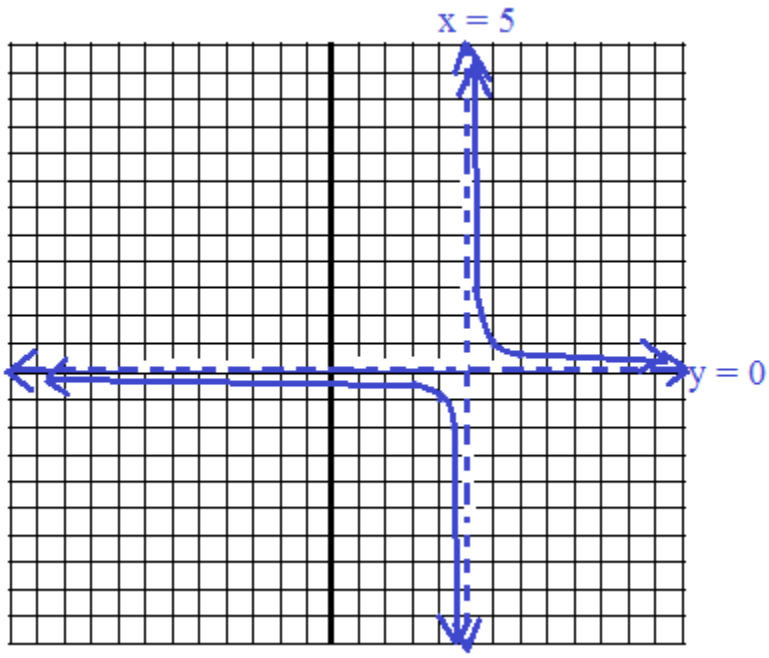
Reciprocal Functions

Notes, examples, and practice (and solutions)



Topics include asymptotes, parent functions, transformations, graphing, intercepts, domain/range, applications, and more…

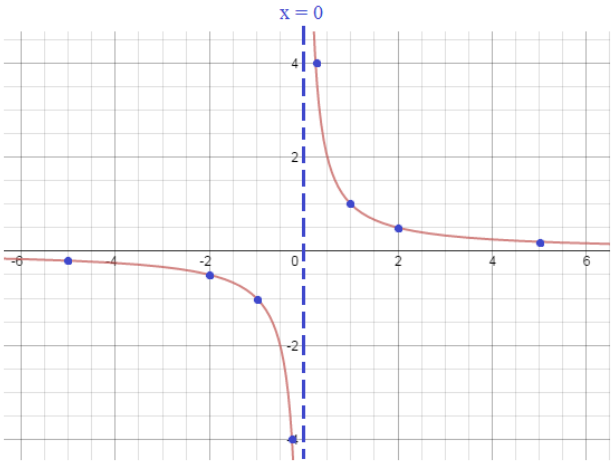
The Reciprocal Function

$f(x) = \frac{1}{x}$

Sketching the 'parent function'

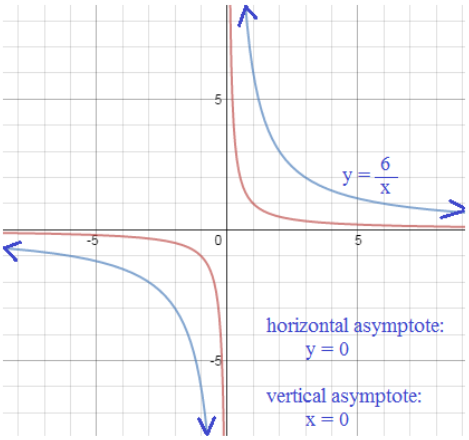
Using a table of values, a pattern emerges....

