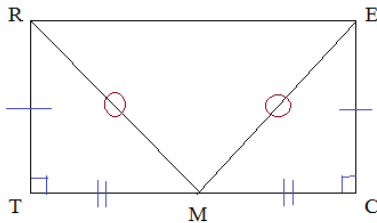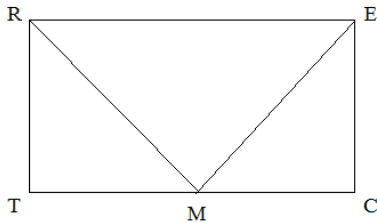# Special Quadrilaterals 2

Notes, examples, practice questions (and, solutions)

Topics include slope, distance, properties of quadrilaterals, proofs, and more...

Rectangle Property proofs

Given: Rectangle RECT
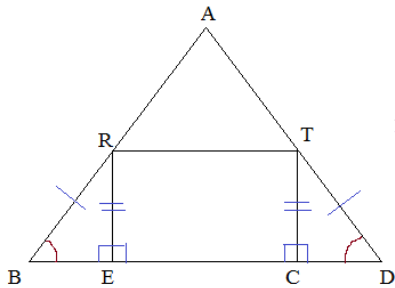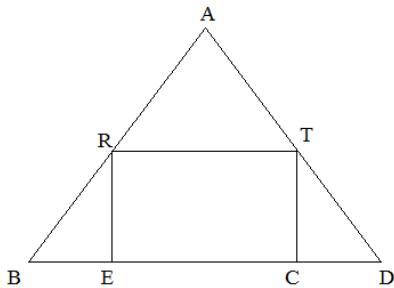   M is midpoint of $\overline{TC}$

Prove: $\triangle$ REM is isosceles

Statements	Reasons
1) Rectangle RECT	1) Given
2) $\overline{RT} \cong \overline{EC}$	2) Definition of Rectangle (opposite sides congruent)
3) M is midpoint of $\overline{TC}$	3) Given
4) $\overline{TM} \cong \overline{CM}$	4) Definiton of midpoint (midpoint divides segment into congruent halves)
5) $\angle T$ and $\angle C$ are right angles	5) Definiton of Rectangle (angles are 90 degrees)
6) $\angle T \cong \angle C$	6) All right angles are congruent
7) $\triangle RTM \cong \triangle ECM$	7) SAS (Side-Angle-Side) 2, 6, 4
8) $\overline{RM} \cong \overline{EM}$	8) CPCTC (corresponding parts of congruent triangles are congruent)
9) $\triangle$ REM is isosceles	9) Definition of Isosceles Triangle (2 or more sides of triangle are congruent)

Given: RECT is a rectangle
        $\overline{RB} \cong \overline{TD}$

Prove: $\triangle$ ABD is isosceles

Statements	Reasons
1) Rectangle RECT	1) Given
2) $\overline{RE} \cong \overline{CT}$	2) Definition of Rectangle (opposite sides are congruent)
3) $\overline{RB} \cong \overline{TD}$	3) Given
4) $\angle REC$ and $\angle TCE$ are right angles	4) Definition of Rectangle (angles are right angles)
5) REB and REC are supplementary TCE and TCD are supplementary	5) Definition of Supplementary (angles that form a straight angle are supplementary)
6) REB and TCD are right angles	6) Subtraction property
7) $\triangle REB = \triangle TCD$	7) RHL (Right Angle- Hypotenuse - Leg) 6, 3, 2
8) $\angle B = \angle D$	8) CPCTC (corresponding parts of congruent triangles are congruent)
9) $\overline{AB} \cong \overline{AD}$	9) If congruent angles, then congruent sides (in triangle, if congruent angles, then opposite sides are congruent)
10) $\triangle$ ABD is isosceles	10) Definition of Isosceles –2 or more congruent sides (Also, base angles of triangle are congruent)

Coordinate Geometry: Verifying/Identifying Special Quadrilaterals

Quadrilaterals and Slope

Parallelogram: opposite sides parallel

Rectangle: opposite sides parallel; adjacent sides are perpendicular

Rhombus: opposite sides parallel; diagonals are perpendicular

square: diagonals are perpendicular; adjacent sides are perpendicular; opposite sides parallel

kite: diagonals are perpendicular; opposite sides are not parallel

trapezoid: one pair of opposite sides are parallel

Quadrilaterals and the Distance Formula

Parallelogram: opposite sides are congruent

Rhombus: all sides are congruent

Rectangle: opposite sides congruent AND diagonals congruent

Square: all sides congruent AND diagonals congruent

Kite: pair of consecutive sides are congruent

Isosoceles trapezoid: one pair of congruent (opposite) sides AND congruent diagonals

---

*Example:* Verify using slope that the quadrilateral is a rhombus.

If the slopes of the opposites sides are equal, then it's a parallelogram...

RH: slope is 4/3
HO: slope is 0
OM: slope is 4/3
RM: slope is 0

Then, if the diagonals are perpendicular, then it's a rhombus...

HM: slope is -2
OR: slope is 1/2

(Note: since 4/3 and 0 are not opposite reciprocals, the sides
are not perpendicular. Therefore, the figure is not a square.)


H(5, 5)   O(10, 5)
R(2, 1)   M (7, 1)
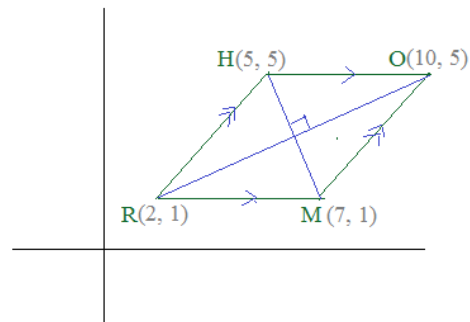
*Example:* Verify the quadrilateral is a rhombus using distance/length only

If the distances between the sides are the same, it's a rhombus or a square.

HO: distance is 5
MO: using distance formula --- $\sqrt{(7-10)^2 + (1-5)^2}$ = 5

RM: length is 5 units
RH: using distance formula --- $\sqrt{(2-5)^2 + (1-5)^2}$ = 5

Then, if the diagonals are congruent, it's a square.. If not, it's only a rhombus..

RO: $\sqrt{(2-10)^2 + (1-5)^2}$ = $4\sqrt{5}$

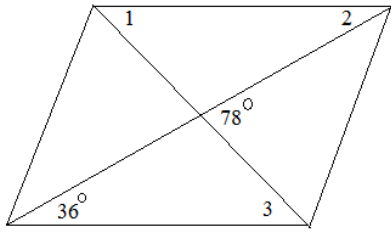HM: $\sqrt{(7-5)^2 + (1-5)^2}$ = $2\sqrt{5}$

