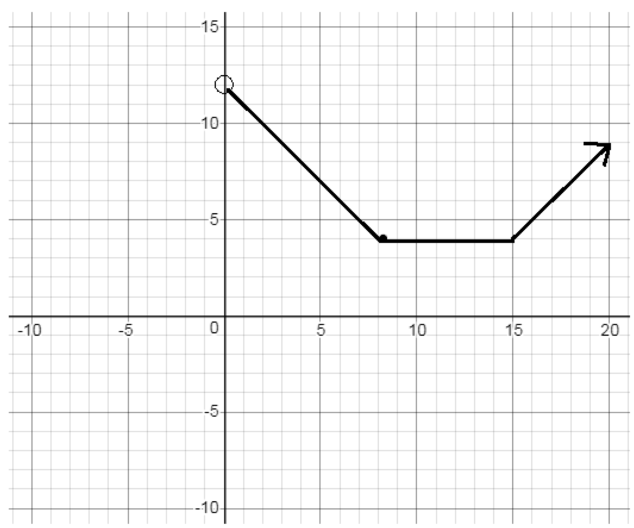


6)

$h(x) = \begin{cases} & \text{if} \\ & \text{if} \\ & \text{if} \end{cases}$

domain:

range:



Piecewise Functions: mixed pieces

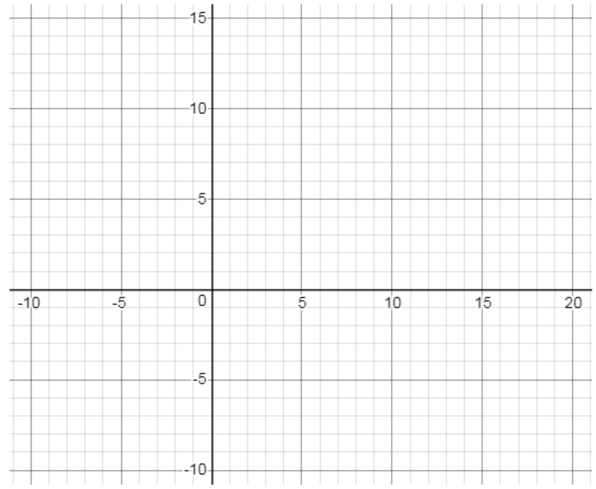
Graph the following piecewise functions. Then, identify the domain and range.

7)

$$f(x) = \begin{cases} x + 15 & \text{if } x \leq -5 \\ -|x| + 2 & \text{if } -5 < x < 5 \\ \sqrt{x-5} + 3 & \text{if } x \geq 5 \end{cases}$$

domain:

range:

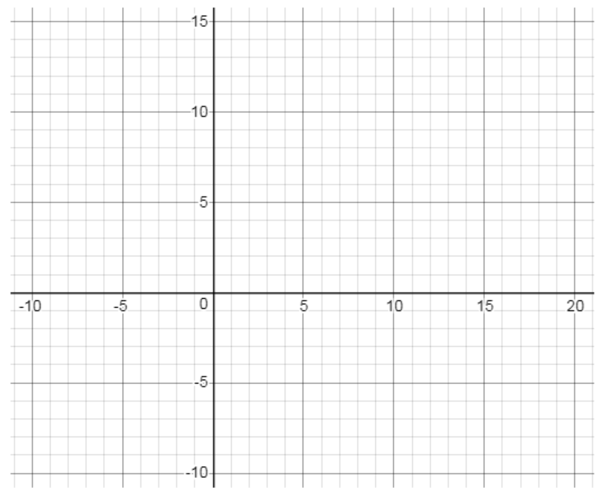


8)

$$g(x) = \begin{cases} \sqrt{-6-x} + 8 & \text{if } x < -6 \\ (x+3)^2 + 1 & \text{if } -6 \leq x \leq 1 \\ \frac{1}{2}x + 7 & \text{if } x > 4 \end{cases}$$

domain:

range:

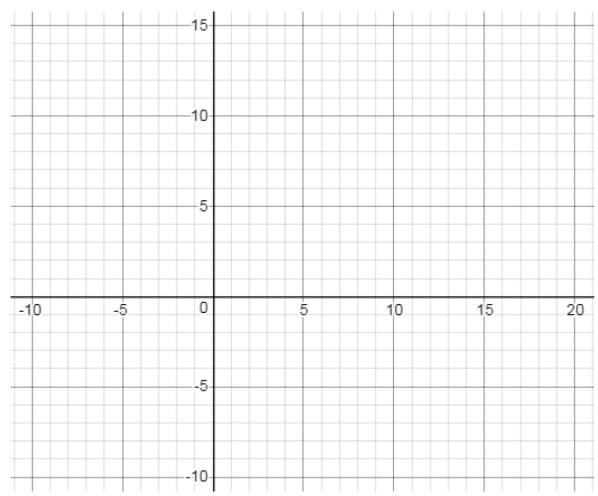


9)

$$h(x) = \begin{cases} x & \text{if } x < 1 \\ \frac{1}{2}(2^x) & \text{if } 1 \leq x < 4 \\ 2|x - 10| - 4 & \text{if } x \geq 4 \end{cases}$$

domain:

range:



VI. Piecewise Models (Word Problems) and Concepts

1) A discount book store charges \$4 per book..
   If a customer buys more than 5, the price drops to \$3.50 per book...

   Write a piecewise function to model the cost of books..
   How much would 20 books cost?

2) A store sells t-shirts...

   It charges \$10 per shirt for the first batch of 50...
   Since the store has the screen design,
   the next batch of 50 would cost \$9 per shirt...
   And, all batches after that would cost \$8 per shirt....

   Write a piecewise function describing the cost of shirts...
   How much would 120 shirts cost?

3) A shop down the street sells hats...

   It charges \$10 per hat.
   If a customer purchases more than 30 hats, the owner offers
   a \$1 discount per hat. (\$9 per hat)
   If a customer buys more than 50 hats, the owner offers another
   \$1 discount (\$8 per hat).

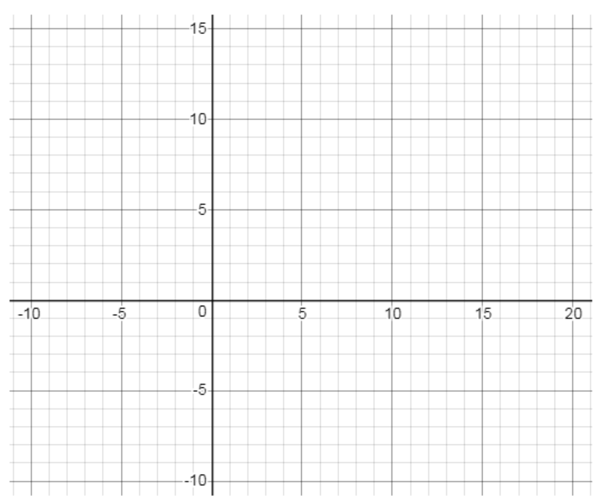
   Write a piecewise function to describe the cost of hats.
   The math club has a budget of \$300. How many hats could it buy?

4) Write and graph a piecewise function with the following characteristics.

