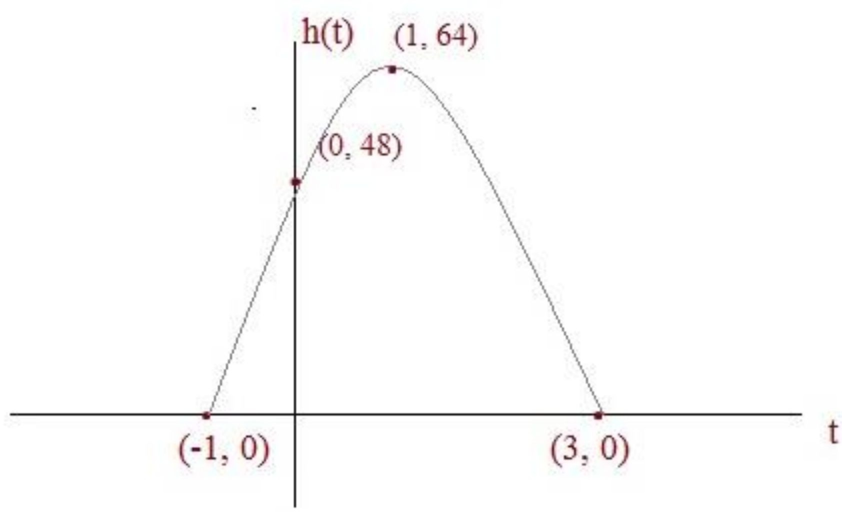
Algebra II:

Quadratics Overview



Includes Quadratic Forms, Intercepts, Graphs, Completing the Square, Word Problems, the Discriminant, and more.

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Quadratics Overview: Forms, Graphs, & Word Problems

Definition of Quadratic: A polynomial of degree 2

Three Forms:

"Standard Form" -- $f(x) = ax^2 + bx + c$ where $a \neq 0$

vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$

$(0, c)$ is the y-intercept

If $a > 0$, the parabola faces up; vertex is minimum value
If $a < 0$, the parabola faces down; vertex is maximum value

"Vertex Form" -- $f(x) = a(x - h)^2 + k$

'Vertex form' displays the vertex: (h, k)

"Intercept Form" or "Factored Form" -- $f(x) = a(x - x_1)(x - x_2)$

'Intercept form' shows the x-intercepts:

$(x_1, 0)$ $(x_2, 0)$

The axis of symmetry is a vertical line through the midpoint of x_1 and x_2

(Algebra techniques can change a quadratic from one form to another.)

Example: $f(x) = x^2 + 10x + 16$

$f(0) = 0 + 0 + 16 = 16$ $\boxed{(0, 16) \text{ is y-intercept}}$

(standard form $\longrightarrow$ intercept form) Factor: $x^2 + 10x + 16$

$(x + 8)(x + 2)$

$(x + 8)(x + 2) = 0$ $\boxed{(-8, 0) \ \& \ (-2, 0) \text{ are x-intercepts}}$

midpoint of -8 & -2 is -5 $\boxed{\text{axis of symmetry: } x = -5}$

(standard form $\longrightarrow$ vertex form) Completing the Square: $x^2 + 10x + 16$

$x^2 + 10x + 25 - 25 + 16$

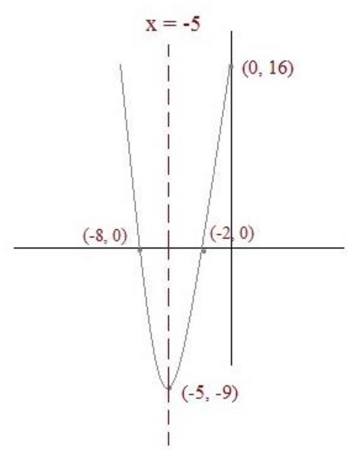
$(x + 5)^2 - 9$

$\uparrow$ h $\uparrow$ k

$\frac{-b}{2a} = \frac{-10}{2(1)} = -5$

$f(-5) = 25 - 50 + 16 = -9$ $\Rightarrow$ $\boxed{\text{vertex is } (-5, -9)}$ $\Leftarrow$

y-intercept: (0, 16)
x-intercepts: (-8, 0) (-2, 0)
vertex: (-5, -9)



Quadratics Overview (continued)

Example: $f(x) = 3(x+4)(x-7)$

Note: the vertex is identified in 3 different ways!

$\boxed{\text{x-intercepts are (-4, 0) and (7, 0)}}$ because $f(-4) = 0$ and $f(7) = 0$

midpoint of (-4, 0) & (7, 0) is (3/2, 0) $\boxed{\text{vertical line of symmetry through } x = \frac{3}{2}}$

Since the line of symmetry goes through the vertex, the vertex is $\boxed{(3/2,\ f(3/2))}$ ✓

(intercept form $\longrightarrow$ standard form) Expand/Distribute: $3(x+4)(x-7)$

$(3x+12)(x-7)$

$3x^2 + 12x - 21x - 84$

$3x^2 - 9x - 84$

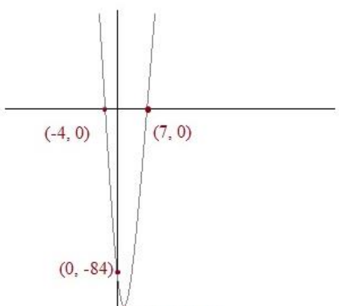
$\boxed{\text{y-intercept is (0, -84)}}$ because $f(0) = -84$

vertex? $\frac{-b}{2a} = \frac{9}{2(3)} = \frac{3}{2}$

$f(3/2) = 3(3/2 + 4)(3/2 - 7) = -363/4$

$\Longrightarrow \boxed{(3/2, -363/4)}$ ✓

(standard form $\longrightarrow$ vertex form) Complete the square: $3x^2 - 9x - 84$



$f(x) = 3(x^2 - 3x - 28)$

$\frac{f(x)}{3} = x^2 - 3x \quad - 28$

$\frac{f(x)}{3} = x^2 - 3x + 9/4 - 9/4 - 28$

$\frac{f(x)}{3} = (x - 3/2)^2 - 121/4$

$f(x) = 3(x - \underbrace{3/2}_{h})^2 - \underbrace{\frac{363}{4}}_{k}$ ✓

Example: $f(x) = (x-1)^2 - 25$

Vertex is $(h, k) \longrightarrow \boxed{(1, -25)}$

Axis of symmetry: $x = h \longrightarrow \boxed{x = 1}$

(vertex form $\longrightarrow$ standard form) Expand: $(x-1)^2 - 25$

$(x-1)(x-1) - 25$

$x^2 - 2x + 1 - 25$

$x^2 - 2x \boxed{-24}$

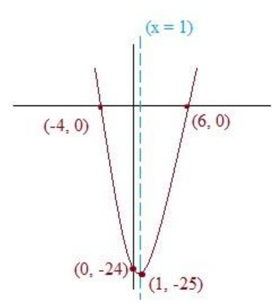
$f(0) = -24$ $\boxed{\text{y-intercept: (0, -24)}}$

(standard form $\longrightarrow$ intercept form) Factor: $x^2 - 2x - 24$

$(x-6)(x+4)$

("roots" are $x = 6$ and $x = -4$)

$\boxed{\text{x-intercepts: (6, 0) (-4, 0)}}$



Quadratics Overview: *Word Problems*

1) The product of two consecutive even numbers is 3024. What are the numbers?

Let x = 1st number
x + 2 = 2nd number

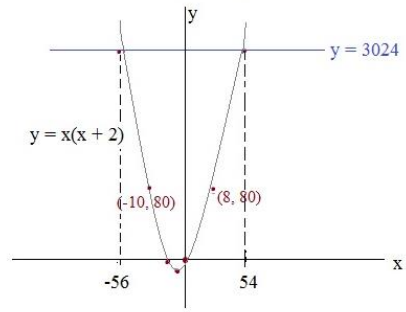
$x(x + 2) = 3024$

$x^2 + 2x - 3024 = 0$

$(x + 56)(x - 54) = 0$

$\boxed{\begin{array}{l} x = 54 \\ x + 2 = 56 \end{array}}$ or $\boxed{\begin{array}{l} x = -56 \\ x + 2 = -54 \end{array}}$

Sketch of x(x + 2)

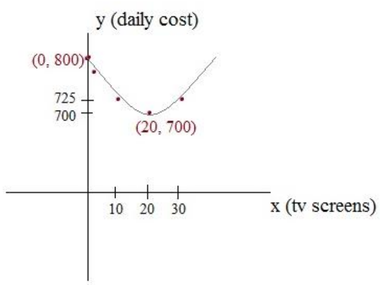


vertex is minimum: (-1, -1)

x-intercepts: (0, 0) (-2, 0)
(x values where "the product of consecutive even numbers equals 0")

2) A manufacturer of television screens has daily production costs of $C = 800 - 10x + \frac{1}{4}x^2$
How many tv screens (x) should he produce each day to minimize costs?

The Cost function consists of inputs (x) and outputs (y)



x	y
0	800
3	772.25
10	725
20	700
30	725

y-intercept: (0, 800)
(represents the fixed costs)

vertex: (20, 700) is the global minimum
(represents minimized costs)

**Find the vertex to find the x value that minimizes costs!

$C = \frac{1}{4}x^2 - 10x + 800$

$\frac{-b}{2a} = \frac{-(-10)}{2(1/4)} = \boxed{20}$

$C = \frac{1}{4}(20)^2 - 10(20) + 800$

$= 100 - 200 + 800 = \boxed{700}$

The manufacturer should produce 20 tv screens each day to minimize costs.

3) A kid launches a water balloon from a 3rd floor window.

The trajectory is $h(t) = -16t^2 + 32t + 48$ where t = time (seconds in the air)
h(t) = height of the water balloon

a) How high is the 3rd floor window?

b) What is the maximum height (above the ground) that the water balloon reaches?

c) When does the water balloon burst on the ground?

