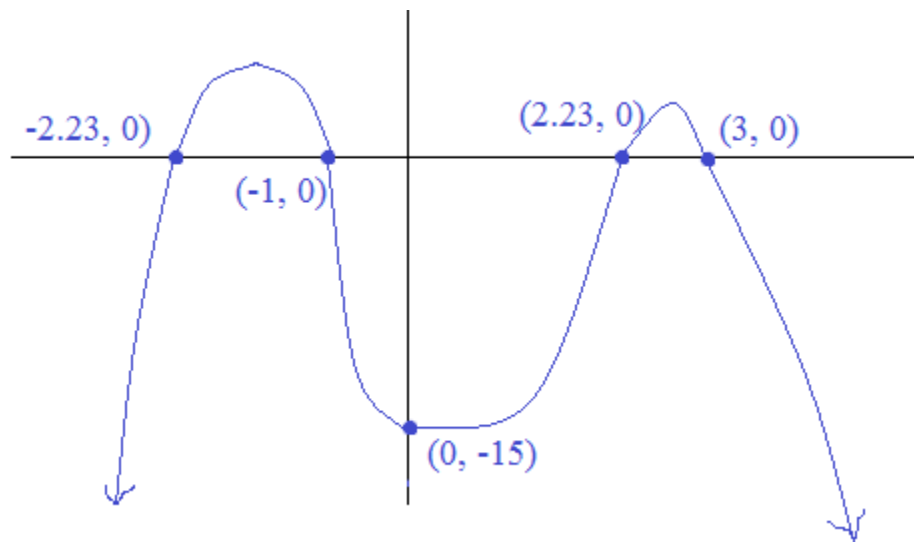


Polynomials: Factors, Roots, and Theorems

Notes, Definitions, Examples, and Practice Test (w/solutions)



Includes intercepts, Factor, Remainder & Rational Root Theorems, conjugates, synthetic division, and more...

Methods of Factoring

Greatest Common Factor

$$6X^2 - 3X = 0$$

$$3X(2X - 1) = 0$$

-- Take out greatest common factor 3X

$$\begin{array}{l} 3X = 0 \\ 2X - 1 = 0 \end{array} \quad \begin{array}{l} X = 0 \\ X = 1/2 \end{array}$$

-- Solve each piece..

$$6(0) - 3(0) = 0 \quad \checkmark$$

-- Check solutions

$$\begin{array}{l} 6(1/2)^2 - 3(1/2) = \\ 6/4 - 3/2 = 0 \quad \checkmark \end{array}$$

Finding 2 Linear Binomials

$$X^2 - 7X + 6 = 0$$

$$(X - 1)(X - 6) = 0$$

-- Find 2 numbers whose product is 6 (the constant) & whose sum is -7 (the middle coefficient)

$$\begin{array}{l} (X - 1) = 0 \\ (X - 6) = 0 \end{array} \quad \begin{array}{l} X = 1 \\ X = 6 \end{array}$$

-- Solve each piece..

$$(1)^2 - 7(1) + 6 = 0 \quad \checkmark$$

-- Check solutions

$$(6)^2 - 7(6) + 6 = 0 \quad \checkmark$$

Completing the Square

$$X^2 + 8X - 84 = 0$$

$$X^2 + 8X = 84$$

-- Isolate the "X terms"

$$X^2 + 8X + 16 = 84 + 16$$

-- Divide the coefficient of the 2nd term by 2 and square it. (Add this number to both sides)

$$(X + 4)(X + 4) = 100$$

-- Factor and solve

$$\sqrt{(X + 4)^2} = \sqrt{100}$$

$$\begin{array}{l} X + 4 = 10 \\ X + 4 = -10 \end{array} \quad \begin{array}{l} X = 6 \\ X = -14 \end{array}$$

-- Check solutions

$$\begin{array}{l} (6)^2 + 8(6) - 84 = 0 \quad \checkmark \\ (-14)^2 + 8(-14) - 84 = 0 \quad \checkmark \end{array}$$

Quadratic Formula

$$X^2 - 7X + 11 = 0$$

$$a = 1 \quad b = -7 \quad c = 11$$

-- Identify coefficients

$$X = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(11)}}{2(1)}$$

-- plug into quadratic formula

$$X = \frac{7 + \sqrt{5}}{2} \cong 4.618$$

-- Simplify

$$X = \frac{7 - \sqrt{5}}{2} \cong 2.382$$

-- check solutions

$$(2.4)^2 - 7(2.4) + 11 = -.04$$

(this "rough check" supports our solutions)

$$(4.6)^2 - 7(4.6) + 11 = -.04$$

$$Y = AX^2 + BX + C$$

(Since a quadratic's lead term has an exponent 2, there will be 2 solutions)

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Difference of Squares

$$X^2 - 16 = 0$$

X^2 and 16
are perfect squares

$$(X - 4)(X + 4) = 0$$

$$X = -4, 4$$

$$(-4)^2 - 16 = 0 \quad \checkmark$$

$$(4)^2 - 16 = 0 \quad \checkmark$$

Identify the perfect squares

Factor: "(Square root of first minus
square root of second) x (square root
of first plus square root of second)"

Solve and check

Sum of squares
DOES NOT FACTOR!!

$$X^2 + 49 = 0$$

X^2 and 49 are perfect squares,
but it does not factor...

$$X^2 + 49 \neq (X + 7)(X + 7)$$

$$(X + 7)(X + 7) = X^2 + 14X + 49$$

Solution: $X^2 + 49 = 0$

$$X^2 = -49$$

$$X = 7i \text{ or } -7i$$

(i is an imaginary number)

Difference of Cubes

Factor: $X^3 - 8$

X^3 and 8 are perfect cubes

X and 2 are the cube roots

$$(X - 2)(X^2 + 2X + 4)$$

Identify perfect cubes.

Determine cube roots. Then,

Factor using "SOAP" (signs are Same, Opposite, Always Positive)

$$(A - B)(A^2 + AB + B^2)$$

S O AP

Sum of Cubes

$$X^3 + 27 = 0$$

X^3 and 27 are perfect cubes

X and 3 are the cube roots

$$(X + 3)(X^2 - 3X + 9) = 0$$

$$(X + 3) = 0 \quad X = -3$$

$$(X^2 - 3X + 9) = 0$$

(Use Quadratic Formula)

Identify perfect cubes.

Determine cube roots. then,

Factor using "SOAP" (signs are Same/Opposite/AlwaysPositive)

$$(A + B)(A^2 - AB + B^2)$$

S O AP

Since it is X^3 ,
we're looking for
3 solutions...

Real Solution

$$\frac{3 \pm \sqrt{9 - 36}}{2} = 0$$

Complex/Imaginary
Solutions

$$X = \frac{3 + 3i\sqrt{3}}{2}$$

$$X = \frac{3 - 3i\sqrt{3}}{2}$$

Factoring (4 term) Polynomials: Grouping

Factor by 'Grouping'

- 1) Separate polynomial into groups
- 2) Factor each group (using Greatest Common Factor)
- 3) Merge and re-group

Example 1: $y^3 + 2y^2 - 81y - 162$

Solution A: $y^3 + 2y^2 - 81y - 162$ *Separate the polynomial*

$y^2(y + 2) - 81(y + 2)$ *Factor each group
(using GCF)*

$(y^2 - 81)(y + 2)$ *Merge and re-group*

$(y - 9)(y + 9)(y + 2)$

Solution B: $y^3 - 81y + 2y^2 - 162$

$y(y^2 - 81) + 2(y^2 - 81)$

$(y + 2)(y^2 - 81)$

$(y + 2)(y + 9)(y - 9)$

Note: Although Solutions A and B approach the polynomial differently, the outcome is the same!

Example 2: $b^3 + b^2 = 64b + 64$

$b^3 + b^2 - 64b - 64 = 0$ *Write equation (setting polynomial equal to zero)*

$b^2(b + 1) - 64(b + 1) = 0$ *Separate into groups and find GCF's*

$(b^2 - 64)(b + 1) = 0$ *Merge and regroup*

$(b + 8)(b - 8)(b + 1) = 0$ *Factor further*

$b = -8, 8, -1$ *Solve*

Then, check your solutions:

$b = -8 : (-8)^3 + (-8)^2 = 64(-8) + 64$

$-512 + 64 = -512 + 64$ ✓

Substitute into the original equation

$b = 8 : (8)^3 + (8)^2 = 64(8) + 64$

$512 + 64 = 512 + 64$ ✓

$b = +1 : (-1)^3 + (-1)^2 = 64(-1) + 64$

$-1 + 1 = -64 + 64$ ✓

Graphing Polynomials: 2 examples

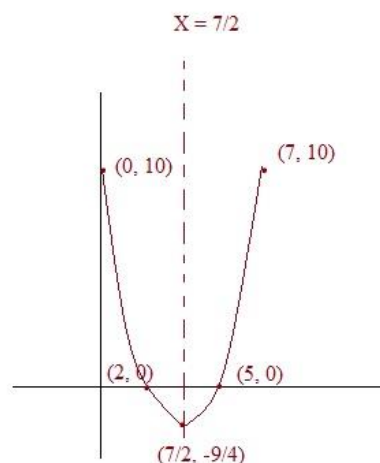
Quadratic Function: $f(x) = x^2 - 7x + 10$

Identify y-intercept (0, ?) is the y-intercept
 $f(0) = 0^2 - 7(0) + 10 = 10$

Find x-intercepts (the roots) (?, 0) are the x-intercepts
 $f(x) = 0 : \quad x^2 - 7x + 10 = 0$
 $(x - 5)(x - 2) = 0$
 $x = 2, 5$ ("roots")

Plot points and recognize the axis of symmetry and vertex Find midpoint of 2 and 5 to determine axis of symmetry.. $x = 7/2$

$f(7/2) = 49/4 - 49/2 + 10 = -9/4$
 Vertex is $(7/2, -9/4)$



(Since the coefficient of the x is positive, the parabola faces up.. The vertex is the function's minimum.. There is no maximum)

Cubic Function:

$f(x) = x^3 - 4x^2 - 11x + 30$

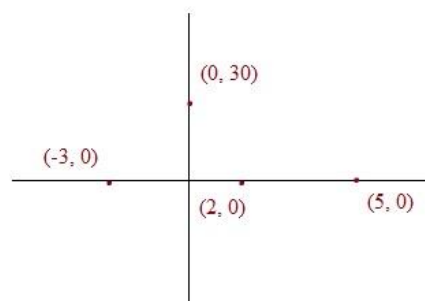
Identify y-intercept $f(0) = 30$ (0, 30) is the y-intercept

Find the x-intercepts. Since it is a cubic, there should be 3 roots --- 3 intercepts...
 $f(x) = 0 : \quad x^3 - 4x^2 - 11x + 30 = 0$

(Using factoring techniques, we find)

$(x + 3)(x - 2)(x - 5) = 0$

$x = -3, 2, 5$ (-3, 0), (2, 0), (5, 0) are the x-intercepts

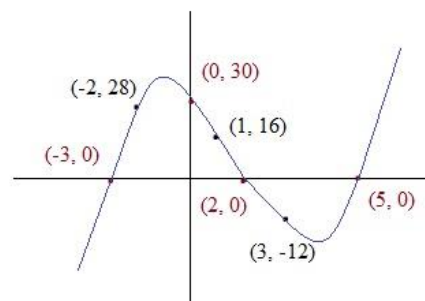


Plot points and determine end behavior

Leading term is x^3
 Therefore, the curve's end behavior will be "up to the right" and "down to the left"

To check our intercepts and make a more accurate graph, we add points:

$f(1) = 16$
 $f(-2) = 28$
 $f(3) = -12$



Fundamental Theorem of Algebra: Any polynomial of degree n will have exactly n roots

What is a root?
For a polynomial $P(x)$, if r is a root, then $P(r) = 0$

$X^2 + 3X + 2$ degree $n = 2$ Two roots: $-1, -2$

$-3X^2 - 10X + 24 + X^3$ degree $n = 3$ (the largest exponent is 3) Three roots: $2, -3, 4$

Factor and find the roots:

$X^4 + 5X^2 - 36$ Recognize that 9 and -4 add up to 5 and multiply to -36

$(X^2 - 4)(X^2 + 9)$ Notice that the first term is "difference of squares"

$(X + 2)(X - 2)(X^2 + 9)$ Set factors equal to zero to find roots

$(X + 2) = 0$
 $(X - 2) = 0$

-2
 2
 $3i$
 $-3i$

where $i^2 = -1$

Since the polynomial is degree 4, there are 4 roots (in this example: 2 are real; 2 are imaginary)

Rational Root Test : A polynomial with leading coefficient 'a' and constant 'b' can have rational roots only of the form

$\pm \frac{p}{q}$ where p is a factor of b and q is a factor of a

Note: the Rational Root Test will identify possible roots. You must test the candidates.