   Domain: all real numbers

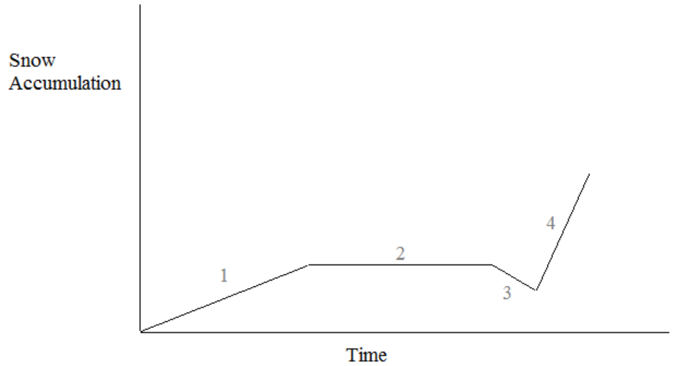
   Range: { -4, 2, 5 }

   $f(3) = 2$ and $f(-3) = 5$



5) Match the events with the piecewise function

a) The snow falls for an hour.
b) The snow stops..
c) A blizzard arrives..
d) The snow melts..



6) Betty and Diane are writing wedding invitations.
Betty starts at 8:00 AM and Diane starts at 10:00 AM.
Betty can write 20 invitations per hour.
And, Diane can write 25 invitations per hour.

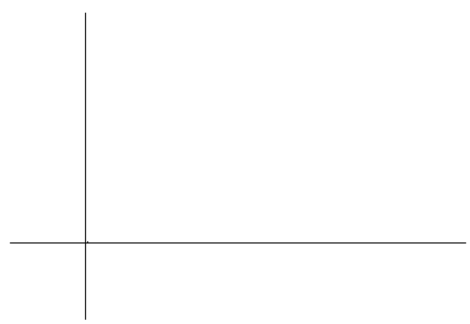
a) How many invitations will be written by 2:00 PM?

b) When will they finish 355 invitations?

c) If they write all day, when will Betty and Diane have the same number?

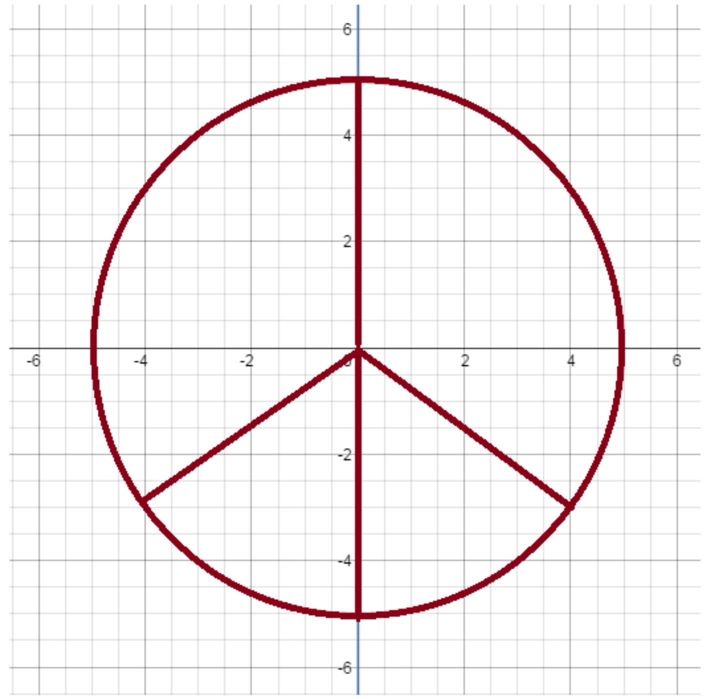
7) You and a friend can each make 1 sandwich every 2 minutes.
At lunchtime, you start making sandwiches for the camp. Ten minutes later, your friend shows up and helps you.

Write and graph a (piecewise) model of sandwiches made as a function of time.



Peace-wise Function

$$g(x) = \begin{cases} \frac{3}{4}x & \text{if } -4 \leq x \leq 0 \\ \pm\sqrt{25 - x^2} & \text{if } -5 \leq x \leq 5 \\ -\frac{3}{4}x & \text{if } 0 \leq x \leq 4 \\ \text{all real numbers between } -5 \text{ and } 5 & \text{if } x = 0 \end{cases}$$



This is a tremendous function... (even if it fails the vertical line test!)

SOLUTIONS -→

Solving and Graphing $f(x)$ Functions

SOLUTIONS

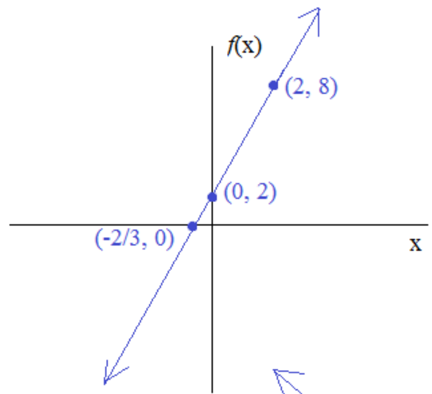
Find the solutions AND graph each function.

1) $f(x) = 3x + 2$

a) $f(2) = 3(2) + 2 = 8$

b) $f(0) = 3(0) + 2 = 2$

c) $f(-6) = 3(-6) + 2 = -16$



2) $f(x) = |x - 4| + 1$

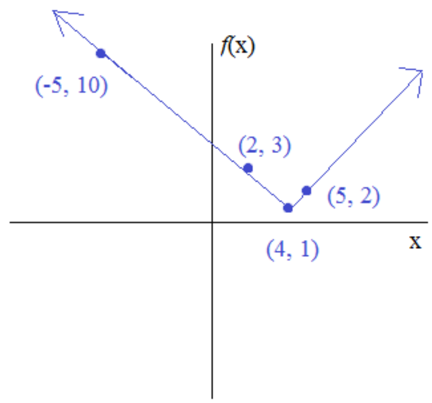
a) $f(5) = |(5) - 4| + 1 = 2$

b) $f(-5) = |(-5) - 4| + 1 = 10$

c) $f(2) = |(2) - 4| + 1 = 3$

vertex: (4, 1)

absolute value
is "v shaped"

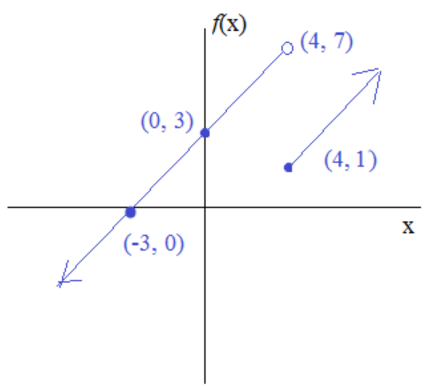


3) $f(x) = \begin{cases} x + 3 & \text{if } x < 4 \\ x - 3 & \text{if } x \geq 4 \end{cases}$

a) $f(0) = (0) + 3 = 3$ (1st piece)

b) $f(7) = (7) - 3 = 4$ (2nd piece)

c) $f(4) = (4) - 3 = 1$ (2nd piece)