$0 = -16t^2 + 32t + 48$

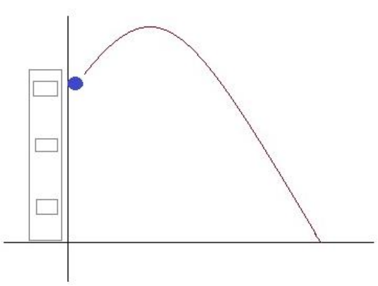
$0 = t^2 - 2t - 3$

$0 = (t - 3)(t + 1)$

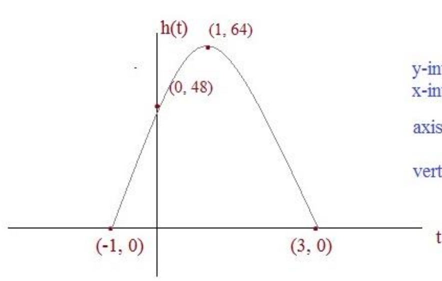
$t = 3, \cancel{-1}$

Since time isn't negative, eliminate all values for $x < 0$
(i.e. domain: $x \geq 0$)

Diagram to illustrate the trajectory of the water balloon



Graph of $h(t) = -16t^2 + 32t + 48$



y-intercept: (0, 48)
x-intercepts: (-1, 0) (3, 0)

axis of symmetry: x = 1

vertex: (1, 64)

Answers:
a) The window is 48 feet high. (the height of the balloon at t = 0)
b) The maximum height is 64 feet (the vertex of the parabola)
c) The balloon hits the ground 3 seconds after it is launched. (the x-intercept)

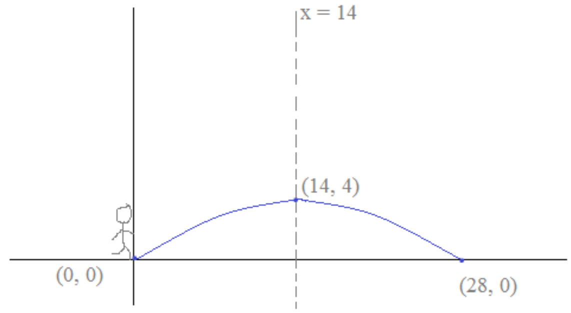
Quadratics Overview: *Word Problems*

4) An athlete makes an incredible jump modeled by the following equation: $y = \frac{-1}{49}(x^2 - 28x)$

where x and y are measured in feet.

How *far* did the athlete jump? How *high* did he jump?

Sketch the trajectory of the jump and label the points:



$$y = \frac{-1}{49}(x^2 - 28x)$$

Find the x-intercepts:

$$0 = \frac{-1}{49}(x^2 - 28x)$$

$$0 = \frac{-1}{49}(x)(x - 28)$$

(0, 0) (28, 0)

$$x = 0,\ 28$$

Find the vertex (to reveal the maximum height):

(Note: the domain is [0, 28], because the jumper doesn't go underground)

Since (0, 0) and (28, 0) are x-intercepts (and the parabola has symmetry), the vertex will go through $x = 14$

$$y = \frac{-1}{49}\left((14)^2 - 28(14)\right) = 4$$

(14, 4) is the vertex

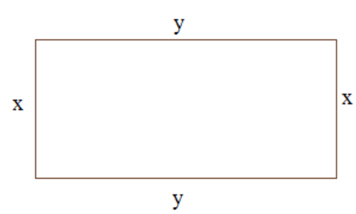
note: $\frac{-b}{2a} = 14$ $\quad \underset{a}{\frac{-1}{49}}x^2 + \underset{b}{\frac{4}{7}}x$

The athlete jumped 28 feet far

and 4 feet high...

5) A rectangular backyard is 1080 square feet. To enclose the backyard, 132 feet of fence are used. What are the dimensions of the fence?

Sketch a diagram and select variables:



$$2x + 2y = 132$$

$$y = \frac{1080}{x}$$

$2x + 2y = 132$ (perimeter)

$xy = 1080$ (area)

Solve and answer:

$$2x + 2\frac{1080}{x} = 132$$

$$2x + \frac{2160}{x} = 132$$

$$2x^2 - 132x + 2160 = 0$$

$$x^2 - 66x + 1080 = 0$$

$$(x - 30)(x - 36) = 0$$

$x = 30,\ 36$

(30, 36)

(36, 30)

dimensions are 30' x 36'

Quadratic Equation Question and Answer

What is the vertex of $f(x) = -3x^2 - 12x + 13$?

Can you find the x-intercepts? The y-intercept?

Graph the function to verify your solutions.

There are 2 ways to find the vertex:

1) The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $\qquad \frac{-b}{2a} = \frac{-(-12)}{2(-3)} = -2 \qquad f(-2) = -3(-2)^2 - 12(-2) + 13 = 25$

Vertex: (-2, 25)

2) Put equation into vertex form to reveal 'h' and 'k':

Vertex form: $a(x - h)^2 + k$

Complete the square!

$-3x^2 - 12x \quad + 13$ ('separate' the equation)

$-3(x^2 + 4x) \quad + 13$ (factor out the -3)

$-3(x^2 + 4x + 4) \quad + 13 + 12$ $(b/2)^2 = 4$, so add 4 to 'complete the square' (Note: adding a 4 on the left, the equation changes by -12. therefore, you must add a 12 to the right side!)

$-3(x + 2)^2 + 25$

h = -2 $\qquad$ k = 25

Vertex: (-2, 25)

To find the y-intercept:

$f(0) = -3(0)^2 - 12(0) + 13 = 13$

y-intercept: (0, 13)

To find x-intercept (or roots):

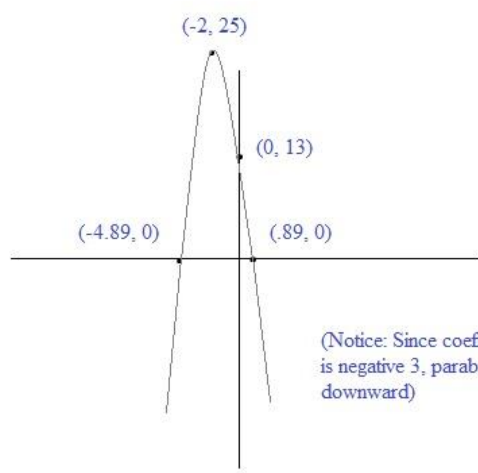
$0 = -3x^2 - 12x + 13$

Use quadratic formula!

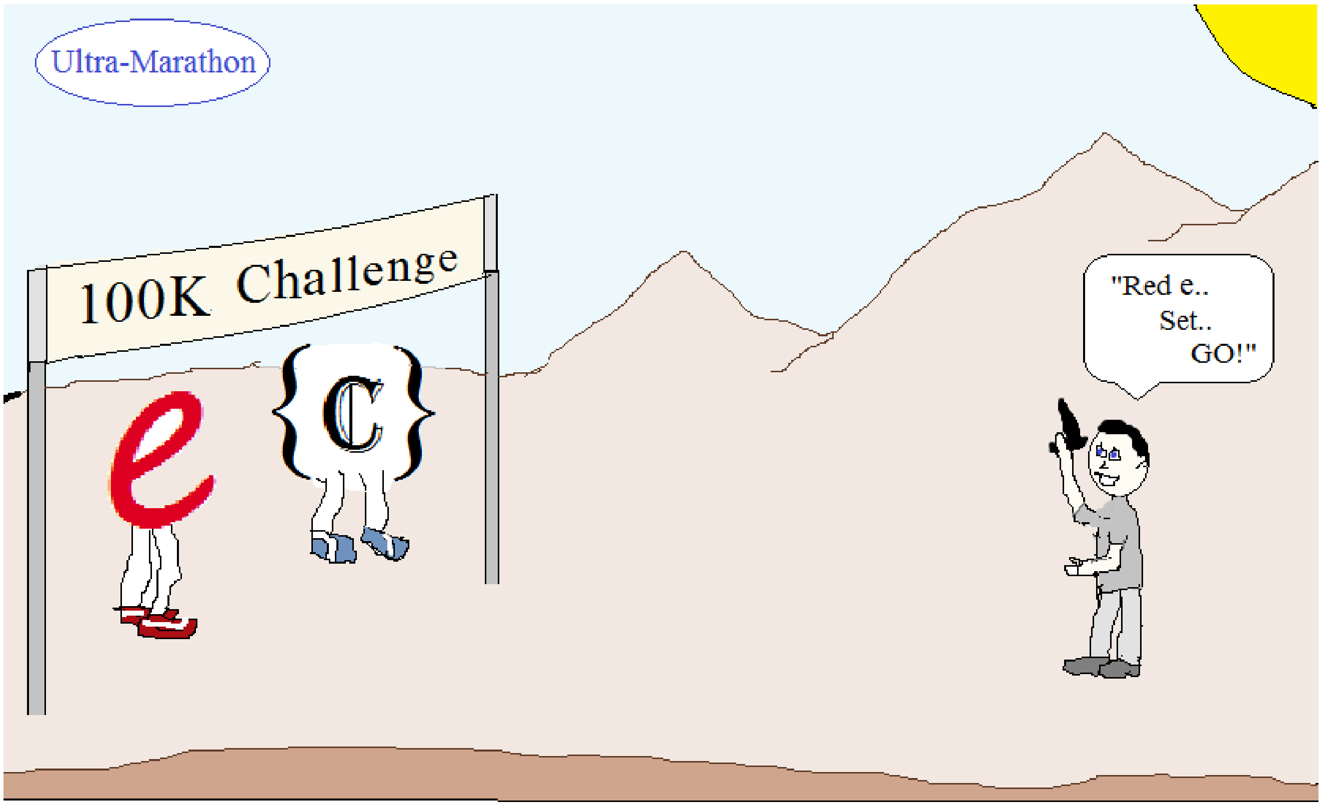
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \frac{-(-12) \pm \sqrt{(-12)^2 - 4(-3)(13)}}{2(-3)}$$

$$\frac{12 \pm \sqrt{300}}{-6} = \frac{6 \pm 5\sqrt{3}}{-3}$$

x-intercepts (approximately) $\qquad$ (-4.89, 0) $\qquad$ (0.89, 0)



(Notice: Since coefficient of x^2 is negative 3, parabola faces downward)



Testing the limits of endurance,
these math figures will run on and on...