x	$\frac{1}{x}$
-5	-1/5
-2	-1/2
-1	-1
-1/4	-4
0	undefined
1/4	4
1	1
2	1/2
5	1/5

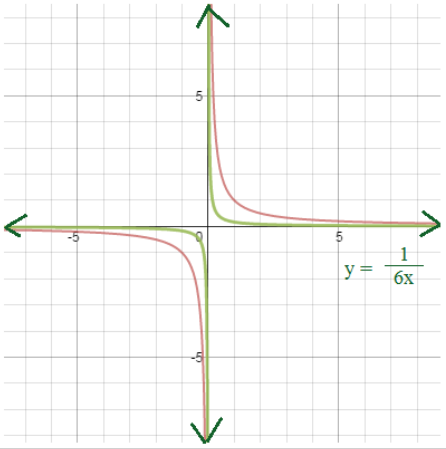


Transforming a reciprocal function: $y = \frac{a \cdot 1}{b(x - c)} + d$

'a': amplitude "stretch" $y = \frac{6}{x}$

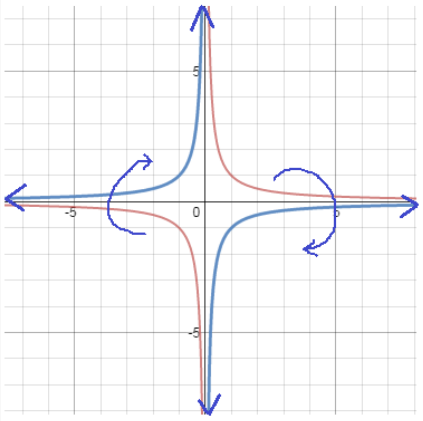


'b': compression $y = \frac{1}{6x}$

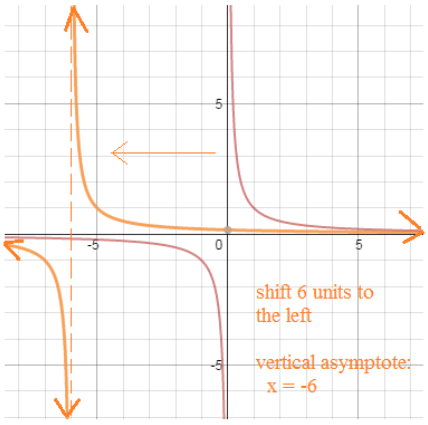


'(-)': reflection

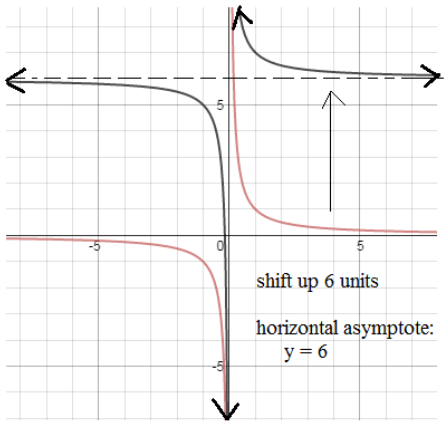
$y = \frac{-1}{x}$



'c': horizontal shift $y = \frac{1}{x + 6}$



'd': vertical shift $y = \frac{1}{x} + 6$



Note: A 'reciprocal' function is not an 'inverse' function

Example: $y = x^2$

Reciprocal: $\frac{1}{x^2}$ because $x^2 \cdot \frac{1}{x^2} = 1$

vs.

Inverse: $\sqrt{x}$ because $\sqrt{x^2} = x$ or $\sqrt{x}^2 = x$

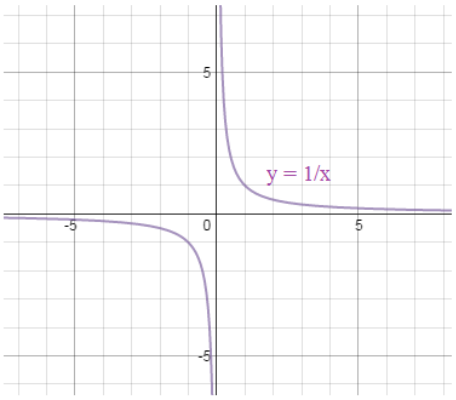
vs.

Opposite: $-x^2$ because $x^2 + -x^2 = 0$

Sketching Reciprocal Functions

Approach 1: Using the parent function and transformations

Example: Sketch the function $y = \frac{2}{(x-3)} + 5$.



Recognize the parent function: $y = \frac{1}{x}$

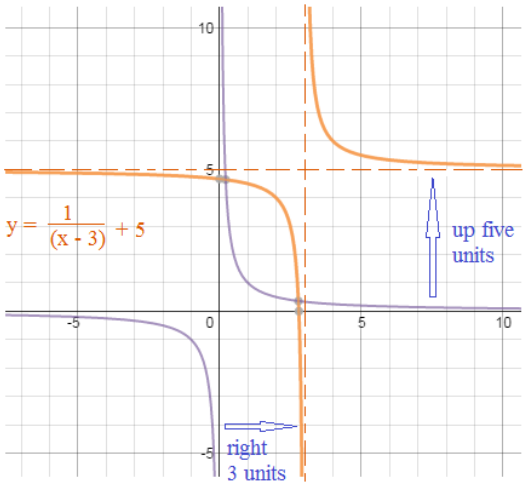
Determine the transformations/shifts:

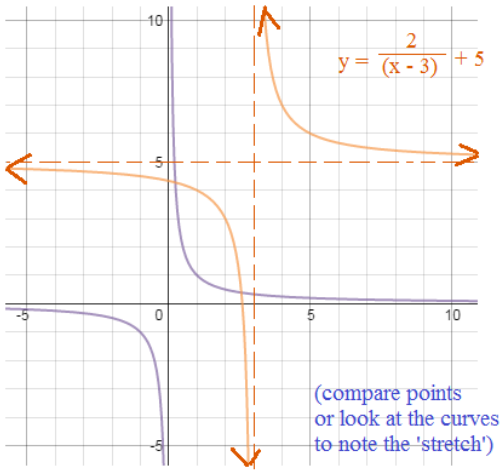
vertical shift (d): up 5 units

horizontal shift (c): shift 3 units to the right

amplitude (a): "stretch" by magnitude of 2

vertical and horizontal shifts





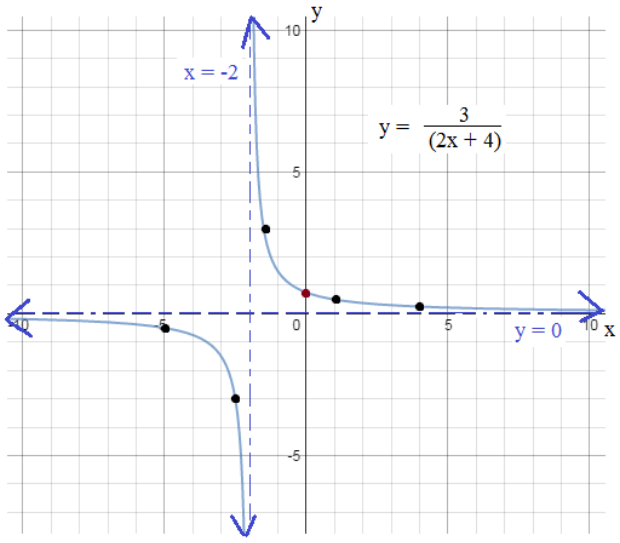
Approach 2: Using asymptotes, intercepts, points, and end behavior

Example: Sketch the function $y = \frac{3}{(2x+4)}$

Asymptotes: rational expression is 'bottom heavy', so the end behavior is y = 0

the expression is undefined when denominator = 0...

