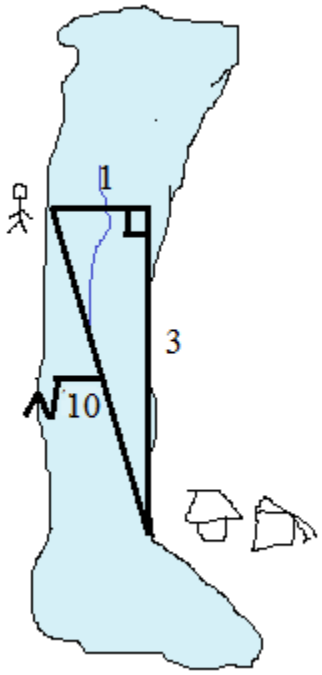
Pythagorean Theorem 2

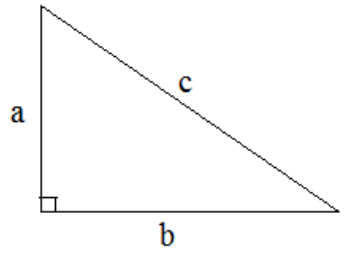
Practice Questions (and Answers)



Topics include Pythagorean Triples, Word Problems, radicals, distance/rate, geometry applications, perimeter, and more.

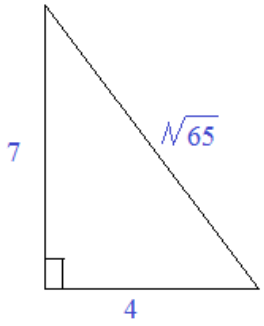
Pythagorean Theorem:

$a^2 + b^2 = c^2$ where a and b are lengths of the legs of a right triangle and c is the length of the hypotenuse



"sum of the squares of the legs is equal to the square of the hypotenuse"

Example:



$(4)^2 + (7)^2 = c^2$

$16 + 49 = 65$

$c = \sqrt{65}$

Identifying triangles by their sides:

$a^2 + b^2 = c^2$ right triangle

$a^2 + b^2 > c^2$ acute triangle

$a^2 + b^2 < c^2$ obtuse triangle

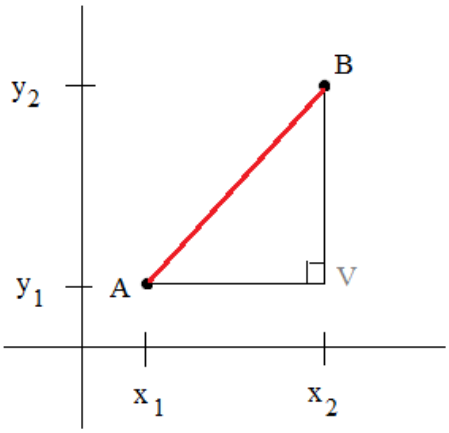
Distance Formula illustrates Pythagorean Theorem!

$AB^2 = AV^2 + BV^2$

point A: (x_1, y_1)

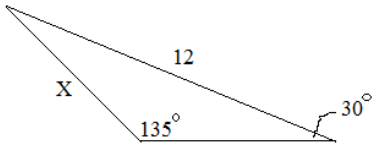
point B: (x_2, y_2)

distance AB $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

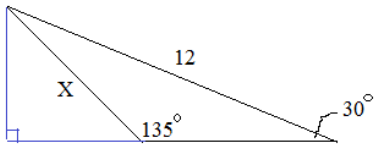


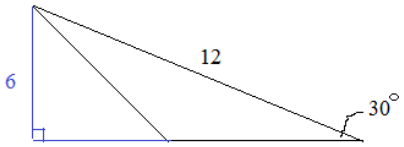
Special Right Triangle Applications

Example: Find X

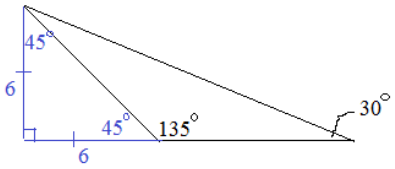


Drop an altitude, creating another triangle....





30-60-90 right triangle: small side is 1/2 of hypotenuse... therefore, side opposite 30 degree angle is 6...



(supplementary angles) We know the left triangle is a 45-45-90 right triangle: hypotenuse is leg $\cdot \sqrt{2}$

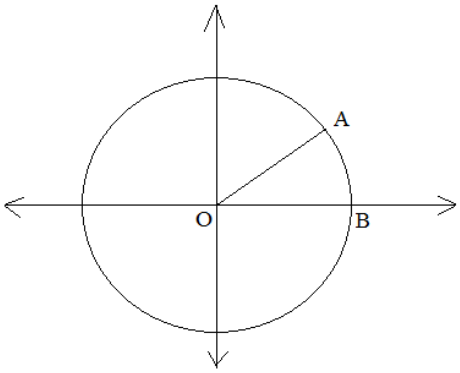
therefore, X = $6\sqrt{2}$

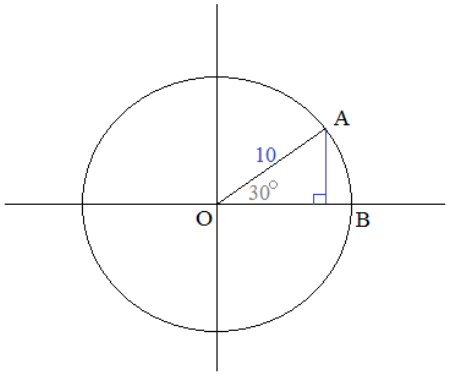
Example: Here is a circle that is centered on the origin. If the radius is 10 and $\angle AOB$ is 30° ,

what is the coordinate of B?

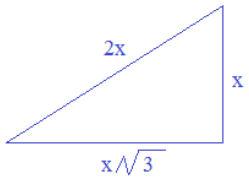
(10, 0)

what is the coordinate of A?





30-60-90 triangle



AB = 5

OB = $5\sqrt{3}$

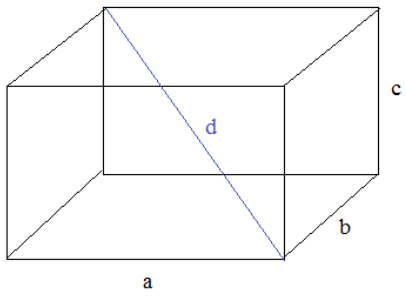
($5\sqrt{3}$, 5)

Finding the diagonal of a rectangular prism

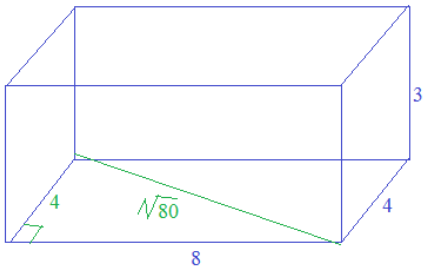
There is a short-cut formula!

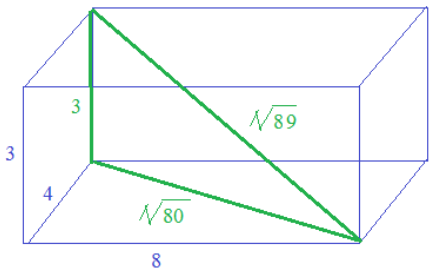
$$\text{diagonal} = \sqrt{a^2 + b^2 + c^2}$$

where a, b, and c are the lengths of the sides



Example: Find the diagonal inside a 3 x 4 x 8 rectangular prism.





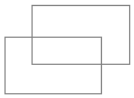
short-cut with the formula ⇒ $\text{diagonal} = \sqrt{3^2 + 4^2 + 8^2}$

$= \sqrt{89}$ ✓

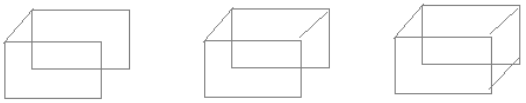
Side-Note: Sketching a rectangular prism

1) draw the rectangle face...

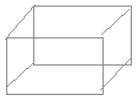
2) copy/draw the same rectangle face in another position



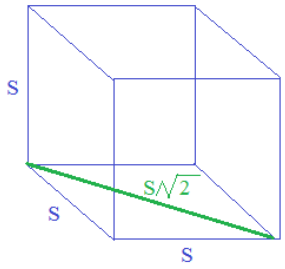
3) draw line segments connecting the corresponding vertices...

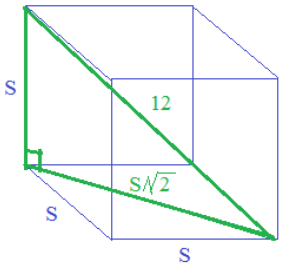


(this process works for any prism!)



Example: The diagonal of a cube is 12.
What is the length of each edge?





Let each side/edge = S

$S^2 + (S\sqrt{2})^2 = 12^2$

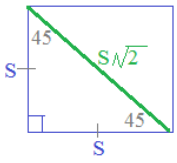
$S^2 + 2S^2 = 144$

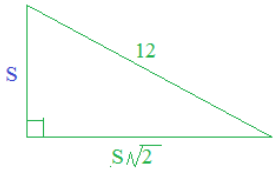
$3S^2 = 144$

$S^2 = 48$

$S = 4\sqrt{3}$

(View of bottom)





(View of side)

Also, diagonal of prism = $\sqrt{a^2 + b^2 + c^2}$

$12 = \sqrt{s^2 + s^2 + s^2}$

$144 = 3S^2 \Rightarrow S = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ ✓

Example: A rectangular prism with diagonal $40\sqrt{2}$ has side edges with a ratio of 3:4:5.
What are the dimensions of the prism?