$f(X) = -X - 30 + X^3 + 6X^2$

$X^3 + 6X^2 - X - 30$

Write polynomial in standard form (order). Then, identify a (coefficient of first term) and b (the constant)

$a = 1$ $b = -30$

factors of 1: $p = 1$
factors of -30: $p = 1, 2, 3, 5, 6, 10, 15, 30$

Determine factors of each term. Then, identify all the possible roots by listing

$\pm \frac{p}{q}$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

(Use synthetic division to) test the candidates to find a root.

1 | 1 6 -1 -30
1 7 6 -24

(1 is not a root)

2 | 1 6 -1 -30
2 2 16 30
1 8 15 0

2 is a root; and, $(X - 2)$ is a factor of the polynomial...

You may factor the remaining polynomial to find the other 2 roots.

$X^2 + 8X + 15$
 $(X + 5)(X + 3)$

-3 and -5 are the other roots...

To check your answer, confirm that $f(2) = f(-3) = f(-5) = 0$

Rational Root Test & Factoring (continued)

$$f(X) = 6X^3 + 11X^2 - 3X - 2$$

$$a = 6$$

$$b = -2$$

factors of a: $q = 1, 2, 3, 6$
 factors of b: $p = 1, 2$

$$\text{possible roots: } \pm \frac{p}{q} \quad +\frac{1}{1}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6}, +\frac{2}{1}, +\frac{2}{2}, +\frac{2}{3}, +\frac{2}{6}$$

Notice: there are 16 candidates. and, three of them are roots.. So, there is a 3/16 chance of randomly selecting a root the first time...

Is 1 a root? $f(1) = 6(1)^3 + 11(1)^2 - 3(1) - 2 = 12 \neq 0$ NO

Is 2 a root? $f(2) = 6(2)^3 + 11(2)^2 - 3(2) - 2 = 84 \neq 0$ NO

Is -2 a root? $f(-2) = 6(-2)^3 + 11(-2)^2 - 3(-2) - 2 = 0$ YES!!

Using the Factor/Remainder Theorems, we search for a root...

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

Using synthetic division, we can break down the polynomial...

$$(X - (-2))(6X^2 - X - 1)$$

$$(X + 2)(3X + 1)(2X - 1) = 0$$

Then, factor and set equal to 0 to find the other roots...

$$X = \boxed{-2, -1/3, 1/2}$$

What is the y-intercept? $f(0) = -2$ (0, -2)

What are the x-intercepts? $f(x) = 0$ (-2, 0), (-1/3, 0), (1/2, 0)

Fundamental Theorem of Algebra (continued): It guarantees that any polynomial of degree n will have exactly n roots.. (** You must "double count" the double roots..)

$$f(X) = X^3 - 3X^2 + 4$$

The polynomial has degree 3, so there will be exactly 3 roots..

$$f(X) = (X - 2)(X - 2)(X + 1)$$

$(X - 2)^2$ produces a double root.

Roots are 2, 2, -1

$$Y = X^3 - 3X^2 + 3X - 1$$

According to Fundamental Theorem of Algebra, there will be 3 roots (i.e. 3 zeros)

$$Y = (X - 1)(X - 1)(X - 1)$$

This is an example of a "triple root"

Roots are 1, 1, 1

Factors and Remainders:

Is 3 a factor of 1284?

Yes, because $1284 \div 3 = 428$

Is 7 a factor of 1284?

No, because there is a remainder..

(It isn't evenly divisible)

$$\begin{array}{r} 183 \text{ remainder } 3 \\ 7 \overline{)1284} \\ \underline{-7} \\ 58 \\ \underline{-56} \\ 24 \\ \underline{-21} \\ 3 \end{array}$$

Is $(X-8)$ a factor of $X^3 - 7X^2 + 14X - 8$?

$$\begin{array}{r} X^2 + X + 22 \text{ remainder } 168 \\ (X-8) \overline{)X^3 - 7X^2 + 14X - 8} \\ \underline{-X^3 + 8X^2} \\ 22X^2 + 14X \\ \underline{-22X^2 + 176X} \\ 168 \end{array}$$

Polynomial Long Division

Synthetic Division

$$\begin{array}{r|rrrr} 8 & 1 & -7 & 14 & -8 \\ & & 8 & 8 & 176 \\ \hline & 1 & 1 & 22 & 168 \end{array}$$

No, $(X - 8)$ is not a factor..

Is $(X - 1)$ a factor?

$$\begin{array}{r} X^2 - 6X + 8 \\ (X-1) \overline{)X^3 - 7X^2 + 14X - 8} \\ \underline{-X^3 + X^2} \\ -6X^2 + 14X \\ \underline{-6X^2 + 6X} \\ 8X - 8 \\ \underline{-8X + 8} \\ 0 \end{array}$$

Polynomial Long Division

Synthetic Division

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 14 & -8 \\ & & 1 & -6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

Yes, $(X - 1)$ is a factor!

$$X^2 - 6X + 8 = (X - 4)(X - 2)$$

What is $f(8)$? $(8)^3 - 7(8)^2 + 14(8) - 8 =$

$$512 - 448 + 112 - 8 = 168$$

It's the same as the remainder!!

What is $f(1)$? $(1)^3 - 7(1)^2 + 14(1) - 8 = 0$

The root has no remainder; It's a factor...

Remainder Theorem: If a polynomial function $f(x)$ is divided by a linear term $(x - a)$ and the remainder is r , then $f(a) = r$

This implies the

Factor Theorem: If a polynomial function $f(x)$ has a factor $(x - a)$, then $f(a) = 0$
In other words, there is no remainder..

Since $(X - 4)$
 $(X - 2)$
 $(X - 1)$ are factors,
 $f(4) = f(1) = f(2) = 0$

⇒ Given the roots 2 and -4
What is the polynomial function?

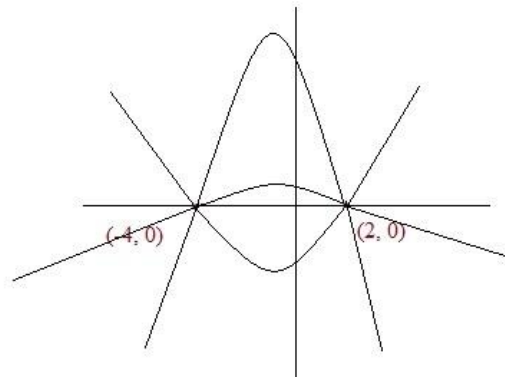
Since the roots are 2 and -4,
the zeros (x-intercepts) are 2 and -4.

So, the factors are $(X - 2)$ and $(X + 4)$

$$\begin{aligned} f(X) &= (X - 2)(X + 4) \\ &= X^2 + 2X - 8 \end{aligned}$$

However, that is only 1 possibility.
The graph shows other curves with
roots of 2 and -4.

So, we need more information --
another point -- to determine the
specific function.



All 3 are parabolas that have zeros at
2 and -4

⇒ Now, suppose you're given roots: 2, -4
and given y-intercept: 3

Use the formula $Y = a(X - r_1)(X - r_2)$

$$Y = a(X + 4)(X - 2)$$

$(2, 0)$ and $(-4, 0)$ both work...

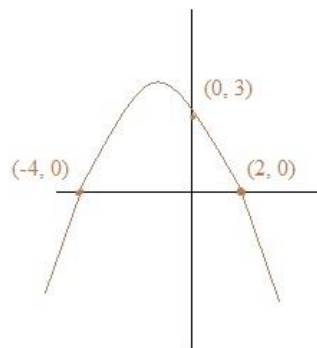
Now, insert $(0, 3)$

$$3 = a(0 + 4)(0 - 2)$$

$$3 = a(4)(-2)$$

$$a = \frac{-3}{8}$$

$$Y = \frac{-3}{8}(X + 4)(X - 2)$$



"Sum & Product of Roots of a Polynomial"

For the Polynomial $P(X) = a_n X^n + a_{n-1} X^{n-1} + \dots + a_1 X^1 + C$

The Sum of the roots is: $-\frac{a_{n-1}}{a_n}$ The Product of the roots is: If n is odd, $-\frac{C}{a_n}$ If n is even, $\frac{C}{a_n}$

$$f(X) = X^2 - 10X + 21$$

$$\text{Sum: } \frac{-(-10)}{1} \quad \text{Product: } \frac{21}{1}$$

$$f(X) = (X - 3)(X - 7) \quad \text{so, roots are } 3, 7$$

$$\text{Sum: } 3 + 7 = 10 \quad \text{Product: } 3 \times 7 = 21$$

$$f(X) = 2X^3 - 9X^2 - 11X + 30$$

$$\text{Sum: } \frac{-(-9)}{2} \quad \text{Product: } \frac{-30}{2}$$

$$f(X) = (2X - 3)(X - 5)(X + 2) \quad \text{so, roots are } 3/2, 5, -2$$

$$\text{Sum: } 3/2 + 5 + -2 = 9/2 \quad \text{Product: } 3/2 \times 5 \times (-2) = -15$$

"Conjugate Pair Theorem"

If a polynomial has real coefficients, then any complex zeros occur in conjugate pairs. In other words, if $a + bi$ is a zero, then, $a - bi$ is a zero..

$$f(X) = X^2 + 4 \quad X^2 + 4 = 0 \quad X = \pm 2i$$

$$X^2 = -4$$

$$f(X) = X^3 - 3X + 52 \quad \text{Suppose we know } (X - (2 + 3i)) \text{ is a factor.}$$

then, $2 + 3i$ is a root...