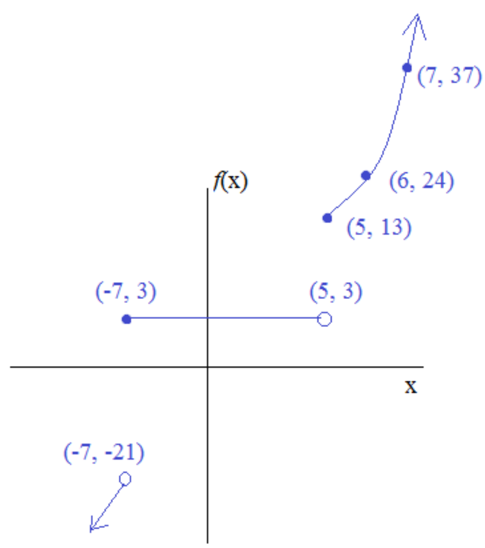


4) $f(x) = \begin{cases} 2x - 7 & \text{if } x < -7 \\ 3 & \text{if } -7 \leq x < 5 \\ x^2 - 12 & \text{if } x \geq 5 \end{cases}$

a) $f(-8) = 2(-8) - 7 = -23$

b) $f(0) = 3$

c) $f(7) = 49 - 12 = 37$



Piecewise Functions Quiz

www.mathplane.com

SOLUTIONS

I. Function notation - answer the following:

a) $f(x) = \begin{cases} x+2, & \text{if } x < 3 \\ x+7, & \text{if } x \geq 3 \end{cases}$

$f(-5) =$ $(-5) + 2 = -3$

$f(3) =$ $(3) + 7 = 10$

$f(5) =$ $(5) + 7 = 12$

b) $g(x) = \begin{cases} 3x+2, & \text{if } x < -6 \\ 5, & \text{if } -6 \leq x < 10 \\ x^2, & \text{if } x \geq 10 \end{cases}$

$g(0) =$ since 0 is between -6 and 10, the output is 5

$g(-6) =$ since -6 is $\geq$ -6, the output is 5

$g(10) =$ $(10)^2 = 100$

c) $j(x) = \begin{cases} -10, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 10, & \text{if } x > 0 \end{cases}$

$j(-25) =$ -10

$j(1/2) =$ 10

$j(0) =$ 0

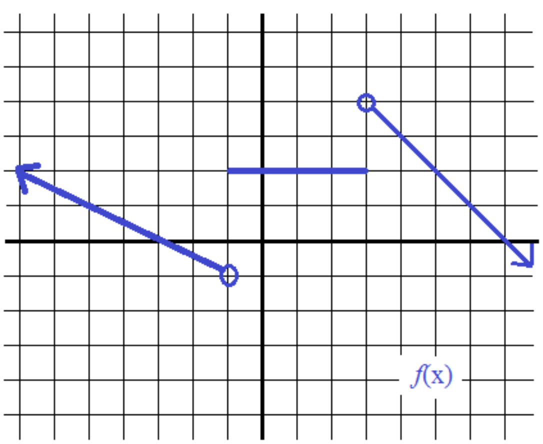
d) $h(t) = \begin{cases} \sqrt{(-t)}, & \text{if } t < 0 \\ 5, & \text{if } 0 \leq t < 5 \\ -2x, & \text{if } 5 \leq t \end{cases}$

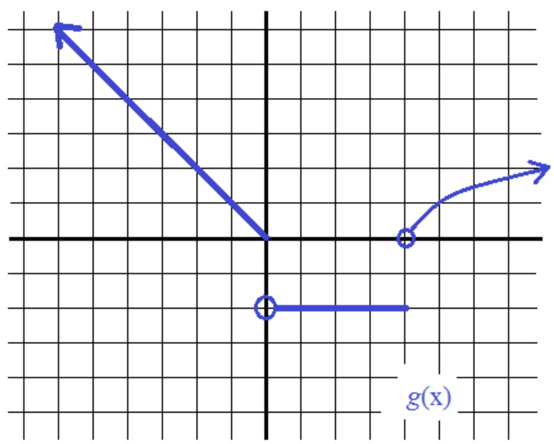
$h(-4) = \sqrt{(-(-4))} = 2$

$h(5) = -2(5) = -10$

$h(10) = -2(10) = -20$

II. Using a graph -- answer the following





$f(-5) =$ 1

$f(-1) =$ 2

$f(1) =$ 2

$f(7) =$ 0

$g(-3) =$ 3

$g(4) =$ -2

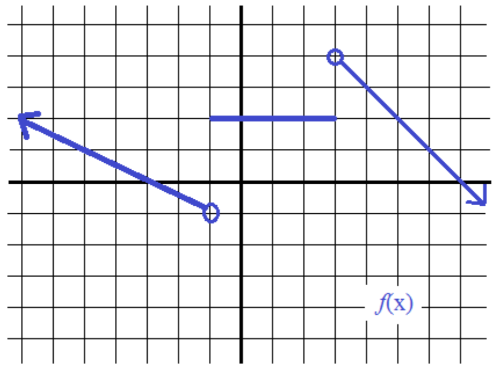
$g(5) =$ 1

$g(x) = -x \quad \text{if } x \leq 0$

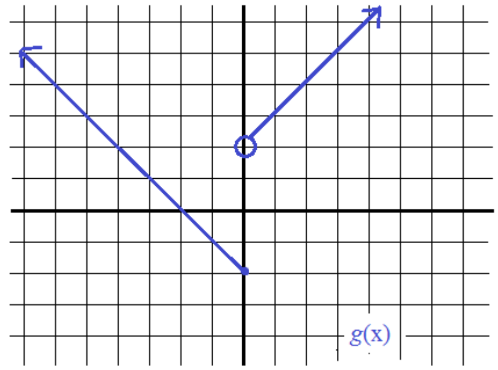
$g(-20) =$ 20

III. Identifying the Piecewise function -- write an expression to describe the graph

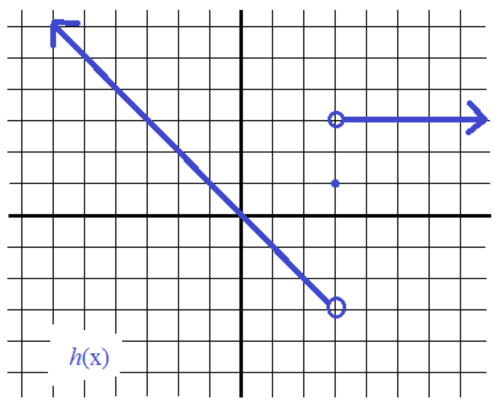
SOLUTIONS



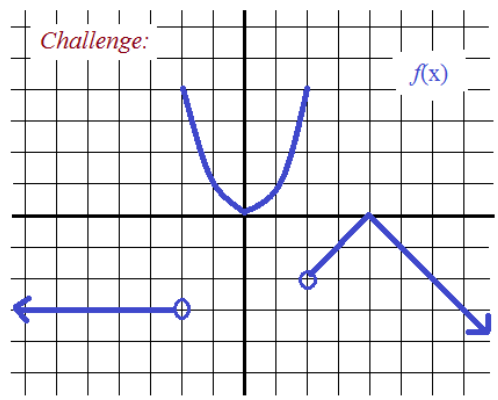
$$f(x) = \begin{cases} -1/2(x) - 3/2 & \text{if } x < -1 \\ 2 & \text{if } -1 \le x \le 3 \\ -x + 7 & \text{if } x > 3 \end{cases}$$



$$g(x) = \begin{cases} x + 2 & \text{if } x > 0 \\ -x - 2 & \text{if } x \le 0 \end{cases}$$