The diagonal of the bottom rectangle is

$(4x)^2 + (5x)^2 = (d)^2$

$41x^2 = d^2$

$\sqrt{41}\ x = d$

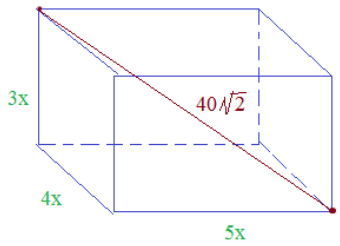
Then, the diagonal of the entire prism is

$(40\sqrt{2})^2 = (\sqrt{41}\ x)^2 + (3x)^2$

$3200 = 41x^2 + 9x^2$

$64 = x^2$

$x = 8$



24 x 32 x 40

$d = \sqrt{(3x)^2 + (4x)^2 + (5x)^2}$

$40\sqrt{2} = \sqrt{50x^2}$

$3200 = 50x^2$

$64 = x^2$

$x = 8$

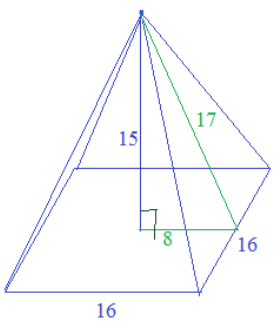
Example: A square pyramid with base perimeter of 64 units has a height of 15.
What is the *slant height*?

Draw the altitude (height) of 15...
then, draw the base of the right triangle, which is half the square.. 8

then, apply Pythagorean Theorem...
(It's a triple)

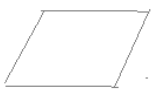
8-15-17

Slant height is 17

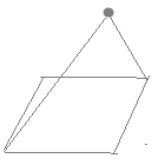


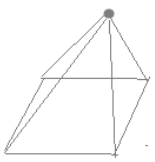
Sketching a rectangular pyramid

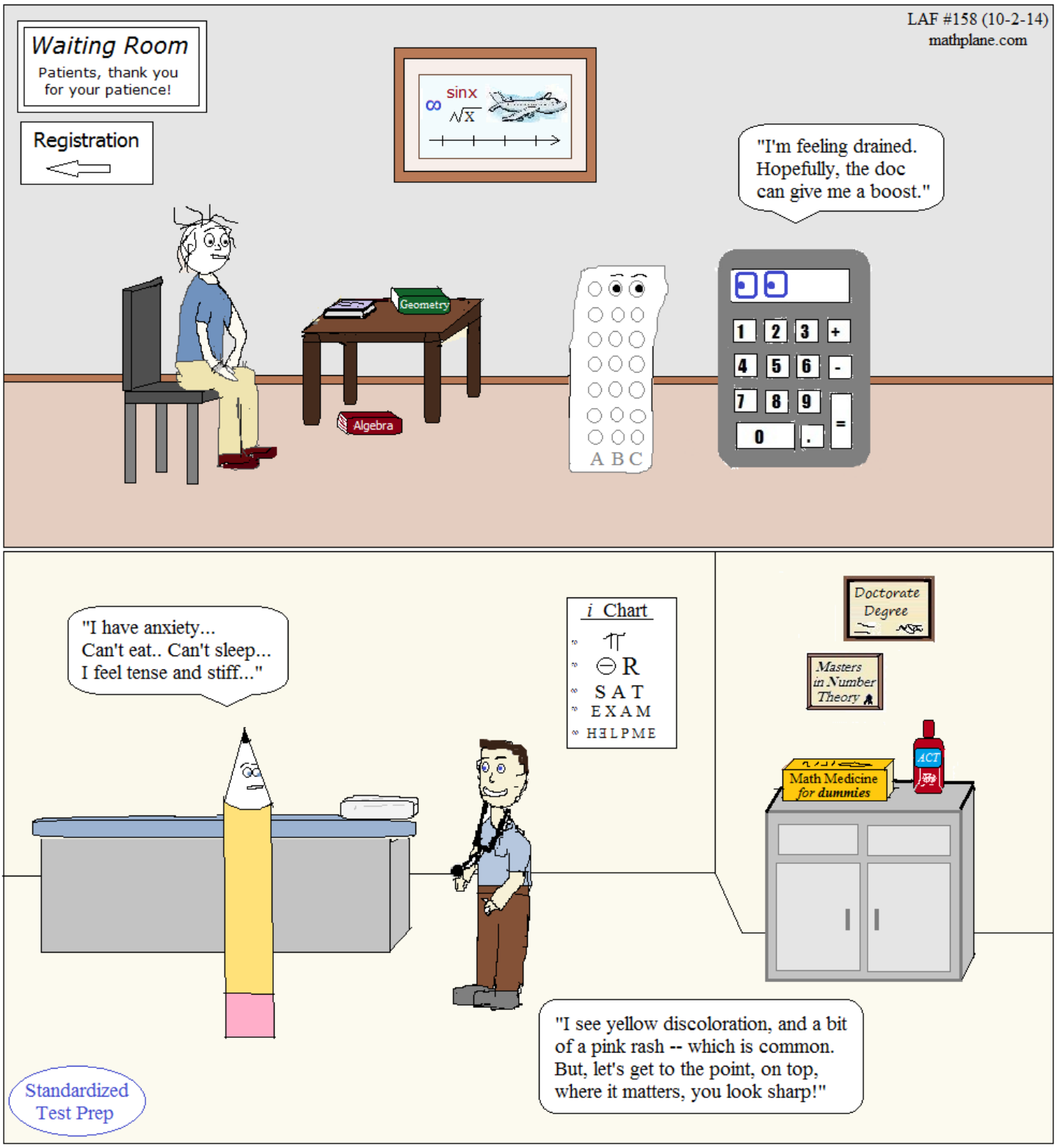
1) draw the rectangle base in the shape of a parallelogram



2) pick a point above the base, and draw 4 segments to each vertex of the parallelogram







Practice Questions-→

Pythagorean Theorem and Distance Practice

1) The following are sides of a triangle.
Determine whether is the triangle is right, obtuse, acute, not possible.

a) 2, 7, 10

b) 4, 5, 8

c) 10, 6, 8

d) 7, 8, 9

e) 11, 11, 11

2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

3) Multiple Choice: Jack traveled through D to get from F to B.
How much shorter is the direct route versus the route he took?

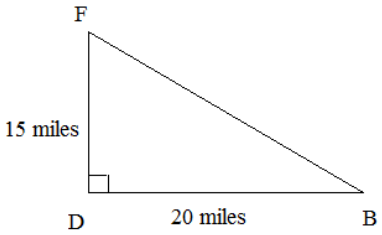
a) 5

b) 10

c) 15

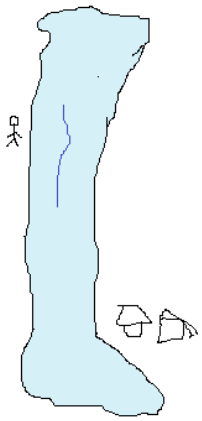
d) 20

e) 25



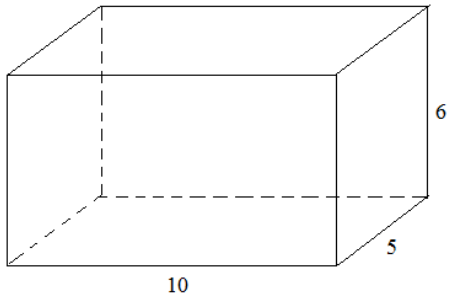
4) Pythagorean Theorem rate question:

A boy stands on the shore of a one-mile wide lake.
He wants to reach camp down shore 3 miles on the opposite side.
He can swim 2mph and walk 4mph.
Is it quicker to swim across and then walk OR swim directly to the camp?

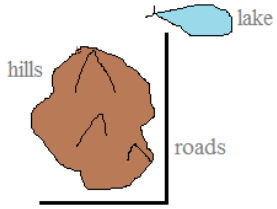


Pythagorean Theorem and Distance Practice

5) Find the length of the diagonal of the rectangular prism.

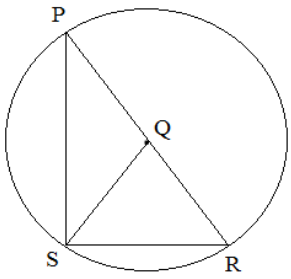


6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake.
(15 miles due East. Then, 25 miles due North)...
Meanwhile, a bird flying at 7 miles per hour can go directly over the hills.
Who would reach the water first?



7) Given: Circle Q
$\overline{PS} \perp \overline{SR}$
$\overline{PS} = 36$
$\overline{SR} = 15$

Find: The area of circle Q

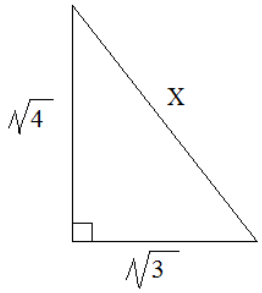


8) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?

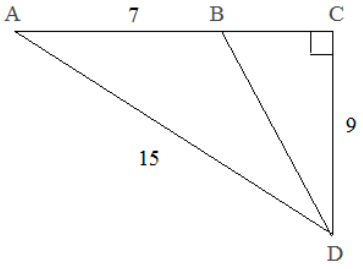
Pythagorean Theorem and Distance Practice

9) What is X?



10) Find the perimeter of a rectangle whose base is 10 and diagonal measure is 16.

11) What is the perimeter of $\triangle$ BCD?

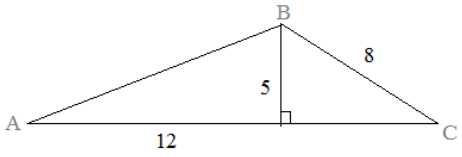


12) $\overline{TM}$ is an altitude of equilateral triangle TRI.

If $\overline{RI} = 7$, what is the measure of $\overline{TM}$?

Pythagorean Theorem and Distance Practice

13) Find the area of the triangle:



14) A boat is tied to a dock by 25 feet of rope.
The dock is 15 feet above the water.

If 8 feet of rope is pulled in, how far will the boat move toward the dock?

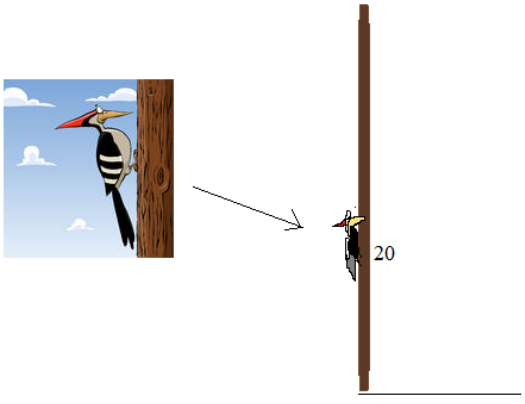
15) Sammy the snail and Ted the turtle have lunch together at the jungle cafe.