H(5, 5)   O(10, 5)
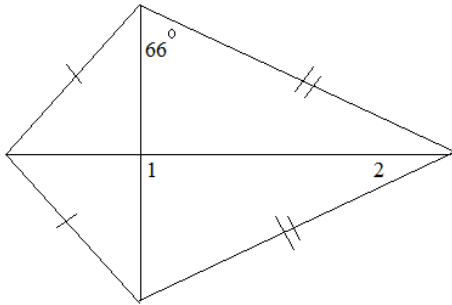R(2, 1)   M (7, 1)
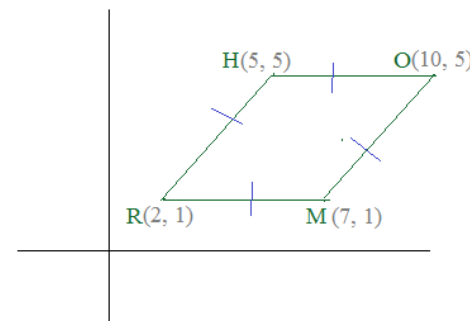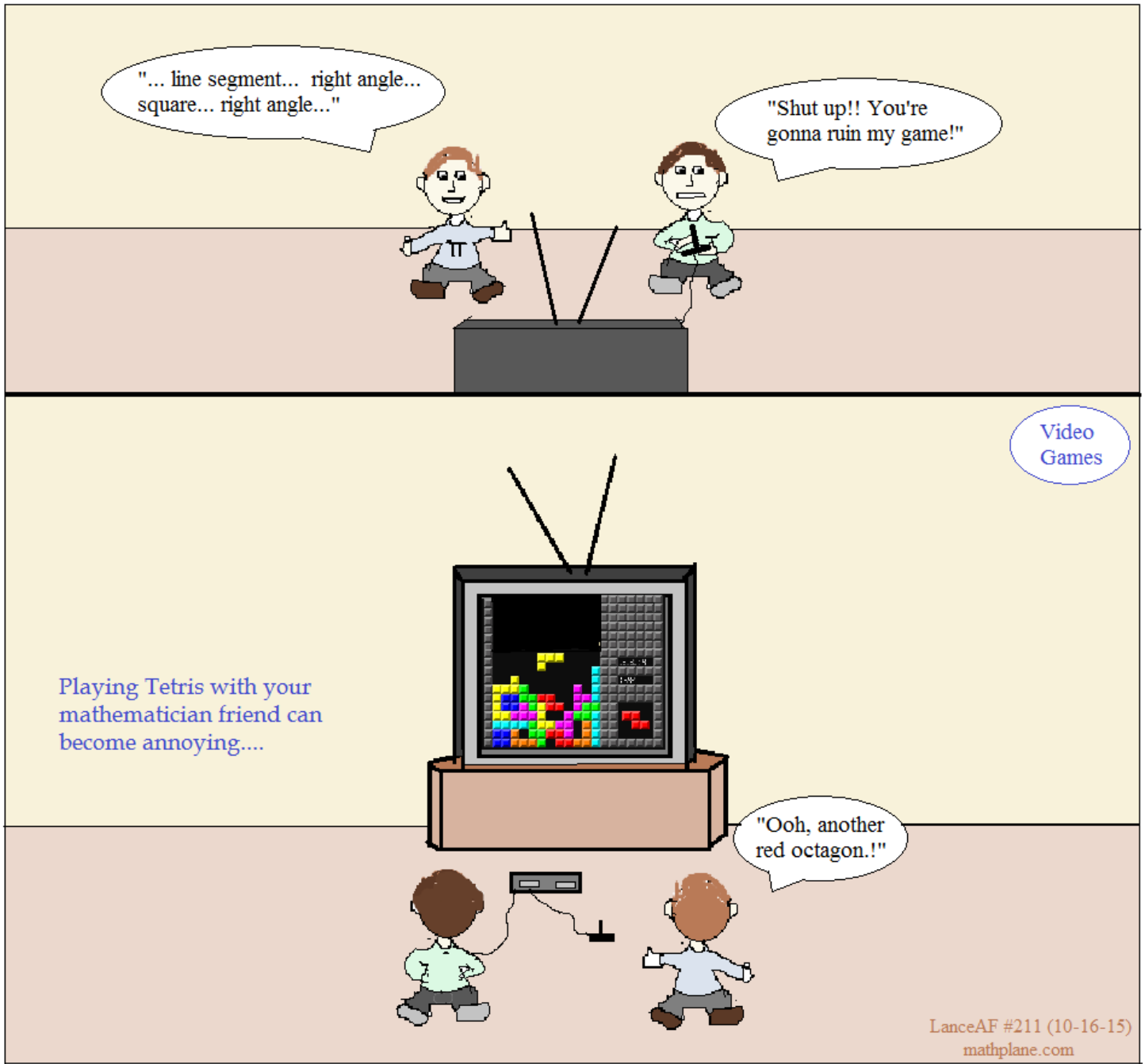
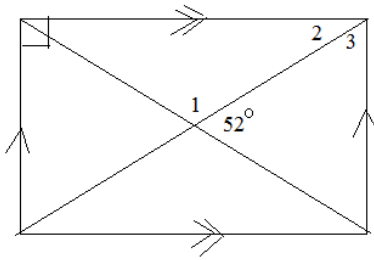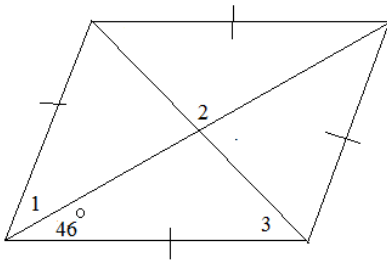Exercises -→

WARM-UP: Determine the angle measures
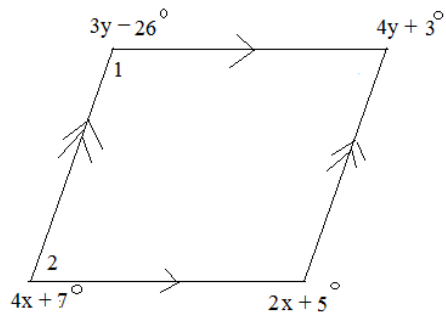
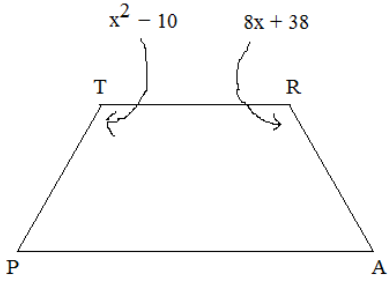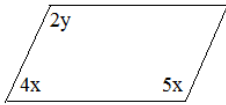1) Parallelogram
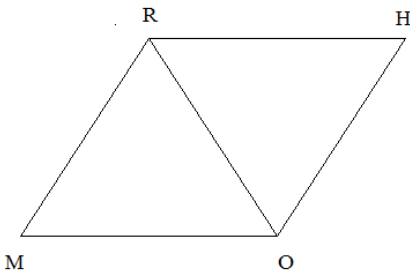

2) Kite

3) Rectangle

4) Rhombus

5) Parallelogram

1) In the isosceles trapezoid TRAP,
   what are the measures of $\angle P$ and $\angle T$?

$x^2 - 10$ $8x + 38$

T   R

P   A

2) Find values for each variable for
   the given parallelogram:

2y

4x   5x

3) Triangles ROM and ROH are equilateral...

If the diagonal HM creates an angle HMO of measure $3x - 6$,
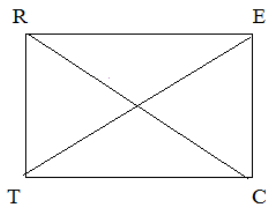and RO $= 10x$,
then what is the perimeter of rhombus RHOM?

R   H

M   O

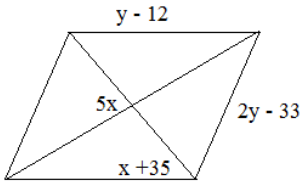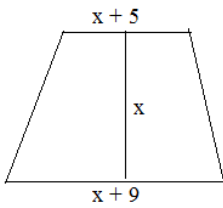4) RECT is a rectangle...

RC $= x^2$

RE $= 4 - x$

ET $= 6x - 5$

Find the possible values of x

R   E

T   C

5) For the following rhombus , determine x and y...

y - 12

5x

2y - 33

x +35

6) If the area is 78 square units,
what is x?

x + 5

x

x + 9

7) In the following parallelogram, find the angle measures of 1, 2, and 3...
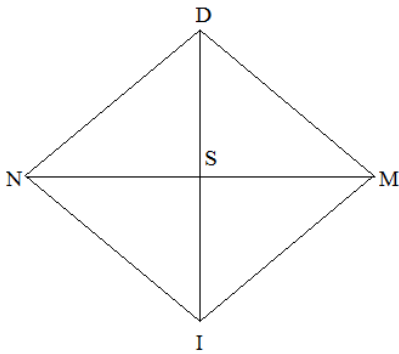
3

34°

1

2

108°

8) In rhombus MIND, $\angle$DSN $= 2x^2 + 5x + 15$ DN $= 2x + 3y$

DM $= 5y + 4$ Find x, y, and $\overline{ID}$

DS $= xy$

D

S

N

M

I

9) Given: The kite MATH

$\overline{MA} = x + 5$

$\overline{AT} = 2x + 1$

$\overline{MH} = 2y + 1$

$\overline{HT} = 3y - 2$

Find: the perimeter

10) Find the perimeter of square SQAR where vertices are Q (-4, 1) and R (-1, 6).
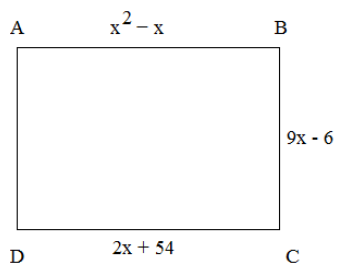
a) 16

b) $4\sqrt{34}$

c) $4\sqrt{17}$

d) 32

e) $16\sqrt{2}$
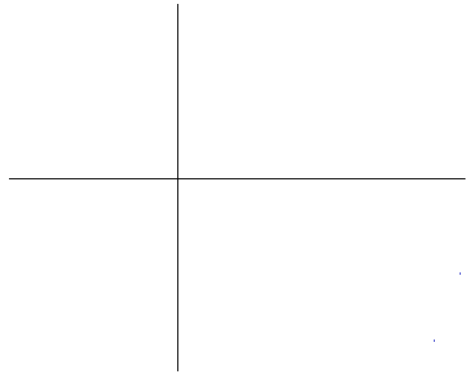
11) If ABCD is a parallelogram, what is x?
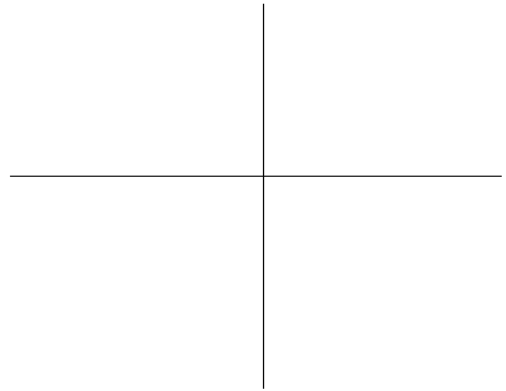
Is ABCD a rhombus?