By the conjugate pair theorem, $2 - 3i$ must be a root, too...

$$(2 - 3i)^3 - 3(2 - 3i) + 52 = (4 - 12i - 9)(2 - 3i) - 6 + 9i + 52$$

$$= (-5 - 12i)(2 - 3i) + 9i + 46$$

(-4 is the other root)

$$= -10 - 24i + 15i - 36 + 9i + 46$$

$$= 0$$

Descartes Rule of Signs

Example: $f(x) = x^5 + 5x^4 - 10x^3 - 50x^2 + 9x + 45$

Use Descartes Rule of Signs to determine possible roots

Positive reals...

Assume $f(x)$... how many change of signs?

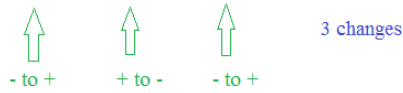
Let's use $f(1) = 1 + 5 - 10 - 50 + 9 + 45$



Negative reals...

Assume $f(-x)$... how many change of signs?

Let's use $f(-1) = -1 + 5 + 10 - 50 - 9 + 45$



What is it? A method for determining the maximum number of positive/negative zeros.

If the terms of a single variable polynomial are arranged in descending order, then the number of positive roots is equal to the number of sign changes of coefficients or less by an even number below...

And, the number of negative roots is equal to the number of sign changes of coefficients after multiplying the variable by -1

Possibilities..

Positive Reals	Negative Reals	Imaginary	Total
2	3	0	5
0	1	4	5
0	3	2	5
2	1	2	5

⇒ The table displays the possible roots. the actual number is 2 positive real roots and 3 negative real roots

Example: Given Polynomial $3x^4 - 10x^2 + x + 5$

Use Decartes Rule of Signs to determine the maximum number of positive zeros.

For positive variables, there would be 2 sign changes...

$3x^4 - 10x^2 + x + 5$

For negative variables, there would be 2 sign changes...

$3x^4 - 10x^2 + x + 5$

Possible zeros....

Positive Reals	Negative Reals	Imaginary	Total
2	2	0	4
0	0	4	4
0	2	2	4
2	0	2	4

⇒ Maximum number of possible positive zeros is 2...

The actual numbers: 2 possible zeros
2 negative zeros

Sketching Polynomials

Generally, when *sketching* a polynomial, we try to provide as much information as possible or demanded:

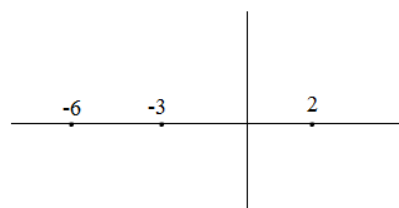
zeros (x-intercepts), end behavior, y-intercept, additional points

Example: Sketch the function $f(x) = -4(x - 2)^3(x + 3)^2(x + 6)$

Step 1: Identify the zeros (x-intercepts)

Since the function is written in "factored form" ("intercept form"), the zeros can be identified easily:

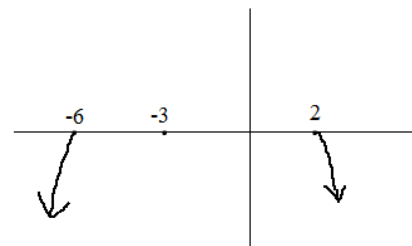
$$f(x) = 0 = -4(x - 2)^3(x + 3)^2(x + 6) \quad \text{at } x = 2, -3, \text{ and } -6$$



Step 2: Recognize the end behavior

What is the "degree" of the polynomial? It is NOT 3; If you were to multiply the terms (i.e. FOIL the parts), the first term would be $-4x^6$

Since the exponent is even, the function's end behavior is the same in either direction. And, since the lead coefficient is *negative* four, the end behavior is "down"

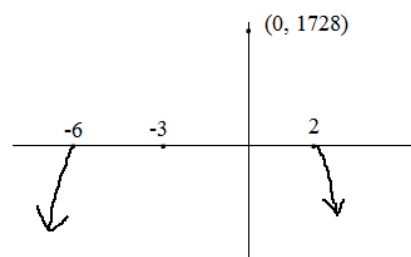


Step 3: Find the y-intercept (or any other "easy" points)

Since the y-intercept is easy to find -- simply plug in 0 -- it's a great way to solidify and check your sketch!

$$f(0) = -4(0 - 2)^3(0 + 3)^2(0 + 6) = -4(-8)(9)(6) = 1728$$

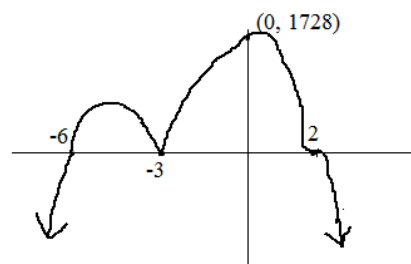
(0, 1728)



Step 4: Fill in the rest of the graph (applying *multiplicity* and "bounces")

since $(x - 2)$ is to the 3rd power, "the zero 2 has multiplicity" of 3

since $(x + 3)$ is to the 2nd power, "the zero -3 has multiplicity" of 2 (it will "bounce")



Step 5: Quick check

You may pick specific points to add to the graph. And, you can do quick checks ---

for example, if we test -10:

$$f(-10) = -4(-10 - 2)^3(-10 + 3)^2(-10 + 6)$$

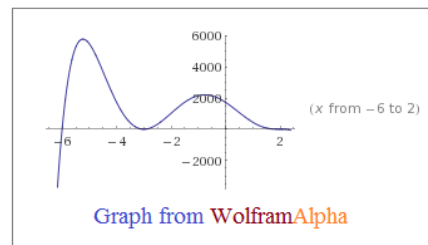
- - + -

the answer will be *negative* so our end behavior on the left is correct...

or, if we test -1: $-4(-1 - 2)^3(-1 + 3)^2(-1 + 6)$

- - + +

the answer will be *positive*, so the point is above the x-axis...



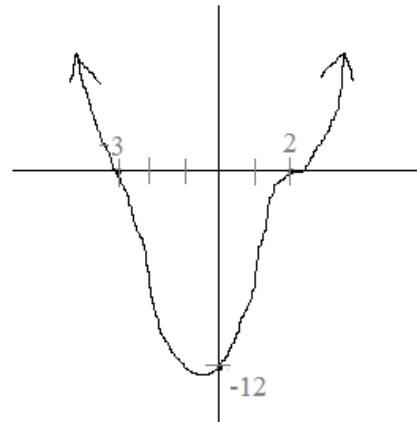
Example: Describe the following 4th degree polynomial:

Step 1: Identify the zeros (x-intercepts)

$(-3, 0)$ and $(2, 0)$ are points on the graph...

therefore, zeros include -3 and 2

$$y = (x + 3)(x - 2)$$



Step 2: Consider degree (and multiplicity)

Since this is a 4th degree polynomial, we need to add more zeros...

Also, note the "pause" in the graph at $x = 2$

Therefore, we need to add $(x - 2)$ terms...

$$y = (x + 3)(x - 2)^3$$

Step 3: Specify the graph by determining the "a" value

There are an infinite number of polynomials that pass through -3 and 2...

But, by adding a coefficient, we express a unique equation (that includes the point $(0, -12)$).

$$y = a(x + 3)(x - 2)^3 \quad \text{plug in } (0, -12)$$

$$-12 = a(0 + 3)(0 - 2)^3$$

$$-12 = a(3)(-8)$$

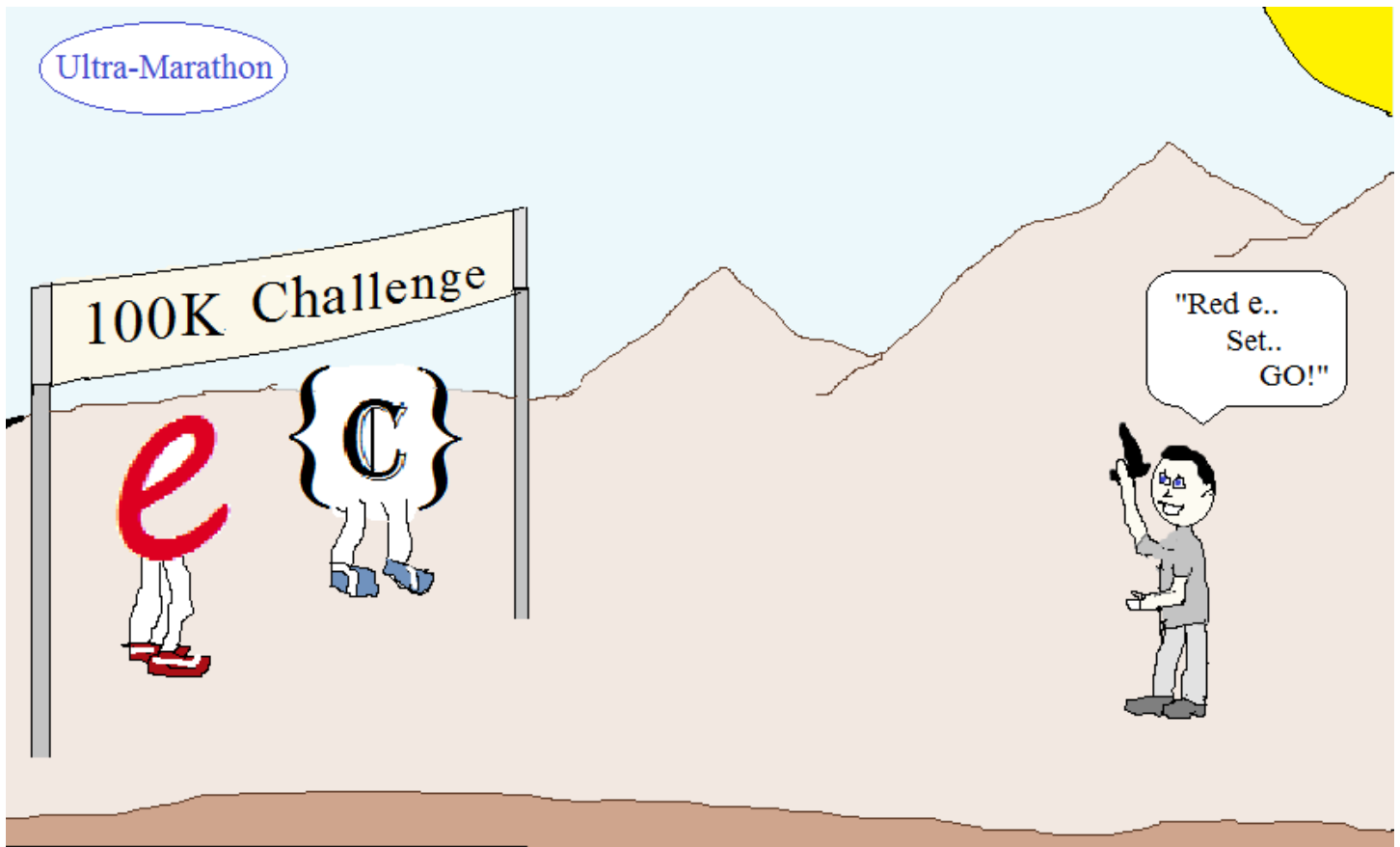
$$-12 = -24a$$

$$a = \frac{1}{2}$$

$$y = \frac{1}{2}(x + 3)(x - 2)^3$$

$$\text{or, } \frac{x^4}{2} - \frac{3x^3}{2} - 3x^2 + 14x - 12$$

Ultra-Marathon



Testing the limits of endurance,
these math figures will run on and on...