At noon, Sammy leaves, heading due north at 15 feet per hour.
Then, at 1:00pm, Ted leaves, heading due east at 8 yards per hour.

How far apart are they at 6:00pm?

16) A woodpecker is perched up against a 20-foot pole, pecking away!
Eventually, he chips away enough of the wood that the pole cracks, buckles, and folds over: the top of the pole landing on the ground 12 feet from the bottom of the pole.
Undeterred, it stands on the top and continues pecking away!

How high off the ground is the woodpecker?



Identify a related Pythagorean Triple. Then, find x.

a) 15 - 20 - x 3 - 4 - 5 x = 25

b) 9 - x - 15

c) x - 30 - 34

d) 24 - 32 - x

e) 10 - x - 26

f) x - 60 - 65

g) 40 - x - 85

h) 18 - 80 - x

i) 14 - x - 50

j) 100 - 105 - x

k) x - 70 - 74

l) 35 - x - 125

m) 2.5 - x - 6.5

Pythagorean Triple (or, Triplet)

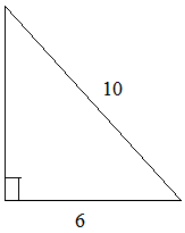
consists of 3 *positive integers* a, b, c that satisfy the Pythagorean Theorem

$$a^2 + b^2 = c^2$$

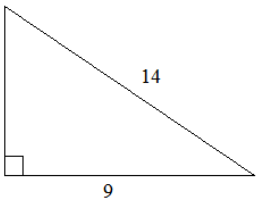
Find the area and perimeter of each figure.

Pythagorean Theorem

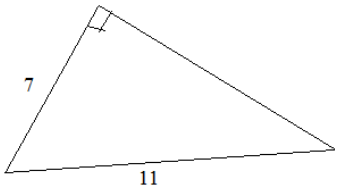
1)



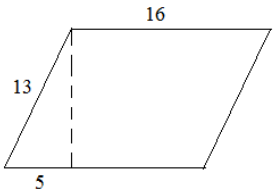
2)



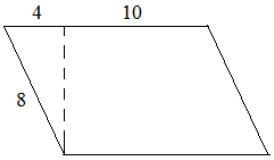
3)



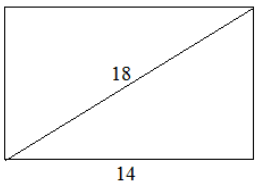
4) (parallelogram)



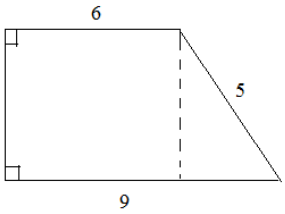
5) (parallelogram)



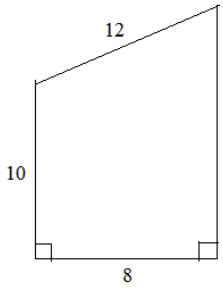
6) (rectangle)



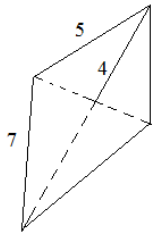
7)



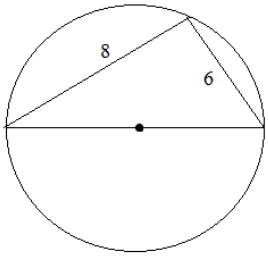
8)



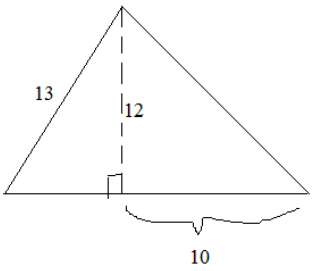
9) (kite)



10) (circle)

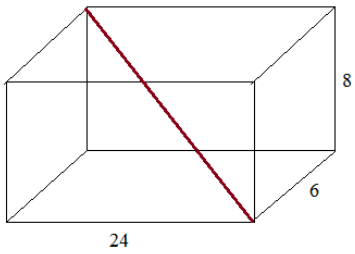


11)

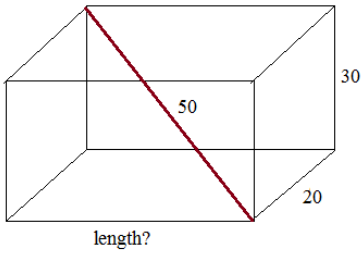


Pythagorean Theorem in 3-D Space

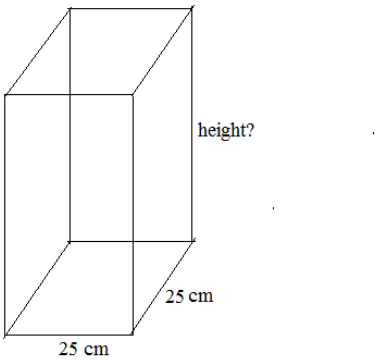
1) Find the diagonal of the rectangular prism
(figure not necessarily drawn to scale)



2) Find the length of the base of the rectangular prism.

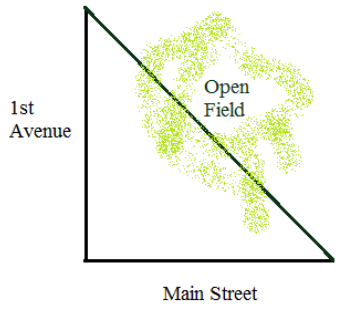


3) Paolo needs to send ski poles to Swen in Sweden.
The poles are 110 cm long, and the shipping box has a square base 25cm x 25cm.
What is the minimum height of the box required to ship the poles?



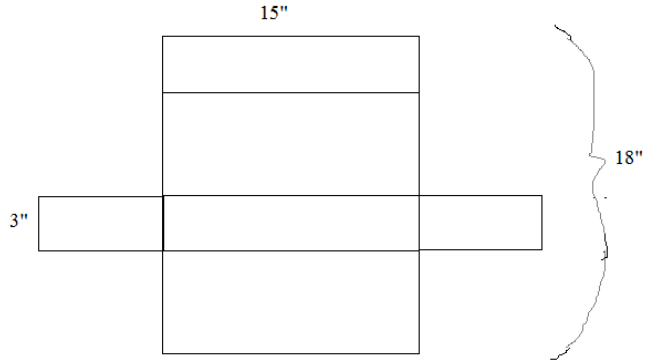
4) A runner ordinarily runs west on Main Street and then north on 1st Avenue. Today, he took a short-cut and ran directly through an open field for 20 km.

If his usual run is 28 km, what are the distances along Main Street and 1st Avenue?



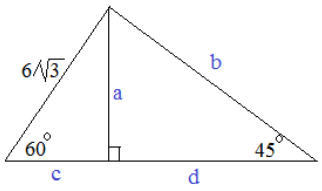
5) You need to send an item in a box constructed from the following cardboard.

What is the longest item that could fit in the constructed box?



Special Right Triangles

1) Find the measures of the labeled sides:

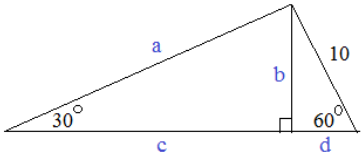


a)

b)

c)

d)

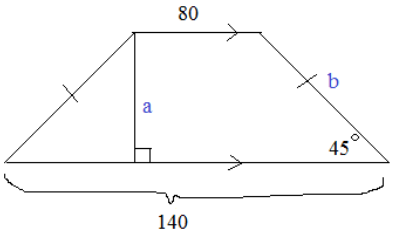


a)

b)

c)

d)



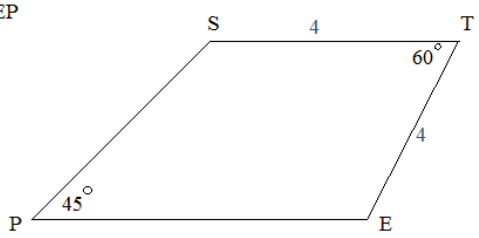
a)

b)

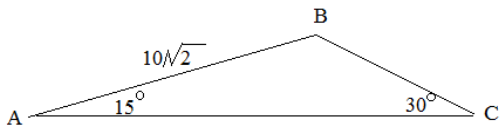
2) Given: Trapezoid STEP

Find $\overline{EP}$

$\overline{SP}$

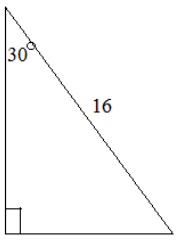


3) ***Challenge: Find the perimeter of ABC...

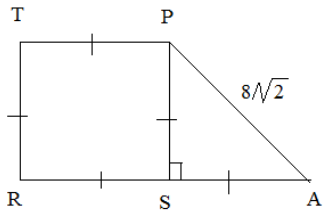


Special Right Triangle Applications

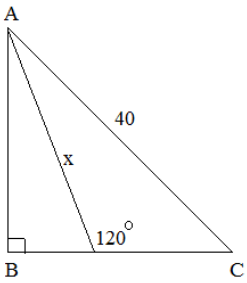
4) What is the area of the triangle?



5) What is the area of TRAP?



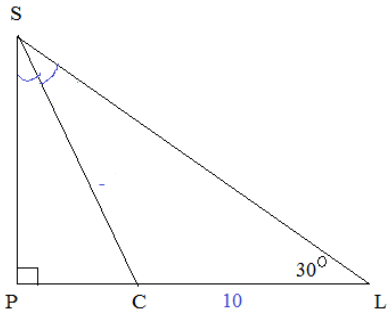
6) If $\overline{AB} = \overline{BC}$, then what is x?



7) $\overline{SC}$ bisects $\angle PSL$

$\overline{CL} = 10$

Find SP and PL



8) Find the perimeter of square SQAR where vertices are Q (-4, 1) and R (-1, 6).

a) 16

b) $4\sqrt{34}$

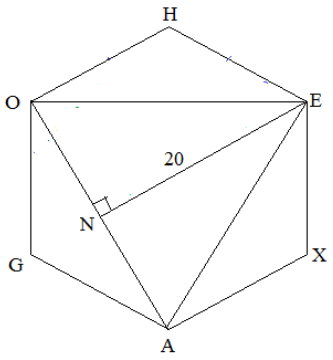
c) $4\sqrt{17}$

d) 32

e) $16\sqrt{2}$

9) What is the perimeter of the regular hexagon, HEXAGO?

