Simplifying Rational Expressions

Notes, Examples, and quiz (with solutions)

$\boxed{\text{Simplifying Rational Expressions}}$

What is a "rational expression"?

An expression that can be written as a *polynomial divided by a polynomial.*

Note: A *rational number* is a number that can be expressed as a fraction

A rational expression may be called a "fractional expression"

Examples:

$$\frac{x+3 \quad \text{— polynomial (linear binomial)}}{x^2+2x+8 \quad \text{— polynomial (trinomial quadratic)}}$$

$$\frac{3 \quad \text{— monomial (degree 0)}}{(x+2)(x-7) \quad \text{— quadratic written in factored form}}$$

To simplify a rational expression, "factor and cancel"...

Examples: $\dfrac{3x+6}{x^2+5x+6}$ factor: $\dfrac{3(x+2)}{(x+2)(x+3)}$

cancel: $\dfrac{3(\cancel{x+2})}{(\cancel{x+2})(x+3)} = \dfrac{3}{(x+3)}$

$\dfrac{2x^2-18}{(x+3)(x+7)}$ factor: $\dfrac{2(x^2-9)}{(x+3)(x+7)} = \dfrac{2(x+3)(x-3)}{(x+3)(x+7)}$

cancel: $\dfrac{2(\cancel{x+3})(x-3)}{(\cancel{x+3})(x+7)} = \dfrac{2(x-3)}{(x+7)}$ OR $\dfrac{2x-6}{x+7}$

***USING (-1)

$\dfrac{x^2+3x-10}{2-x}$ factor: $\dfrac{(x-2)(x+5)}{(2-x)}$ (x - 2) and (2 - x) are very similar!

$\dfrac{(x-2)(x+5)}{(-1)(x-2)}$ $(-1)(x-2)=(2-x)$

cancel: $\dfrac{(\cancel{x-2})(x+5)}{(-1)(\cancel{x-2})} = \dfrac{(x+5)}{(-1)}$ OR $-x-5$

Multiplying and Dividing Rational Expressions

The process of multiplying/dividing rational expressions is similar to ordinary fractions.

When multiplying rational expressions, try to factor, cancel and , combine.
When dividing rational expressions, "invert and multiply"....

Example: $\dfrac{x^2 + 6x + 5}{x^2 - 1} \cdot \dfrac{2x - 2}{3x^2 + 7}$

Factor the polynomials: $\dfrac{(x + 1)(x + 5)}{(x + 1)(x - 1)} \cdot \dfrac{2(x - 1)}{3x^2 + 7}$

Cancel terms: $\dfrac{(x + 1)(x + 5)}{(x + 1)(x - 1)} \cdot \dfrac{2(x - 1)}{3x^2 + 7}$

Combine: $\dfrac{2(x + 5)}{3x^2 + 7}$

Example: $\dfrac{x^2 - 25}{x^2 - 4x} \cdot \dfrac{x^2 + x - 20}{x^2 + 10x + 25}$

Factor the polynomials: $\dfrac{(x + 5)(x - 5)}{x(x - 4)} \cdot \dfrac{(x + 5)(x - 4)}{(x + 5)(x + 5)}$

Cancel terms: $\dfrac{(x + 5)(x - 5)}{x(x - 4)} \cdot \dfrac{(x + 5)(x - 4)}{(x + 5)(x + 5)}$

Combine: $\dfrac{x - 5}{x}$

Example: $\dfrac{5x^2 + 15x + 10}{(x + 3)(x - 4)} \div \dfrac{(x + 2)^2}{2x - 8}$

"Invert and Multiply": $\dfrac{5x^2 + 15x + 10}{(x + 3)(x - 4)} \cdot \dfrac{2x - 8}{(x + 2)^2}$

Factor the polynomials: $\dfrac{5(x^2 + 3x + 2)}{(x + 3)(x - 4)} \cdot \dfrac{2(x - 4)}{(x + 2)(x + 2)}$

Cancel the terms: $\dfrac{5(x + 2)(x + 1)}{(x + 3)(x - 4)} \cdot \dfrac{2(x - 4)}{(x + 2)(x + 2)}$

Combine: $\dfrac{10(x + 1)}{(x + 3)(x + 2)}$ OR $\dfrac{10x + 10}{x^2 + 5x + 6}$

As with any group of fractions, look for the least common denominator....