LanceAF #87 5-24-13
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PRACTICE Exercises (w/SOLUTIONS)-→

Classifying Polynomials

For each polynomial, determine the degree, the lead coefficient, and classify:

a) $2x^3 + 3x + 6$

Degree: 3

Lead Coefficient: 2

Classification: Cubic Trinomial

b) x

Degree:

Lead Coefficient:

Classification:

c) $3 - 4x^8$

Degree:

Lead Coefficient:

Classification:

d) $p^3 - 2p + 2p^3$

Degree:

Lead Coefficient:

Classification:

e) $t^3 - 3t^2 + t^5 - t^6$

Degree:

Lead Coefficient:

Classification:

f) $3x^4y^3 + 5x^3y + x^2 + y^3 + 9$

Degree:

Lead Coefficient:

Classification:

Why are these NOT polynomials?

a) $5n^2 + \sqrt{n} + 6 - 3mn$

d) $\frac{x^2 - 4}{x + 3}$

b) $3x + 5x^3 - \frac{7}{x} + xy$

e) $\frac{xy}{z} + 3xz$

c) $6t^5 + 4t^3 + \sqrt{2}t + 3^t$

f) $3x^5 + 14x^3 + \pi x + x^{-1} + 5x^{-2}$

How many zeros are possible in each function? (Optional: Sketch examples)

A) Line

B) Quadratic

C) Cubic

D) Exponential

E) Logarithmic

F) Polynomial with degree 4

G) Polynomial with degree 5

Determine the end behavior, identify the intercepts, and then sketch the polynomial...

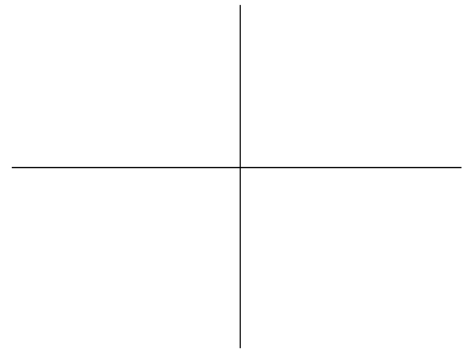
1) $f(x) = (x - 1)(x + 5)^2$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



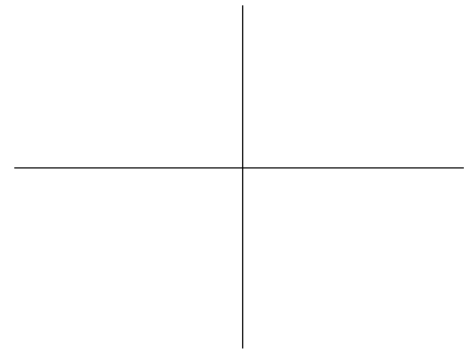
2) $y = x^3 - 10x^2 - 11x$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



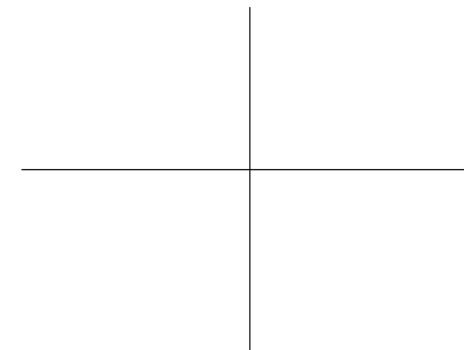
3) $g(x) = -(x + 2)^3(x - 3)$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



Determine the end behavior, identify the intercepts, and then sketch the polynomial...

Polynomials Sketching Exercises

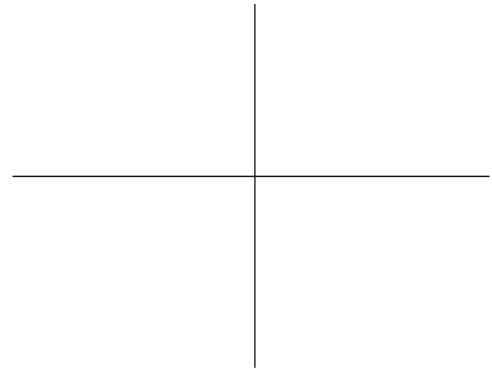
4) $y = 2x^3 + 6x^2 - x - 3$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



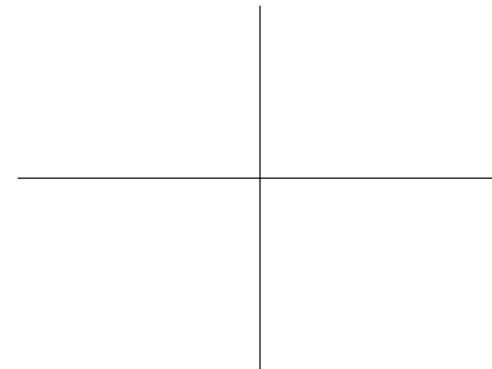
5) $y = (x + 2)^2(x - 3)(x - 1)$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



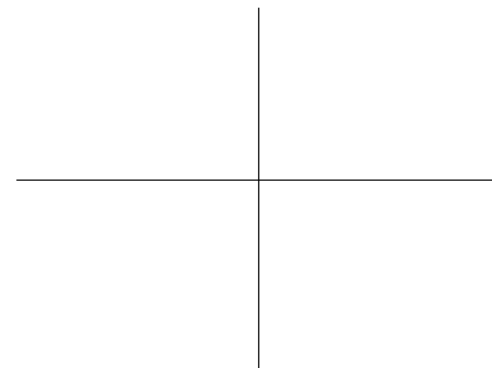
6) $y = (x - 4)^2(x^2 + 4)$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



Determine the end behavior, identify the intercepts, and then sketch the polynomial...

Polynomials Sketching Exercises

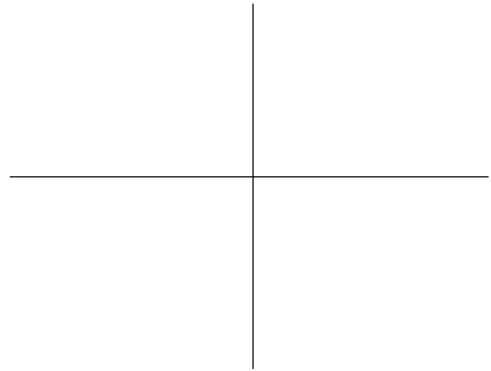
7) $h(x) = (x + 3)^3(2x - 5)$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



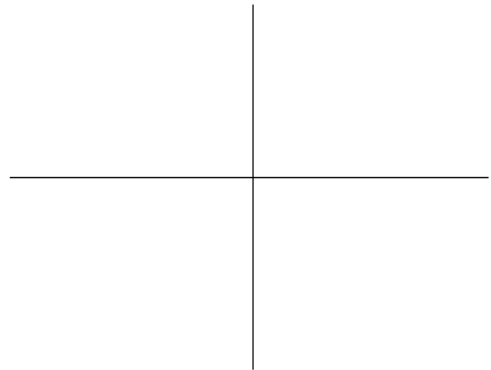
8) $y = -\frac{3}{4}x^4 + \frac{3}{4}$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



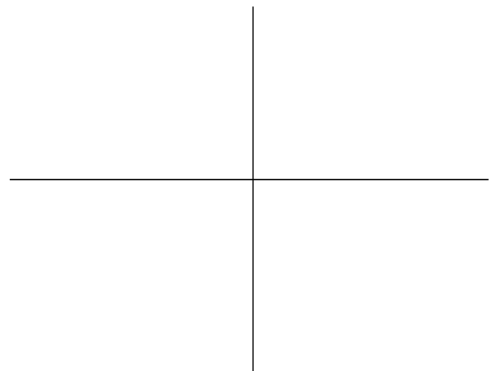
9) $y = (x + 1)(x - 3)^2(x + 5)^3$

end behavior:
(degree and lead coefficient)

y-intercept:

x-intercept(s):

multiplicity:
(‘bounces’ and ‘pauses’)



Polynomials and Roots Test

I. General Topics

A) $f(x) = x^3 - 3x^2 - 6x + 8$

Classify the polynomial:

What are the 'p' values?

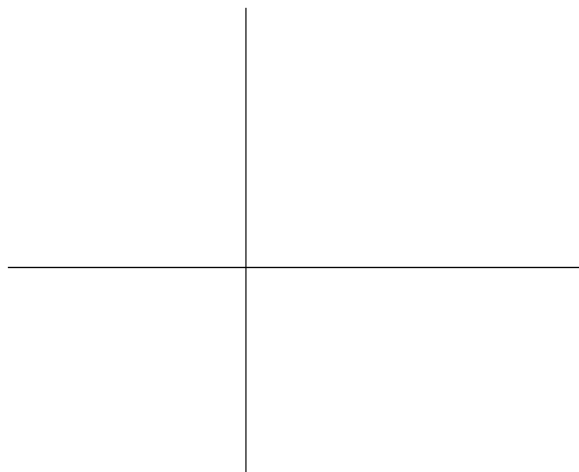
What are the 'q' values?

List all possible rational zeros:

What are the x-intercepts?

What is the y-intercept?

Sketch the function:



B) $g(x) = 2x^3 + 13x^2 + 5x - 6$

What is the degree of the polynomial?

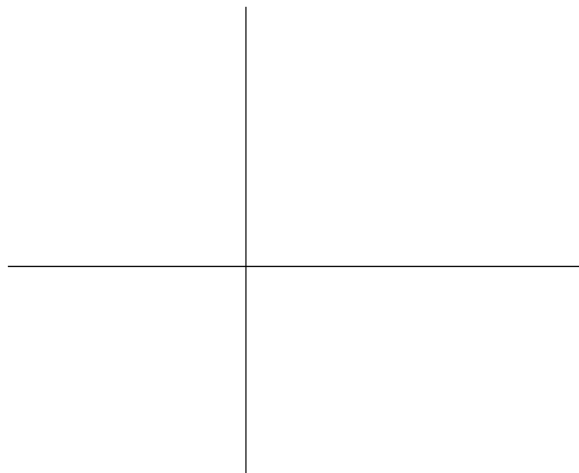
List all possible rational zeros:

Identify the zeros:

What are the factors?

What is the remainder of $g(x) \div (x + 5)$?

Sketch the function:



C) $h(x) = -x^4 + 2x^3 + 8x^2 - 10x - 15$

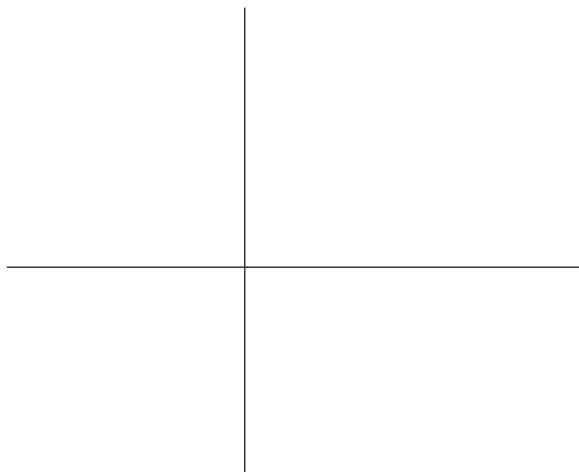
Classify the polynomial:

Describe the end behavior:

What are the factors?