A $x^2 - x$ B

9x - 6

D   2x + 54   C

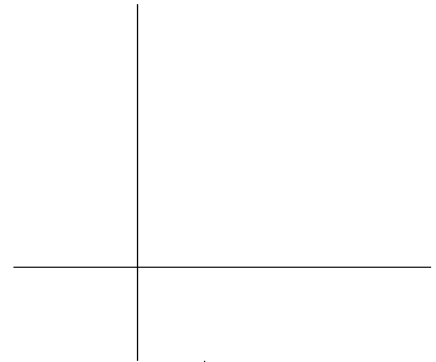Describe the most exact quadrilateral *using distance formula only.*

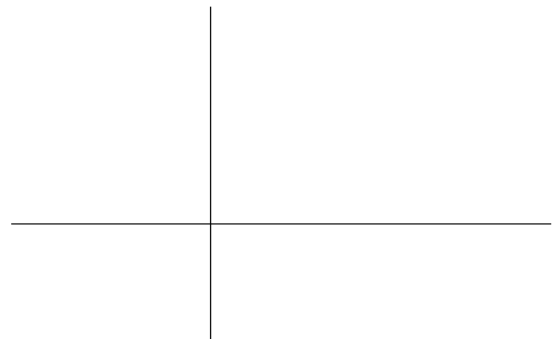1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

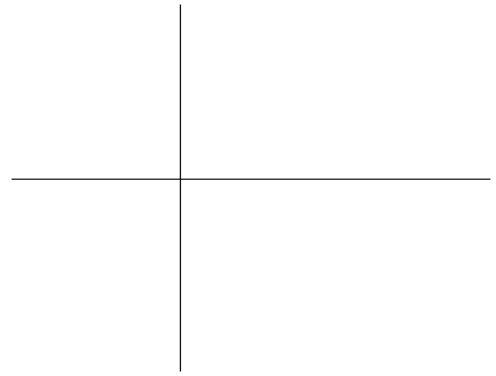3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

Describe the most exact quadrilateral *using distance formula only.*

5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)

6) Consecutive Vertices: (-1, 8) (5, 2) (-3, 3) (7, -7)

7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

8) Consecutive Vertices: (0, 0) (-3, 4) (8, 4) (5, 0)

Describe the most exact quadrilateral *using slope only.*

1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

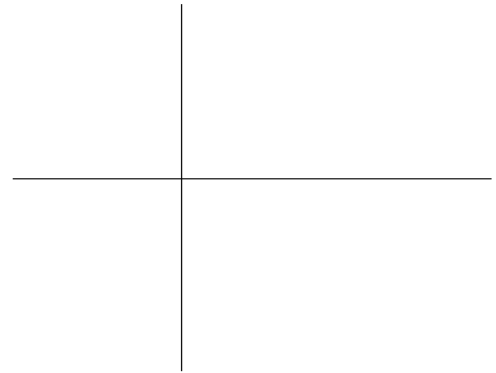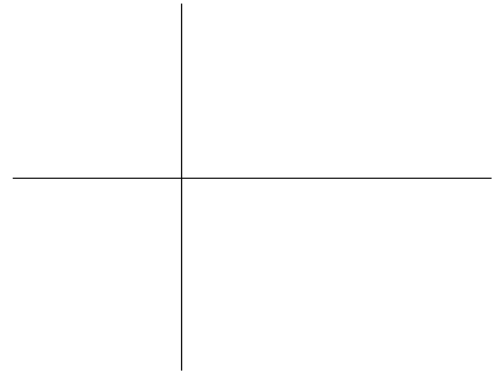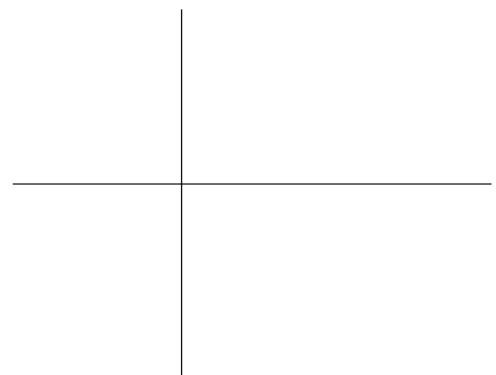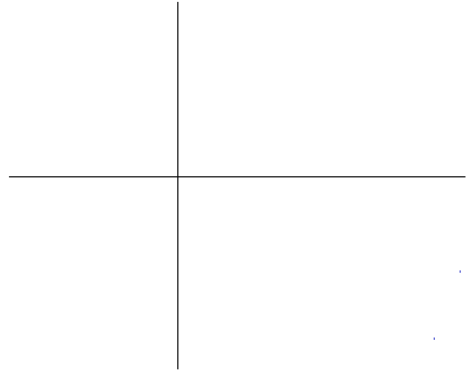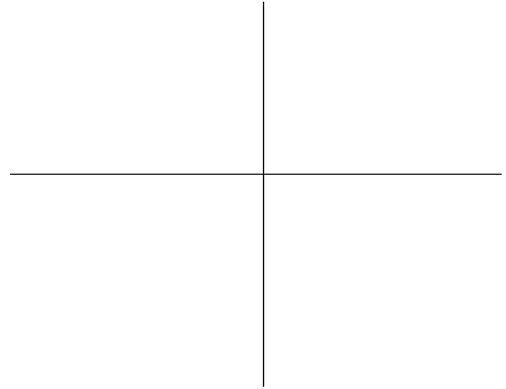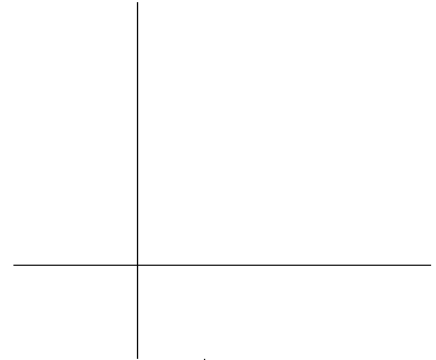4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

Describe the most exact quadrilateral *using slope only.*
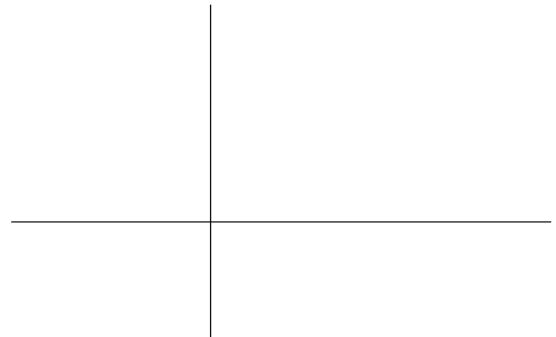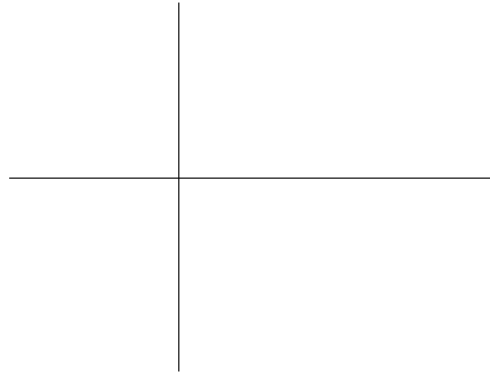
5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)

6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)

7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)
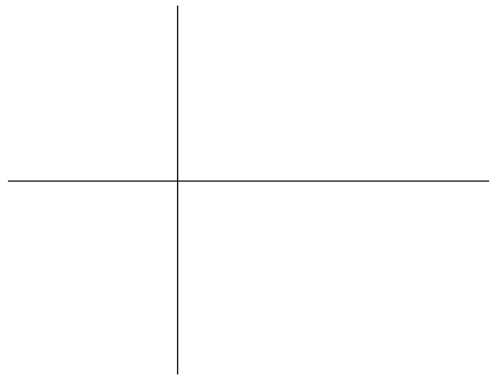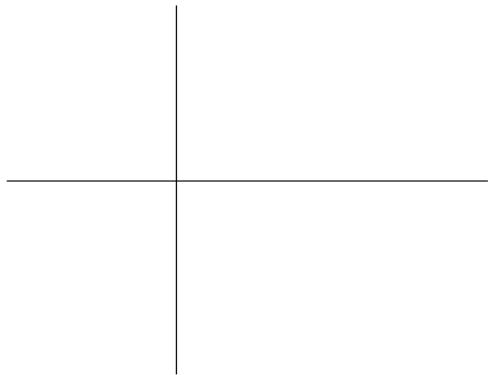
8) Consecutive Vertices: (0, 0) (-3, 4) (8, 4) (5, 0)

Determine the name of the figure formed by *connecting the midpoints of the sides* of each quadrilateral.

0) Quadrilateral

Connecting the midpoints...