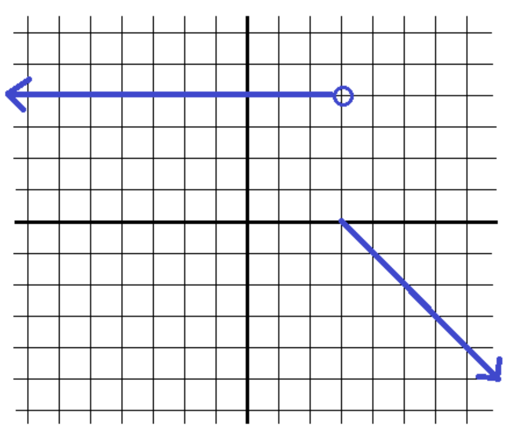
$$h(x) = \begin{cases} 3 & \text{if } x > 3 \\ 1 & \text{if } x = 3 \\ -x & \text{if } x < 3 \end{cases}$$



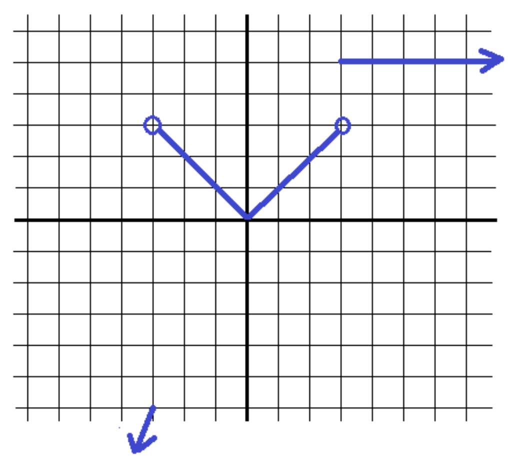
$$f(x) = \begin{cases} -3 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -|x - 4| & \text{if } x > 2 \end{cases}$$

IV: Graphing Piecewise functions

$$f(x) = \begin{cases} 4, & \text{if } x < 3 \\ -x + 3, & \text{if } x \ge 3 \end{cases}$$

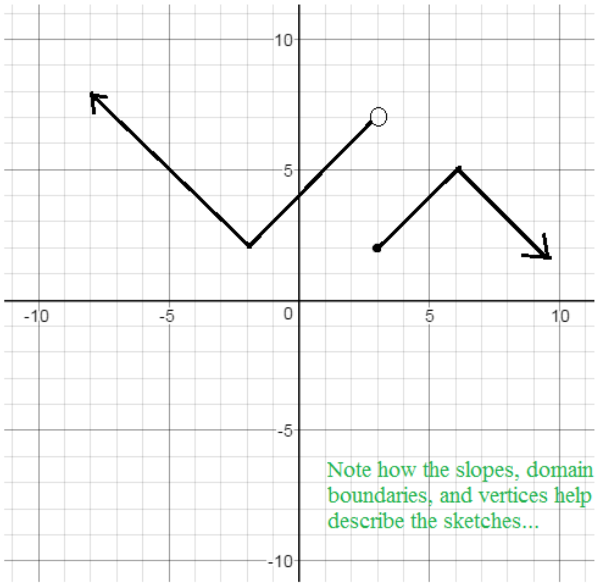


$$g(x) = \begin{cases} 2x, & \text{if } x < -3 \\ |x|, & \text{if } -3 \le x < 3 \\ 5, & \text{if } x \ge 3 \end{cases}$$



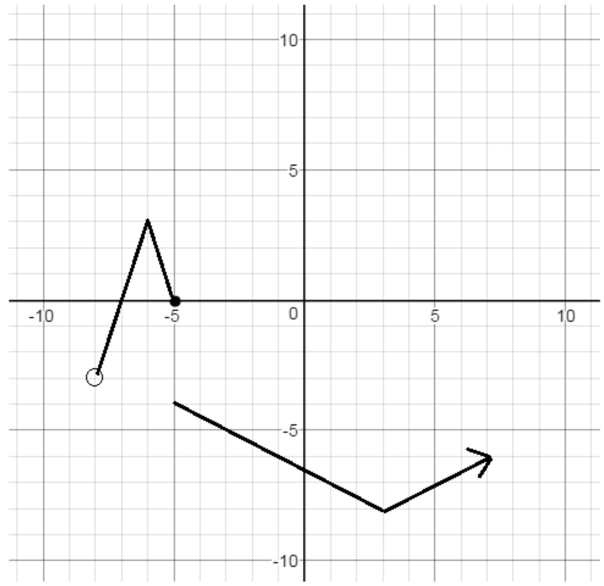
V. Use a minimal number of "pieces" to describe the graphs... SOLUTIONS

1)



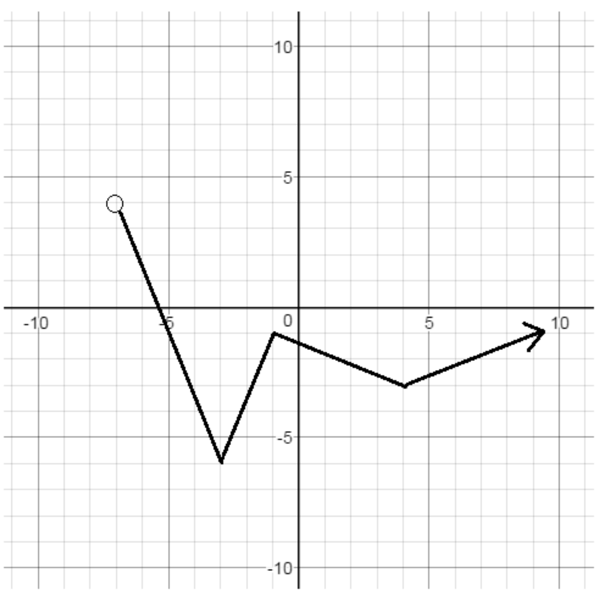
$$f(x) = \begin{cases} |x+2| + 2 & \text{if } x < 3 \\ -|x-6| + 5 & \text{if } x \geq 3 \end{cases}$$

2)



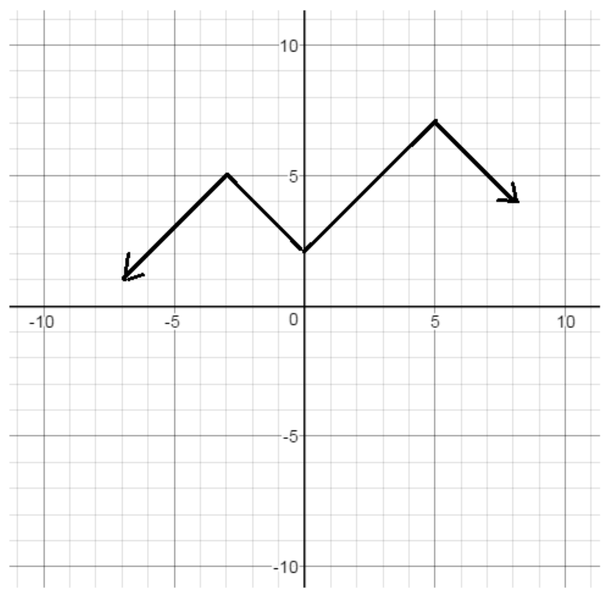
$$g(x) = \begin{cases} -3|x+6| + 3 & \text{in the interval } (-8, -5] \\ \frac{1}{2}|x-3| - 8 & \text{in the interval } [-5, \infty) \end{cases}$$