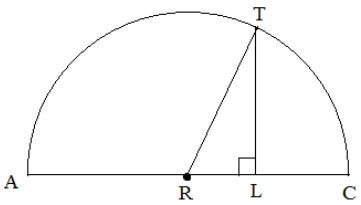
What is the perimeter of the triangle AOE?



10) The figure is a semicircle.

The 'diameter' AC is 16

If TL = 7, what is RL?



11) EDTA is a square.

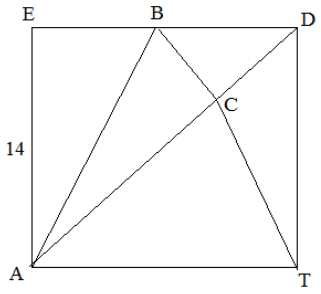
B is the midpoint of $\overline{ED}$

$\overline{BC}$ is perpendicular to the diagonal $\overline{AD}$

$\overline{AB}$ = _____

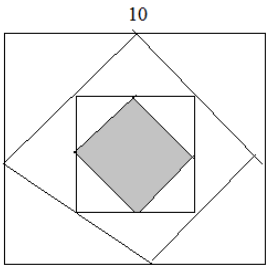
$\overline{CD}$ = _____

$\overline{AC}$ = _____

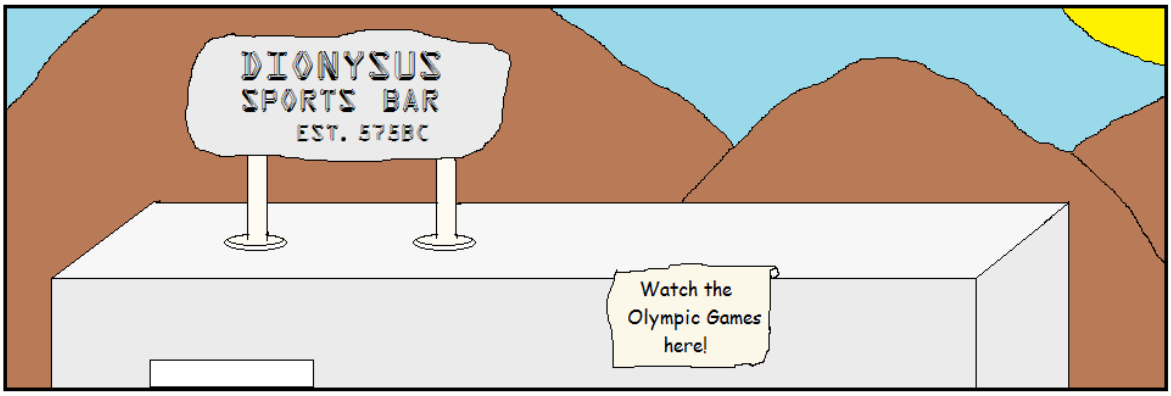


12) The following figure contains inscribed squares.

What is the area and perimeter of the shaded square?



Math Mixer in Ancient Greece



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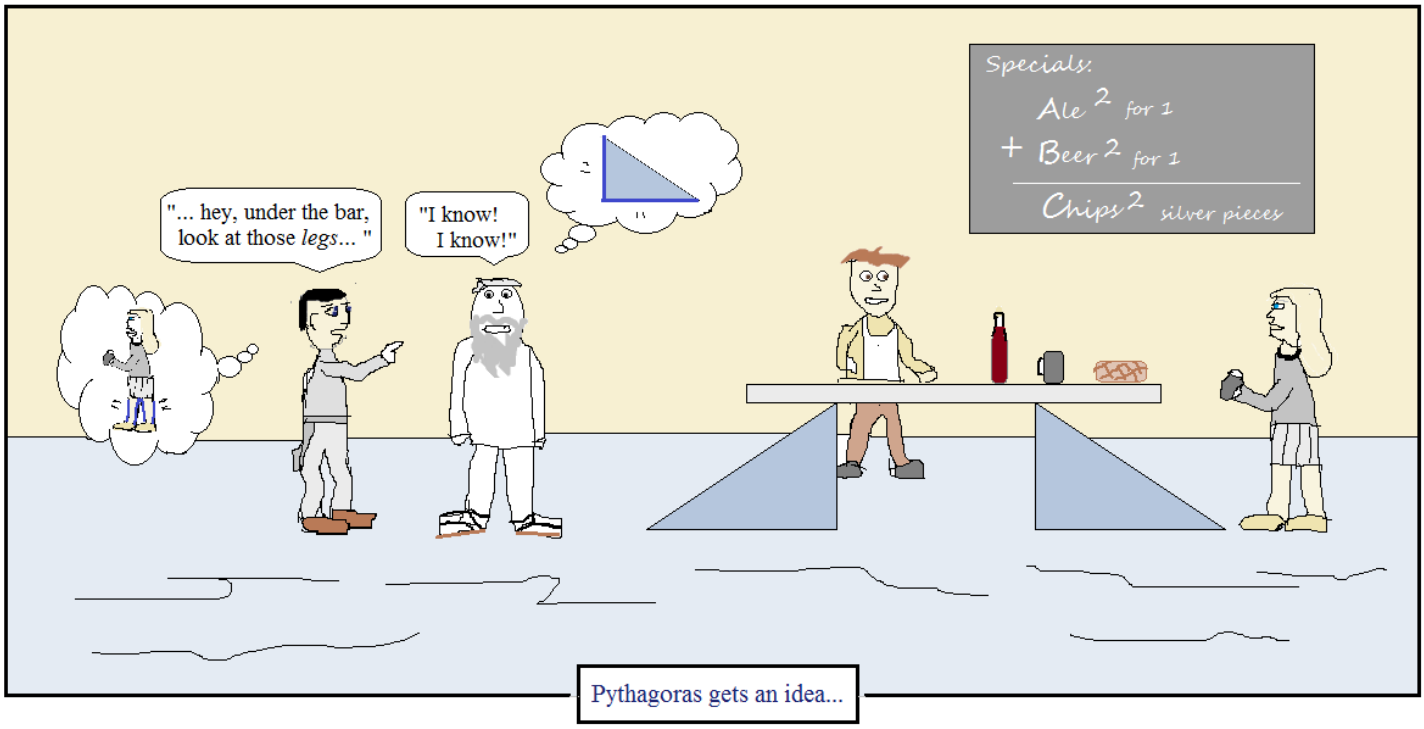
www.mathplane.com



Pythagoras gets an idea...

ANSWERS-→

SOLUTIONS

Pythagorean Theorem and Distance Practice

1) The following are sides of a triangle.
Determine whether is the triangle is right, obtuse, acute, not possible.

a) 2, 7, 10 not possible $2 + 7 < 10$

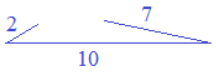
b) 4, 5, 8 obtuse $16 + 25 < 64$

c) 10, 6, 8 right ('3-4-5' triangle) $36 + 64 = 100$

d) 7, 8, 9 acute $49 + 64 > 81$

e) 11, 11, 11 acute (equilateral triangle)

$121 + 121 > 121$



$a^2 + b^2 = c^2$ right

$a^2 + b^2 > c^2$ acute

$a^2 + b^2 < c^2$ obtuse

$a + b < c$ not possible

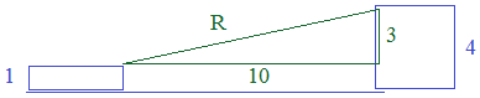
2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

Ramp $(R)^2$ = $(\text{height})^2$ + $(\text{distance})^2$

$R^2 = 9 + 100$

$R = \sqrt{109}$

Diagram:



3) Multiple Choice: Jack traveled through D to get from F to B.
How much shorter is the direct route versus the route he took?

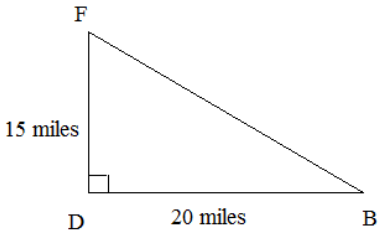
a) 5

b) 10

c) 15

d) 20

e) 25



long route = 15 miles + 20 miles = 35 miles

short route: $15^2 + 20^2 = \overline{FB}^2$

$\overline{FB} = 25$ miles

FB is 10 miles shorter than FDB

4) Pythagorean Theorem rate question:

A boy stands on the shore of a one-mile wide lake.
He wants to reach camp down shore 3 miles on the opposite side.
He can swim 2mph and walk 4mph.
Is it quicker to swim across and then walk OR swim directly to the camp?

distance = rate(time)

$\text{time} = \frac{\text{distance}}{\text{rate}}$

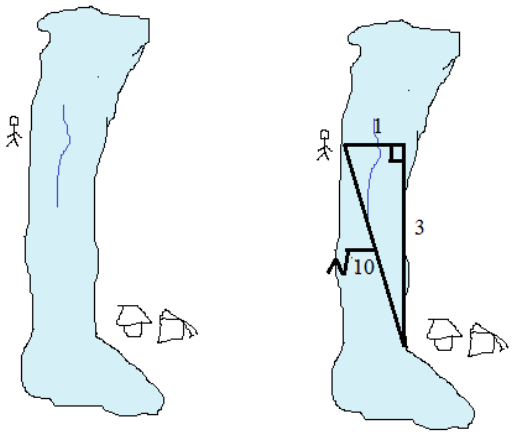
swim directly:

$\text{time} = \frac{\sqrt{10} \text{ miles}}{2 \text{ mph}} = 1.58 \text{ hours}$

swim and walk:

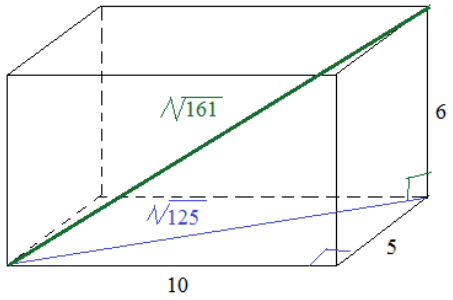
$\text{time (swim)} = \frac{1 \text{ mile}}{2 \text{ mph}} = .5 \text{ hours}$

$\text{time (walk)} = \frac{3 \text{ miles}}{4 \text{ mph}} = .75 \text{ hours}$



It's faster to swim across and then walk....