(These midsegments
are 1/2 the length of
the horizontal diagonal)

The inside is a parallelogram...
(opposite sides are congruent)

1) Rhombus:

2) Kite:

3) Square:

4) Rectangle:

5) Parallelogram:

6) Trapezoid

7) Isosceles Trapezoid

Prove: The connected midpoints of a rectangle form a parallelogram.
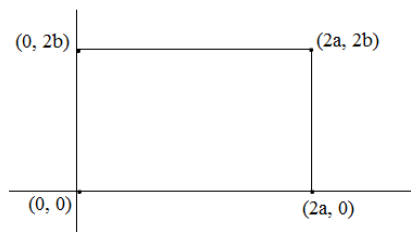
(0, 2b) _____ (2a, 2b)

(0, 0) (2a, 0)

Prove: The diagonals of a rectangle bisect each other.

Prove: In an isosceles right triangle,
         the median from the vertex to the hypotenuse is also an altitude.

Prove: A quadrilateral formed by connecting the midpoints of a rectangle's sides is a rhombus.

Prove: The midpoints of a quadrilateral form a parallelogram.
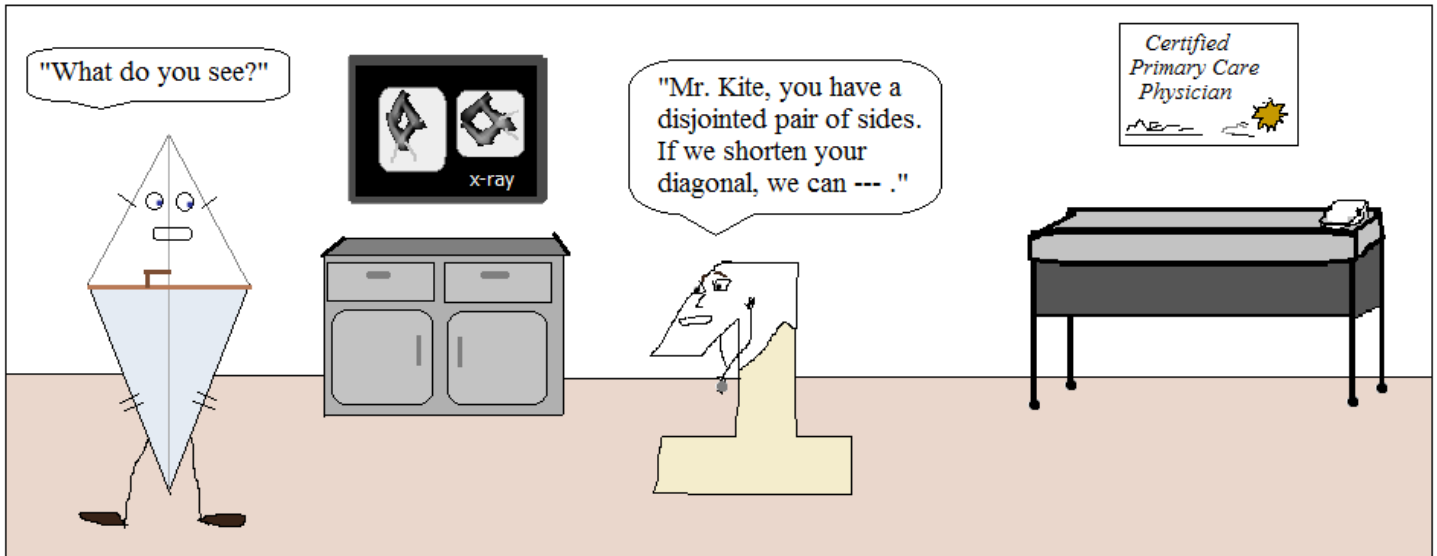
SOLUTIONS→

WARM-UP: Determine the angle measures

SOLUTIONS

Special Quadrilateral Angle Properties

1) Parallelogram
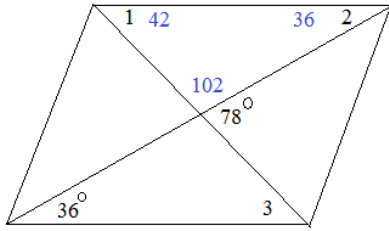
angle 2 = 36   (because alternate interior angles are congruent)

and, since 1 + 2 + 102 = 180, angle 1 = 42

and, therefore, angle 3 = 42

2) Kite
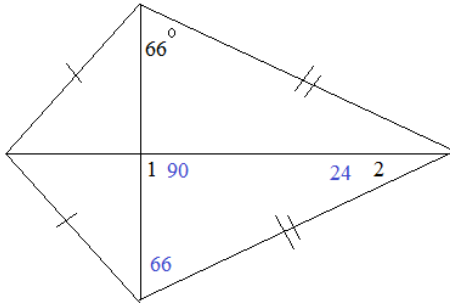
angle 1 = 90 (because diagonals of kite are perpendicular)

therefore, angle 2 = 24 (because sum of angles in triangle is 180)

3) Rectangle
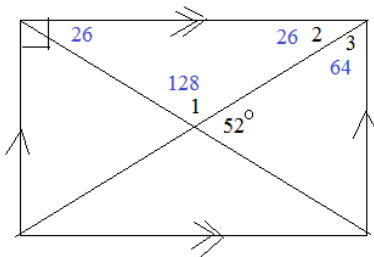
angle 1 is supp to 52 ⟹ 128 degrees

angle 2 + angle 1 + (angle 2) = 180
x   +   128   +   x   = 180
                  2x   =   52
                  x = 26

angle 3 + 52 + (angle 3) = 180
         angle 3 = 64

4) Rhombus
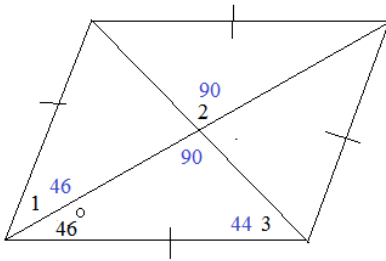
Rhombus:

angle 1: since diagonals are angle bisectors, 46

angle 2: since diagonals are perpendicular, 90

angle 3: since triangles interior equal 180,   44

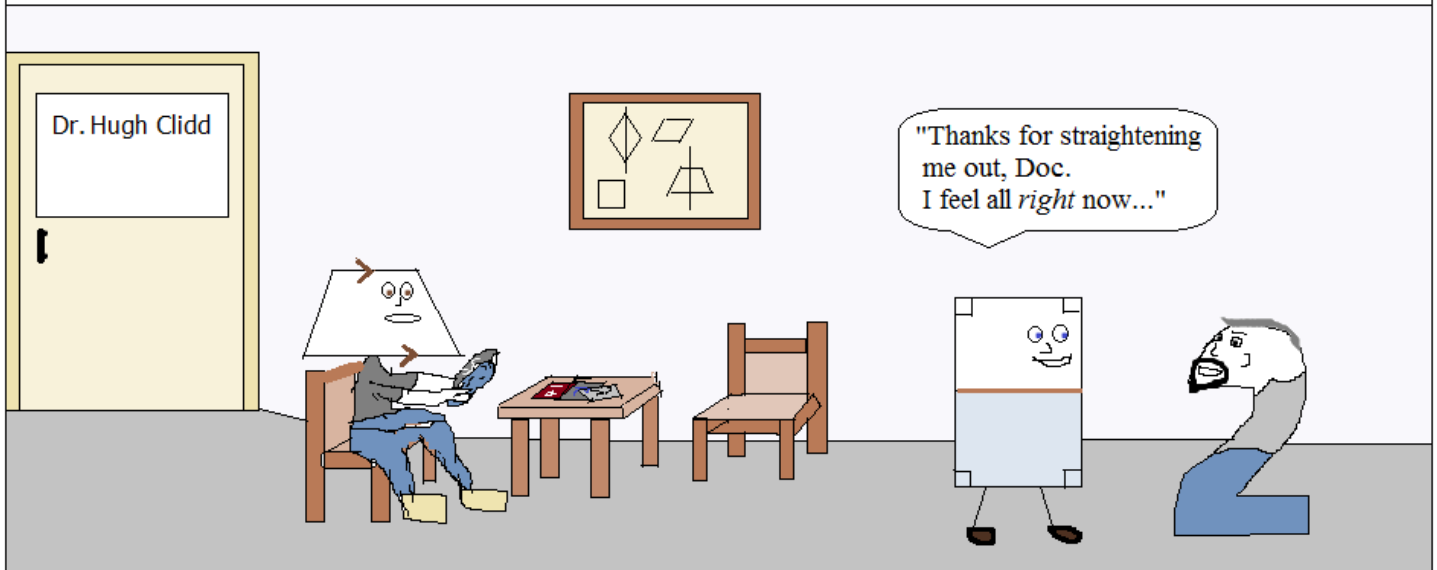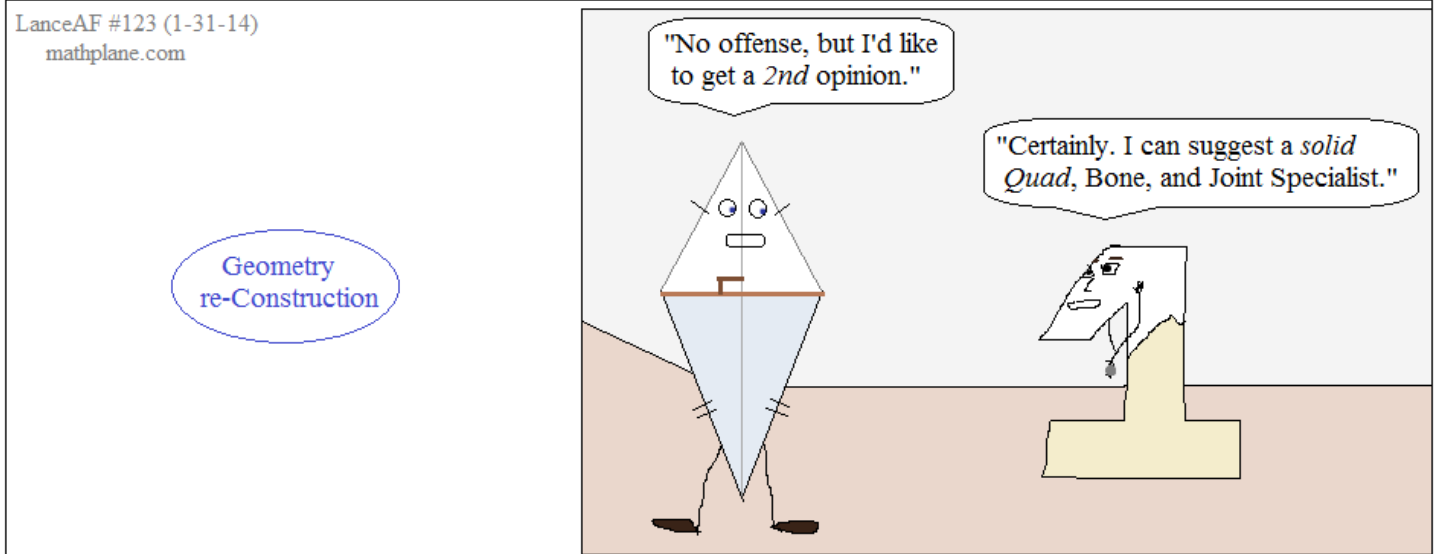5) Parallelogram
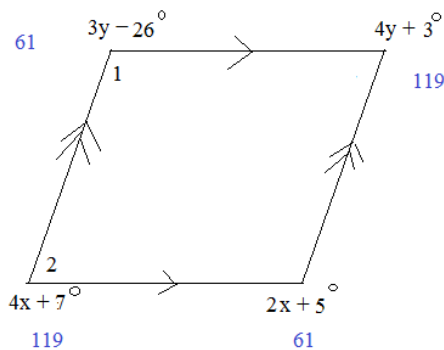
method 1:
consecutive
angles are
supplementary