3)



$$h(x) = \begin{cases} \frac{5}{2}|x+3| - 6 & \text{if } -7 < x \leq -1 \\ \frac{2}{5}|x-4| - 3 & \text{if } x > -1 \end{cases}$$

4)



$$p(x) = \begin{cases} -|x+3| + 5 & \text{in the interval } (-\infty, 0) \\ -|x-5| + 7 & \text{in the interval } (0, \infty) \end{cases}$$

Graph the following piecewise functions. Then, identify the domain and range.

SOLUTIONS

Piecewise Functions: Linear pieces

1)

$$f(x) = \begin{cases} x+3 & \text{if} \quad x < 2 \\ -1 & \text{if} \quad 2 \leq x < 6 \\ -x+10 & \text{of} \quad x \geq 6 \end{cases}$$

All x values (i.e. places on the graph "left to right")

domain: all real numbers

$(-\infty \ , \ \infty \ )$

All $f(x)$ values (i.e. places on the graph "bottom to top")

range: $f(x) < 5$

$(-\infty \ , 5 \ )$

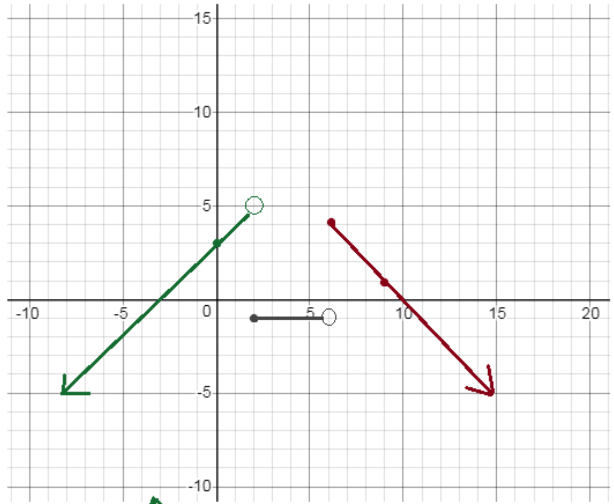
Since the 3 "pieces" will be lines segments or rays, we can use an 'endpoint method' to graph.

The endpoint (boundary) of the first piece occurs at (2, 5). Since it is $x < 2$, it's an open circle.

Then, pick a point left of 2 --- such as (0, 3)... Put that point on the graph and extend a ray from (2, 5) through (0, 3)

The endpoints of the second piece are (2, -1) and (6, -1). Since inputs are $2 \leq x < 6$, the left endpoint is closed and the right endpoint is open...

Finally, the boundary of the 3rd piece occurs at (6, 4)... Then, we can draw a ray through (9, 1)...



2)

$$g(x) = \begin{cases} 3 - 4x & \text{if} \quad x < 0 \\ 5 & \text{if} \quad 3 \leq x \leq 7 \\ -x + 10 & \text{if} \quad x \geq 12 \end{cases}$$

domain: it's described in the list of "if" statements!!

$x < 0$ or $3 \leq x \leq 7$ or $x \geq 12$

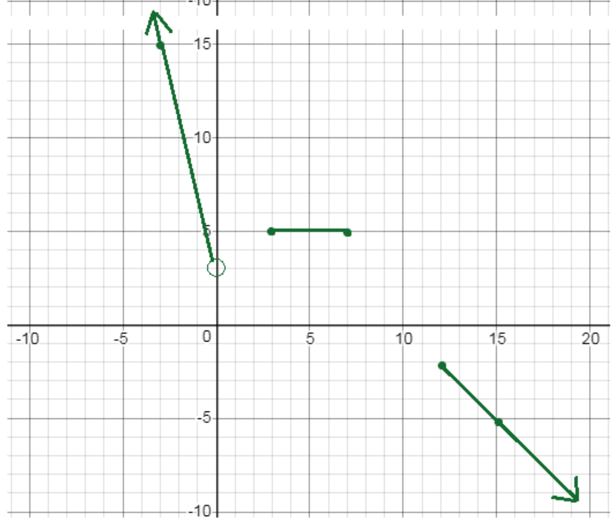
range: $g(x) \leq -2$ or $g(x) > 3$

$(-\infty \ , -2 \,]$ U $(3, \ \infty \ )$

The 1st piece, we can use the boundary $x = 0$ (0, 3) open circle and, pick a point less than 0.. (-3, 15) and extend the ray...

The 2nd piece, we can use the endpoints (3, 5) and (7, 5)

The 3rd piece, we can use the endpoint $x = 12$ (12, -2) closed circle and, pick a point greater than 12.. (15, -5) and extend the ray..



3)

$$h(t) = \begin{cases} 2t + 2 & \text{if} \quad 0 < t \leq 4 \\ t + 6 & \text{if} \quad 4 < t \leq 8 \\ 14 & \text{if} \quad t > 8 \end{cases}$$

domain: $t > 0$

$(0 \ , \ \infty \ )$

range: $2 < h(t) \leq 14$

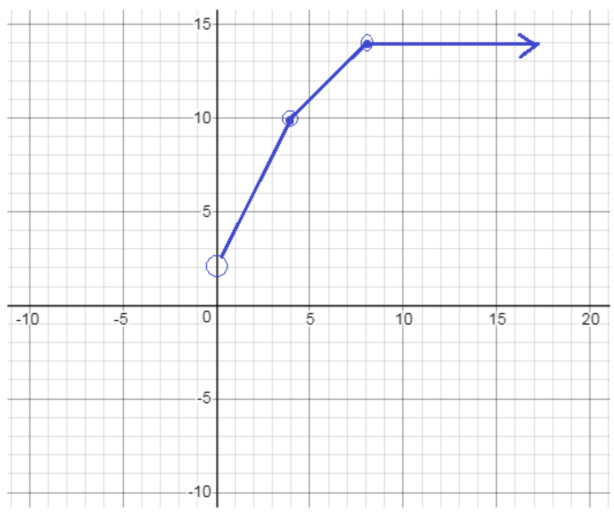
$(2 \ , \ 14 \,]$

Again, we have 3 straight pieces...

Endpoints of the 1st piece:
(0, 2) open circle
(4, 10) closed circle

Endpoints of 2nd piece:
(4, 10) open circle
(8, 14) closed circle

Endpoint of 3rd piece:
(8, 14) open circle..
then, horizontal ray extending to the right...