5) Find the length of the diagonal of the rectangular prism.



SOLUTIONS

Pythagorean Theorem and Distance Practice

one method: first, find diagonal of bottom:

$d^2 = a^2 + b^2$

$d^2 = 10^2 + 5^2 = 125$

$d = 5\sqrt{5}$

then, find the prism's diagonal:

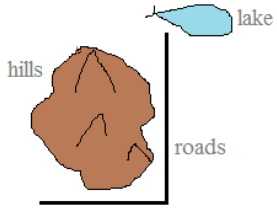
$D^2 = d^2 + c^2$

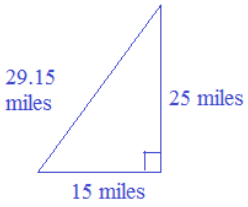
$D^2 = 125 + 6^2$

$D = \sqrt{161}$

shortcut: $\sqrt{10^2 + 5^2 + 6^2} = \sqrt{161}$

6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake.
(15 miles due East. Then, 25 miles due North)...
Meanwhile, a bird flying at 7 miles per hour can go directly over the hills.
Who would reach the water first?





Pythagorean Theorem:

$a^2 + b^2 = c^2$

$15^2 + 25^2 = c^2$

$c = 5\sqrt{34}$ or approx. 29.15 miles

distance = rate x time

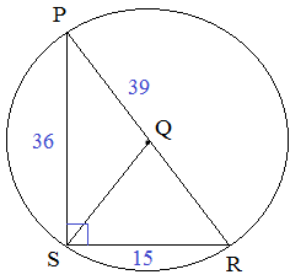
biker: 40miles = (10m/hr)(time)
time = 4 hours

bird: 29.15 miles = (7m/hr)(time)
time = 4.16 hours (approx.)

The biker will reach the lake first!

7) Given: Circle Q
$\overline{PS} \perp \overline{SR}$
$\overline{PS} = 36$
$\overline{SR} = 15$

Find: The area of circle Q



15 - 36 - X X = 39

(5 - 12 - 13 right triangle)

diameter: 39 radius: 19.5

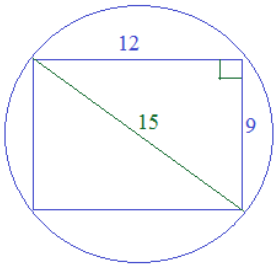
Area = π (radius)^2

Area = $\pi (19.5)^2$

$= 380.25\pi$ square units

8) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?



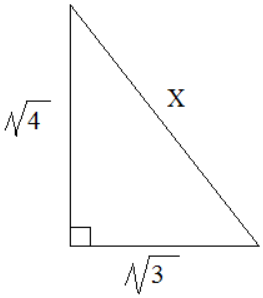
diameter of circle is 15

circumference = π (diameter)

15π

Pythagorean Theorem and Distance Practice

9) What is X?



SOLUTIONS

This is NOT a 3-4-5 Pythagorean Triple!

$X \neq \sqrt{5}$

$(\sqrt{3})^2 + (\sqrt{4})^2 = X^2$

$3 + 4 = X^2$

$X = \sqrt{7}$

Since X is a side length, it cannot be negative...

10) Find the perimeter of a rectangle whose base is 10 and diagonal measure is 16.

Use Pythagorean Theorem...

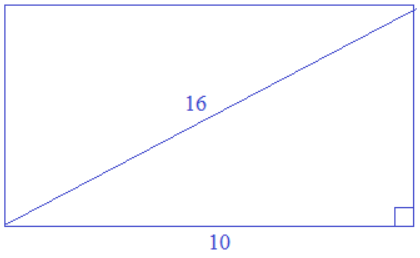
$a^2 + 10^2 = 16^2$

$a^2 = 156$

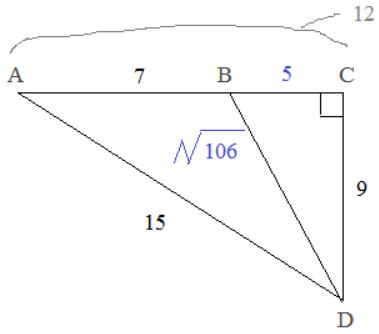
$a = 2\sqrt{39}$

So, the perimeter is

$10 + 10 + 2\sqrt{39} + 2\sqrt{39} = 20 + 4\sqrt{39}$



11) What is the perimeter of △ BCD?



9-12-15 right triangle

then, Pythagorean Theorem:

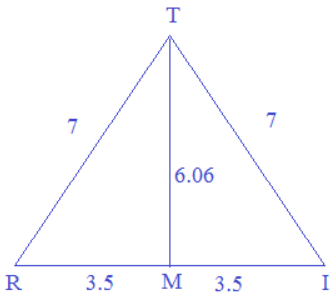
$5^2 + 9^2 = \overline{BD}^2$

$BD = \sqrt{106}$

Perimeter = $14 + \sqrt{106}$

12) $\overline{TM}$ is an altitude of equilateral triangle TRI.

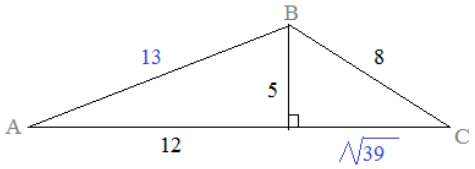
If $\overline{RI} = 7$, what is the measure of $\overline{TM}$?



$\overline{TM} = 6.06$

Pythagorean Theorem and Distance Practice

13) Find the area of the triangle:



$a^2 + b^2 = c^2$

$25 + b^2 = 64$

$b = \sqrt{39}$

SOLUTIONS

Area $= \frac{1}{2}$ (base)(height)

$= \frac{1}{2}(12 + \sqrt{39})(5)$

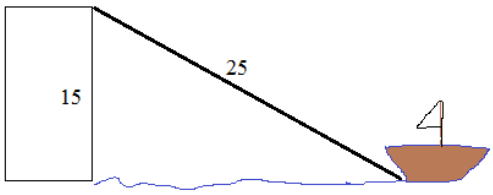
$=$ 45.6 (approx)

14) A boat is tied to a dock by 25 feet of rope.
The dock is 15 feet above the water.

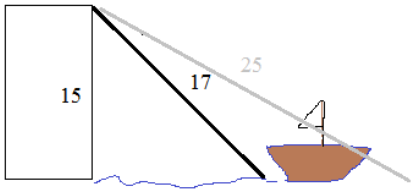
If 8 feet of rope is pulled in, how far will the boat move toward the dock?

Step 1: Sketch a picture

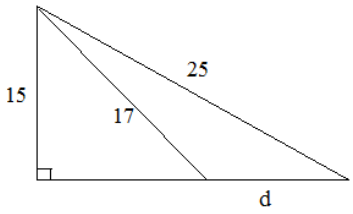
Before...



After 8 feet of rope is pulled in...

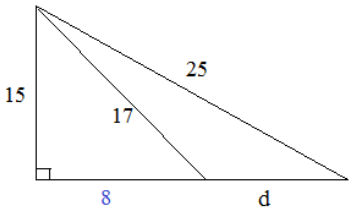


Step 2: Diagram with Right Triangles



We want to find d
(the distance the boat moved)

Step 3: Solve



8-15-17 Pythagorean Triplet

Then,

$(8 + d)^2 + 15^2 = 25^2$

$(8 + d)^2 = 400$

$d = 12$

The boat moved 12 feet toward the dock

Pythagorean Theorem and Distance Practice

15) Sammy the snail and Ted the turtle have lunch together at the jungle cafe.

At noon, Sammy leaves, heading due north at 15 feet per hour.
Then, at 1:00pm, Ted leaves, heading due east at 8 yards per hour.

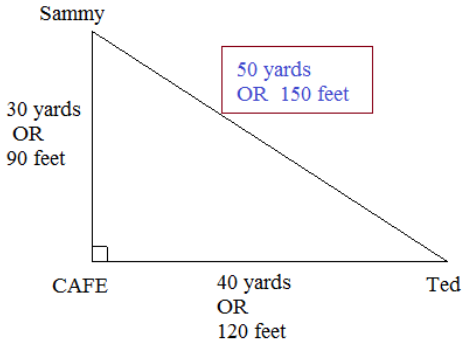
How far apart are they at 6:00pm?

6 hours at
15 feet/hour
OR
5 yds/hour

CAFE

5 hours at
8 yards/hour
OR
24 feet/hour

SOLUTIONS



Pythagorean Theorem

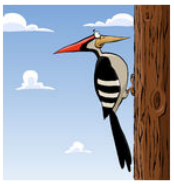
$30^2 + 40^2 = 50^2$ Yards

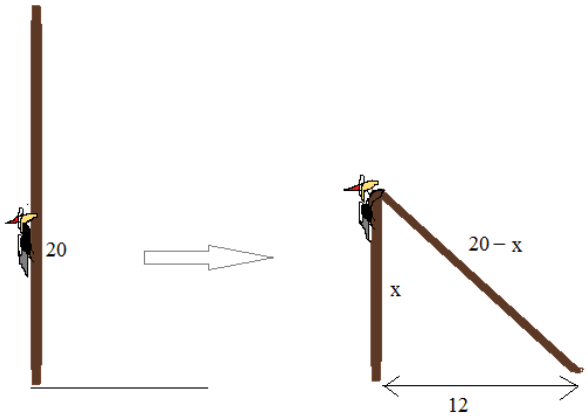
OR

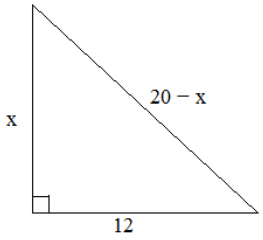
$90^2 + 120^2 = 150^2$ Feet

16) A woodpecker is perched up against a 20-foot pole, pecking away!
Eventually, he chips away enough of the wood that the pole cracks, buckles, and folds over:
the top of the pole landing on the ground 12 feet from the bottom of the pole.
Undeterred, it stands on the top and continues pecking away!

How high off the ground is the woodpecker?