7y − 23 = 180

7y = 203

y = 29

6x + 12 = 180

6x = 168

x = 28

method 2:
opposite angles
congruent

4x + 7 = 4y + 3

3y - 26 = 2x + 5

4x - 4y = -4

2x - 3y = -31

4x - 6y = -62

61 and 119

2y = 58
y = 29

1) In the isosceles trapezoid TRAP,
what are the measures of $\angle P$ and $\angle T$?
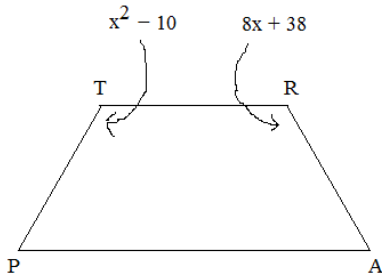
$x^2 - 10 = 8x + 38$

$x^2 - 8x - 48 = 0$

base and upper angles are congruent
(isosceles trapezoid)

$(x - 12)(x + 4) = 0$

$x = 12$ or -4

$x^2 - 10$ $8x + 38$

T   R

P   A

If $x = 12$, then angle T = 134, angle R = 134, and so, angle P and angle A = 46

If $x = -4$, then angle T = 6, angle R = 6, and so, angle P and angle A = 174...

It doesn't look like the diagram, but it is a possiblity!!

2) Find values for each variable for
the given parallelogram:
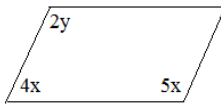
Consecutive angles are supp.
$4x + 5x = 180$
$x = 20$

2y

4x   5x

Opposite angles are congruent
$5x = 100..$
So, $2y = 100$
$y = 50$

3) Triangles ROM and ROH are equilateral...

If the diagonal HM creates an angle HMO of measure $3x - 6$,
and RO = 10x,
then what is the perimeter of rhombus RHOM?

R   10x   H

60
60

10x

10x

30°   3x - 6

M   O
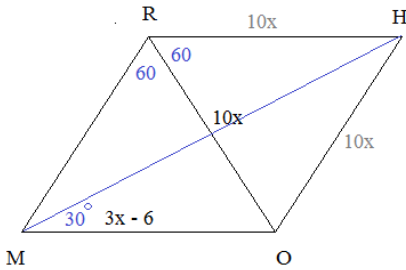
since angle M is 60 degrees, the diagonal bisects it ----> 30 degrees

$3x - 6 = 30$ $x = 12$

Then, RO = 12(10) = 120

therefore, perimeter is 480 units

4) RECT is a rectangle...

RC = x^2

RE = $4 - x$

ET = $6x - 5$

Find the possible values of x
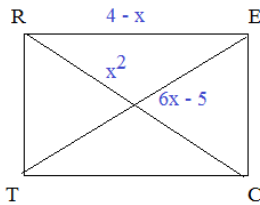
$x^2 = 6x - 5$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$

$x = 1, 5$

no solution

x cannot be 5, because RE would
have a negative length!

x cannot be 1, because ET and RC
would have length 1...
And, RE would have length 3...
That's not possible! because the
hypotenuse cannot be smaller than
the side!

R   4 - x   E

x^2

6x - 5

T   C

5) For the following rhombus, determine x and y...

y - 12

5x   2y - 33

x + 35
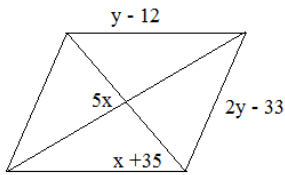
y - 12 = 2y - 33   (all sides congruent)

5x = 90   (diagonals are perpendicular)

(x + 35 is irrelevant to solving)

x = 18

y = 21

SOLUTIONS

6) If the area is 78 square units, what is x?

x + 5

x

x + 9
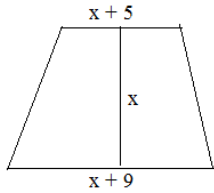
$78 = 1/2(x)(2x + 14)$

$78 = (x)(x + 7)$

$78 = x^2 + 7x$

$x^2 + 7x - 78 = 0$

$(x + 13)(x - 6) = 0$

x = -13 or 6

since side length must be positive, we eliminate -13..

Therefore, x = 6

7) In the following parallelogram, find the angle measures of 1, 2, and 3...

3

34°

1

2   108°
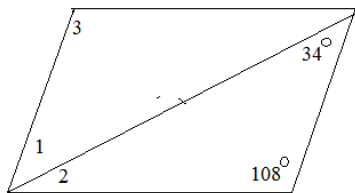
3 = 108   opposite angles are congruent

since consecutive angles are supplementary,
1 + 2 = 72....

1 = 34   and   2 = 38

8) In rhombus MIND, $\angle DSN = 2x^2 + 5x + 15$ DN = 2x + 3y

DM = 5y + 4

DS = xy

Find x, y, and $\overline{ID}$

D

S

N   M

I
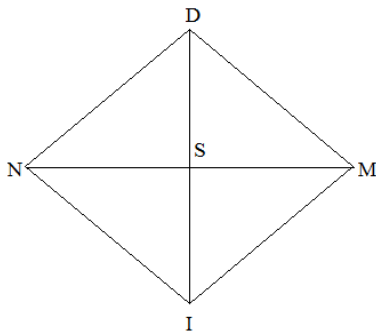
Diagonals are perpendicular

$DSN = 2x^2 + 5x + 15 = 90$

$2x^2 + 5x - 75 = 0$

$(2x + 15)(x - 5) = 0$

x = -15/2 or 5

Note: DS = xy.. so, if y > 0, then x must be > 0...

therefore, x = 5,   y = 3

and, ID = 2(xy) = 30

Diagonals bisect each other...