What is the y-intercept?

Sketch the function:



Polynomials and Roots Test

II. Factoring, Synthetic Division, and Roots

Factor and identify all roots:

A) $x^5 - x^4 + 9x^3 - 9x^2$

B) $x^4 - 2x^2 + 1$

C) $x^3 + 4x^2 + 9x + 36$

III. Determining the Polynomial

A) Write a cubic function whose graph passes through
 $(-2, 0)$ $(2, 0)$ $(-4, 0)$ $(-1, 3)$

B) Write at least two quadratic functions that have
x-intercepts $(3, 0)$ and $(8, 0)$

Write a polynomial of least degree that has real coefficients, lead coefficient of 1, and the given zeros:

A) $-2, -2, 2$

B) $2, 5, i$

C) $4, 2, -3i$

D) $2 + i, 2 - i, 3$

E) $0, 1, -3$

Polynomials and Roots Test

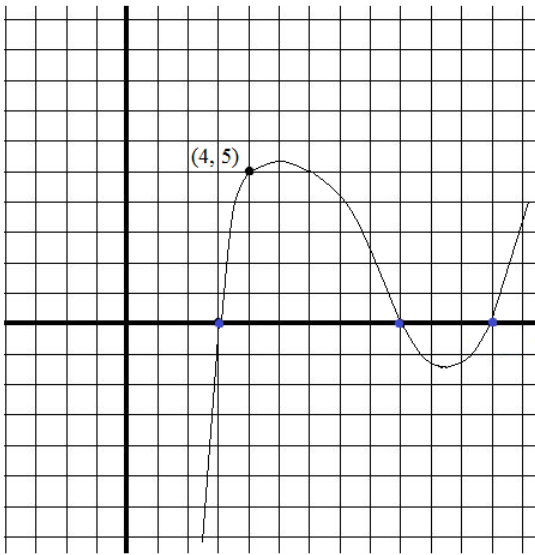
IV. Applying Concepts and Theorems

A) What is the remainder of $10x^5 + 3x^4 - 7x^3 + 2x - 6 \div (x - 1)$?
(Remainder Theorem)

B) Verify that $x^{20} - 1$ has a factor of $(x + 1)$.
(Factor Theorem)

C) Three roots of $x^4 - 5x^3 - 33x^2 + 113x + 140$ are $-1, -5, 7$... What is the 4th root?
(Sum and Products of Roots)

D) Find the y-intercept of the following sketch:



Classifying Polynomials

SOLUTIONS

For each polynomial, determine the degree, the lead coefficient, and classify:

a) $2x^3 + 3x + 6$

Degree: 3

Lead Coefficient: 2

Classification: Cubic Trinomial

b) x

Degree: 1

Lead Coefficient: 1

Classification: Linear Monomial

$1x^1$

c) $3 - 4x^8$

Degree: 8

$-4x^8 + 3$

Lead Coefficient: -4

Classification: Binomial of degree 8

d) $p^3 - 2p + 2p^3$

Degree: 3

Lead Coefficient: 3

Classification: Cubic Binomial

$3p^3 - 2p$

e) $t^3 - 3t^2 + t^5 - t^6$

Degree: 6

Lead Coefficient: -1

Classification: Polynomial of degree 6

$-t^6 + t^5 + t^3 - 3t^2$

f) $3x^4y^3 + 5x^3y + x^2 + y^3 + 9$

Degree: 7

Lead Coefficient: 3

Classification: Polynomial of degree 7

$4 + 3 = 7$

$\begin{matrix} \setminus & / \\ | & | \end{matrix}$

$3x^4y^3 + 5x^3y + x^2 + y^3 + 9$

Why are these NOT polynomials?

a) $5n^2 + \sqrt{n} + 6 - 3mn$ $\sqrt{n} = n^{\frac{1}{2}}$ (exponent cannot be a fraction)

b) $3x + 5x^3 - \frac{7}{x} + xy$ $\frac{7}{x} = 7x^{-1}$ (exponent cannot be negative)

c) $6t^5 + 4t^3 + \sqrt{2}t + 3^t$ 3^t (exponent cannot be a variable)

Exponent must be a whole number...

d) $\frac{x^2 - 4}{x + 3}$ $x - 3 + \frac{5}{x + 3}$ (variable doesn't have whole number exponent)

e) $\frac{xy}{z} + 3xz$ variable in denominator -- i.e. variable with negative exponent

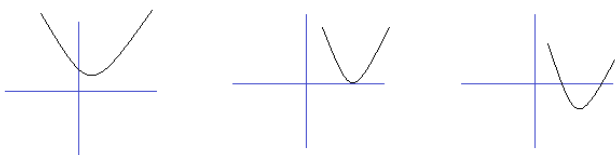
f) $3x^5 + 14x^3 + \pi x + x^{-1} + 5x^{-2}$ negative exponents of variables

How many zeros are possible in each function? (Optional: Sketch examples)

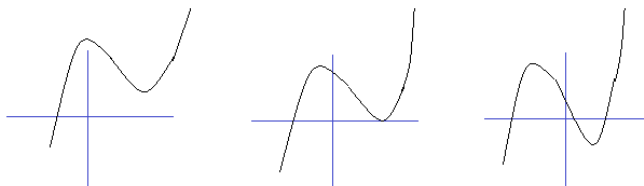
SOLUTIONS

A) Line 0 or 1 (0 if the line is horizontal)

B) Quadratic 0, 1, or 2

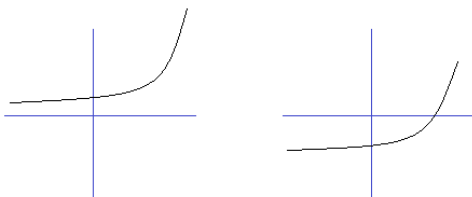


C) Cubic 1, 2, or 3

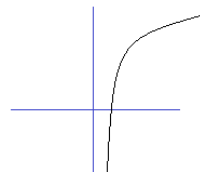


(Can't be 0, because end behaviors in opposite direction)

D) Exponential 0 or 1



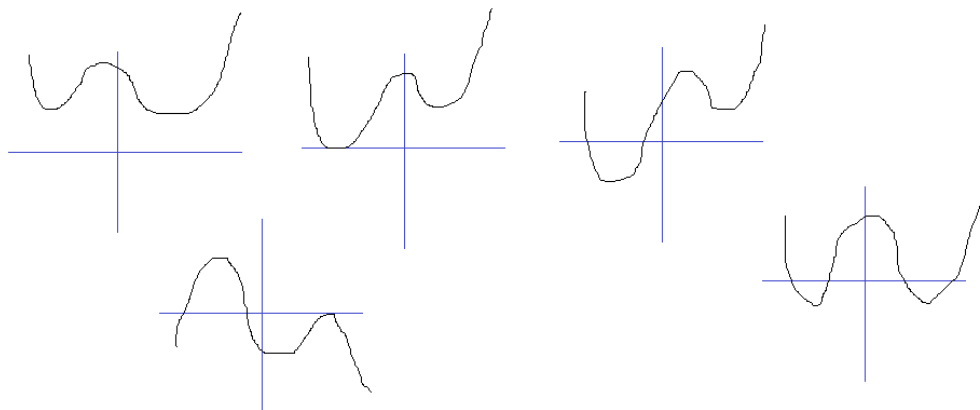
E) Logarithmic 1



(Note: logarithmic functions are inverses of exponential functions... Since the domain of exponential functions is all real numbers, then the range of logarithmic functions must be all real numbers...)

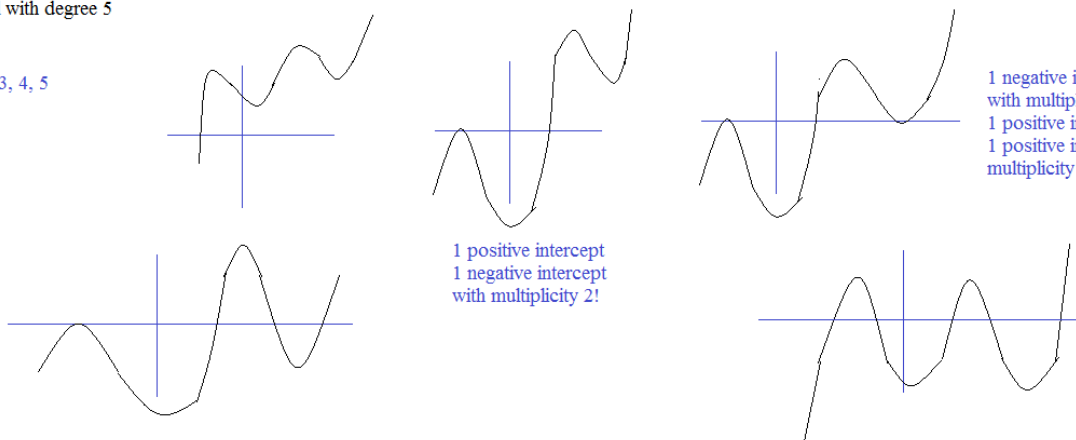
F) Polynomial with degree 4

0, 1, 2, 3, 4



G) Polynomial with degree 5

1, 2, 3, 4, 5



1 negative intercept with multiplicity 2, 1 positive intercept, and 1 positive intercept with multiplicity 2

1 positive intercept 1 negative intercept with multiplicity 2!

Determine the end behavior, identify the intercepts, and then sketch the polynomial...