$x^2 + 12^2 = (20 - x)^2$

$x^2 + 144 = 400 - 40x + x^2$

$40x = 256$

$x = 6.4$ feet

Identify a related Pythagorean Triple. Then, find x.

a) 15 - 20 - x 3 - 4 - 5 x = 25

b) 9 - x - 15 3 - 4 - 5 x = 12

c) x - 30 - 34 8 - 15 -17 x = 16

d) 24 - 32 - x 3 - 4 - 5 x = 40

e) 10 - x - 26 5 - 12 -13 x = 24

f) x - 60 - 65 5 - 12 -13 x = 25

g) 40 - x - 85 8 - 15 - 17 x = 75

h) 18 - 80 - x 9 - 40 - 41 x = 82

i) 14 - x - 50 7 - 24 - 25 x = 48

j) 100 - 105 - x 20 - 21 - 29 x = 145

k) x - 70 - 74 12 - 35 - 37 x = 24

l) 35 - x - 125 7 - 24 - 25 x = 120

m) 2.5 - x - 6.5 5 - 12 - 13 x = 6

SOLUTIONS

A few Pythagorean Triples:

3, 4, 5

5, 12, 13

8, 15, 17

7, 24, 25

9, 40, 41

12, 35, 37

20, 21, 29

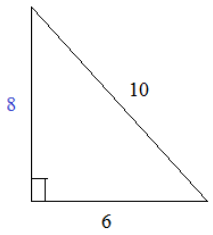
Find the area and perimeter of each figure.

mathplane.com

SOLUTIONS

Pythagorean Theorem

1) $10^2 - 6^2 = b^2 \quad b = 8$



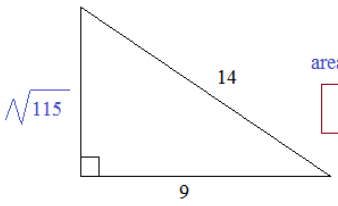
area $= \frac{1}{2}$ (base)(height)

$= \frac{1}{2}(6)(8)$

$= 24$

perimeter: 24

2) $14^2 - 9^2 = b^2 \quad b = \sqrt{115}$

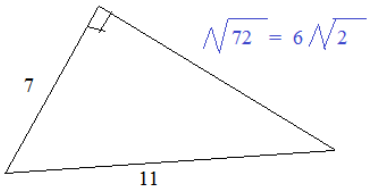


area $= \frac{9}{2}\sqrt{115}$

approx 48.26

perimeter: $23 + \sqrt{115}$

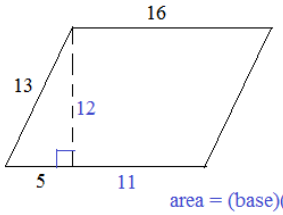
3)



area = $21\sqrt{2}$

perimeter: $18 + 6\sqrt{2}$

4) (parallelogram)



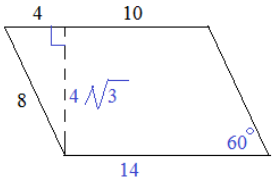
area = (base)(height)

$5^2 + 12^2 = 13^2$

$= (16)(12) = 192$

perimeter: 58

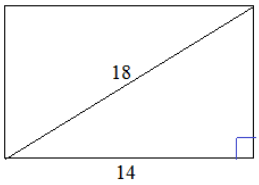
5) (parallelogram)



area = $56\sqrt{3}$

perimeter: 44

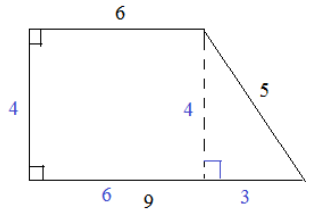
6) (rectangle)



area = $112\sqrt{2}$

perimeter = $28 + 16\sqrt{2}$

7)

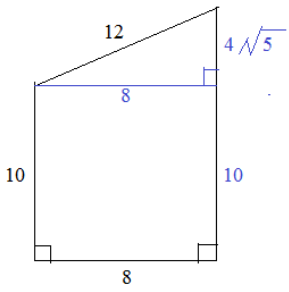


perimeter: 24

area $= \frac{1}{2}$ (base1 + base2)(height)

$= \frac{1}{2}(9 + 6)(4) = 30$

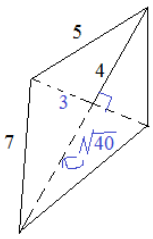
8)



area = $80 + 16\sqrt{5}$

perimeter = $40 + 4\sqrt{5}$

9) (kite)

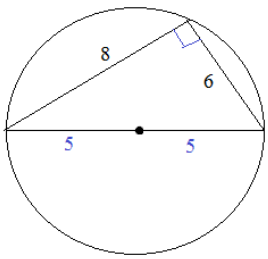


perimeter = 24

area $= \frac{1}{2}$ (diagonal 1)(diagonal 2)

$\frac{1}{2}(6)(4 + 2\sqrt{10}) \approx$ 40.97

10) (circle)

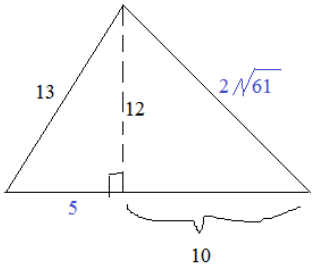


Note: triangle inscribed in semicircle is right triangle..

perimeter/circumference: 10π

area = 25π

11)



area = (1/2)(15)(12) = 90

perimeter = $28 + 2\sqrt{61}$

Pythagorean Theorem: $a^2 + b^2 = c^2$

rectangle: area = (length)(width)
perimeter = 2(length) + 2(width)

triangle: area = (1/2)(base)(height)
perimeter = (side) + (side) + (side)

circle: area = π (radius)^2

circumference = 2π(radius)

kite: area = (1/2)(diagonal 1) (diagonal 2)
(or, find area of each triangle)

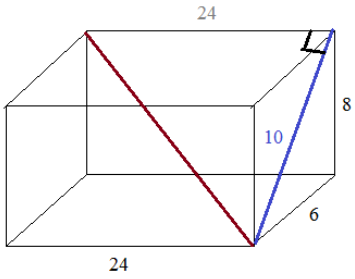
trapezoid: area = (1/2)(base1 + base2)(height)

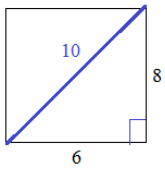
Pythagorean Theorem in 3-D Space

1) Find the diagonal of the rectangular prism
(figure not necessarily drawn to scale)

SOLUTIONS

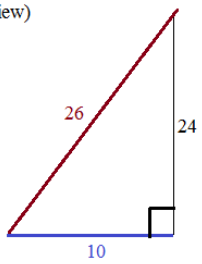
For ease, we'll apply Pythagorean Theorem to the side first (because it's a Triple)



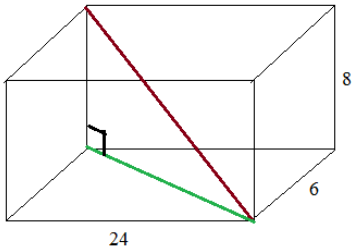


Then, we'll apply Pythagorean Theorem to other part of prism to get the diagonal...

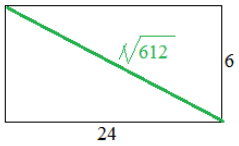
(top view)

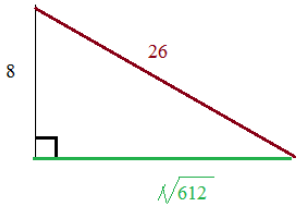


The diagonal is 26.

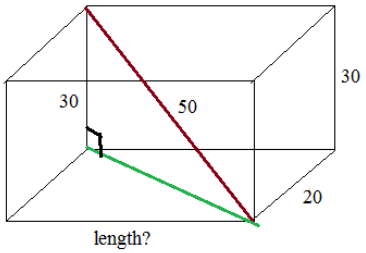


The more difficult route: apply Pythagorean Theorem to bottom first...

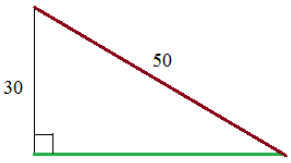




2) Find the length of the base of the rectangular prism.



(side view)

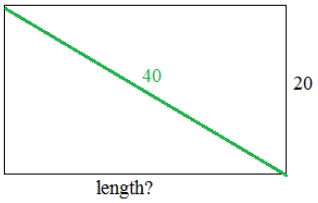


Using Pythagorean Theorem,

$d^2 + 30^2 = 50^2$

we can see the base diagonal is 40 cm...

(top view)

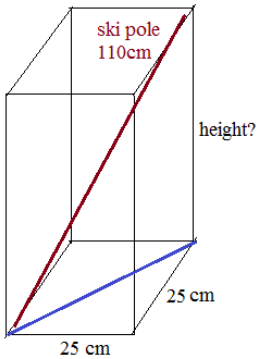


Then, using Pythagorean Theorem,

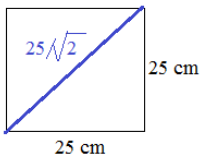
$20^2 + (\text{length})^2 = 40^2$

we can see the length is $20\sqrt{3}$

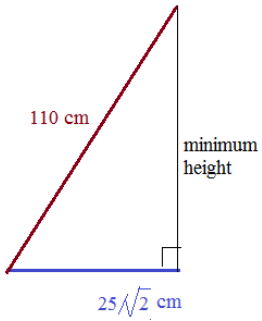
3) Paolo needs to send ski poles to Swen in Sweden.
The poles are 110 cm long, and the shipping box has a square base 25cm x 25cm.
What is the minimum height of the box required to ship the poles?



(top view)



(side view)



$(\text{base diagonal})^2 + (\text{height})^2 > (110\text{ cm})^2$

$1250 + (\text{height})^2 > 12{,}100$

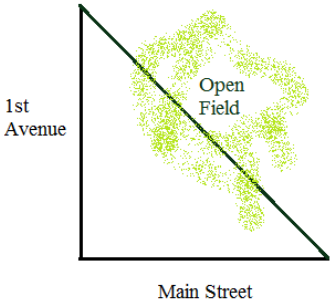
$\text{height} > 104.16$ cm

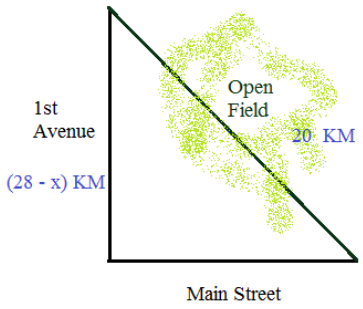
The height must be greater than 104.16, in order to fit the 110 cm ski poles..