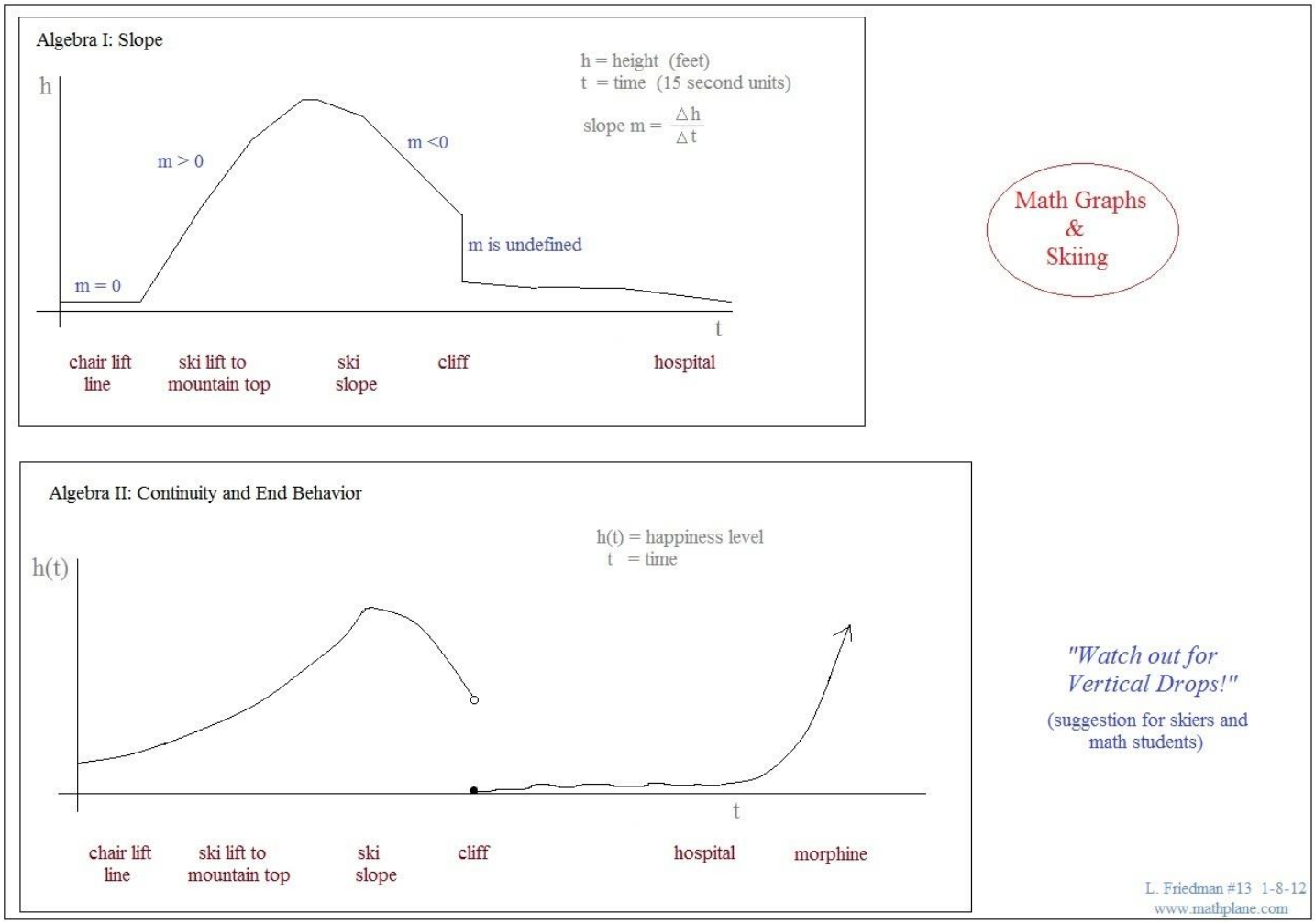
This occurs at x = -2



Intercepts: x-intercepts (zeros)? They occur when y = 0... There are none..

y-intercept -- when x = 0: $y = \frac{3}{(2(0)+4)} = 3/4$

A few points: (1, 1/2) (4, 1/4) (-5, -1/2) (-2.5, -3) (-3/2, -3)



PRACTICE EXERCISES-→

Reciprocal Functions

Answer the following and sketch the graph:

1) $y = \frac{3}{x}$

Domain:

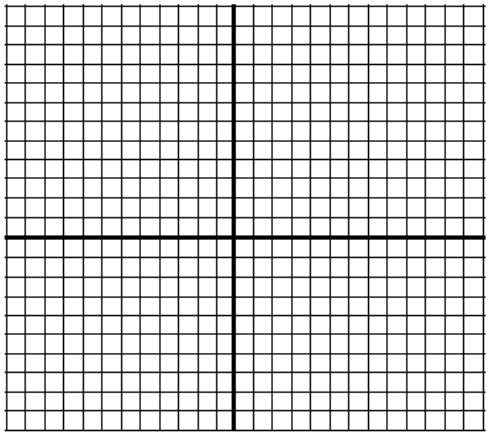
Range:

Vertical Asymptote:

Horizontal Asymptote:

y-intercept:

x-intercept(s):



2) $y = \frac{1}{x - 5}$

Domain:

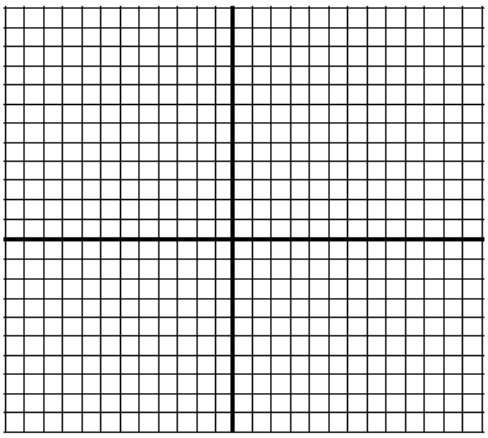
Range:

Vertical Asymptote:

Horizontal Asymptote:

y-intercept:

x-intercept(s):



3) $f(x) = \frac{3}{x + 4} + 2$

Domain:

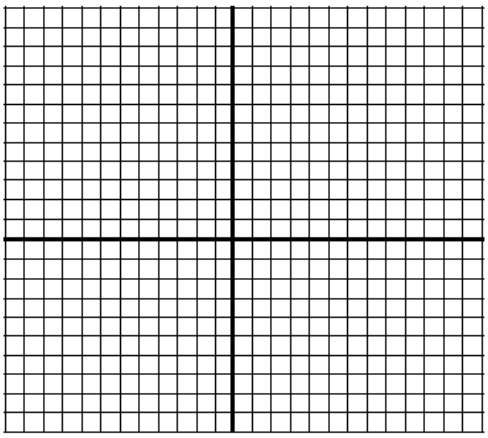
Range:

Vertical Asymptote:

Horizontal Asymptote:

y-intercept:

x-intercept(s):



4) $y = \frac{3x + 10}{x}$

Domain:

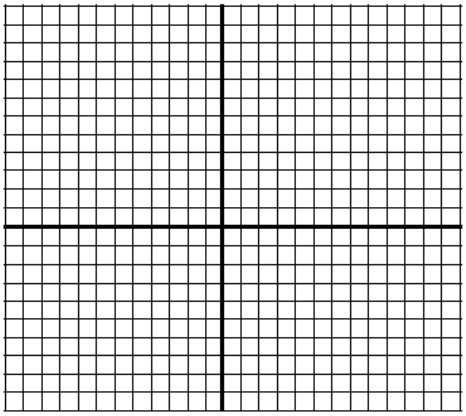
Range:

Vertical Asymptote:

Horizontal Asymptote:

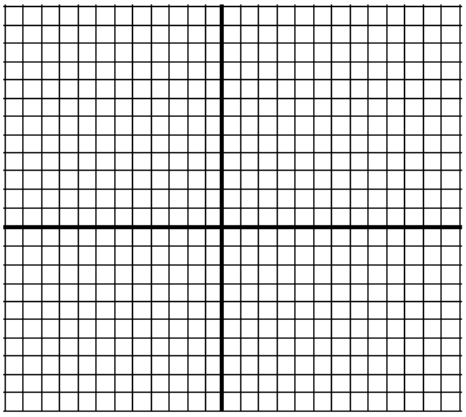
y-intercept:

x-intercept(s):



5) Write a rational function that has asymptotes $x = -3$ and $y = 5$
(bonus: write a 2nd function that has the same asymptotes)

Graph the function(s).



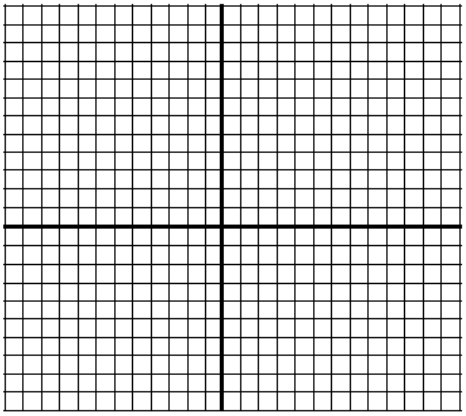
6) A tennis club has a $100 membership and charges $5 per hour of court time. Write a model that expresses the *average* cost of playing tennis at the club. (label the variables)

What do the asymptotes represent?

Identify and discuss the domain and range:

Graph the model:

What is the average cost of playing 10 hours?
(Show algebraically and graphically)



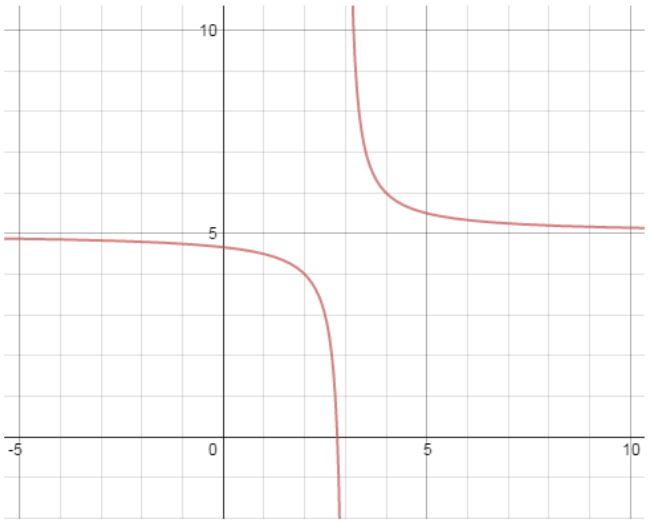
Reciprocal Functions

7) For the following reciprocal functions:

a) Draw and name the asymptotes

b) What is the equation?

c) Find the x and y-intercepts



a) Draw and name the asymptotes

b) What is the equation?