Describe the following piecewise functions. Determine the domain and range. SOLUTIONS

Piecewise Functions: Linear pieces

4)

$$f(x) = \begin{cases} -3 & \text{if } x \leq -2 \\ 2 & \text{if } -2 < x \leq 7 \\ x - 1 & \text{if } x > 7 \end{cases}$$

Looking at the domain ('movement from left to right') & the endpoints of each piece we can determine the "if" statements..

$<$ for open circles
$\leq$ for closed circles

domain: all real numbers

$(-\infty\ ,\ \infty\ )$

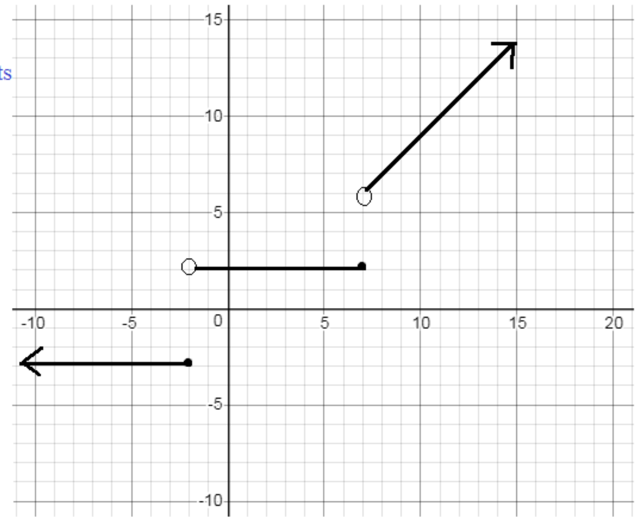
range: { -3, 2, and all reals > 6 }

$[-3] \cup [2] \cup (6,\ \infty)$

1st piece is a horizontal line " y = -3 " where every output is -3

2nd piece is a horizontal line " y = 2 " where every output is 2

3rd piece is ray where the slope is 1..
"y = 1x + b"
To find b, plug in another point... We'll use (10, 9)..
9 = 1(10) + b b = -1



5)

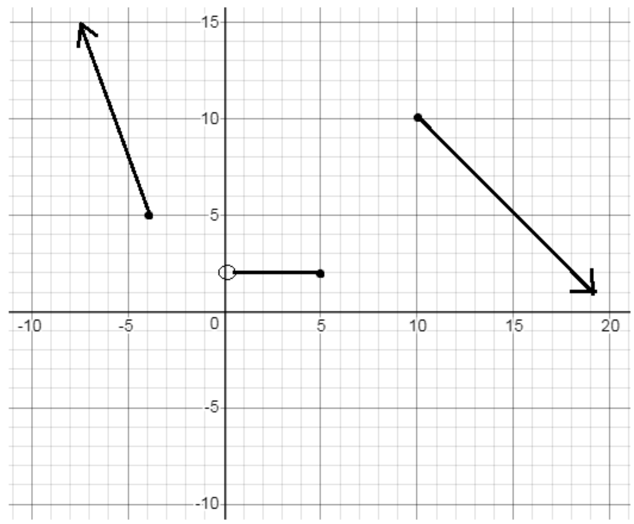
$$g(x) = \begin{cases} -3x - 7 & \text{if } x \leq -4 \\ 2 & \text{if } 0 < x \leq 5 \\ -x + 20 & \text{if } x \geq 10 \end{cases}$$

recognizing the slopes and plugging in points into y = mx + b, we can identify the 1st and 3rd pieces...

domain: (the "if" statements) $x \leq -4$ or $0 < x \leq 5$ or $x \geq 10$

range: since left piece extends indefinitely upward, and right piece extends indefinitely downward, AND they overlap, the range is all real numbers

$(-\infty\ ,\ \infty\ )$



6)

$$h(x) = \begin{cases} -x + 12 & \text{if } 0 < x \leq 8 \\ 4 & \text{if } 8 < x \leq 15 \\ x - 11 & \text{if } x > 15 \end{cases}$$

NOTE: since this is a function, there is no overlap in the "if" statements!!

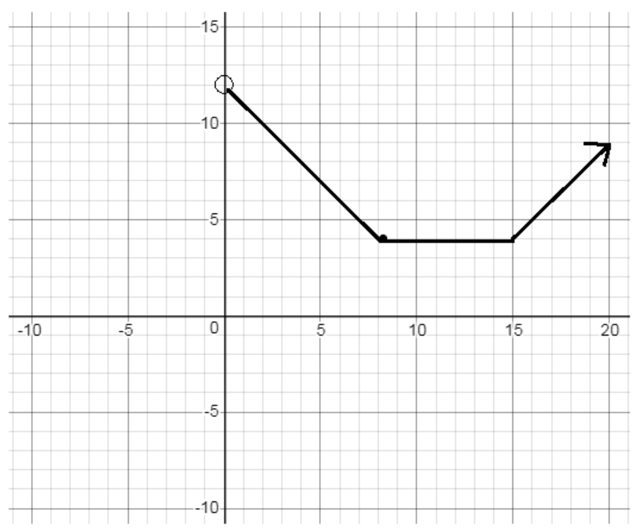
domain: $x > 0$

$(0,\ \infty)$

range: $h(x) \geq 4$

(the minimum value is 4, and it extends indefinitely upward)

$[4,\ \infty)$



Graph the following piecewise functions. Then, identify the domain and range. SOLUTIONS

Piecewise Functions: mixed pieces

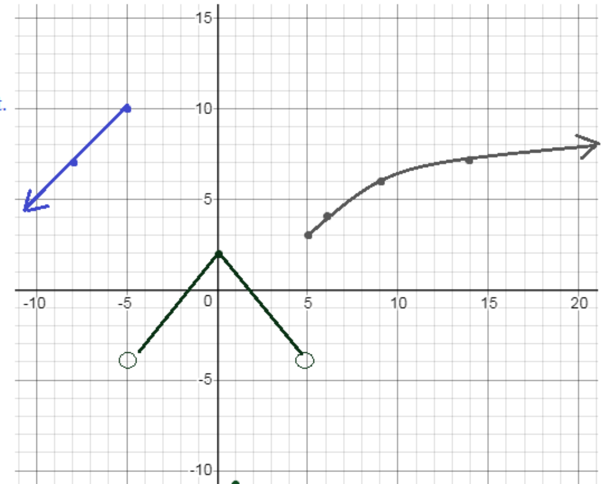
7)
$$f(x) = \begin{cases} x + 15 & \text{if } x \le -5 \\ -|x| + 2 & \text{if } -5 < x < 5 \\ \sqrt{x-5} + 3 & \text{if } x \ge 5 \end{cases}$$

Since piece 1 is linear, we'll use endpoint method... Right boundary is (-5, 10).. And, point (-8, 7) is a point to the left.

Piece 2 is an absolute value function opening downward. The vertex is (0, 2).. And, the endpoints occur at (-5, -3) and (5, -3)... since < and <. we use open circles

Piece 3 is a square root function that starts at (5, 3) and opens to the right... We'll plot a few points..

(6, 4) (9, 5) (14, 6)



domain: all real numbers

$(-\infty, \infty)$

range: Since the 1st piece extends to negative infinity and the 3rd piece extends to positive infinity, the range is all real numbers...