LanceAF #87 5-24-13
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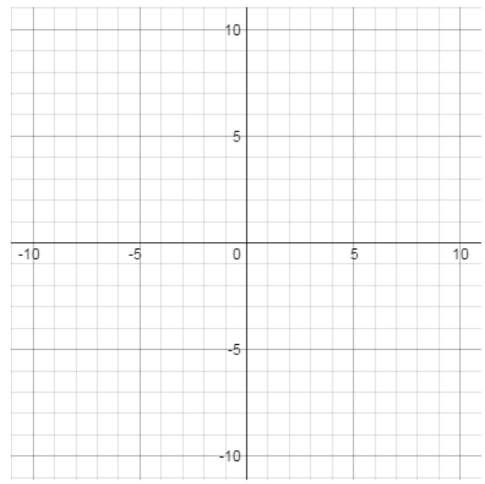
Practice Test -→

Quadratics Practice Quiz

I. Understanding Quadratic equations

Identify the quadratic form, axis of symmetry, vertex, x-intercept(s), and y-intercept for each equation. Then, sketch the graph.

A) $x^2 - 6x + 5$



Form:

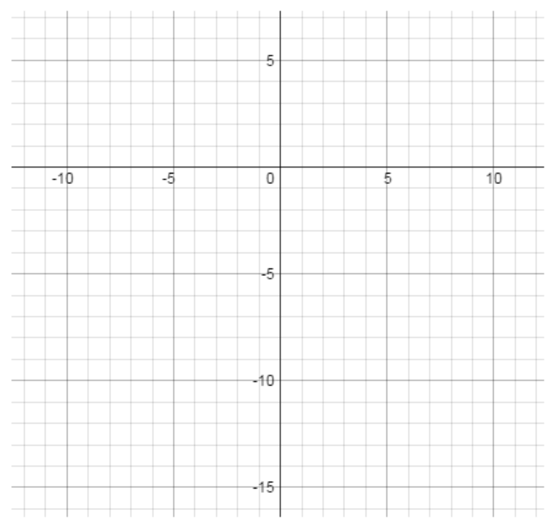
Axis of Symmetry:

Vertex:

x-intercepts:

y-intercept:

B) $y = -2(x+3)^2 + 2$



Form:

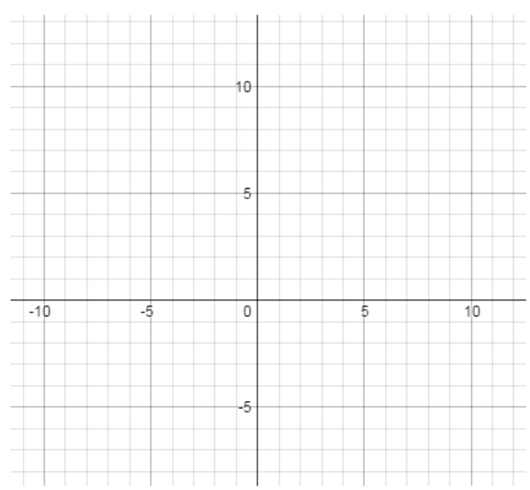
Axis of Symmetry:

Vertex:

x-intercepts:

y-intercept:

C) $f(x) = 3(x+1)(x+4)$



Form:

Axis of Symmetry:

Vertex:

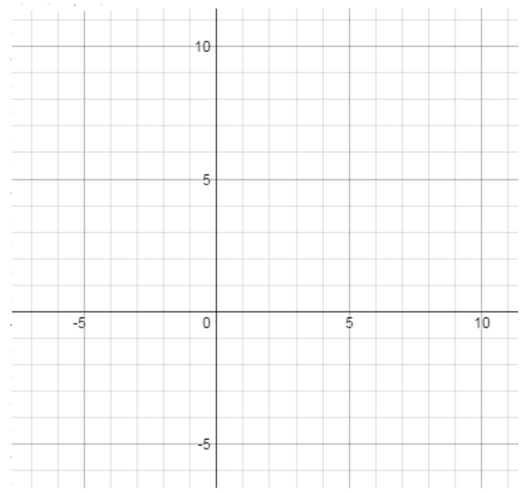
x-intercepts:

y-intercept:

Identify the quadratic form, axis of symmetry, vertex, x-intercept(s), and y-intercept for each equation.
Then, sketch the graph.

Quadratics Practice Quiz

D) $y = -2x^2 + 5x + 7$



Form:

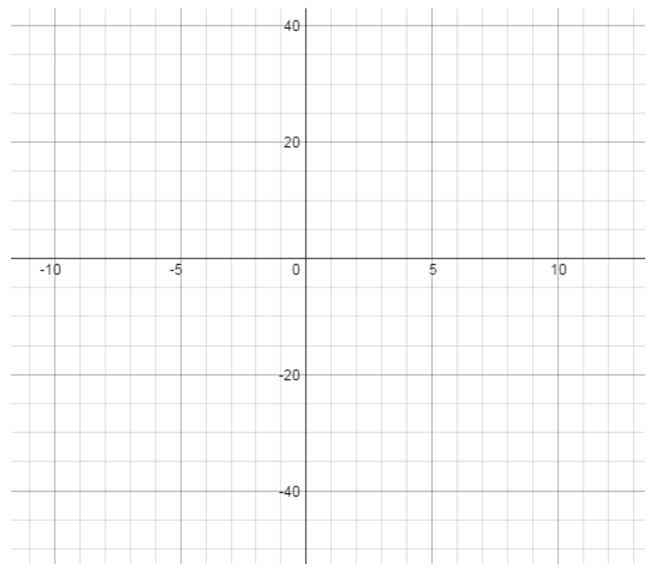
Axis of Symmetry:

Vertex:

x-intercepts:

y-intercept:

E) $y = 3(x + 7)(x - 1)$



Form:

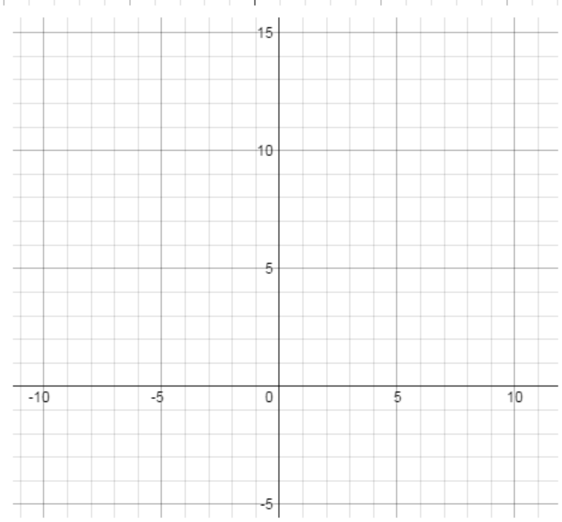
Axis of Symmetry:

Vertex:

x-intercepts:

y-intercept:

F) $y = \frac{1}{2}(x + 5)^2 + 1$



Form:

Axis of Symmetry:

Vertex:

x-intercepts:

y-intercept:

II. Changing forms

A) Change to Vertex form

$x^2 + 10x - 24$ $3x^2 + 6x + 7$ $-2(x + 1)(x + 3)$

B) Change to Intercept Form

$y = 3x^2 + 18x + 15$ $\frac{1}{4}(2x + 13)^2 - \frac{1}{4}$

C) Change to Standard Form

$y = 2(x + 4)(x - 7)$ $y = (x + 5)^2 - 3$ $y = (x + \frac{1}{2})^2 + 4$

III. Discriminant

Using the discriminant, determine how many x-intercepts (i.e. real zeros) exist. Then, identify the x-intercept(s).

$x^2 + 14x + 33$ $x^2 + 2x + 1$ $x^2 + 8x + 40$ $2x^2 - 5x - 4$

IV: Application/Word Problem

A kid throws a tennis ball from a balcony. The trajectory is described by the following equation:

$h(t) = -16t^2 + 48t + 64$ where t is time (seconds)
h(t) is height of the ball (feet)

a) What is the height of the balcony?

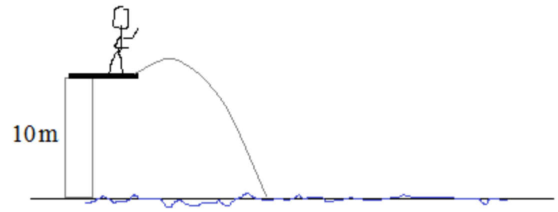
b) What is the <u>maximum</u> height the tennis ball reaches?

c) When does the ball hit the ground?

The position of a diver above the water is modeled by

$$h(t) = -4.9t^2 + 3t + 10$$

where t = time in seconds
$h(t)$ = height above water in meters



a) When will the diver hit the water?

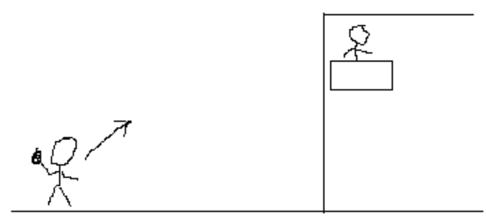
b) What is the domain of this function?

c) What is the range of this function?