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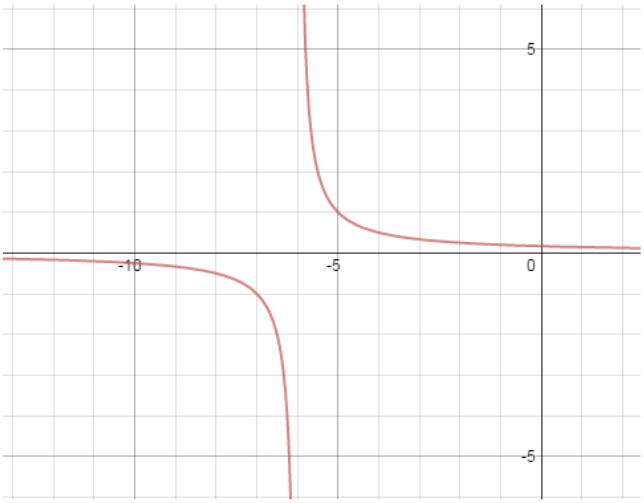
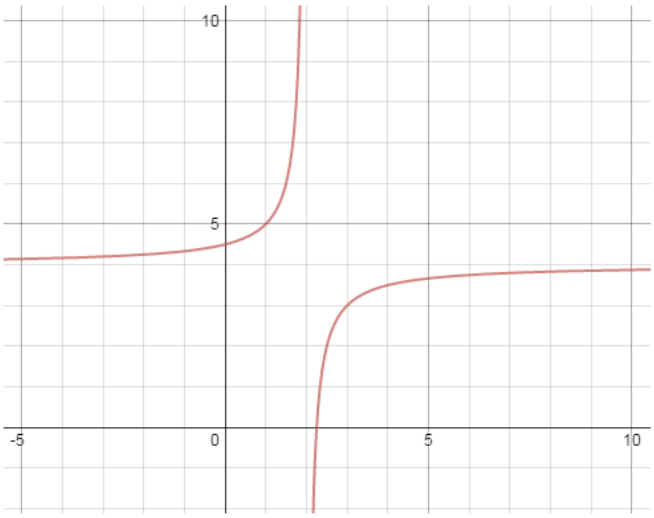
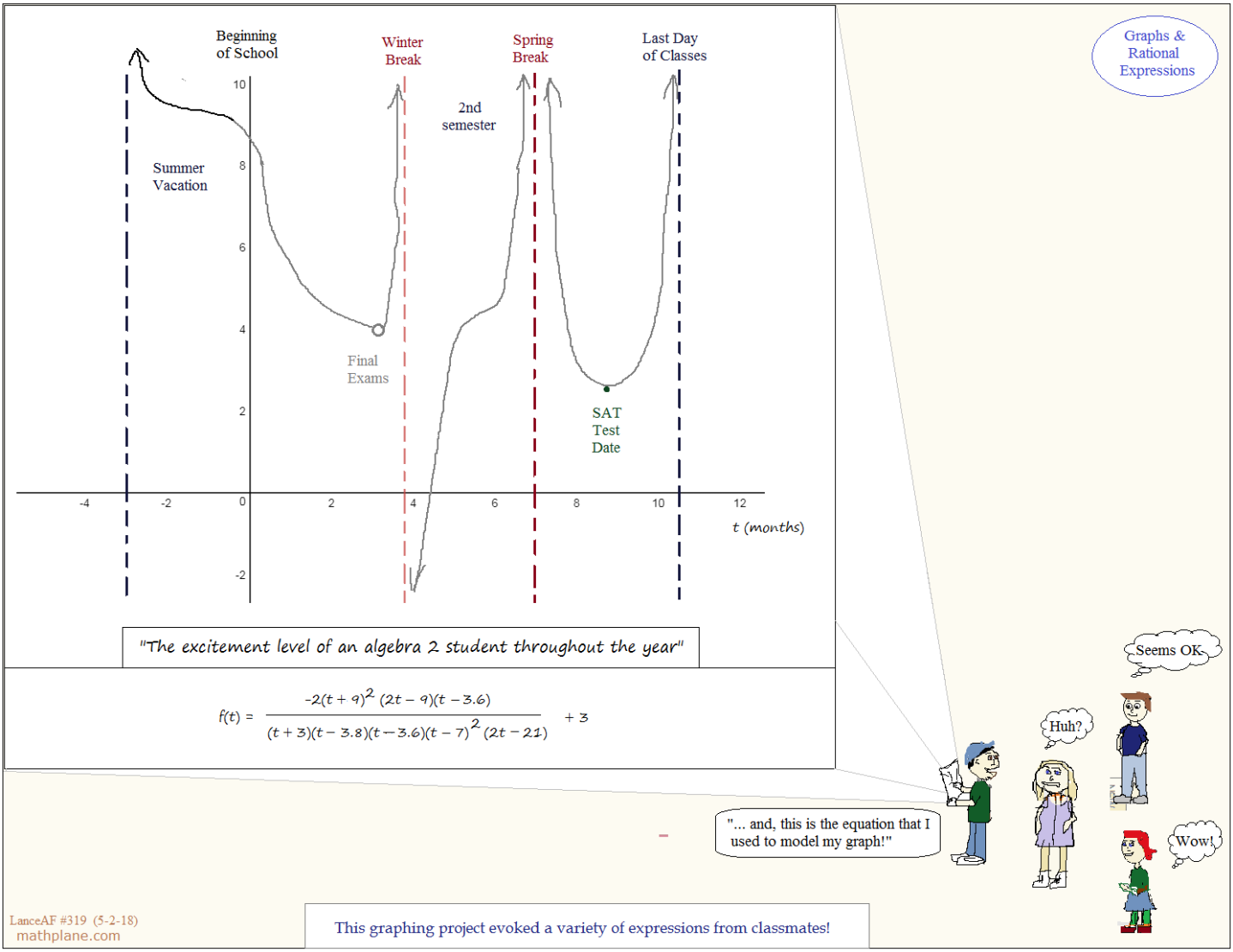


a) Draw and name the asymptotes

b) What is the equation?

c) Find the x and y-intercepts




Graphs & Rational Expressions
Beginning of School
Winter Break
Spring Break
Last Day of Classes
2nd semester
Summer Vacation
Final Exams
SAT Test Date
10
8
6
4
2
-2
-4
-2
0
2
4
6
8
10
12
t (months)
"The excitement level of an algebra 2 student throughout the year"
$f(t) = \frac{-2(t+9)^2(2t-9)(t-3.6)}{(t+3)(t-3.8)(t-3.6)(t-7)^2(2t-21)} + 3$
Seems OK.
Huh?
"... and, this is the equation that I used to model my graph!"
Wow!
LanceAF #319 (5-2-18)
mathplane.com
This graphing project evoked a variety of expressions from classmates!