All sides are congruent

2x + 3y = 5y + 4

2x - 2y = 4

x - y = 2

so, if x = 5, then y = 3

OR

if x = -15/2, then y = -19/2

(**However, this is impossible because the sides would be negative!)

quick check: sides = 19
                     angle = 90
                     and xy = 15

9) Given: The kite MATH

$\overline{MA} = x + 5$

$\overline{AT} = 2x + 1$

$\overline{MH} = 2y + 1$

$\overline{HT} = 3y - 2$

Find: the perimeter

Step 1: Sketch the figure
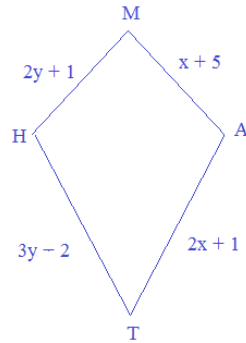
Step 2: Use properties of kite
(consecutive side pairs congruent)

step 3: "turn the kite" and assume
different consecutive pairs

$MH = MA$ $HT = AT$

$2y + 1 = x + 5$ $3y - 2 = 2x + 1$

$x - 2y = -4$ $2x - 3y = -3$

solve the system

$2x - 3y = -3$

$-2x + 4y = 8$

_____

$y = 5$

$x = 6$

perimeter: 13 + 13 + 11 + 11

48

$MA = AT$ $MH = HT$

$x + 5 = 2x + 1$ $2y + 1 = 3y - 2$

$x = 4$

$y = 3$

perimeter: 9 + 9 + 7 + 7

32

10) Find the perimeter of square SQAR where vertices
are Q (-4, 1) and R (-1, 6).

a) 16

b) $4\sqrt{34}$

c) $4\sqrt{17}$

d) 32

e) $16\sqrt{2}$

(-1, 6)

S   R

(-4, 1) Q   A
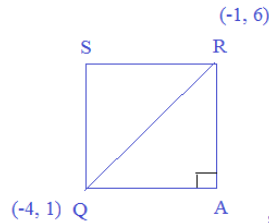
length of diagonal
is   (distance formula)

$\sqrt{34}$

since 45-45-90 triangle,
each side is

$\dfrac{\sqrt{34}}{\sqrt{2}} = \sqrt{17}$

perimeter is $4\sqrt{17}$

11) If ABCD is a parallelogram, what is x?

Is ABCD a rhombus?

A $x^2 - x$ B

9x - 6

D   2x + 54   C
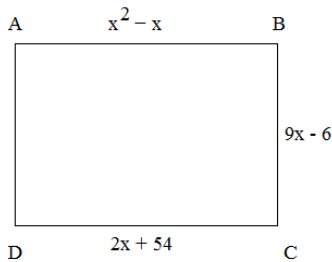
Since it is a parallelogram, opposite sides are congruent...

$x^2 - x = 2x + 54$

$x^2 - 3x - 54 = 0$

$(x - 9)(x + 6) = 0$

$x = 9, -6$

if x = 9, then the sides are 72, 75, 72, 75

if x = -6, then the sides are 42, -60, 42, -60
doesn't exist!

Describe the most exact quadrilateral *using distance formula only.*

1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

Parallelogram

(1, 1) to (4, 5) $\sqrt{(1-4)^2 + (1-5)^2} = 5$

(4, 5) to (9, -7) $\sqrt{(9-4)^2 + (-7-5)^2} = 13$

Since opposite sides are congruent, it must be a parallelogram...

(Since *not all* sides are same, we can eliminate square and rhombus)

(9, -7) to (6, -11) $\sqrt{(9-6)^2 + (-7--11)^2} = 5$

(6, -11) to (1, 1) $\sqrt{(1-6)^2 + (1--11)^2} = 13$

diagonals: (1, 1) to (9, -7) $8\sqrt{2}$

(4, 5) to (6, -11) $2\sqrt{65}$

Since the diagonals are NOT congruent, it cannot be a rectangle..
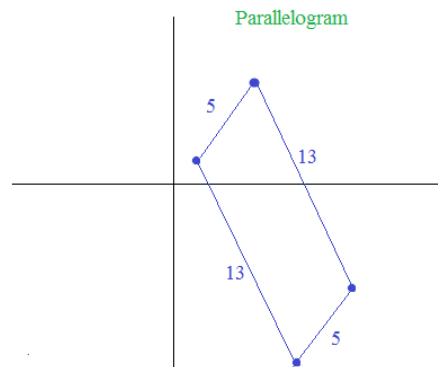
2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

(0, 3) to (3, 0) $3\sqrt{2}$

Rectangle

(3, 0) to (-6, -9) $9\sqrt{2}$

Since opposite sides are congruent, the quadrilateral is a parallelogram...

(-6, -9) to (-9, -6) $3\sqrt{2}$

(Since the 4 sides are not all the same, it eliminates rhombus and square)

(-9, -6) to (0, 3) $9\sqrt{2}$

diagonals:

Then, since the diagonals are congruent, it is a rectangle...

(0, 3) to (-6, -9) $6\sqrt{5}$

(3, 0) to (-9, -6) $6\sqrt{5}$
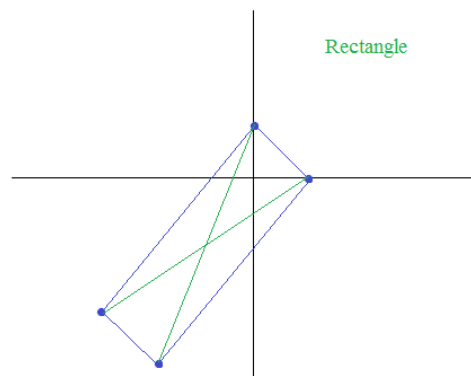
3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

(0, 8) to (3, 4)   5

Rhombus

(3, 4) to (3, 9)   5

Since all the sides are the same, it's a rhombus or a square...

(3, 9) to (0, 13)   5

(0, 13) to (0, 8)   5

Then, since the diagonals are NOT congruent, then it cannot be a square..

diagonals:

The quadrilateral is a rhombus...

(0, 8) to (3, 9) $\sqrt{10}$

(3, 4) to (0, 13) $3\sqrt{10}$
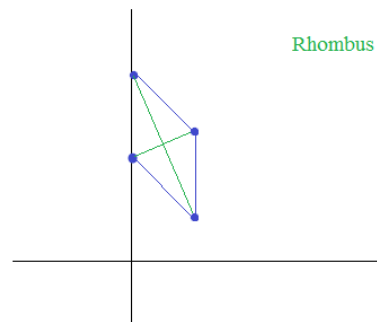
4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

Kite

(2, 5) to (3, -2) $5\sqrt{2}$

(3, -2) to (8, 3) $5\sqrt{2}$

Since there are 2 pairs of *consecutive* congruent sides, the figure is a kite!
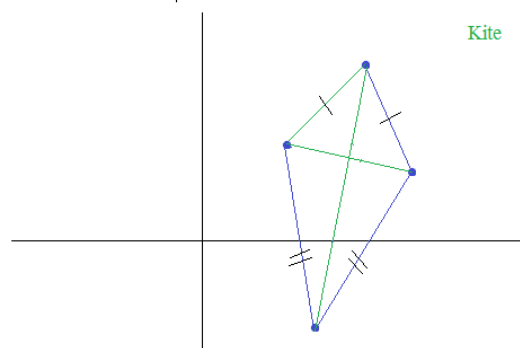
(8, 3) to (6, 7) $2\sqrt{5}$

(6, 7) to (2, 5) $2\sqrt{5}$

mathplane.com

Describe the most exact quadrilateral *using distance formula only.*

5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)

(3, 5) to (7, 4) $\sqrt{(3-7)^2 + (5-4)^2} = \sqrt{17}$

(7, 4) to (6, 0) $\sqrt{(7-6)^2 + (4-0)^2} = \sqrt{17}$

(6, 0) to (2, 1) $\sqrt{(6-2)^2 + (0-1)^2} = \sqrt{17}$

(2, 1) to (3, 5) $\sqrt{(2-3)^2 + (1-5)^2} = \sqrt{17}$

diagonals: (3, 5) to (6, 0) distance = $\sqrt{34}$

(7, 4) to (2, 1) distance = $\sqrt{34}$

since the distances between consecutive points are all the same, sides are congruent...

It's a rhombus... But, is it a square?

the lengths of the diagonals are congruent...
Therefore, it is a square!

Square
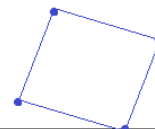
6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)

(-3, 3) to (7, -7) $10\sqrt{2}$

(7, -7) to (5, 2) $\sqrt{85}$

(5, 2) to (1, 6) $4\sqrt{2}$

(1, 6) to (-3, 3) 5

diagonals: (-3, 3) to (5, 2) $\sqrt{65}$

(7, -7) to (1, 6) $\sqrt{205}$

the lengths of all 4 sides are different, so it's either a quadrilateral or trapezoid..

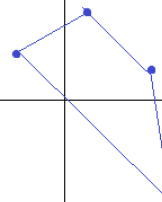Midpoint distances: (2, -2) to (3, 4)

$\sqrt{37}$

(6, -5/2) to (-1, 9/2)

this length is middle, so it's a midsegment..

$7\sqrt{2}$

Trapezoid

7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

(-10, 5) to (0, 3) $2\sqrt{26}$

(0, 3) to (-10, -10) $\sqrt{269}$

(-10, -10) to (-20, 3) $\sqrt{269}$

(-20, 3) to (-10, 5) $2\sqrt{26}$

2 pairs of congruent sides...
It's a kite..