Example: Simplify $\dfrac{3x + 2}{x^2\,y} + \dfrac{2y - 7}{4xy}$

Determine the least common denominator

If we multiply the left denominator by 4
and multiply the right denominator by x,
we have a common denominator...

$$4x^2y$$

$$\frac{4\,(3x + 2)}{4 \cdot x^2\,y} + \frac{(2y - 7)\,x}{4xy \cdot x} \;=\; \frac{12x + 8}{4\,x^2\,y} + \frac{2xy - 7x}{4\,x^2\,y}$$

$$=\; \boxed{\frac{5x + 2xy + 8}{4\,x^2\,y}}$$

Example: Simplify $\dfrac{5x}{x^2 - 9} - \dfrac{6}{2x + 6}$

Factor the parts to help find common denominator

$$\frac{5x}{(x + 3)(x - 3)} - \frac{6}{2(x + 3)}$$

If we multiply the left by 2
and multiply the right by (x - 3),
we have a common denominator...

$$2(x + 3)(x - 3)$$

$$\frac{2 \cdot}{2}\,\frac{5x}{(x + 3)(x - 3)} - \frac{6}{2(x + 3)}\,\frac{(x - 3)}{(x - 3)}$$

$$\frac{10x}{2\,(x + 3)(x - 3)} - \frac{(6x - 18)}{2\,(x + 3)(x - 3)} \;=\; \boxed{\frac{4x + 18}{2\,(x + 3)(x - 3)}}$$

Example: Simplify $\dfrac{3x}{x^2 + 9} + \dfrac{4}{x - 3}$

Both denominators are prime (i.e. cannot be broken down)

$$\frac{(x - 3)}{(x - 3)} \cdot \frac{3x}{x^2 + 9} + \frac{4}{x - 3} \cdot \frac{(x^2 + 9)}{(x^2 + 9)}$$

$$\frac{3x^2 - 9x + 4x^2 + 36}{(x^2 + 9)(x - 3)} \;=\; \boxed{\frac{7x^2 - 9x + 36}{(x^2 + 9)(x - 3)}}$$

Example: $\dfrac{2y}{y^2 - 9} - \dfrac{1}{y - 3}$

$$\frac{2y}{(y + 3)(y - 3)} - \frac{1\,(y + 3)}{(y - 3)(y + 3)}$$

$$\frac{2y - (y + 3)}{(y - 3)(y + 3)} \;=\; \frac{y - 3}{(y - 3)(y + 3)} \;=\; \boxed{\frac{1}{y + 3}}$$

Simplifying Complex Rational Expressions

Example:

Approach 1:

$$\frac{\dfrac{m-1}{m^2} - \dfrac{2}{m}}{m^2}$$

combine numerator...

$$\frac{\dfrac{m-1-2m}{m^2}}{m^2}$$

simplify

$$\frac{-m-1}{m^4}$$

where m is not 0

Approach 2:

$$\frac{\dfrac{m-1}{m^2} - \dfrac{2}{m}}{m^2}$$

separate terms

$$\frac{\dfrac{m-1}{m^2}}{m^2} - \frac{\dfrac{2}{m}}{m^2}$$

simplify

$$\frac{m-1}{m^4} - \frac{2}{m^3}$$

combine terms

$$\frac{m-1}{m^4} - \frac{2m}{m^4}$$

$$\frac{-m-1}{m^4}$$

where m is not 0

Example:

$$\frac{\dfrac{7}{5-x}}{\dfrac{2}{5-x} + \dfrac{1}{3x-15}}$$

combine the denominator expressions

$$\frac{2}{5-x} + \frac{1}{3(x-5)}$$

$$\frac{2}{5-x} + \frac{1}{-3(5-x)}$$

$$\frac{(-3)2}{(-3)(5-x)} + \frac{1}{-3(5-x)} = \frac{-5}{-3(5-x)}$$

then, simplify the overall expression...

$$\frac{\dfrac{7}{5-x}}{\dfrac{-5}{-3(5-x)}} = \frac{7}{\dfrac{5}{3}} = \boxed{\frac{21}{5}}$$