Jim throws a snack up to his friend standing on a balcony 30 feet above the ground. The path of the snack is modeled by the equation

$$h(t) = -16t^2 + 32t + 5$$

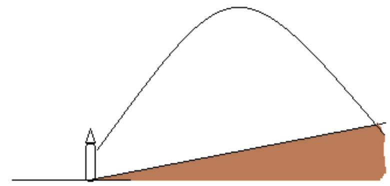
where t = time in seconds
$h(t)$ = height in feet



Will the snack reach Jim's friend?

A rocket is fired into a hill.
The path of the rocket is modeled by the formula

$h(x) = -16x^2 + 1500x$ where $h(x)$ is the "vertical distance" traveled by the rocket and x is the "horizontal distance" traveled.



If the rocket is fired from the base of the hill, and the slope of the hill is 1/5,

a) what is the maximum height of the rocket?

b) where does the rocket hit the ground (in the hill)?

A super dog's leap can be modeled by the function $h(t) = -16t^2 + 24t$ where t is time (seconds) and h(t) is height in feet.

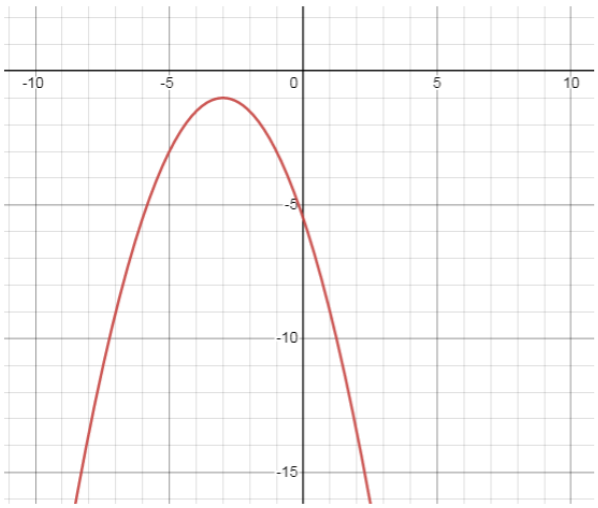
a) What is the maximum height of this dog?

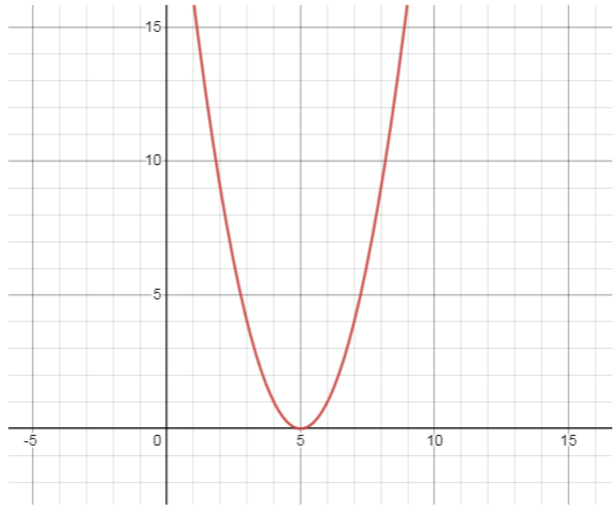
b) When is the dog at least 8 feet high?

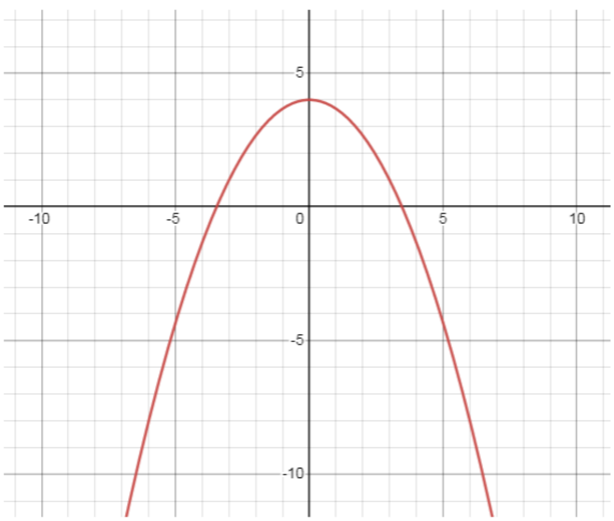
c) How long is the dog over 6 feet high?

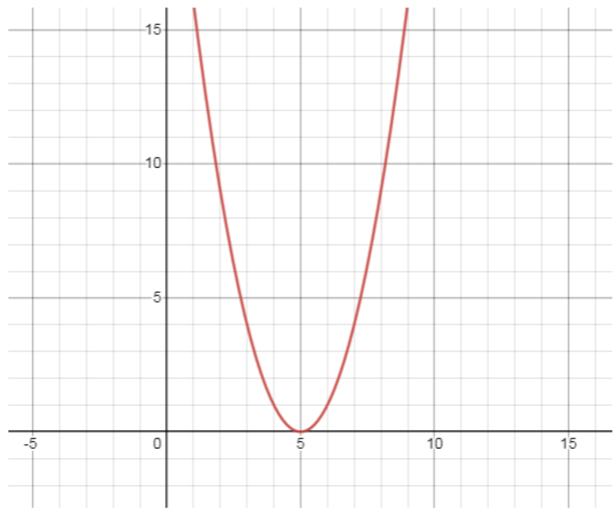
1) For the following graphs, determine whether the discriminant is positive, negative, or zero…

Discriminants and Graphs









2) Match the graphs with the number/types of solutions.

A (blue)

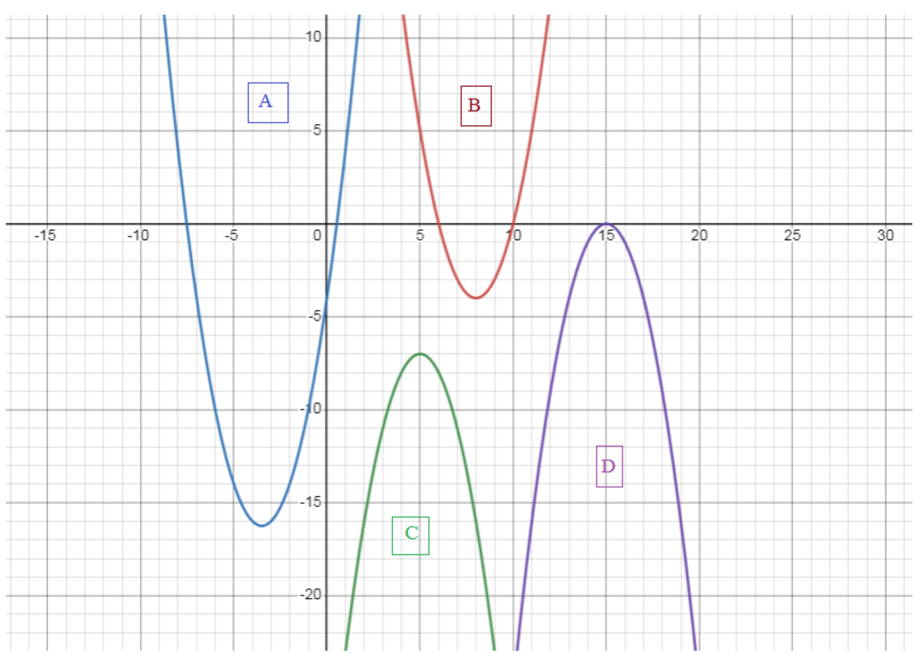
B (red)

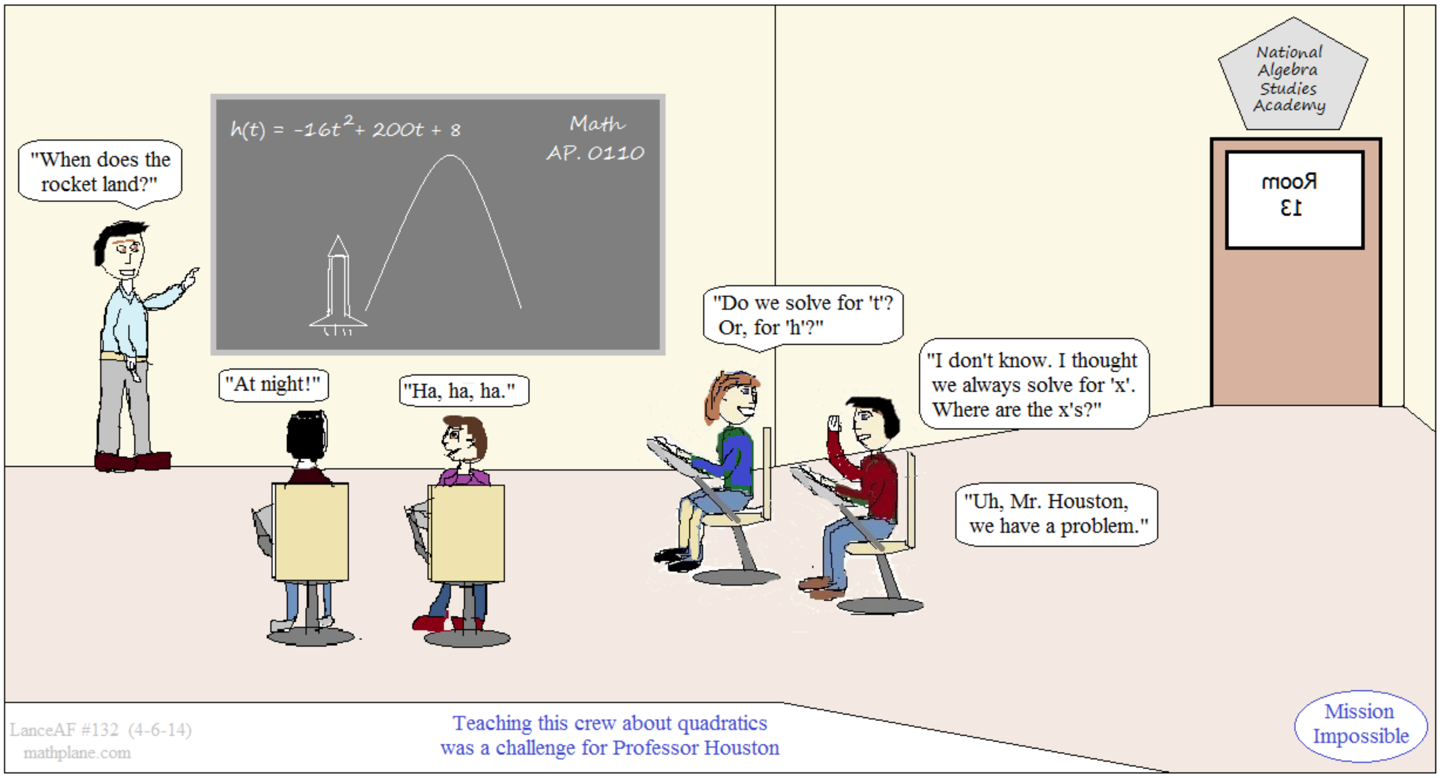
C (green)