Kite
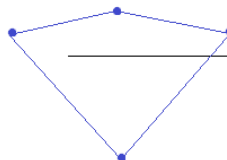
8) Consecutive Vertices (0, 0) (-3, 4) (8, 4) (5, 0)

(0, 0) to (-3, 4) 5

(-3, 4) to (8, 4) 11

(8, 4) to (5, 0) 5

(5, 0) to (0, 0) 5

diagonals: (0, 0) to (8, 4) $4\sqrt{5}$

(-3, 4) to (5, 0) $4\sqrt{5}$

distances between midpoints:

(-3/2, 2) to (6.5, 2) 8

(5/2, 4) to (5/2, 0) 4

the length of the segment connecting midpoints is the average of sides..
therefore, it is a midsegment...

diagonals are congruent: either a rectangle or isosceles trapezoid

Isosceles Trapezoid

Describe the most exact quadrilateral *using slope only.*

1) Consecutive Vertices: (1, 1) (4, 5) (9, -7) (6, -11)

(1, 1) to (4, 5) $\frac{5-1}{4-1} = \frac{4}{3}$

(4, 5) to (9, -7) $\frac{-7-5}{9-4} = \frac{-12}{5}$

(9, -7) to (6, -11) $\frac{-11--7}{6-9} = \frac{-4}{-3} = \frac{4}{3}$

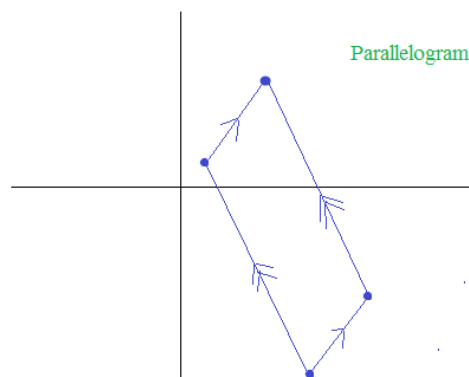(6, -11) to (1, 1) $\frac{1--11}{1-6} = \frac{12}{-5}$

diagonals: (1, 1) (9, -7)   slope: -1

(4, 5) (6, -11)   slope: -8

since opposite sides have the same slopes, it must be a parallelogram...

Then, since the slopes are not opposite reciprocals, it cannot be square or rectangle...

And, since diagonals are not perpendicular, it cannot be a rhombus

Parallelogram

2) Consecutive Vertices: (0, 3) (3, 0) (-6, -9) (-9, -6)

Slopes of sides:

(0, 3) to (3, 0)   -1

(3, 0) to (-6, -9)   1

(-6, -9) to (-9, -6) -1

(-9, -6) to (0, 3)   1

Slopes of diagonals:

(0, 3) to (-6, -9)   2

(3, 0) to (-9, -6)   1/2

Since opposite sides are parallel, the quadrilateral is a parallelogram...

Then, since the consecutive sides have slopes that are opposite reciprocals, the sides are perpendicular; right angles...
So, it's a rectangle or square...

Then, since the diagonals are NOT perpendicular (slopes are not opposite reciprocals), the figure can't be a square...

So, it's a rectangle...

Rectangle
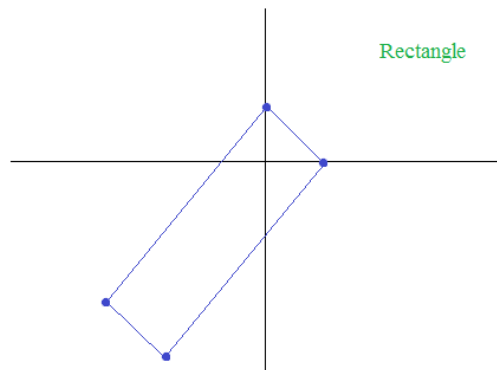
3) Consecutive Vertices: (0, 8) (3, 4) (3, 9) (0, 13)

Slopes of sides:

(0, 8) to (3, 4)   -4/3

(3, 4) to (3, 9)   undefined

(3, 9) to (0, 13)   -4/3

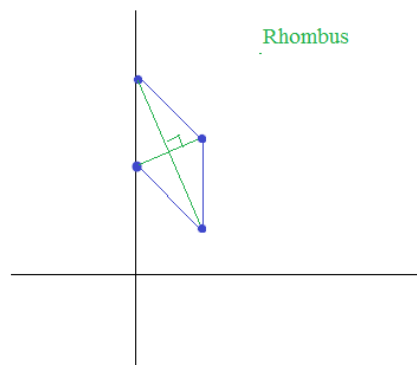(0, 13) to (0, 8)   undefined

Slopes of the diagonals:

(0, 8) to (3, 9)   1/3

(3, 4) to (0, 13)   -3

Since slopes of opposite sides are the same, the quadrilateral is a parallelogram...

(and, since the slopes are not opposite reciprocals, the corners are NOT right angles..)

Then, since the slopes of the diagonals are opposite reciprocals, the diagonals are perpendicular...

Therefore, it is a rhombus...

Rhombus

4) Consecutive Vertices: (2, 5) (3, -2) (8, 3) (6, 7)

(2, 5) to (3, -2)   -7

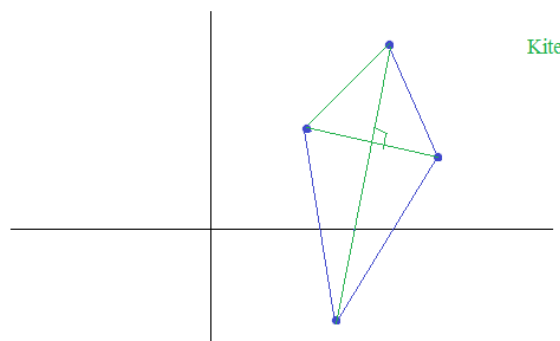(3, -2) to (8, 3)   1

(8, 3) to (6, 7)   -2

(6, 7) to (2, 5)   1/2

Slopes of diagonals:

(2, 5) to (8, 3)   -1/3

(3, -2) to (6, 7)   3

Since none of the side slopes are congruent, the figure is not a trapezoid or parallelogram...

Then, since the slopes of the diagonals are opposite reciprocals, the diagonals are perpendicular...

Therefore, the quadrilateral is a kite

Kite

Describe the most exact quadrilateral *using slope only*.

5) Consecutive Vertices: (3, 5) (7, 4) (6, 0) (2, 1)
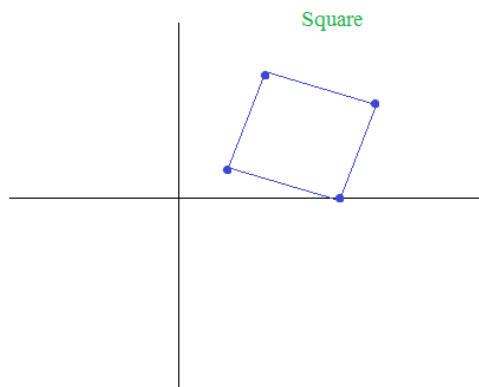
(3, 5) to (7, 4)   -1/4
(7, 4) to (6, 0)   4
(6, 0) to (2, 1)   -1/4
(2, 1) to (3, 5)   4

since opposite sides have same slope, they are parallel.. (parallelogram)

And, since consecutive sides have opposite reciprocals, the sides are perpendicular..

(rectangle)

slope of diagonals:

(3, 5) to (6, 0)   -5/3

(7, 4) to (2, 1)   3/5

since diagonals are opposite reciprocals, they are perpendicular...

Therefore, figure is a square

Square

6) Consecutive Vertices: (-3, 3) (7, -7) (5, 2) (1, 6)
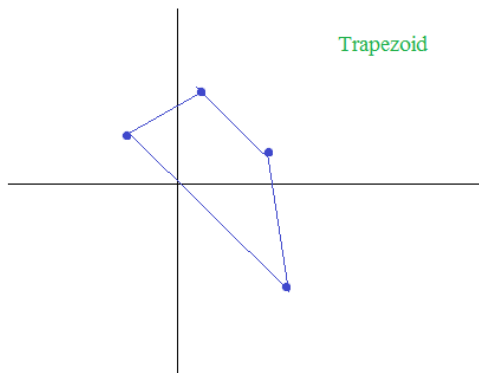
(-3, 3) to (7, -7)   -1
(7, -7) to (5, 2)   -9/2
(5, 2) to (1, 6)   -1
(1, 6) to (-3, 3)   3/4

since one pair of opposite sides have the same slope, then there are only 2 parallel sides...   (trapezoid)

since slopes of other opposite sides are NOT reciprocals, it is not isosceles...

slope of diagonals

(-3, 3) to (5, 2)   -1/8

(7, -7) to (1, 6)   -13/6

since slopes of diagonals are not opposites, this is NOT an isosceles trapezoid...