SOLUTIONS-→

Reciprocal Functions

SOLUTIONS

Answer the following and sketch the graph:

1) $y = \frac{3}{x}$

Domain: all real numbers except 0 $(-\infty, 0) \cup (0, \infty)$

Range: all real numbers except 0 $(-\infty, 0) \cup (0, \infty)$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 0$

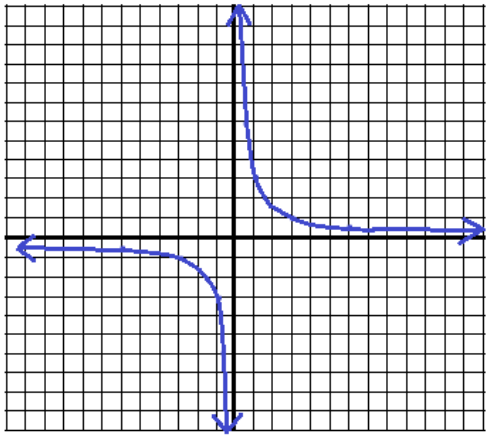
y-intercept: none

x-intercept(s): none

points include:

(1, 3) (3, 1)
(1/2, 6) (6, 1/2)

(-1, -3) (-3, -1)
(-1/2, -6) (-6, -1/2)



2) $y = \frac{1}{x - 5}$ reciprocal function (1/x) shifted 5 units to the right

Domain: all real numbers except 5 $(-\infty, 5) \cup (5, \infty)$

Range: all real numbers except 0 $(-\infty, 0) \cup (0, \infty)$

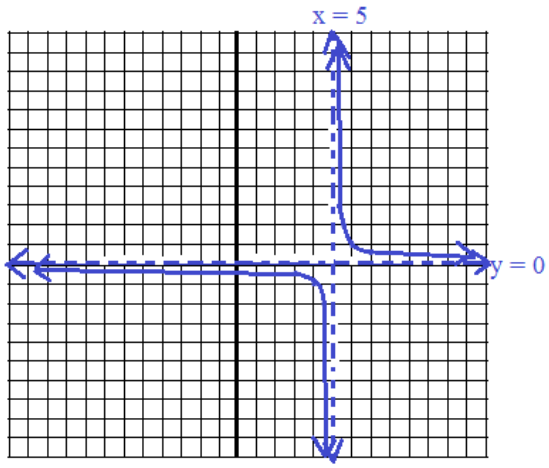
Vertical Asymptote: $x = 5$

Horizontal Asymptote: $y = 0$ ('bottom heavy')

y-intercept: (0, -1/5)

x-intercept(s): none

points include:
(6, 1) (5.5, 2) (5.1, 10)
(4, -1) (4.5, -2) (3, -1/2)



3) $f(x) = \frac{3}{x + 4} + 2$ reciprocal function (1/x) shifted to the left 4 units and shifted up 2 units

Domain: all real numbers except $(-\infty, -4) \cup (-4, \infty)$

Range: all real numbers except 2 $(-\infty, 2) \cup (2, \infty)$

Vertical Asymptote: $x = -4$

Horizontal Asymptote: $y = 2$

y-intercept: (0, 11/4)

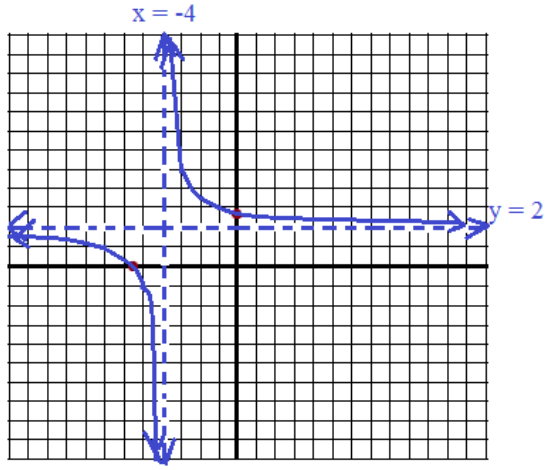
x-intercept(s): (-11/2, 0)

$\frac{3}{x + 4} + 2 = 0$

$\frac{3}{x + 4} = -2$

$-2x - 8 = 3$

$x = -11/2$



SOLUTIONS

Reciprocal Functions

4) $y = \dfrac{3x + 10}{x}$ points include: (2, 8) (5, 5) (10, 4) (-2, -2) (-5, 1) (-10, 2)

Domain: all real numbers except 0 $(-\infty, 0) \cup (0, \infty)$

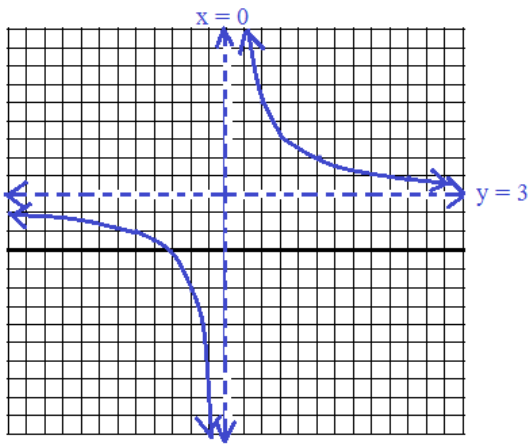
Range: all real numbers except $(-\infty, 3) \cup (3, \infty)$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 3$ degree numerator = degree denominator so look at lead coefficients: $\frac{3}{1} = 3$

y-intercept: none (undefined at x = 0)

x-intercept(s): (-10/3, 0)