SOLUTIONS

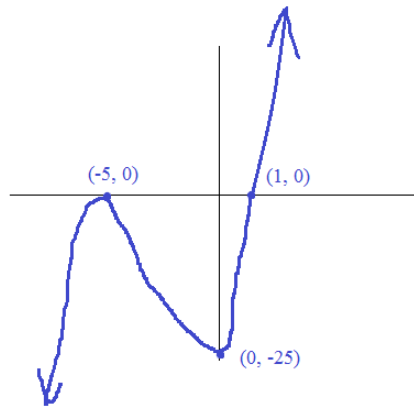
1) $f(x) = (x - 1)(x + 5)^2$

end behavior: (degree and lead coefficient) degree is 3 (odd), lead coefficient is 1 (positive) so "up to the right and down to the left"

y-intercept: If $x = 0$, then $f(0) = (-1)(5)^2 = -25$
(0, -25)

x-intercept(s): If $f(x) = 0$, then $x = 1$ or $x = -5$

multiplicity: ('bounces' and 'pauses') Since $(x + 5)$ is squared, the root -5 has a multiplicity of 2 ----> a "bounce" off the x-axis



2) $y = x^3 - 10x^2 - 11x$

$y = x(x^2 - 10x - 11)$

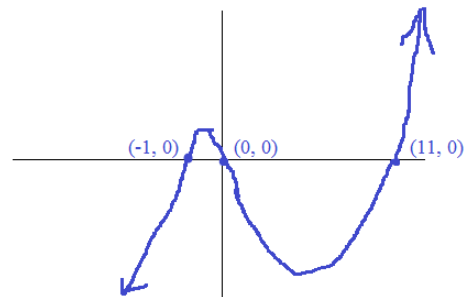
$y = x(x - 11)(x + 1)$

end behavior: (degree and lead coefficient) degree is 3 and lead coefficient is positive 1... So, as $x \rightarrow \infty$, $y \rightarrow \infty$

y-intercept: (0, 0) as $x \rightarrow -\infty$, $y \rightarrow -\infty$

x-intercept(s): (0, 0) (11, 0) (-1, 0)

multiplicity: ('bounces' and 'pauses') None



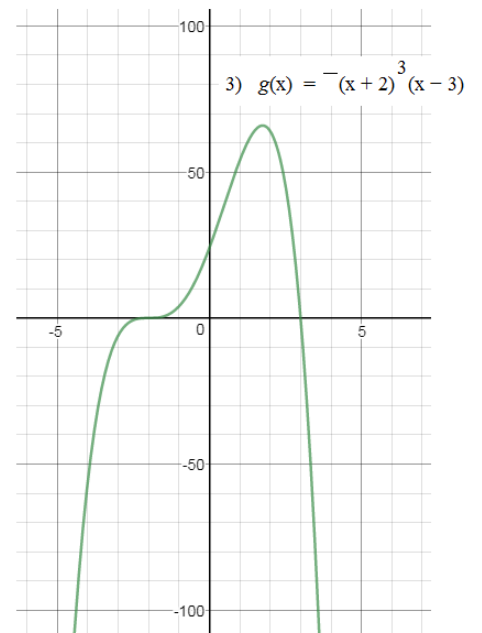
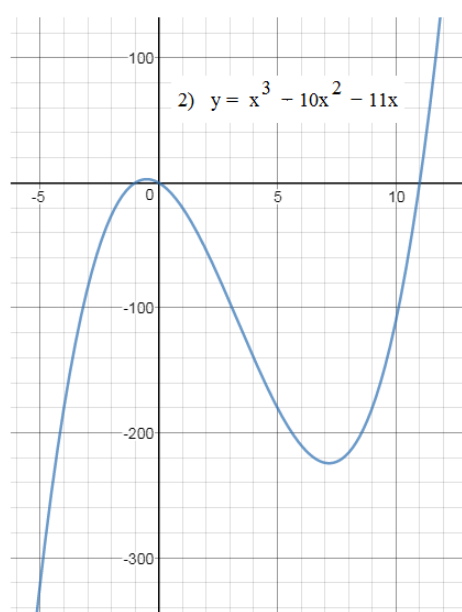
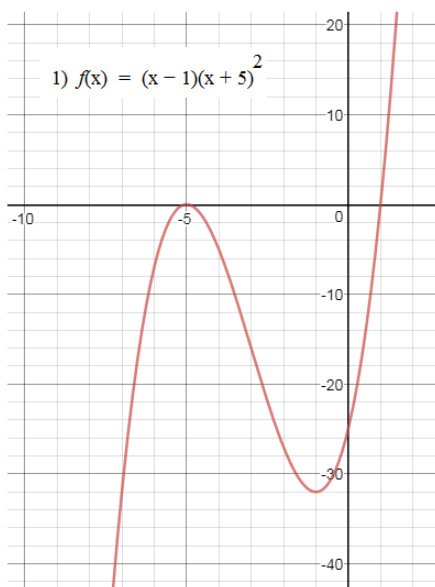
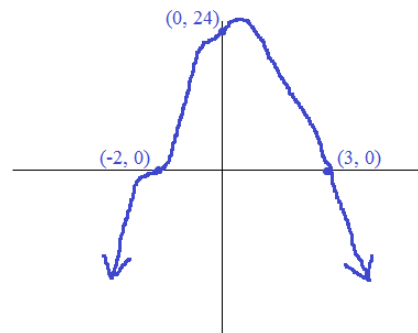
3) $g(x) = -(x + 2)^3(x - 3)$

end behavior: (degree and lead coefficient) Degree is 4, and lead coefficient is NEGATIVE 1 so, end behavior is "down and down"..

y-intercept: when $x = 0$, $g(0) = -(8)(-3) = 24$

x-intercept(s): when $g(x) = 0$?? when $x = -2$ or $x = 3$

multiplicity: ('bounces' and 'pauses') the x-intercept (or zero) is -2, there is a "pause" (multiplicity of 3)



Determine the end behavior, identify the intercepts, and then sketch the polynomial...

SOLUTIONS

4) $y = 2x^3 + 6x^2 - x - 3$ factor by grouping $2x^2(x+3) - 1(x+3)$
 $(2x^2 - 1)(x+3)$

end behavior:
 (degree and lead coefficient)

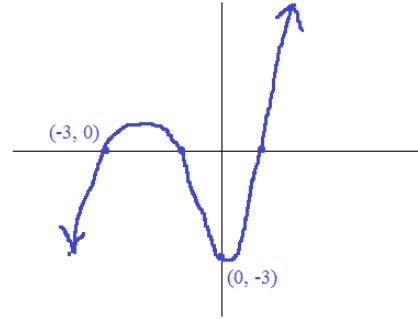
odd degree (3); positive lead coefficient (2)
 therefore,

y-intercept: $(0, -3)$

x-intercept(s): $(\sqrt{\frac{1}{2}}, 0)$ $(-\sqrt{\frac{1}{2}}, 0)$ and $(-3, 0)$

multiplicity:
 ('bounces' and 'pauses') None

as $x \rightarrow \infty$, $y \rightarrow \text{positive infinity}$
 as $x \rightarrow \text{negative infinity}$, $y \rightarrow \text{negative infinity}$



5) $y = (x+2)^2(x-3)(x-1)$

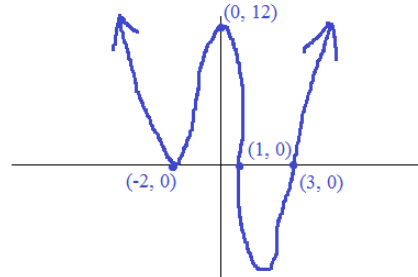
end behavior:
 (degree and lead coefficient)

(if you multiply the terms, and put into standard descending order, degree is 4 and lead coefficient is 1)

y-intercept: plug in 0 for x... $(0, 12)$

x-intercept(s): plug in 0 for y... $(-2, 0)$ $(3, 0)$ $(1, 0)$

multiplicity:
 ('bounces' and 'pauses') since $(x+2)$ has degree 2, the zero -2 has multiplicity of 2... (a "bounce" in the graph)



6) $y = (x-4)^2(x^2+4)$

end behavior:
 (degree and lead coefficient)

Degree: 4 Lead Coefficient: 1

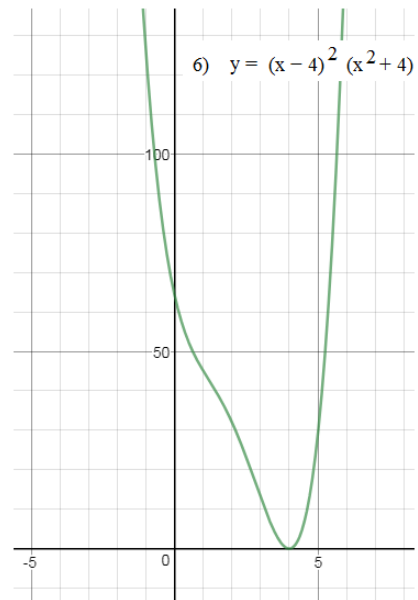
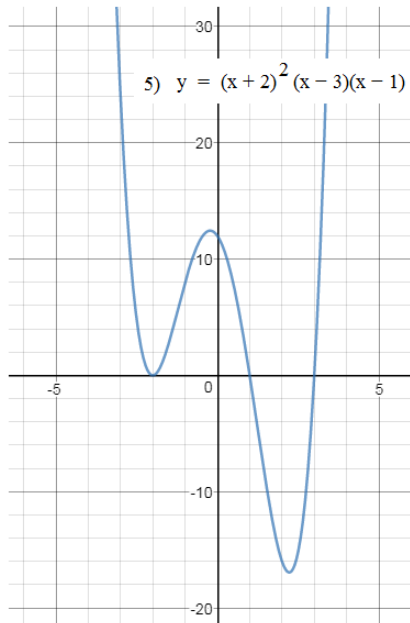
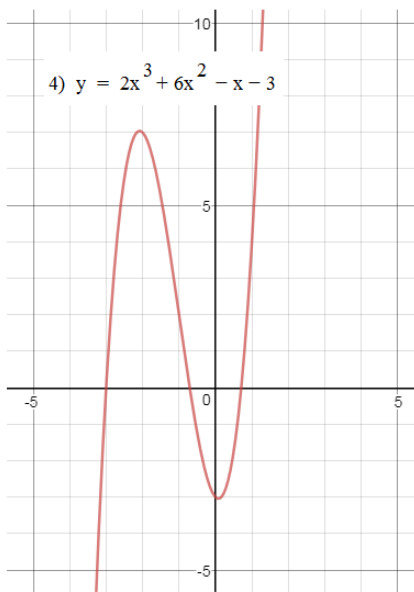
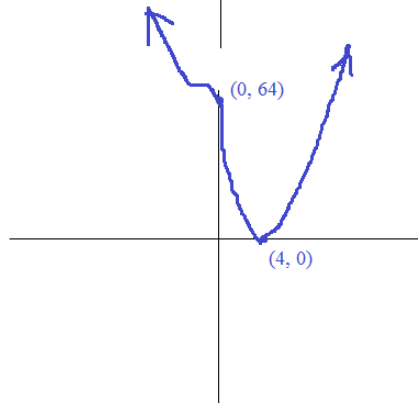
y-intercept: $(0, 64)$

as $x \rightarrow \text{inf.}$, $y \rightarrow \text{inf.}$
 as $x \rightarrow \text{neg. inf.}$, $y \rightarrow \text{inf.}$

x-intercept(s): $(4, 0)$

$$\begin{aligned} x^2 + 4 = 0 & \quad 2 \text{ imaginary roots!!} \\ x^2 = -4 & \quad x = -2i \text{ and } 2i \end{aligned}$$

multiplicity:
 ('bounces' and 'pauses') $(x-4)$ has a degree of 2, so the root 4 has a multiplicity of 2



Determine the end behavior, identify the intercepts, and then sketch the polynomial...