4) A runner ordinarily runs west on Main Street and then north on 1st Avenue.
Today, he took a short-cut and ran directly through an open field for 20 km.

If his usual run is 28 km, what are the distances along Main Street and 1st Avenue?

SOLUTIONS





$$x^2 + (28 - x)^2 = 20^2$$

$$x^2 + 784 - 56x + x^2 = 400$$

$$2x^2 - 56x + 384 = 0$$

$$x^2 - 28x + 192 = 0$$

$$x = 12 \text{ or } 16$$

Two possible answers:

Main Street length: 12 km
1st Avenue length: 16 km

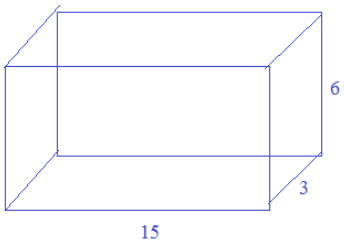
or

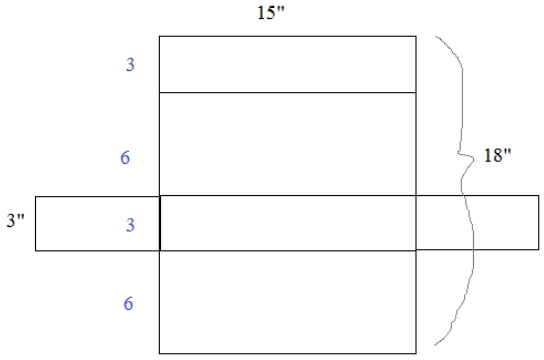
Main Street length: 16 km
1st Avenue length: 12 km

5) You need to send an item in a box constructed from the following cardboard.

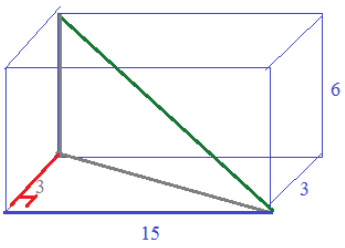
What is the longest item that could fit in the box?

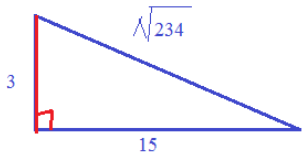
First, let's construct the box....



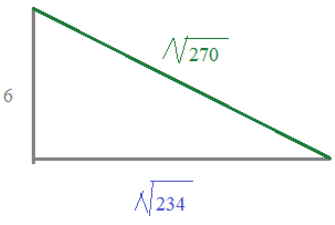


Then, extract the triangles that you'll use to get the diagonal...





(right triangle from the floor/base of the rectangular prism)

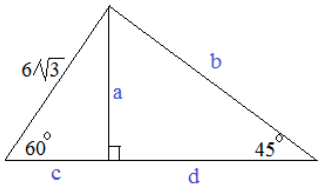


(the diagonal's right triangle.)

1) Find the measures of the labeled sides:

SOLUTIONS

Special Right Triangles

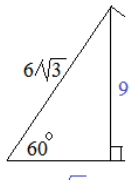


a) 9

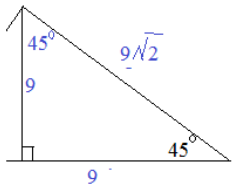
b) $9\sqrt{2}$

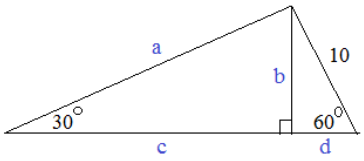
c) $3\sqrt{3}$

d) 9



$3\sqrt{3}$ (half the hypotenuse)



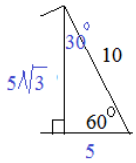


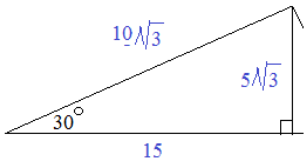
a) $10\sqrt{3}$

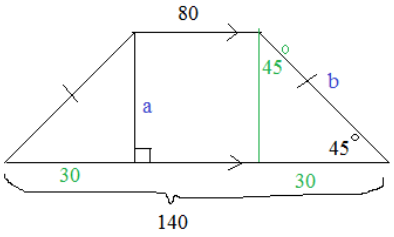
b) $5\sqrt{3}$

c) 15

d) 5

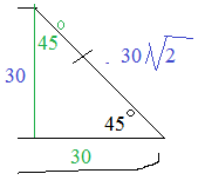






a) 30

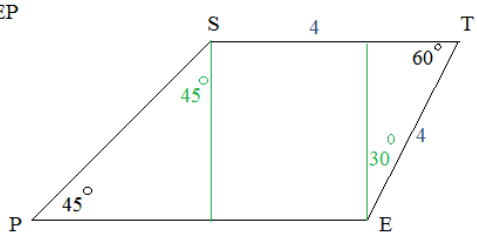
b) $30\sqrt{2}$



2) Given: Trapezoid STEP

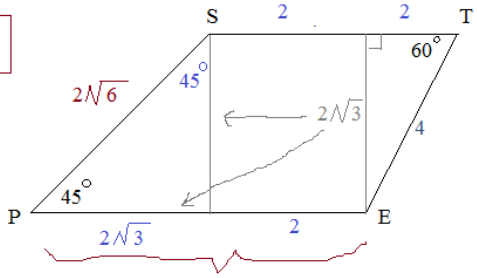
Find $\overline{EP}$

$\overline{SP}$

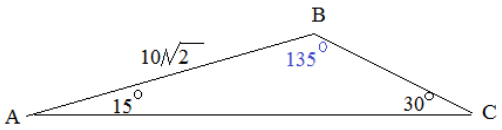


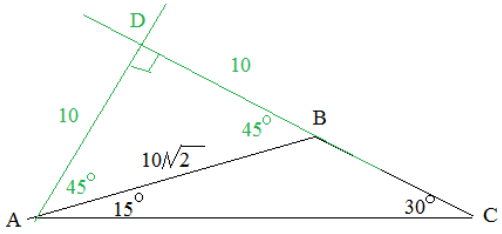
$EP = 2 + 2\sqrt{3}$

$SP = 2\sqrt{6}$



3) ***Challenge: Find the perimeter of ABC...





Auxilary lines create a 30-60-90 right triangle ACD

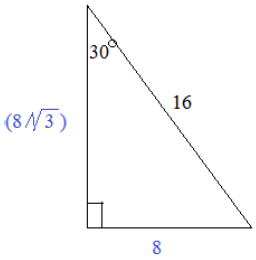
Since $\overline{AD} = 10$, $\overline{AC} = 20$ and $DC = 10\sqrt{3}$

perimeter of ABC is $20 + 10\sqrt{2} + (10\sqrt{3} - 10)$

Special Right Triangle Applications

SOLUTIONS

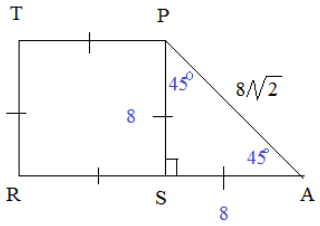
4) What is the area of the triangle?

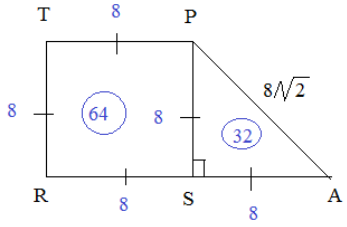


Area of triangle = $\frac{1}{2}$(base)(height)

$\frac{1}{2}(8)(8\sqrt{3}) = 32\sqrt{3}$

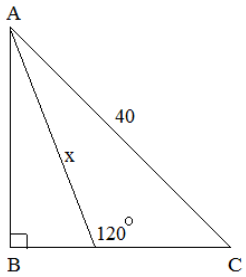
5) What is the area of TRAP?

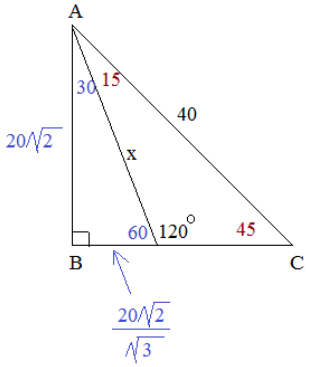




Area of TRAP is 96 square units

6) If $\overline{AB} = \overline{BC}$, then what is x?





$\overline{AB} = \overline{BC} = 20\sqrt{2}$

$\frac{20\sqrt{2}}{\sqrt{3}} \cdot 2 = x$

$x = \frac{40\sqrt{6}}{3}$

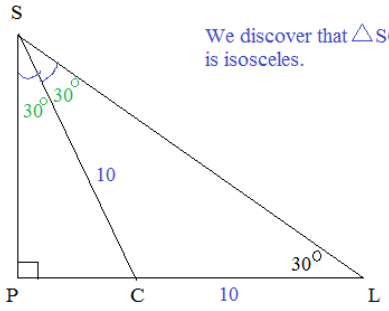
7) $\overline{SC}$ bisects $\angle PSL$

$\overline{CL} = 10$

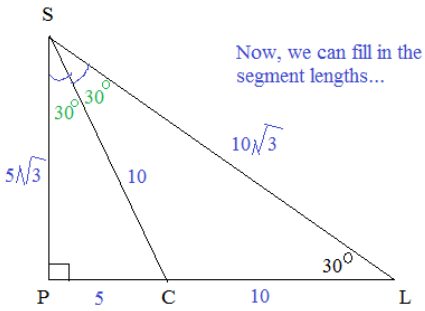
Find SP and PL

$SP = 5\sqrt{3}$

$PL = 15$



We discover that $\triangle SCL$ is isosceles.



Now, we can fill in the triangle segment lengths...

8) Find the perimeter of square SQAR where vertices are Q (-4, 1) and R (-1, 6).