D (purple)

1) 2 real solutions (2 rational)
2) 2 real solutions (2 irrational)
3) 1 real solutions
4) 0 real solutions

5) discriminant $= 0$
6) discriminant $= 65$
7) discriminant $= 16$
8) discriminant $= -28$





"When does the rocket land?"
$h(t) = -16t^2 + 200t + 8$
Math AP. 0110
"At night!"
"Ha, ha, ha."
"Do we solve for 't'? Or, for 'h'?"
"I don't know. I thought we always solve for 'x'. Where are the x's?"
"Uh, Mr. Houston, we have a problem."
National Algebra Studies Academy
Room 13
Teaching this crew about quadratics was a challenge for Professor Houston
LanceAF #132 (4-6-14)
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Mission Impossible

Solutions -→

Quadratics Practice Quiz

SOLUTIONS

I. Understanding Quadratic equations

Identify the quadratic form, axis of symmetry, vertex, x-intercept(s), and y-intercept for each equation. Then, sketch the graph.

A) $x^2 - 6x + 5$

Form: Standard $(ax^2 + bx + c)$

Axis of Symmetry: $x = 3$

Vertex: (3, -4)

x-intercepts: (5, 0) (1, 0)
(1 and 5 are zeros)

y-intercept: (0, 5)

vertex
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
axis of symmetry

$a = 1$
$b = -6$

$\frac{-b}{2a} = 3$

$f(3) = (3)^2 - 6(3) + 5 = -4$

$0 = x^2 - 6x + 5$

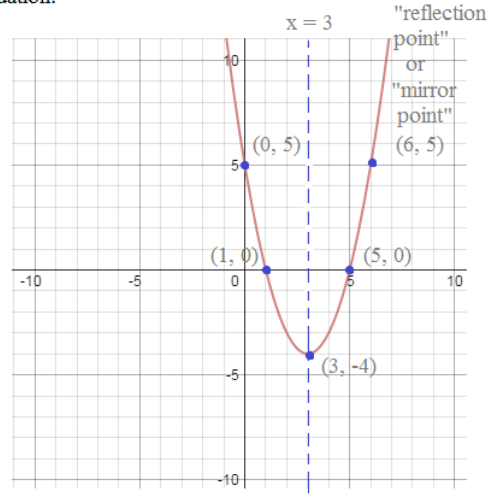
$0 = (x - 5)(x - 1)$

(5, 0) and (1, 0)

When $x = 0$,

$(0)^2 + 6(0) + 5$

$y = 5$



B) $y = -2(x + 3)^2 + 2$

Form: Vertex $(a(x - h)^2 + k)$

Axis of Symmetry: $x = -3$

Vertex: $(-3, 2)$

x-intercepts: (-4, 0) (-2, 0)

y-intercept: (0, -16)

$-2(x - (-3))^2 + 2$

$h = -3$ $k = 2$

axis of symmetry goes through the vertex!

$x = h$ $x = -3$

y-intercept: (0, ?)

$y = -2((0) + 3)^2 + 2 = -16$

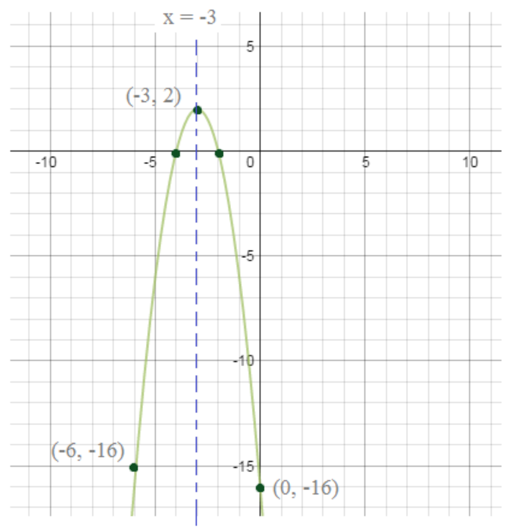
x-intercept: (?, 0)

$0 = -2(x + 3)^2 + 2$

$-2 = -2(x + 3)^2$

$1 = (x + 3)^2$

$x = -4$ or -2



C) $f(x) = 3(x + 1)(x + 4)$

Form: Intercept $a(x - x_1)(x - x_2)$

Axis of Symmetry: $x = \frac{-5}{2}$

Vertex: (-5/2, -27/4)

x-intercepts: (-1, 0) (-4, 0)

y-intercept: (0, 12)

$f(x) = 3(x + 1)(x + 4)$

$x_1 = -1$ $x_2 = -4$

(zeros/x-intercepts)

$f(0) = 3(0 + 1)(0 + 4) = 12$

(0, 12) is y-intercept

Since a quadratic (parabola) is symmetric, we know the midpoint of the zeros will be the axis of symmetry!

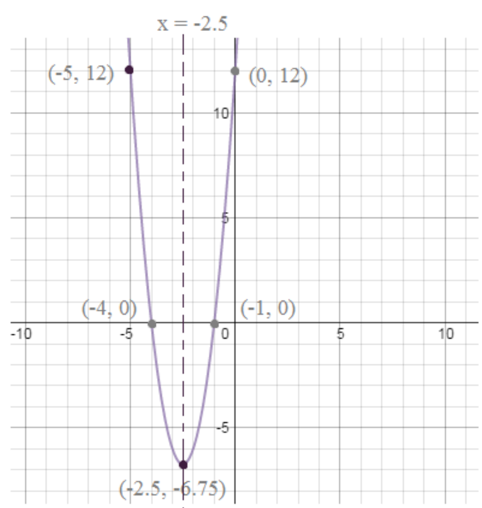
midpoint of -1 and -4 is $\frac{-5}{2}$

axis of symmetry: $x = -5/2$

and, the vertex lies on the axis of symmetry...

$f(-5/2) = 3(-5/2 + 1)(-5/2 + 4)$
$= -27/4$

vertex: (-5/2, -27/4)



Identify the quadratic form, axis of symmetry, vertex, x-intercept(s), and y-intercept for each equation. Then, sketch the graph.

Quadratics Practice Quiz

SOLUTIONS

D) $y = -2x^2 + 5x + 7$

$y = ax^2 + bx + c$

Form: Standard form

Axis of Symmetry: $x = \frac{5}{4}$

Vertex: $\left(\frac{5}{4}, \frac{81}{8}\right)$

x-intercepts: (7/2, 0) (-1, 0)

y-intercept: (0, 7)

y-intercept occurs when x = 0

$\frac{-b}{2a} = \frac{-(5)}{2(-2)} = \frac{5}{4}$

vertex lies on axis of symmetry, so vertex is at x = 5/4

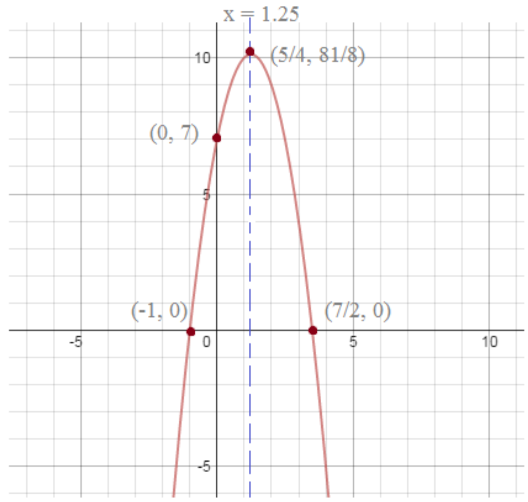
$-2\left(\frac{5}{4}\right)^2 + 5\left(\frac{5}{4}\right) + 7 = \frac{81}{8}$

x-intercepts occur when y = 0

$0 = -1(2x^2 - 5x - 7)$

$= -1(2x - 7)(x + 1)$

x = 7/2 and x = -1



E) $y = 3(x + 7)(x - 1)$

$y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are intercepts

Form: Intercept Form (or factored form)

Axis of Symmetry: x = -3

Vertex: (-3, -48)

x-intercepts: (-7, 0) (1, 0)

y-intercept: (0, -21)

y-intercept occurs when x = 0:

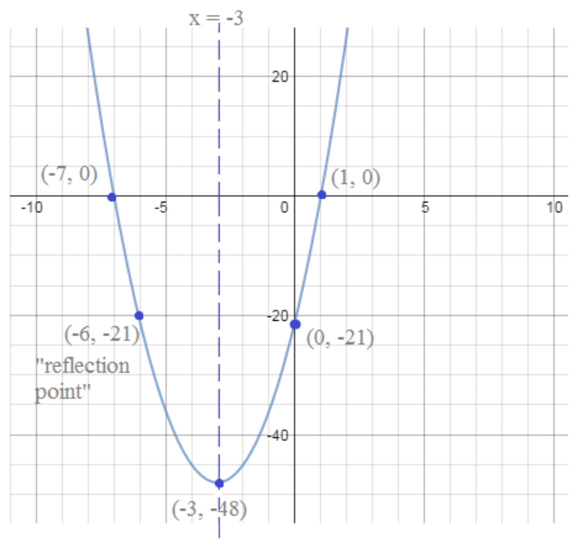
$y = 3(0 + 7)(0 + -1) = -21$

since it is intercept form, we can see the intercepts... -7 and 1

Also, we know the axis of symmetry splits the parabola in half.. So, the A of S occurs at the midpoint of -7 and 1..