SOLUTIONS

Polynomials Sketching Exercises

7) $h(x) = (x + 3)^3(2x - 5)$

end behavior:
(degree and lead coefficient)

If multiplied/converted to standard form,
lead term would be $2x^4$

y-intercept:
(0, -135)

since degree (4) is even and coefficient (2) is positive,

x-intercept(s):
(-3, 0) and (5/2, 0)

multiplicity:
(‘bounces’ and ‘pauses’) the root -3 has a multiplicity of 3 (‘pause’ in the graph)

factored form

8) $y = -\frac{3}{4}x^4 + \frac{3}{4}$

$y = \frac{-3}{4}(x^4 - 1)$

$y = \frac{-3}{4}(x + 1)(x - 1)(x^2 + 1)$

end behavior:
(degree and lead coefficient)

lead degree is 4 (even) lead coefficient is negative

y-intercept:
(0, 3/4)

x-intercept(s):
(1, 0) and (-1, 0) (The other roots i and $-i$ are imaginary)

multiplicity:
(‘bounces’ and ‘pauses’) None

9) $y = (x + 1)(x - 3)^2(x + 5)^3$

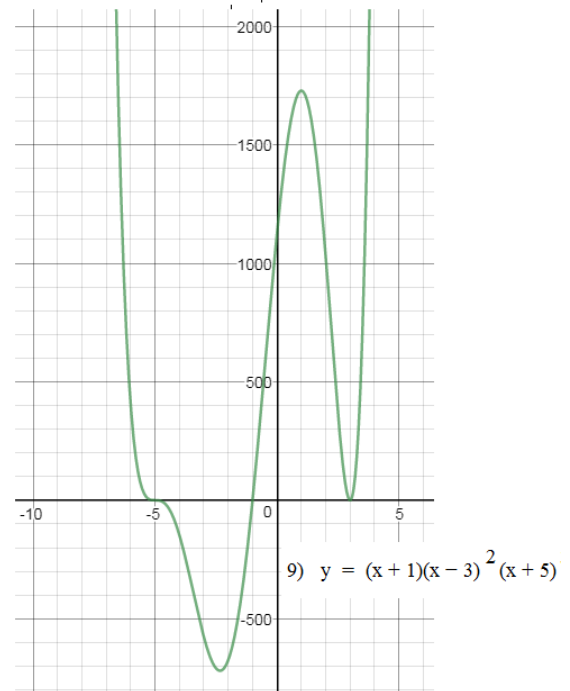
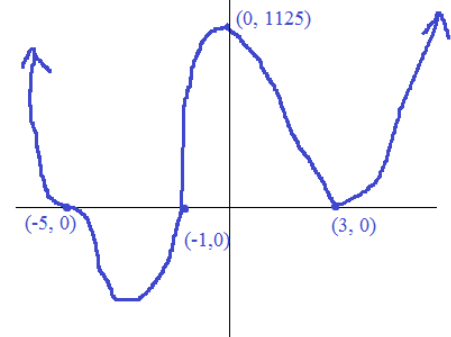
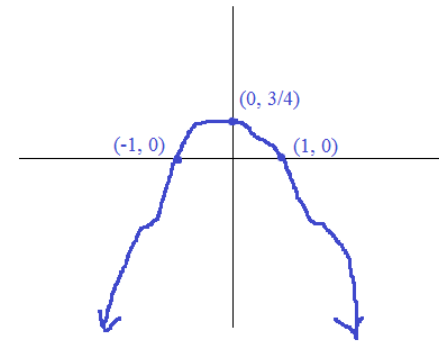
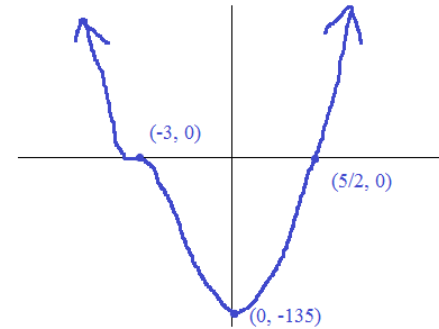
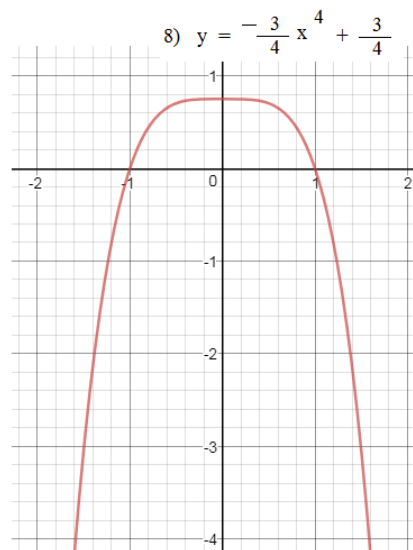
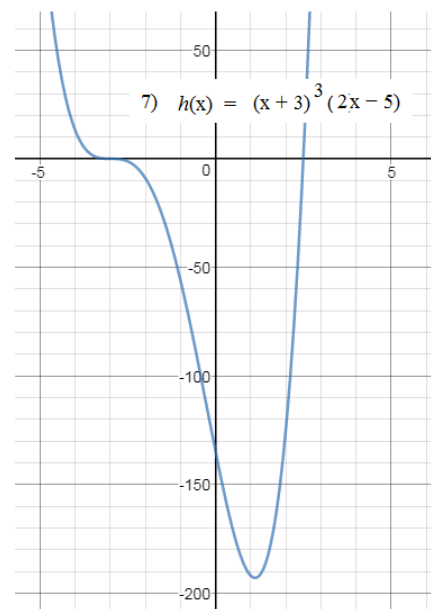
end behavior:
(degree and lead coefficient)

degree is 6 and lead coefficient is 1...

y-intercept:
 $y = (1)(9)(125) = 1125$ (0, 1125)

x-intercept(s):
(-1, 0) (3, 0) (-5, 0)

multiplicity:
(‘bounces’ and ‘pauses’) "bounce" at zero 3
"pause" at zero -5



I. General Topics

A) $f(x) = x^3 - 3x^2 - 6x + 8 = (x - 1)(x - 4)(x + 2)$

Classify the polynomial: (lead degree is 3) -- Cubic polynomial of 4 terms

What are the 'p' values? (factors of the constant) 1, 2, 4, 8

What are the 'q' values? (factors of lead coefficient) 1

List all possible rational zeros: $\pm \frac{p}{q}$ 1, -1, 2, -2, 4, -4, 8, -8

What are the x-intercepts? factor the polynomial: $f(1) = 1 - 3 - 6 + 8 = 0$
 (1, 0) (4, 0) (-2, 0) 1 is a root!

What is the y-intercept?

find $f(0) = 0 - 0 - 0 + 8 = 8$ (0, 8)

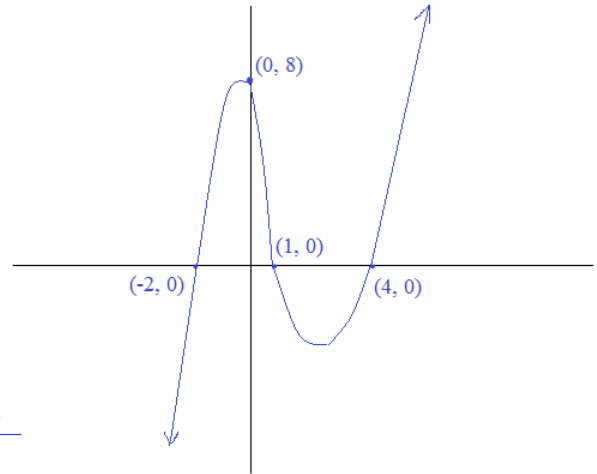
Sketch the function:

Note: you can check your points by plugging them into the original polynomial...

synthetic division

1	1	-3	-6	8
		1	-2	-8
	1	-2	-8	0

$x^2 - 2x - 8$ factor
 $(x - 4)(x + 2)$



B) $g(x) = 2x^3 + 13x^2 + 5x - 6$

What is the degree of the polynomial? 3 (exponent of the lead term)

List all possible rational zeros: $\pm \frac{p}{q}$ p: 1, 2, 3, 6 q: 1, 2

$\pm 1, \pm 2, \pm 3, \pm 6$
$\pm \frac{1}{2}, \pm \frac{3}{2}$

Identify the zeros: factor the polynomial using factor theorem and synthetic division

What are the factors? $(x + 1)(2x - 1)(x + 6)$

What is the remainder of $g(x) \div (x + 5)$?

$g(-5) = -250 + 325 - 25 - 6 = 44$
 using remainder theorem

synthetic division

-5	2	13	5	-6
		-10	-15	50
	2	3	-10	44

using synthetic division

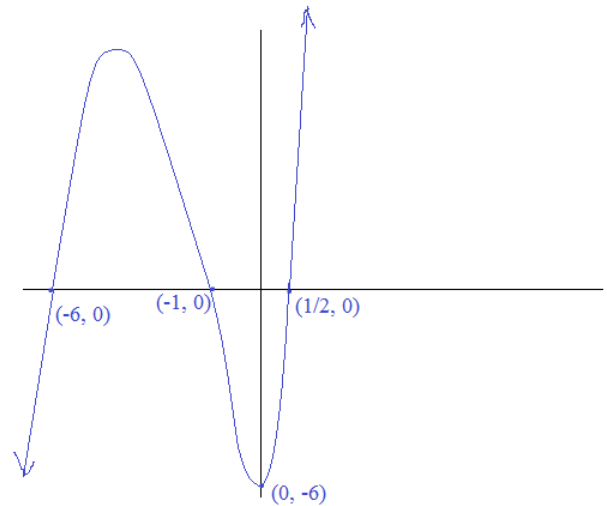
$g(1) = 14$ (not a factor)
 $g(-1) = 0$ (factor!)

synthetic division

-1	2	13	5	-6
		-2	-11	6
	2	11	-6	0

$2x^2 + 11x - 6$
 $(2x - 1)(x + 6)$

Sketch the function:



C) $h(x) = -x^4 + 2x^3 + 8x^2 - 10x - 15$

Classify the polynomial: (lead degree is 4) quartic polynomial (of 5 terms)

Describe the end behavior: degree is 4, lead coefficient is negative:
 as x goes to $-\infty$, $h(x)$ goes to $-\infty$
 as x goes to $+\infty$, $h(x)$ goes to $-\infty$

What are the factors? $(x + 1)(x - 3)(x^2 - 5)$ "polynomial faces down"

What is the y-intercept?

$h(0) = 0 + 0 + 0 - 0 - 15 = -15$
 (0, -15)

Sketch the function:

$-1(x + \sqrt{5})(x - \sqrt{5})$ 3 is a root:

roots are $-1, 3, \sqrt{5}, -\sqrt{5}$

3	-1	3	5	-15
		-3	0	15
	-1	0	5	0

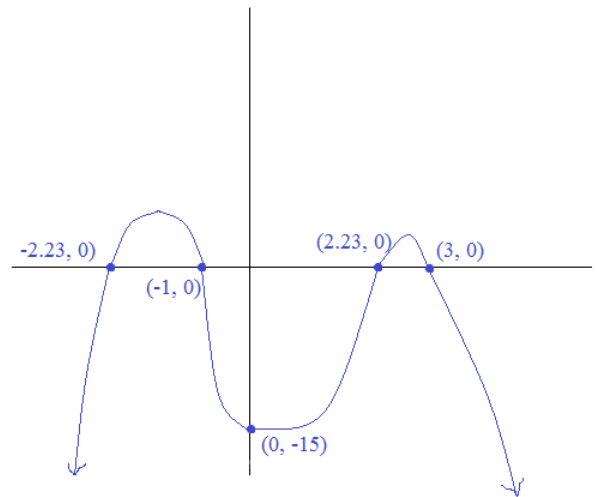
$-x^2 + 5$

possible real zeros: 1, -1, 3, -3, 5, -5, 15, -15
 since $h(-1) = 0$, -1 is a root

synthetic division

-1	-1	2	8	-10	-15
		1	-3	-5	15
	-1	3	5	-15	0

$-x^3 + 3x^2 + 5x - 15$



Polynomials and Roots Test

SOLUTIONS

II. Factoring, Synthetic Division, and Roots

Factor and identify all roots:

A) $x^5 - x^4 + 9x^3 - 9x^2$

Greatest common factor: x^2

$$x^2 (x^3 - x^2 + 9x - 9)$$

"factor by grouping"

$$x^2 (x^2 (x - 1) + 9(x - 1))$$

$$x^2 (x^2 + 9)(x - 1)$$

roots: 0, 0, 3i, -3i, 1

degree is 5; 5 roots...