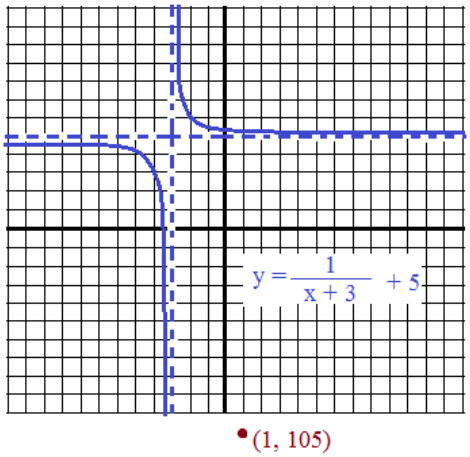
5) Write a rational function that has asymptotes $x = -3$ and $y = 5$
(bonus: write a 2nd function that has the same asymptotes)

Graph the function(s).

$\frac{1}{x}$ asymptote at x = -3 ⇒ $\frac{1}{x+3}$ end behavior horizontal asymptote at y = 5 ⇒ $\frac{1}{x+3} + 5$

a second function may be

$$y = \frac{2}{x+3} + 5$$



6) A tennis club has a $100 membership and charges $5 per hour of court time. Write a model that expresses the *average* cost of playing tennis at the club. (label the variables)

$$AC = \frac{100 + 5h}{h}$$ where h is *natural number*

What do the asymptotes represent? $h \neq 0$ (cannot have 0 hours)

AC = 5 (average cost approaches 5)

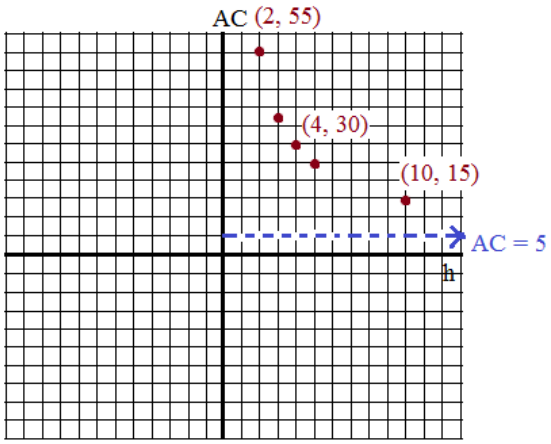
Identify and discuss the domain and range: domain is all natural numbers (there are no "negative hours")

Graph the model: Assuming the club charges per hour (WITHOUT partial hours), the range is {105, 55, 38 1/3, 30, 25, 21 2/3...}

Note: If club charges partial hours, the model's domain/range would differ.

What is the average cost of playing 10 hours? (Show algebraically and graphically)

$$AC = \frac{100 + 5(10)}{(10)} = 15 \text{ dollars/hour}$$



Reciprocal Functions

SOLUTIONS

7) For the following reciprocal functions:

a) Draw and name the asymptotes

$x = 3$

$y = 5$

b) What is the equation?

$$y = 5 + \frac{1}{(x - 3)}$$

c) Find the x and y-intercepts

y-intercept (when x = 0)

$$y = 5 + \frac{1}{(0 - 3)}$$

$y = 4\ 2/3$

$(0, 4\frac{2}{3})$

x-intercept (when y = 0)

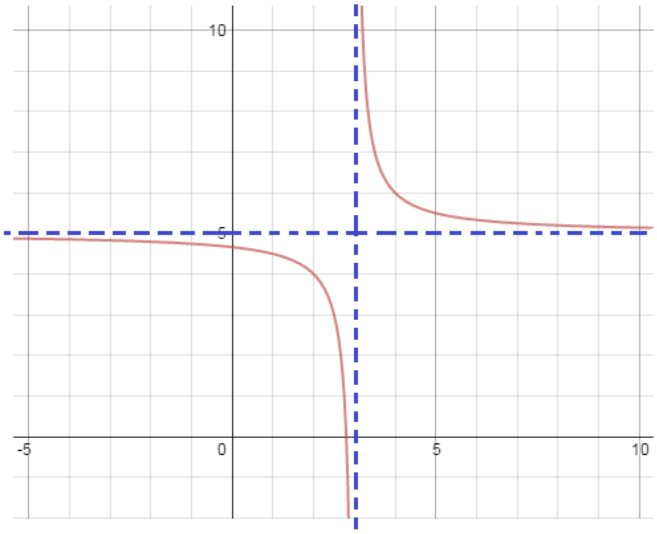
$$0 = 5 + \frac{1}{(x - 3)}$$

$$-5 = \frac{1}{(x - 3)}$$

$-5x + 15 = 1$

$x = 14/5$

$(\frac{14}{5}, 0)$



a) Draw and name the asymptotes

$x = -6$

$y = 0$

b) What is the equation?