Then, to find vertex: x = -3

$y = 3(-3 + 7)(-3 - 1) = -48$



F) $y = \frac{1}{2}(x + 5)^2 + 1$

Form: Vertex Form

Axis of Symmetry: x = -5

Vertex: (-5, 1)

x-intercepts: NONE

y-intercept: (0, 27/2)

Since the equation is in vertex form, we can see the vertex is (-5, 1)

$y = a(x - h)^2 + k$

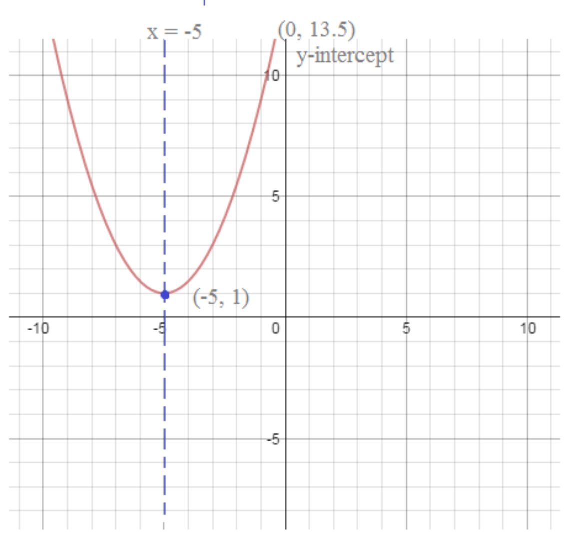
and, therefore, the axis of symmetry is x = -5

x-intercepts occur when y = 0

$0 = \frac{1}{2}(x + 5)^2 + 1$

$-2 = (x + 5)^2$ NO solutions

NOTE: we know the vertex is (-5, 1) AND we know the parabola opens upward... therefore, the curve never crosses the x-axis!



II. Changing forms

SOLUTIONS

A) Change to Vertex form

$x^2 + 10x - 24$

(complete the square)

$25 + y = x^2 + 10x + 25 - 24$

$y + 25 = (x + 5)^2 - 24$

$y = (x + 5)^2 - 49$

$3x^2 + 6x + 7$

$y = 3(x^2 + 2x) + 7$

$y + 3 = 3(x^2 + 2x + 1) + 7$

$y + 3 = 3(x + 1)^2 + 7$

$y = 3(x + 1)^2 + 4$

$-2(x + 1)(x + 3)$

$y = -2x^2 - 8x - 6$

$y = -2(x^2 + 4x) - 6$

$y + (-8) = -2(x^2 + 4x + 4) - 6$

$y = -2(x + 2)^2 + 2$

B) Change to Intercept Form

$y = 3x^2 + 18x + 15$

(factor to reveal intercepts)

$3(x^2 + 6x + 5)$

$3(x + 1)(x + 5)$

$\frac{1}{4}(2x + 13)^2 - \frac{1}{4}$

$\frac{1}{4}(4x^2 + 52x + 169) - \frac{1}{4}$

$x^2 + 13x + \frac{169}{4} - \frac{1}{4}$

$x^2 + 13x + 42$

$(x + 6)(x + 7)$

C) Change to Standard Form

$y = 2(x + 4)(x - 7)$

$2(x^2 - 3x - 28)$

$2x^2 - 6x - 56$

$y = (x + 5)^2 - 3$

$(x + 5)(x + 5) - 3$

$x^2 + 10x + 25 - 3$

$x^2 + 10x + 22$

$y = (x + \frac{1}{2})^2 + 4$

$(x + \frac{1}{2})(x + \frac{1}{2}) + 4$

$x^2 + x + \frac{1}{4} + 4$

$x^2 + x + \frac{17}{4}$

III. Discriminant

Using the discriminant, determine how many x-intercepts (i.e. real zeros) exist. Then, identify the x-intercept(s).

discriminant: $b^2 - 4ac$

$x^2 + 14x + 33$

$a = 1 \quad b = 14 \quad c = 33$

discriminant: $196 - 132 = 64$

since discriminant > 0,
there are two x-intercepts:

$0 = (x + 11)(x + 3) \quad x = -3, -11$

$(-3, 0) \quad (-11, 0)$

$x^2 + 2x + 1$

$a = 1 \quad b = 2 \quad c = 1$

discriminant: $4 - 4 = 0$

since discriminant $= 0$,
there is one x-intercept:

$0 = (x + 1)(x + 1) \quad x = -1$

$(-1, 0)$

$x^2 + 8x + 40$

$a = 1 \quad b = 8 \quad c = 40$

discriminant: $64 - 160 = -96$

since discriminant < 0,
there are
NO REAL x-intercepts

$2x^2 - 5x - 4$

$a = 2 \quad b = -5 \quad c = -4$

discriminant: $25 - (-32) = 57$

since discriminant > 0,
there are two x-intercepts:

(use quadratic formula)

$\frac{-(-5) \pm \sqrt{(-5)^2 + 4(2)(-4)}}{2(2)}$

$\left(\frac{5 + \sqrt{57}}{4}, 0\right)$

$\left(\frac{5 - \sqrt{57}}{4}, 0\right)$

IV: Application/Word Problem

A kid throws a tennis ball from a balcony. The trajectory is described by the following equation:

$h(t) = -16t^2 + 48t + 64$ where t is time (seconds)
h(t) is height of the ball (feet)

a) What is the height of the balcony? 64 feet

b) What is the maximum height the tennis ball reaches? 100 feet

c) When does the ball hit the ground? 4 seconds

a) the height of the balcony is where the kid is standing with the ball at $t = 0$

$h(t) = -16(0)^2 + 48(0) + 64 = 64$ feet

b) the maximum height would be the vertex of the parabola (i.e. the tennis ball trajectory)

$\frac{-b}{2a} = \frac{-48}{2(-16)} = \frac{3}{2}$ the ball reaches max. height in 1 1/2 seconds

$h(3/2) = -16(3/2)^2 + 48(3/2) + 64 = -36 + 72 + 64 = 100$ feet

c) the ball hits the ground when $h(t) = 0$ (height is zero)

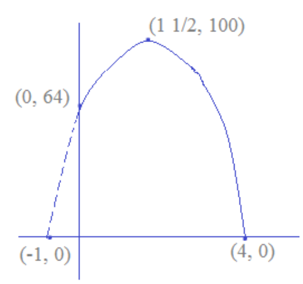
$0 = -16t^2 + 48t + 64$

$0 = 16(t^2 - 3t - 4)$

$0 = 16(t - 4)(t + 1)$

$t = \cancel{-1}, 4$ (since time cannot be negative, we eliminate -1)

Therefore, the ball hits the ground after 4 seconds!

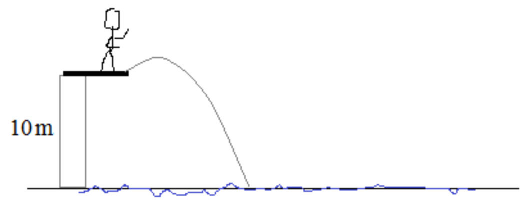


SOLUTIONS

The position of a diver above the water is modeled by

$$h(t) = -4.9t^2 + 3t + 10$$

where t = time in seconds
$h(t)$ = height above water in meters



a) When will the diver hit the water?

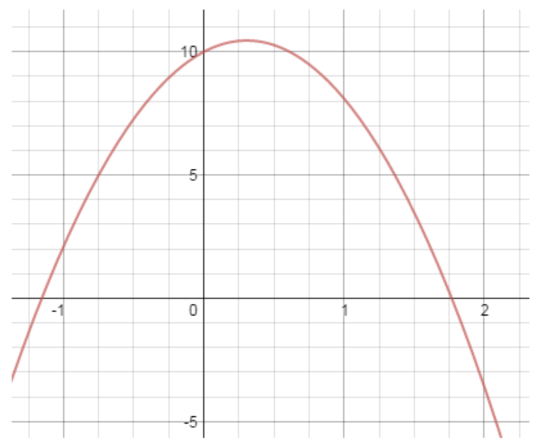
The diver hits the water when the height $h(t) = 0$

$$0 = -4.9t^2 + 3t + 10$$

$$t = \frac{-3 \pm \sqrt{(3)^2 - 4(-4.9)(10)}}{2(-4.9)}$$

$$t = \frac{-3 \pm \sqrt{205}}{-9.8} \longrightarrow t = -1.15,\ 1.77$$

Since time (t) cannot be negative,
the diver hits the water 1.77 seconds after the jump



b) What is the domain of this function?

The domain of the model is [0, 1.77]

Note: one could argue the domain continues after the diver hits the water and goes under the surface... However, after 1.77 seconds, the shape of the curve would likely be different than the smooth parabola!

c) What is the range of this function?