B) $x^4 - 2x^2 + 1$

$$(x^2 - 1)(x^2 - 1)$$

"difference of squares"

$$(x + 1)(x - 1)(x + 1)(x - 1)$$

roots: -1, -1, 1, 1

C) $x^3 + 4x^2 + 9x + 36$

factor by grouping:

$$x^3 + 4x^2 + 9x + 36$$

$$x^2(x + 4) + 9(x + 4)$$

regroup:

$$(x^2 + 9)(x + 4)$$

roots: -4, -3i, +3i

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = 3i, -3i$$

III. Determining the Polynomial

A) Write a cubic function whose graph passes through

(-2, 0) (2, 0) (-4, 0) (-1, 3)

$$y = a(x - x_1)(x - x_2)(x - x_3)$$

$$y = a(x + 2)(x - 2)(x + 4)$$

plug in (-1, 3) to find a

$$3 = a(-1 + 2)(-1 - 2)(-1 + 4)$$

$$3 = -9a$$

$$a = -1/3$$

$$f(x) = -\frac{1}{3}(x + 2)(x - 2)(x + 4) \quad \text{factored form}$$

general form

$$\text{test points to check solution: } = -\frac{x^3}{3} - \frac{4x^2}{3} + \frac{4x}{3} + \frac{16}{3}$$

B) Write at least two quadratic functions that have x-intercepts (3, 0) and (8, 0)

a general equation will be

$$y = a(x - 3)(x - 8)$$

if a = 1, then $y = (x - 3)(x - 8)$

$$= x^2 - 11x + 24$$

if a = 2, then $y = 2(x - 3)(x - 8)$

$$= 2x^2 - 22x + 48$$

the equations have the same x-intercepts, but are shaped differently.. (for example, note: different y-intercepts)

Write a polynomial of least degree that has real coefficients, lead coefficient of 1, and the given zeros:

A) -2, -2, 2

$$(x - (-2))(x - (-2))(x - 2) = (x + 2)^2(x - 2)$$

B) 2, 5, i (according to the conjugate pairs rule) since i is a zero, then -i is also a zero.

$$(x - 2)(x - 5)(x - i)(x + i)$$

$$(x - 2)(x - 5)(x^2 + 1)$$

C) 4, 2, -3i +3i is also a zero...

$$(x - 4)(x - 2)(x + 3i)(x - 3i)$$

$$(x - 4)(x - 2)(x^2 + 9)$$

D) 2 + i, 2 - i, 3

$$(x - 3)(x - (2 + i))(x - (2 - i))$$

$$(x - 3)[x^2 - 2x + ix - 2x + 4 - 2i - ix + 2i - i^2]$$

$$(x - 3)[x^2 - 4x + 5]$$

test 2 + i

$$(2 + i - 3)[(2 + i)(2 + i) - 4(2 + i) + 5]$$

$$(i - 1)[4 + 4i + i^2 - 8 - 4i + 5]$$

$$(i - 1)[4 + (-1) - 3]$$

$$= 0 \checkmark$$

E) 0, 1, -3

$$(x - 0)(x - 1)(x - -3)$$

$$x(x - 1)(x + 3)$$

IV. Applying Concepts and Theorems

- A) What is the remainder of $10x^5 + 3x^4 - 7x^3 + 2x - 6 \div (x - 1)$?
 (Remainder Theorem)

Using remainder theorem: find $f(1)$: $10 + 3 - 7 + 2 - 6 = 2$

remainder is 2

check with synthetic division:

$$\begin{array}{r|rrrrrr} 1 & 10 & 3 & -7 & 2 & -6 \\ & & 10 & 13 & 6 & 8 \\ \hline & 10 & 13 & 6 & 8 & 2 \end{array}$$

- B) Verify that $x^{20} - 1$ has a factor of $(x + 1)$.
 (Factor Theorem)

Using factor/remainder theorem: find $f(-1) = (-1)^{20} - 1 = 0$

remainder is 0, therefore -1 is a root
 and $(x + 1)$ is a factor!

- C) Three roots of $x^4 - 5x^3 - 33x^2 + 113x + 140$ are -1, -5, 7... What is the 4th root? **The 4th root is 4
 (Sum and Products of Roots)

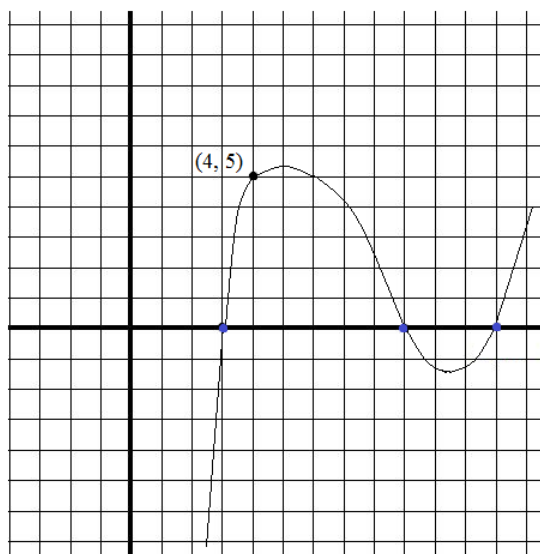
sum of the roots: $-\frac{-5}{1}$ (2nd coefficient)
 (1st coefficient)

product of the roots: $\frac{140}{1}$ (constant)
 (1st coefficient)

$-1 + -5 + 7 + r = 5$ root = 4

$-1 \times -5 \times 7 \times r = 140$
 $r = 4$

- D) Find the y-intercept of the following sketch:



the cubic goes through

(3, 0)

$y = a(x - 3)(x - 9)(x - 12)$

(9, 0)

substitute (4, 5) into (x, y) to find a

(12, 0)

$5 = a(4 - 3)(4 - 9)(4 - 12)$

(4, 5)

$5 = 40a$

$a = 1/8$

$y = 1/8(x - 3)(x - 9)(x - 12)$

to find y-intercept, find $x = 0$

$y = 1/8(0 - 3)(0 - 9)(0 - 12)$

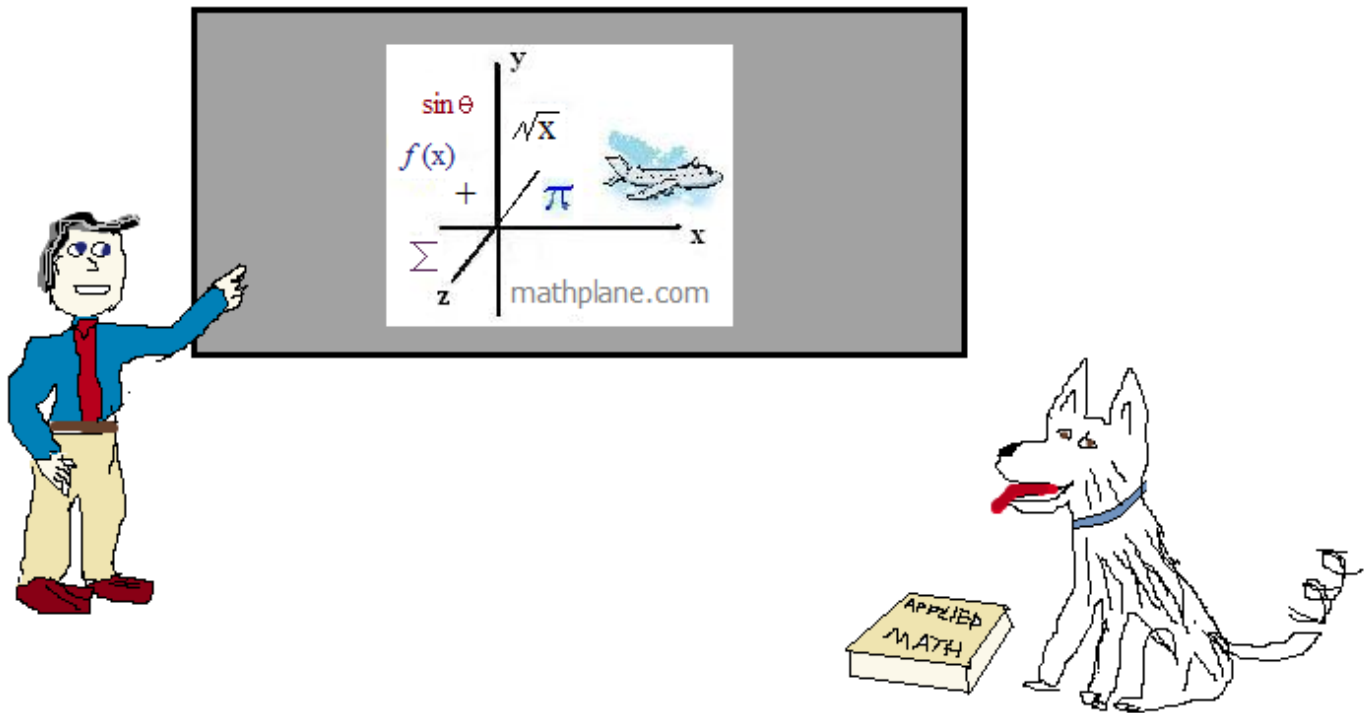
$= 1/8(-324)$

$-\frac{324}{8} = \frac{-81}{2}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, our stores at TeachersPayTeachers and TES

And, we're at Mathplane Express for mobile at Mathplane.ORG

One more question

If $(x + 1)$ is a factor of $x^4 + 6x^3 + 13x^2 + Kx + 4$
then $K =$

ANSWER- →

If $(x + 1)$ is a factor of $x^4 + 6x^3 + 13x^2 + Kx + 4$

then $K =$

ANSWER

Utilizing the factor theorem:

(if $(x + 1)$ is a factor, then)

$$f(-1) = (-1)^4 + 6(-1)^3 + 13(-1)^2 + K(-1) + 4 \quad (\text{must equal zero})$$

$$1 - 6 + 13 - K + 4 = 0$$

$$12 - K = 0$$

$$K = 12$$

Using Synthetic Division:

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 13 & K & 4 \\ & & -1 & -5 & -8 & (-K + 8) \\ \hline & 1 & 5 & 8 & (K - 8) & (12 - K) \end{array}$$

Since the remainder must be 0,

$$12 - K = 0$$

therefore, $K = 12$

To continue factoring the polynomial...

$$(x + 1)(x^3 + 5x^2 + 8x + 4)$$

Using the rational root theorem,
possible roots are 1, 2, 4, -1, -2, -4..
And, since all the terms are positive
we can eliminate 1, 2, 4... There is
no way $f(x) = 0$ if x is positive...

Since $f(-1) = 0$...

$(x + 1)$ is a factor again....

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 8 & 4 \\ & & -1 & -4 & -4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$$(x + 1)(x + 1)(x^2 + 4x + 4)$$

then, the factored form is

$$(x + 1)^2(x + 2)^2$$