$(-\infty, \infty)$

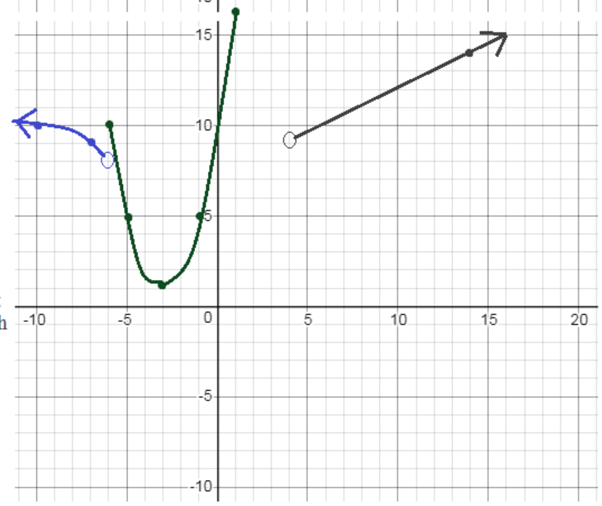
8)
$$g(x) = \begin{cases} \sqrt{-6-x} + 8 & \text{if } x < -6 \\ (x+3)^2 + 1 & \text{if } -6 \le x \le 1 \\ \frac{1}{2}x + 7 & \text{if } x > 4 \end{cases}$$

Piece 1 is a square root function opening to the left... We'll plot easy points..

(-6, 8) (-7, 9) (-10, 10)

Piece 2 is a parabola with vertex (-3, 1) that opens up... We'll identify the endpoints (-6, 10) and (1, 17)...

Piece 3 is a ray that starts at (4, 9) and extends to the right So, we can draw the ray through (14, 14)



domain: domain is the "if statements"...

$x \le 1$ or $x > 4$

$(-\infty, 1] \cup (4, \infty)$

range: The minimum point is the parabola's vertex...

range is $g(x) \ge 1$

$[1, \infty)$

9)
$$h(x) = \begin{cases} x & \text{if } x < 1 \\ \frac{1}{2}(2^x) & \text{if } 1 \le x < 4 \\ 2|x - 10| - 4 & \text{if } x \ge 4 \end{cases}$$

The first piece is the line $y = x$ that stops at $x = 1$

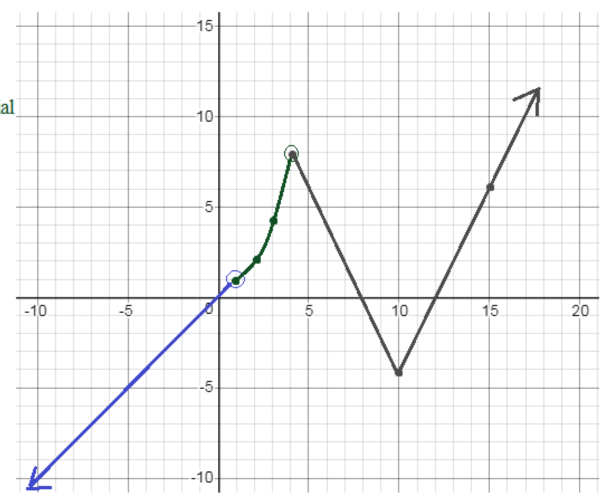
The second piece is a exponential growth function... We'll plot the points $x = 1, 2, 3,$ and 4

The third piece is an absolute value function (that opens up). The vertex is (10, -4).. Then, we can find the piece boundary: $x = 4$

(4, 8)

(10, -4)

(15, 6)



domain: all real numbers

$(-\infty, \infty)$

range: all real numbers

$(-\infty, \infty)$

VI. Piecewise Models (Word Problems) and Concepts

SOLUTIONS

1) A discount book store charges $4 per book..
If a customer buys more than 5, the price drops to $3.50 per book...

Write a piecewise function to model the cost of books..
How much would 20 books cost?

$$c(b) = \begin{cases} 4b & \text{if } b \le 5 \\ 3.5b & \text{if } b > 5 \end{cases}$$

where b is number of books
and
c(b) is cost of books

$c(20) = 70$ dollars

2) A store sells t-shirts...

It charges $10 per shirt for the first batch of 50...
Since the store has the screen design,
the next batch of 50 would cost $9 per shirt...
And, all batches after that would cost $8 per shirt....

Write a piecewise function describing the cost of shirts...
How much would 120 shirts cost?

$$f(x) = \begin{cases} 10x & \text{if } x \le 50 \\ 9x + 50 & \text{if } 50 < x \le 100 \\ 8x + 100 + 50 & \text{if } x > 100 \end{cases}$$

$f(120) = 1110$

this represents the 1st 50 shirts
which were not discounted

$$f(x) = \begin{cases} 10x & \text{if } x \le 50 \\ 9x + \boxed{50} & \text{if } 50 < x \le 100 \\ 8x + \boxed{100} + \boxed{50} & \text{if } x > 100 \end{cases}$$

$2 difference
in price for
first 50 shirts

$1 difference
in price for
2nd 50 shirts

3) A shop down the street sells hats...

It charges $10 per hat.
If a customer purchases more than 30 hats, the owner offers
a $1 discount per hat. ($9 per hat)
If a customer buys more than 50 hats, the owner offers another
$1 discount ($8 per hat).

Write a piecewise function to describe the cost of hats.
The math club has a budget of $300. How many hats could it buy?

$$c(h) = \begin{cases} 10h & \text{if } h \le 30 \\ 9h & \text{if } 30 < x \le 50 \\ 8h & \text{if } x > 50 \end{cases}$$

If the math club buys 30 hats, it would cost 30 x $10...
HOWEVER, if it bought a few more hats, the cost drops to $9!!

33 hats would cost $297... (34 hats would cost $306)

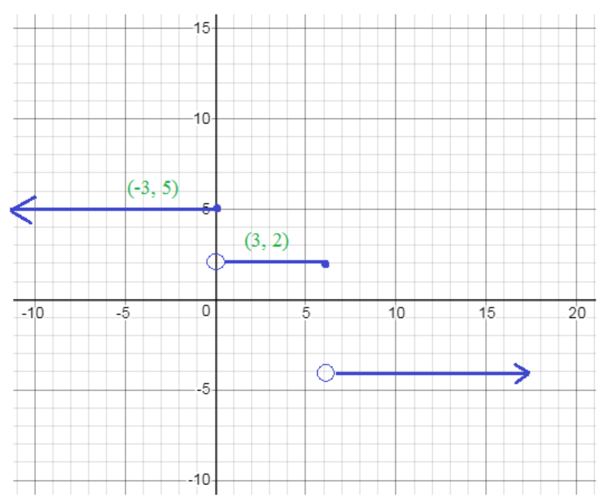
4) Write and graph a piecewise function with the following characteristics.