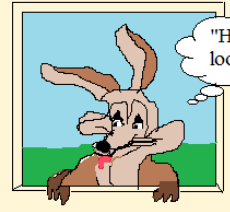$$\frac{x^2 + 11x - 12}{x^2 - 1} \cdot \frac{5x + 5}{3x + 1}$$

$$(2x + 16) \cdot \frac{5}{2x^2 + 17x + 8}$$

$$\frac{y - 6}{10} \div \frac{6 - y}{5}$$

$$\frac{2x^2 - 5x - 3}{4x^2 - 12x - 7} \div \frac{4x + 9}{2x - 7}$$

$$\frac{4s^2 + 8s - 12}{5 - 6s + s^2} \div \frac{2s^2 + 7s + 3}{4s^2 - 8s - 5}$$

$$\frac{\dfrac{x^2 + 3x + 2}{x^2 - 1}}{\dfrac{4x^2 + 8x}{5(x - 1)}}$$

Problem 1

$$\frac{x^2 + 11x - 12}{x^2 - 1} \cdot \frac{5x + 5}{3x + 1}$$

Factor: $\dfrac{(x - 1)(x + 12)}{(x + 1)(x - 1)} \cdot \dfrac{5(x + 1)}{3x + 1}$

Cancel: $\dfrac{(x - 1)(x + 12)}{(x + 1)(x - 1)} \cdot \dfrac{5(x + 1)}{3x + 1}$

Combine: $\boxed{\dfrac{5(x + 12)}{3x + 1}}$ OR $\dfrac{5x + 60}{3x + 1}$

Problem 2

$$(2x + 16) \cdot \frac{5}{2x^2 + 17x + 8}$$

Factor: $\dfrac{2(x + 8)}{1} \cdot \dfrac{5}{(2x + 1)(x + 8)}$

Cancel: $\dfrac{2(x + 8)}{1} \cdot \dfrac{5}{(2x + 1)(x + 8)}$

Combine: $\boxed{\dfrac{10}{2x + 1}}$

Problem 3

$$\frac{y - 6}{10} \div \frac{6 - y}{5}$$

Invert and multiply $\dfrac{y - 6}{10} \cdot \dfrac{5}{6 - y}$

(use -1 as factor) $\dfrac{y - 6}{10} \cdot \dfrac{5}{(-1)(y - 6)}$

cancel and combine: $\dfrac{y - 6}{10} \cdot \dfrac{5}{(-1)(y - 6)}$

simplify/reduce: $\dfrac{5}{(-1)10} = \boxed{\dfrac{-1}{2}}$

Problem 4

$$\frac{2x^2 - 5x - 3}{4x^2 - 12x - 7} \div \frac{4x + 9}{2x - 7}$$

invert and multiply: $\dfrac{2x^2 - 5x - 3}{4x^2 - 12x - 7} \cdot \dfrac{2x - 7}{4x + 9}$

factor: $\dfrac{(2x + 1)(x - 3)}{(2x - 7)(2x + 1)} \cdot \dfrac{2x - 7}{4x + 9}$

cancel: $\dfrac{(2x + 1)(x - 3)}{(2x - 7)(2x + 1)} \cdot \dfrac{2x - 7}{4x + 9}$

combine: $\boxed{\dfrac{x - 3}{4x + 9}}$

Problem 5

$$\frac{4s^2 + 8s - 12}{5 - 6s + s^2} \div \frac{2s^2 + 7s + 3}{4s^2 - 8s - 5}$$

Invert and multiply: $\dfrac{4s^2 + 8s - 12}{s^2 - 6s + 5} \cdot \dfrac{4s^2 - 8s - 5}{2s^2 + 7s + 3}$

factor: $\dfrac{4(s^2 + 2s - 3)}{(s - 1)(s - 5)} \cdot \dfrac{(2s + 1)(2s - 5)}{(2s + 1)(s + 3)}$

cancel: $\dfrac{4(s + 3)(s - 1)}{(s - 1)(s - 5)} \cdot \dfrac{(2s + 1)(2s - 5)}{(2s + 1)(s + 3)}$

combine: $\boxed{\dfrac{4(2s - 5)}{(s - 5)}}$

Problem 6

$$\frac{\dfrac{x^2 + 3x + 2}{x^2 - 1}}{\dfrac{4x^2 + 8x}{5(x - 1)}}$$

note: this is simply a division problem.