First, find the maximum height of the diver. (i.e. the vertex)

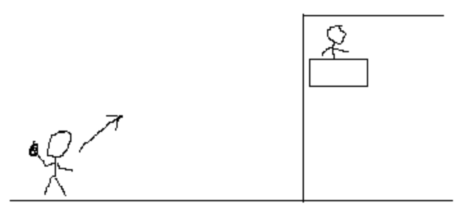
$$\frac{-b}{2a} = \frac{-3}{2(-4.9)} = .306 \longrightarrow \text{The max. height occurs at } t = .306 \text{ seconds}$$

$$h(.306) = -4.9(.306)^2 + 3(.306) + 10 = 10.46 \text{ meters}$$

The range of the model is [0, 10.46]

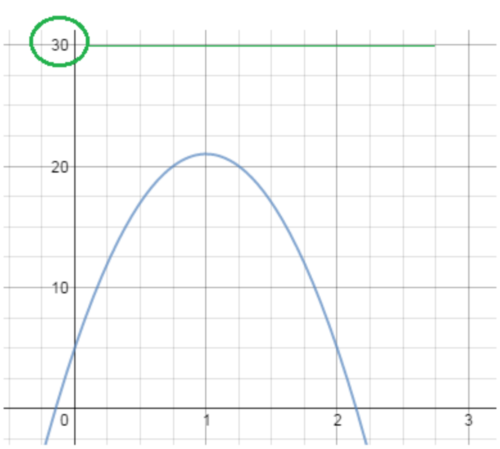
Note: Again, one could argue the range extends to the diver under water. However, we would have to modify the model!

Jim throws a snack up to his friend standing on a balcony 30 feet above the ground. The path of the snack is modeled by the equation



$$h(t) = -16t^2 + 32t + 5$$

where t = time in seconds
$h(t)$ = height in feet



Will the snack reach Jim's friend?

$$h(t) = 30 \longrightarrow 30 = -16t^2 + 32t + 5$$

$$-16t^2 + 32t - 25 = 0$$

Looking at the discriminant: $b^2 - 4ac$

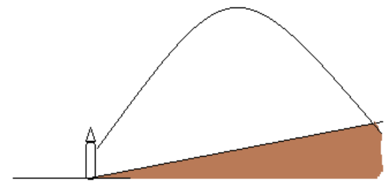
$$(32)^2 - 4(-16)(-25) = -576$$

*Since the discriminant is negative, there is no real solution! (i.e. there is no time (t) where the snack will reach a height of 30 feet)

A rocket is fired into a hill.
The path of the rocket is modeled by the formula

SOLUTIONS

$h(x) = -16x^2 + 1500x$ where $h(x)$ is the "vertical distance" traveled by the rocket and x is the "horizontal distance" traveled.



If the rocket is fired from the base of the hill, and the slope of the hill is 1/5,

a) what is the maximum height of the rocket?

b) where does the rocket hit the ground (in the hill)?

a) rocket height increases from 0 to 46.875 feet (horizontal)

$g(x) = h(x) - \text{slope of hill}$

$g(x) = -16x^2 + 1500x - \frac{1}{5}x$

$-16x^2 + 1499.8x$

$g(46.875) = \boxed{35{,}146.9}$

(Note: without the hill, the rocket would travel 93.75 horizontally, with a max height of 35,156.3)

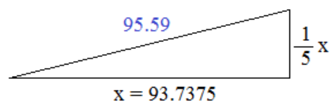
b) First, we must find where the hill and the path of the rocket intersect..

$y = \frac{1}{5}x$

$h(x) = -16x^2 + 1499.8x$

$-16x^2 + 1500x = \frac{1}{5}x$

$x = 0$ or 93.7375 feet (horizontally)



18.7475 feet higher than launch..

$18.7475^2 + 93.7375^2 = \text{distance up hill}^2$

$\boxed{95.59}$

A super dog's leap can be modeled by the function $h(t) = -16t^2 + 24t$ where t is time (seconds) and h(t) is height in feet.

a) What is the maximum height of this dog?

$\frac{-b}{2a} = \frac{-24}{-32} = \frac{3}{4}$ the maximum height occurs when t = 3/4

$h(3/4) = -16(3/4)^2 + 24(3/4)$

$-9 + 18 = \boxed{9 \text{ feet}}$

b) When is the dog at least 8 feet high?

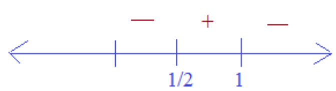
$h(t) \geq 8$

$-16t^2 + 24t \geq 8$

$-16t^2 + 24t - 8 \geq 0$

$-8(2t^2 - 3t + 1) \geq 0$

$-8(2t - 1)(t - 1) \geq 0$



[1/2, 1] between 1/2 and 1 second

c) How long is the dog over 6 feet high?

$h(t) \geq 6$

$-16t^2 + 24t \geq 6$

$-16t^2 + 24t - 6 \geq 0$

find critical values (where equation = 0)

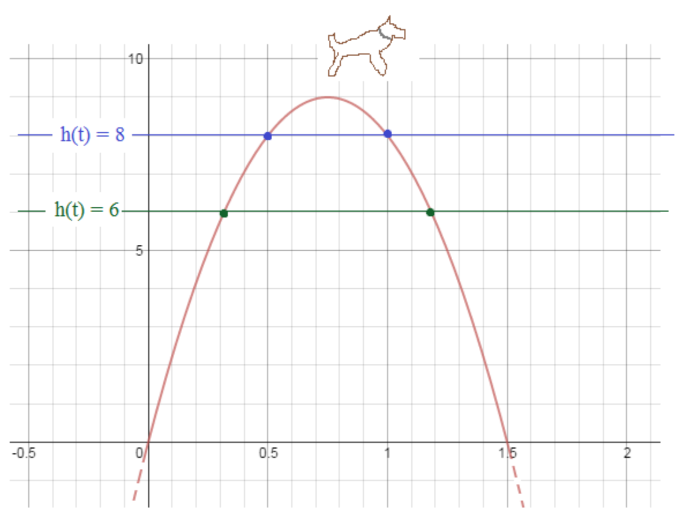
$-2(8t^2 - 12t + 3) = 0$

discriminant = $b^2 + 4ac = 144 - 4(8)(3) = 48$

since 48 is not a perfect square, the solution is not rational...
must use quadratic formula...

$t = \frac{12 \pm \sqrt{(-12)^2 + 4(8)(3)}}{2(8)} = \frac{12 \pm \sqrt{48}}{16} = \frac{12 \pm 4\sqrt{3}}{16}$

$\frac{3 + \sqrt{3}}{4}$ or $\frac{3 - \sqrt{3}}{4}$

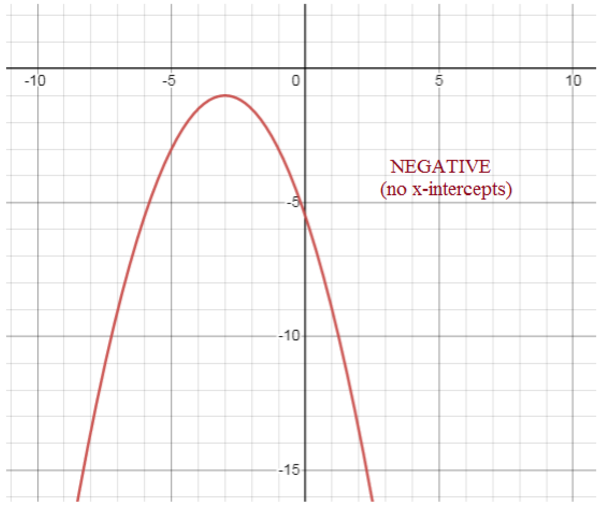


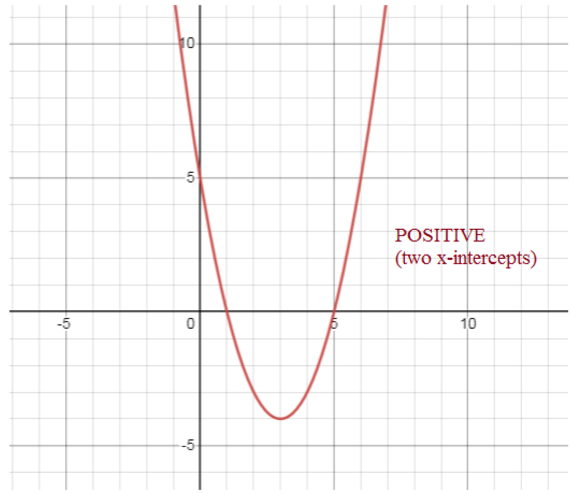
.317 to 1.18
so, the dog is 6 feet high for .86 seconds...

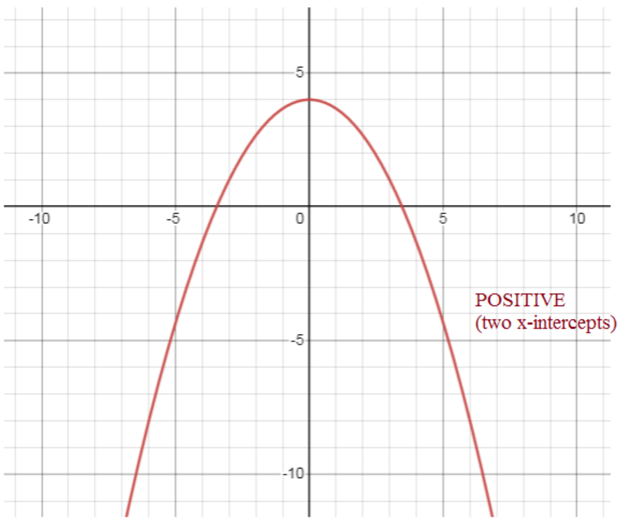
1) For the following graphs, determine whether the discriminant is positive, negative, or zero...