Domain: all real numbers

Range: { -4, 2, 5 }

$f(3) = 2$ and $f(-3) = 5$

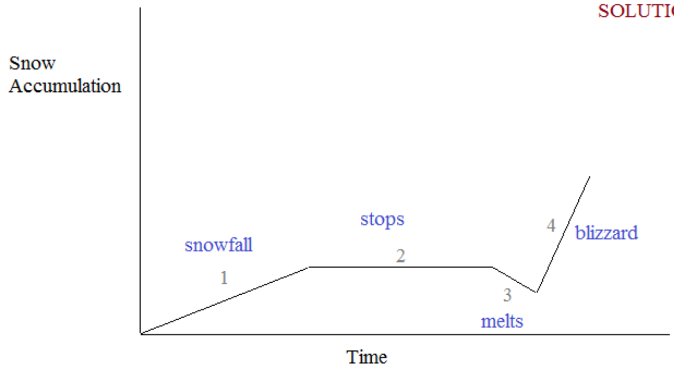
$$f(x) = \begin{cases} 5 & \text{if } x \le 0 \\ 2 & \text{if } 0 < x \le 6 \\ -4 & \text{if } x > 6 \end{cases}$$



5) Match the events with the piecewise function

SOLUTIONS

a) The snow falls for an hour. 1
b) The snow stops.. 2
c) A blizzard arrives.. 4
d) The snow melts.. 3



6) Betty and Diane are writing wedding invitations.
Betty starts at 8:00 AM and Diane starts at 10:00 AM.
Betty can write 20 invitations per hour.
And, Diane can write 25 invitations per hour.

a) How many invitations will be written by 2:00 PM?

b) When will they finish 355 invitations?

c) If they write all day, when will Betty and Diane have the same number?

"distance = rate x time"

Betty: $y = 20t$ $t = 6$ hours $y = 120$ invitations

Diane: $y = 25(t - 2)$ $y = 100$ invitations

$\boxed{\text{220 invitations by 2:00 PM}}$

$20t + 25(t - 2) = 355$

$45t = 405$

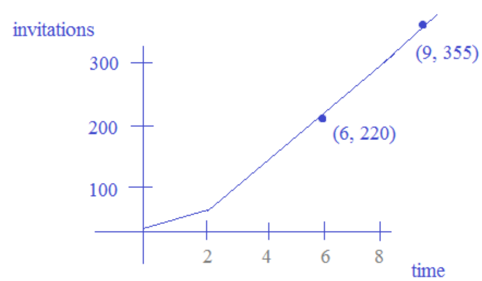
$t = 9 \Rightarrow \boxed{\text{5:00 PM}}$

$20t = 25(t - 2)$

$20t = 25t - 50$

$t = 10 \Rightarrow \boxed{\text{6:00 PM}}$

200 invitations each...



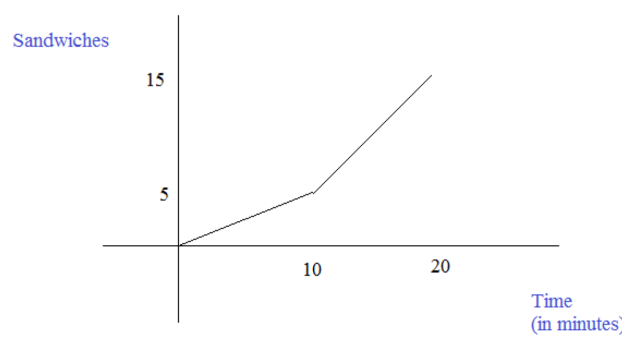
7) You and a friend can each make 1 sandwich every 2 minutes.
At lunchtime, you start making sandwiches for the camp. Ten minutes later, your friend shows up and helps you.

Write and graph a (piecewise) model of sandwiches made as a function of time.

Let S = # of sandwiches

T = Time (in minutes)

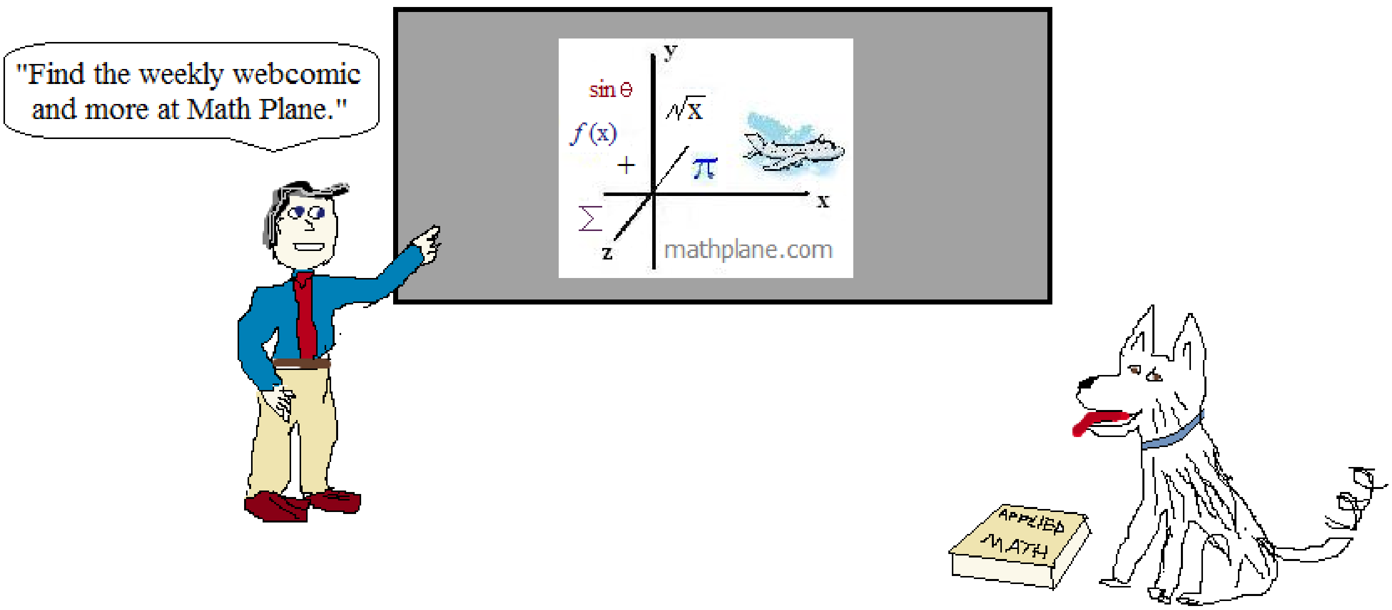
$$S = \begin{cases} \frac{1}{2}T & \text{if } T \leq 10 \\ T + 10 & \text{if } T > 10 \end{cases}$$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Find us at Mathplane.com

Also, TES, and TeachersPayTeachers

Plus, Mathplane *Express* for mobile at mathplane.ORG

One more question:

If $c(x) = 3x^3 + 5x^2 + 4$,

what is $4c(2b)$?

Answer on the next page…

If $c(x) = 3x^3 + 5x^2 + 4$,

what is $4c(2b)$?

SOLUTION

reminder: $c(1) = 3(1)^3 + 5(1)^2 + 4 = 12$

FIRST, $c(2b) = 3(2b)^3 + 5(2b)^2 + 4$

$= 3(8b^3) + 5(4b^2) + 4$

$= 24b^3 + 20b^2 + 4$

THEN, $4 \cdot c(2b) = 4 \cdot (24b^3 + 20b^2 + 4)$

$\boxed{= 96b^3 + 80b^2 + 16}$