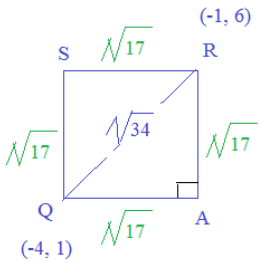
a) 16

b) $4\sqrt{34}$

c) $4\sqrt{17}$

d) 32

e) $16\sqrt{2}$



length of diagonal is $\sqrt{34}$

since 45-45-90 triangle, each side is $\frac{\sqrt{34}}{\sqrt{2}}$

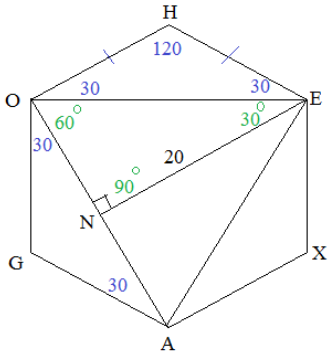
SOLUTIONS

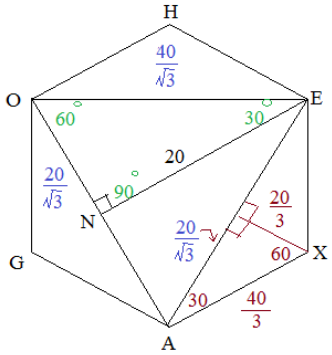
9) What is the perimeter of the regular hexagon, HEXAGO?

What is the perimeter of the triangle AOE?

Interior angles of regular hexagon are 120° ...

From there, we fill out the angle measures in the figure -- notice the 30-60-90 triangles..





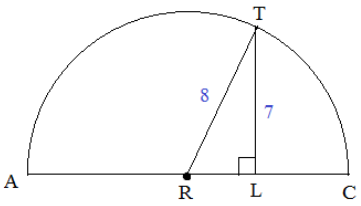
△ AOE is equilateral ----> perimeter = $\frac{120}{\sqrt{3}} = 40\sqrt{3}$

Side $\overline{AX}$ is $\frac{40}{3}$ ----> perimeter of hexagon = 80

10) The figure is a semicircle.

The 'diameter' AC is 16

If TL = 7, what is RL?



Since the diameter is 16, all radii are 8....

RT = 8

TL = 7 (given) then, use Pythagorean Theorem

$RL^2 + 7^2 = 8^2$ $\quad \overline{RL} = \sqrt{15}$

11) EDTA is a square.

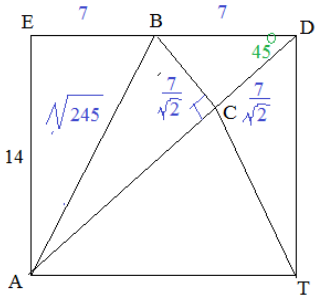
B is the midpoint of $\overline{ED}$

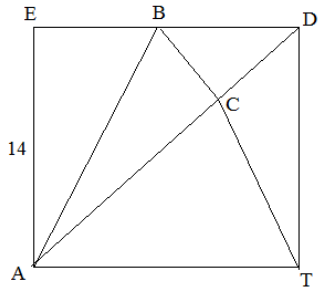
$\overline{BC}$ is perpendicular to the diagonal $\overline{AD}$

$\overline{AB}$ = $\sqrt{245}$

$\overline{CD}$ = $\frac{7}{\sqrt{2}}$

$\overline{AC}$ = $\frac{21\sqrt{2}}{2}$





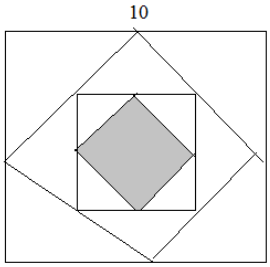
$14\sqrt{2} - \frac{7}{\sqrt{2}} = 14\sqrt{2} - \frac{7}{2}\sqrt{2} = \frac{21\sqrt{2}}{2}$

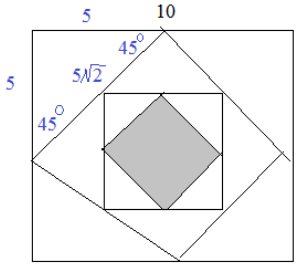
12) The following figure contains inscribed squares.

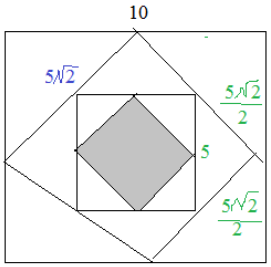
What is the area and perimeter of the shaded square?

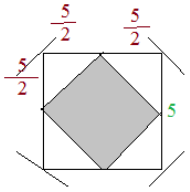
perimeter of shaded square = $10\sqrt{2}$

area of shaded square = $\frac{50}{4}$







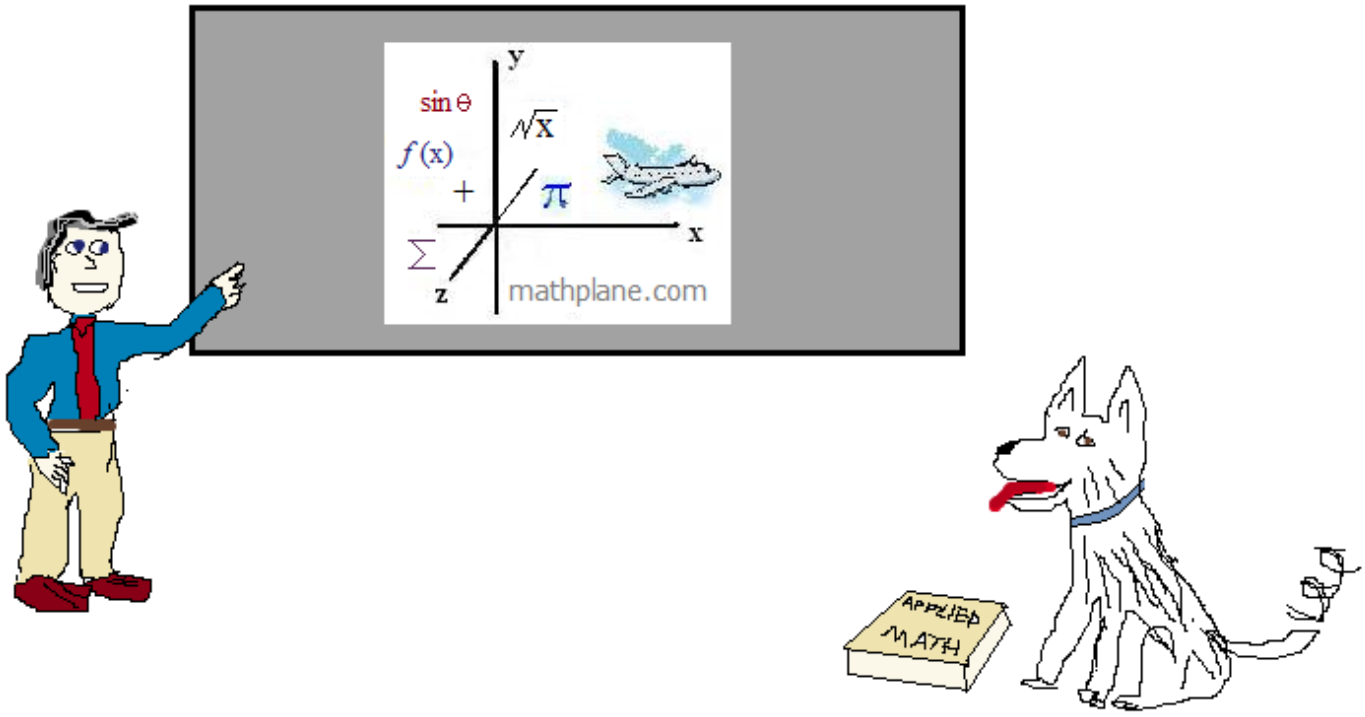


so, each side of the shaded square is $\frac{5\sqrt{2}}{2}$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at TES and TeachersPayTeachers

And, Mathplane *Express* for mobile at mathplane.ORG

One more question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10. What is the measure of each leg and base?

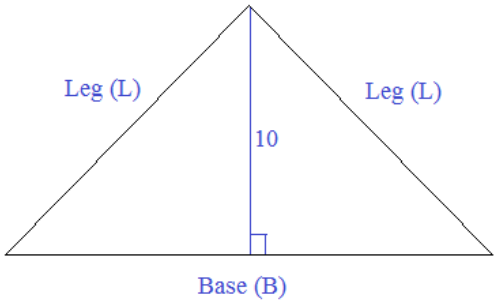
ANSWER-→

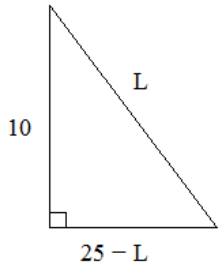
One More Question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10.

What is the measure of each leg and base?

Step 1: Draw a picture and label parts





Step 2: Solve (applying Pythagorean Theorem)

base + leg + leg = 50

Therefore, base = 50 − 2L

and half the base is $\frac{1}{2}(50 - 2L) = 25 - L$

$a^2 + b^2 = c^2$

$10^2 + (25 - L)^2 = L^2$

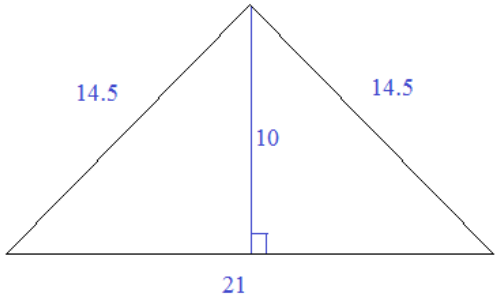
$100 + 625 - 50L + L^2 = L^2$

$725 = 50L$

$L = 14.5$

Each leg is 14.5

Step 3: Check answers



$B = 50 - 2L$

$= 50 - 29 = 21$

The base is 21

Perimeter of triangle = 14.5 + 14.5 + 21 = 50 ✓

Since legs are congruent, it's an isosceles triangle. ✓

$(10.5)^2 + (10)^2 = (14.5)^2$

$110.25 + 100 = 210.25$ ✓

Pythagorean Theorem confirms right and left triangles