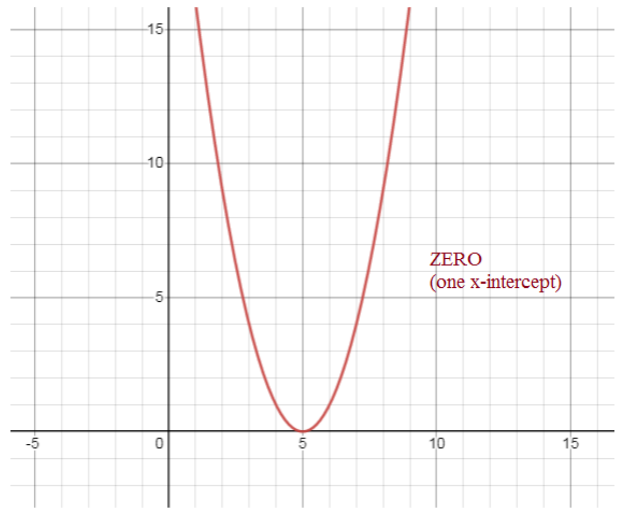
SOLUTIONS

Discriminants and Graphs









2) Match the graphs with the number/types of solutions.

A (blue)	2) and 6)
B (red)	1) and 7)
C (green)	4) and 8)
D (purple)	3) and 5)

1) 2 real solutions (2 rational)

2) 2 real solutions (2 irrational)

3) 1 real solutions

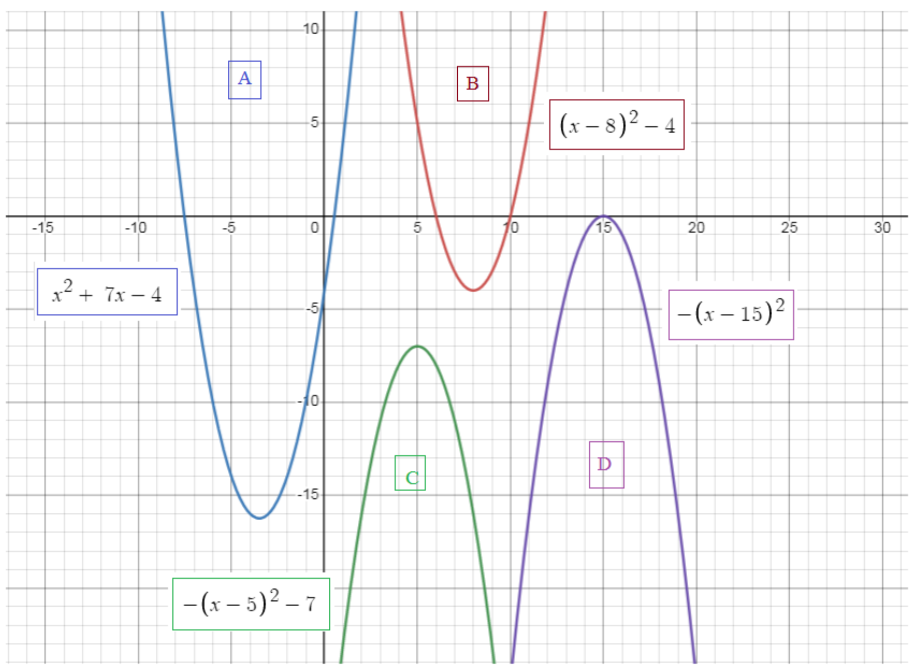
4) 0 real solutions

5) discriminant $= 0$

6) discriminant $= 65$

7) discriminant $= 16$

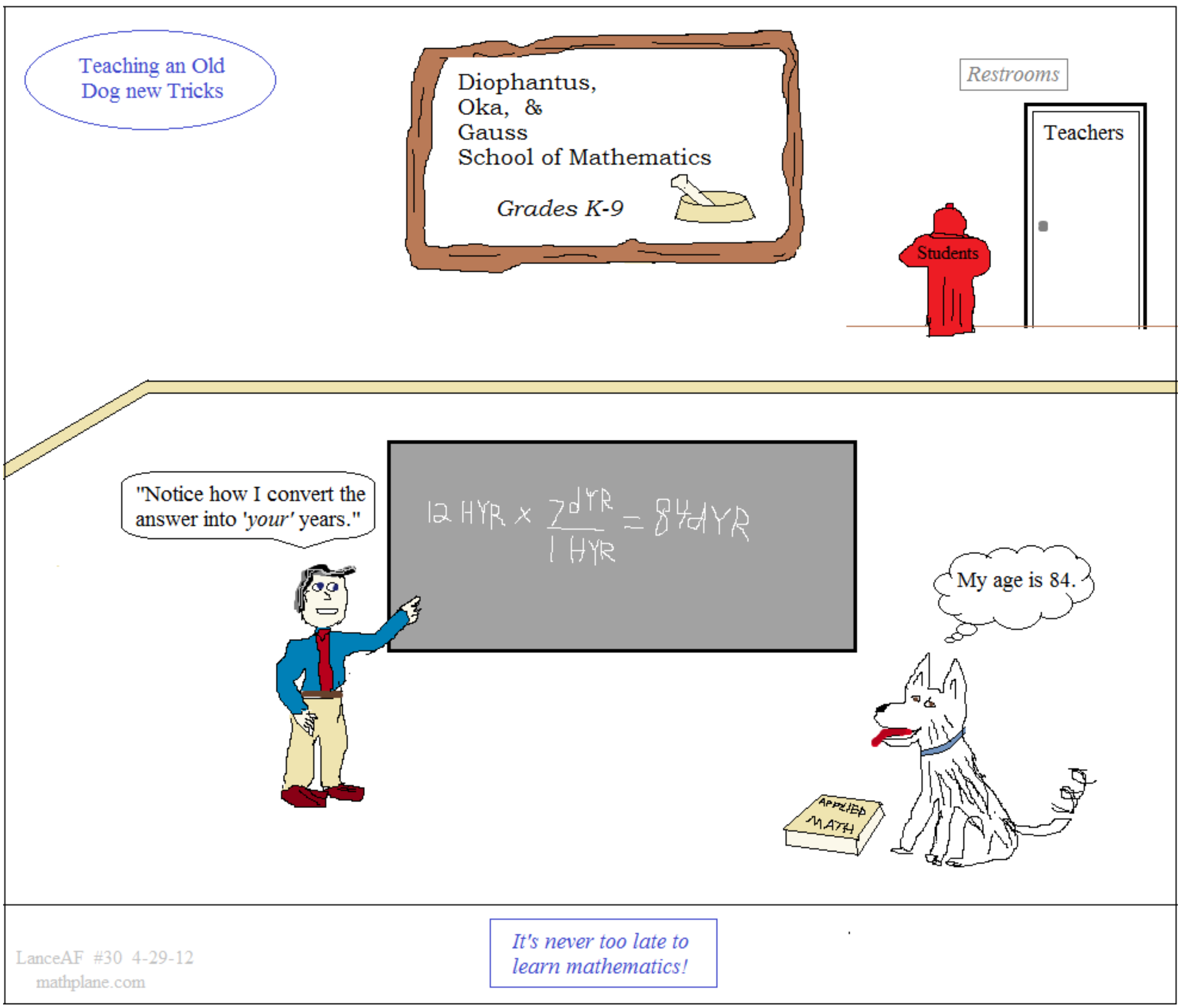
8) discriminant $= -28$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



$$\frac{\boxed{-b} \pm \sqrt{b^2 - 4ac}}{\boxed{2a}}$$

$\frac{-b}{2a}$ is the *axis of symmetry*

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Observation:

Components of the quadratic formula reveal characteristics of a parabola (quadratic).

$$\frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

$b^2 - 4ac$ is the *discriminant*

If $b^2 - 4ac > 0$ then there are 2 real zeros (2 x-intercepts)

If $b^2 - 4ac = 0$ then there is 1 real zero (1 x-intercept)

If $b^2 - 4ac < 0$ then there is no *real* zero (0 x-intercepts)

A basketball player's shot leaves his hand 4 feet off the ground.
The initial velocity of the shot is 15 feet/second.
The height of the rim is 10 feet.

Topic for Discussion

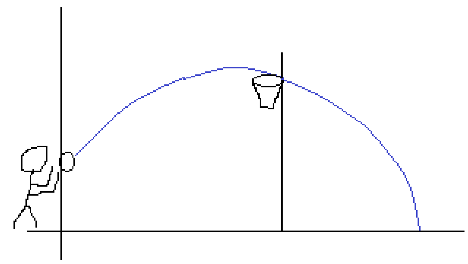
a) Will the player's shot reach the rim? If so, when?

b) Will the shot go in? If not, what velocity is needed?

$$\boxed{h(t) = -16t^2 + v_0 t + h_0}$$

Determine the model of the ball's path:

$$h(t) = -16t^2 + 15t + 4$$



The maximum height occurs at the vertex.

$$\text{Vertex: } \left(\frac{-b}{2a},\ h(-b/2a)\right)$$

$$\frac{-b}{2a} = \frac{-15}{(2)(-16)} = \frac{15}{32} = .46875$$

$$h(.46875) = -16(.46875)^2 + 15(.46875) + 4 \approx 7.51$$

$$-3.52 \quad + \quad 7.03 \quad + \quad 4$$

Since the maximum height is 7.51 feet, the ball never reaches the rim.

What initial velocity is needed to reach the rim?

When $h(t) = 10$

$$10 = -16(t)^2 + 15(t) + 4$$

$$-16t^2 + v_0 t - 6 = 0$$

15 isn't large enough ----> the discriminant is negative

$$t = \frac{-v_0 \pm \sqrt{\boxed{v_0^2 - 4(-16)(-6)}}}{2(-16)}$$

$$v_0^2 - 4(-16)(-6) \geq 0$$

$$v_0^2 \geq 384$$

Initial velocity must be $>$ 19.6 to reach the rim.

However,
(assuming the diameter of the basketball is 9.5 inches), the ball must reach a height (over) 10' 9.5" or 10.792 feet

$$-16t^2 + v_0 t + 4 = 10.792$$

$$-16t^2 + v_0 t - 6.792 = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(-16)(-6.792)}}{2(-16)}$$

$$v_0^2 - 4(-16)(-6.792) \geq 0$$

$$v_0^2 \geq 434.688$$

Initial velocity of 20.85 (or more)