$$y = \frac{1}{(x + 6)}$$

c) Find the x and y-intercepts

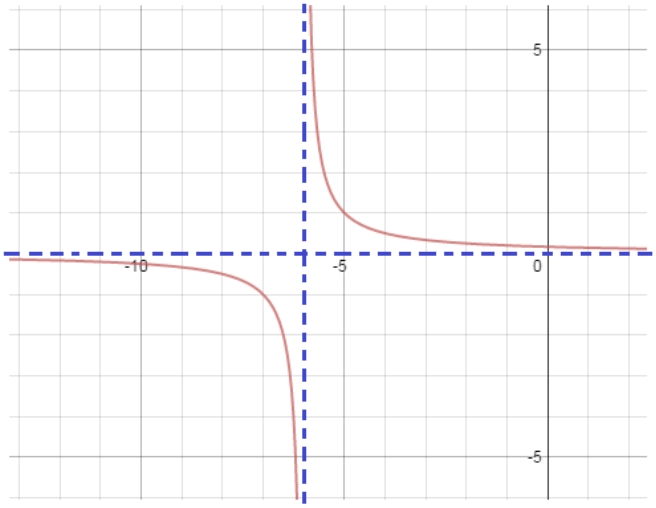
$$y = \frac{1}{(0 + 6)}$$

$y = 1/6$

$(0, \frac{1}{6})$

$$0 = \frac{1}{(x + 6)}$$

NO x-intercept!!



a) Draw and name the asymptotes

$x = 2$

$y = 4$

b) What is the equation?

$$y = 4 - \frac{1}{(x - 2)}$$

c) Find the x and y-intercepts

NOTE: the reciprocal function is reflected over the x-axis, so it is negative

y-intercept:

$(0, 4\frac{1}{2})$

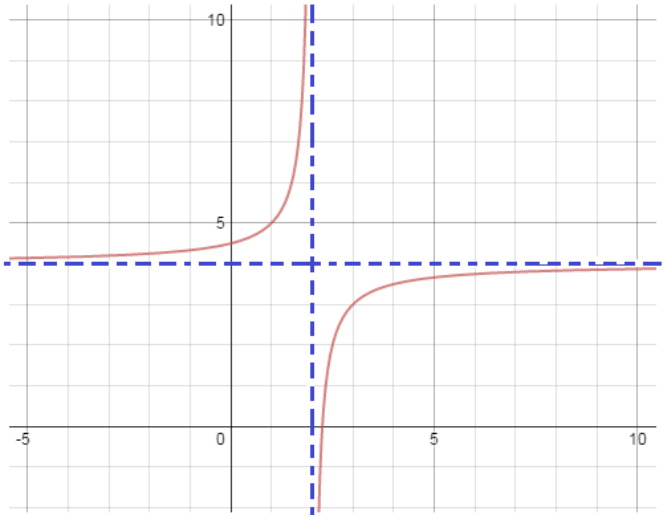
x-intercept:

$$0 = 4 - \frac{1}{(x - 2)}$$

$4x - 8 = 1$

$x = 9/4$

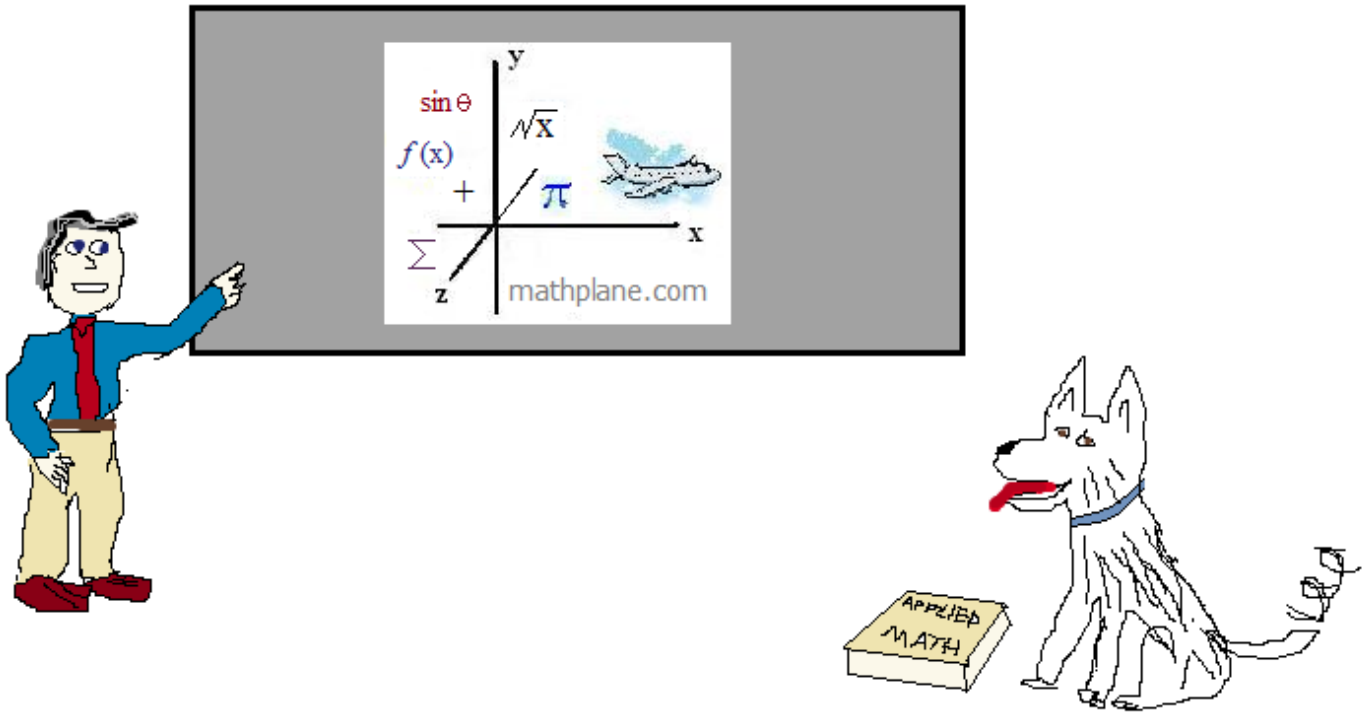
$(\frac{9}{4}, 0)$



Thanks for visiting (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.



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And, our store at TeachersPayTeachers.com