Invert and multiply: $\dfrac{x^2 + 3x + 2}{x^2 - 1} \cdot \dfrac{5(x - 1)}{4x^2 + 8x}$

factor: $\dfrac{(x + 2)(x + 1)}{(x + 1)(x - 1)} \cdot \dfrac{5(x - 1)}{4x(x + 2)}$

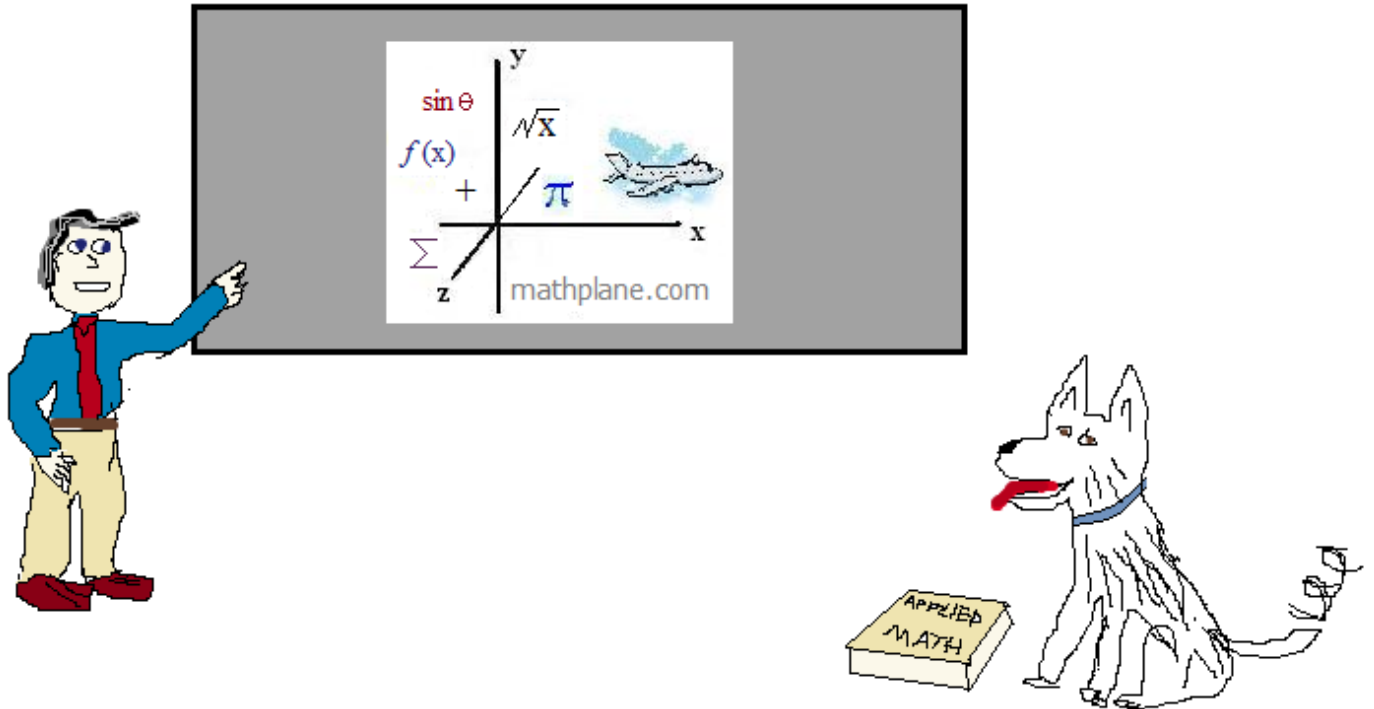cancel: $\dfrac{(x + 2)(x + 1)}{(x + 1)(x - 1)} \cdot \dfrac{5(x - 1)}{4x(x + 2)}$

combine: $\boxed{\dfrac{5}{4x}}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.

Also, at mathplane.ORG for mobile and tablets.

And, our store at TeachersPayTeachers