Trapezoid

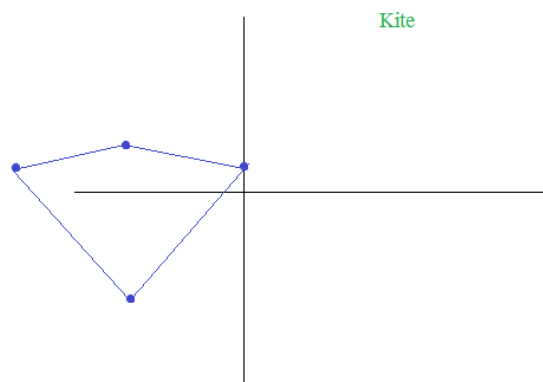7) Consecutive Vertices: (-10, 5) (0, 3) (-10, -10) (-20, 3)

(-10, 5) to (0, 3)   -1/5

(0, 3) to (-10, -10)   13/10

(-10, -10) to (-20, 3) -13/10

(-20, 3) to (-10, 5)   1/5

The slopes are opposites... Suspect it's a kite....

slope of diagonals:

(-10, 5) to (-10, -10)   undefined

(0, 3) to (-20, 3)   0

Diagonals are perpendicular...   KITE

Kite

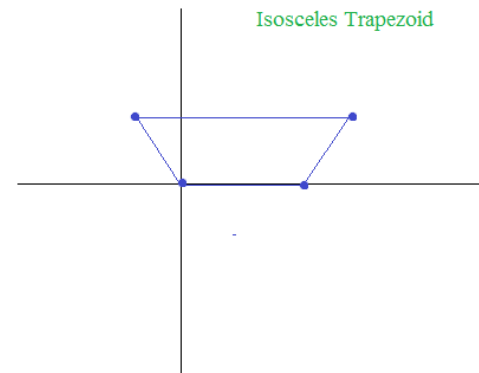8) Consecutive Vertices   (0, 0) (-3, 4) (8, 4) (5, 0)

(0, 0) to (-3, 4)   -4/3

(-3, 4) to (8, 4)   0

(8, 4) to (5, 0)   4/3

(5, 0) to (0, 0)   0

One pair of parallel sides so, trapezoid...

and, non-parallel sides are opposites... isosceles trapezoid

diagonal slopes:

(0, 0) to (8, 4)   1/2
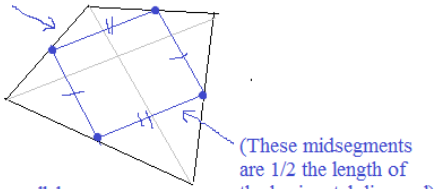
(-3, 4) to (5, 0)   -1/2

diagonal slopes are opposites...

Isosceles Trapezoid

Identifying Special Quadrilaterals: Triangle Congruency and Midsegment Theorems

Determine the name of the figure formed by *connecting the midpoints of the sides* of each quadrilateral.
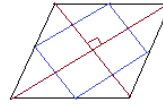
0) Quadrilateral

Connecting the midpoints...

The inside is a parallelogram...
(opposite sides are congruent)

(These midsegments
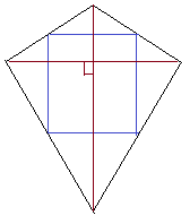are 1/2 the length of
the horizontal diagonal)

1)   Rhombus:

Rectangle

Diagonals of rhombus are perpendicular.
And, each segment is parallel to a diagonal.
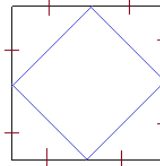Therefore, consecutive sides are
perpendicular.

2) Kite:

Rectangle

(Triangle) Midsegment Theorem:
If a segment joins the *midpoints
of angle sides* of a triangle, then
the segment is parallel to the base
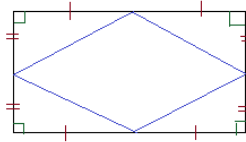and 1/2 the length of the base.

3)   Square:

Square

Four congruent triangles
(side-angle-side) - interior quadrilateral sides congruent

Since triangles are isosceles w/ vertex 90 degrees.
Then, base angles are 45 degrees...
So, interior quadrilateral angles are all 90 degrees
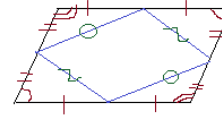
4)   Rectangle:

Rhombus

For congruent triangles
(side-angle-side) - interior quadrilateral sides congruent

However, only the opposite angles are congruent, so
the interior quadrilateral is not equiangular.
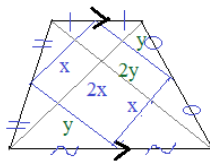
5)   Parallelogram:

Parallelogram

Opposite triangles are congruent, so
opposite sides of interior quadrilateral are congruent

6) Trapezoid

Parallelogram

Using midsegment theorem,
we know that opposite sides are
congruent..
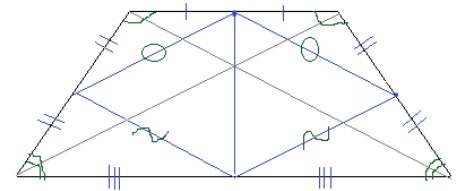Therefore, this is a parallelogram

(Note: Since the diagonals are not
necessarily congruent or perpendicular,
the interior figure is not necessarily
a rectangle or rhombus)

7) Isosceles Trapezoid

Rhombus

Since the base angles are congruent..
And the sides are congruent, the bisectors
form 2 pairs of congruent triangles.

(Using CPCTC, we have 2 pairs of
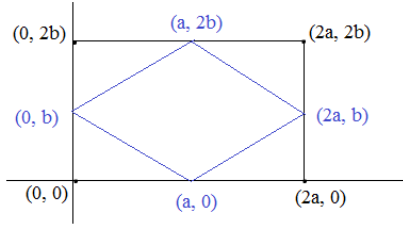consecutive congruent sides)

Kite?

But, remember, the diagonals are congruent...
Therefore, the 4 midsegments (which are 1/2
the length of the diagonals) are all congruent!
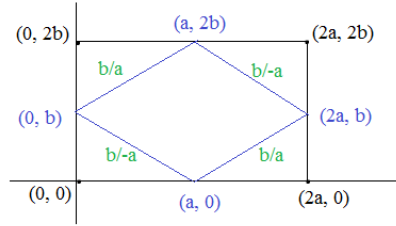
It's a rhombus..

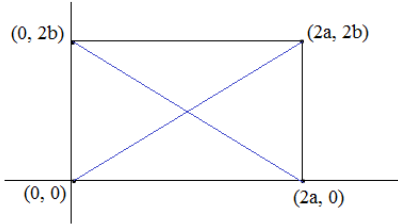Prove: The connected midpoints of a rectangle form a parallelogram.

After labeling the midpoints, we can find the slopes.

(0, 2b)   (a, 2b)   (2a, 2b)

(0, b)   (2a, b)

(0, 0)   (a, 0)   (2a, 0)

(0, 2b)   (a, 2b)   (2a, 2b)

b/a   b/-a

(0, b)   (2a, b)

b/-a   b/a

(0, 0)   (a, 0)   (2a, 0)

Since the opposite sides have same slopes, they are parallel.

Opposite sides parallel ---> parallelogram

Prove: The diagonals of a rectangle bisect each other.

(0, 2b)   (2a, 2b)

(0, 0)   (2a, 0)

find the midpoint of each diagonal:

Since each midpoint is (a, b), they bisect each other.

Prove: In an isosceles right triangle,
the median from the vertex to the hypotenuse is also an altitude.

(0, a)

m = -1

(a/2, a/2)

m = 1

(a, 0)
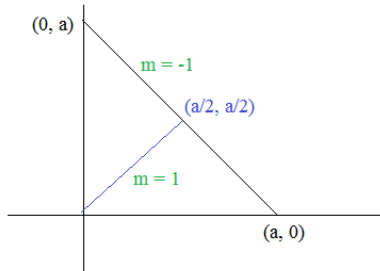
since the slopes are 1 and -1,
the median is perpendicular to the hypotenuse...
Therefore, it is an altitude...

Prove: A quadrilateral formed by connecting the midpoints of a rectangle's sides is a rhombus.

(0, 2b)   (a, 2b)   (2a, 2b)

(0, b)   (2a, b)

(a, 0)   (2a, 0)
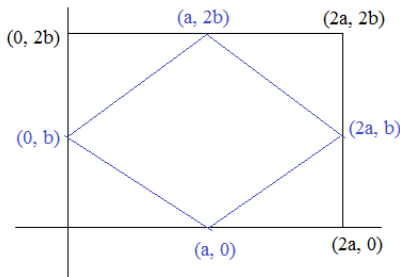
(from above, we proved the midpoints form a parallelogram.... Now, we'll go further to show it's a rhombus!)

distance/length of each side..

$$\sqrt{a^2 + b^2}$$

The diagonals are perpendicular.
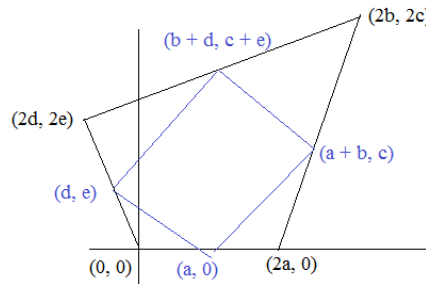(slopes are opposite reciprocals)

One diagonal is vertical; one diagonal is horizontal

Prove: The midpoints of a quadrilateral form a parallelogram.

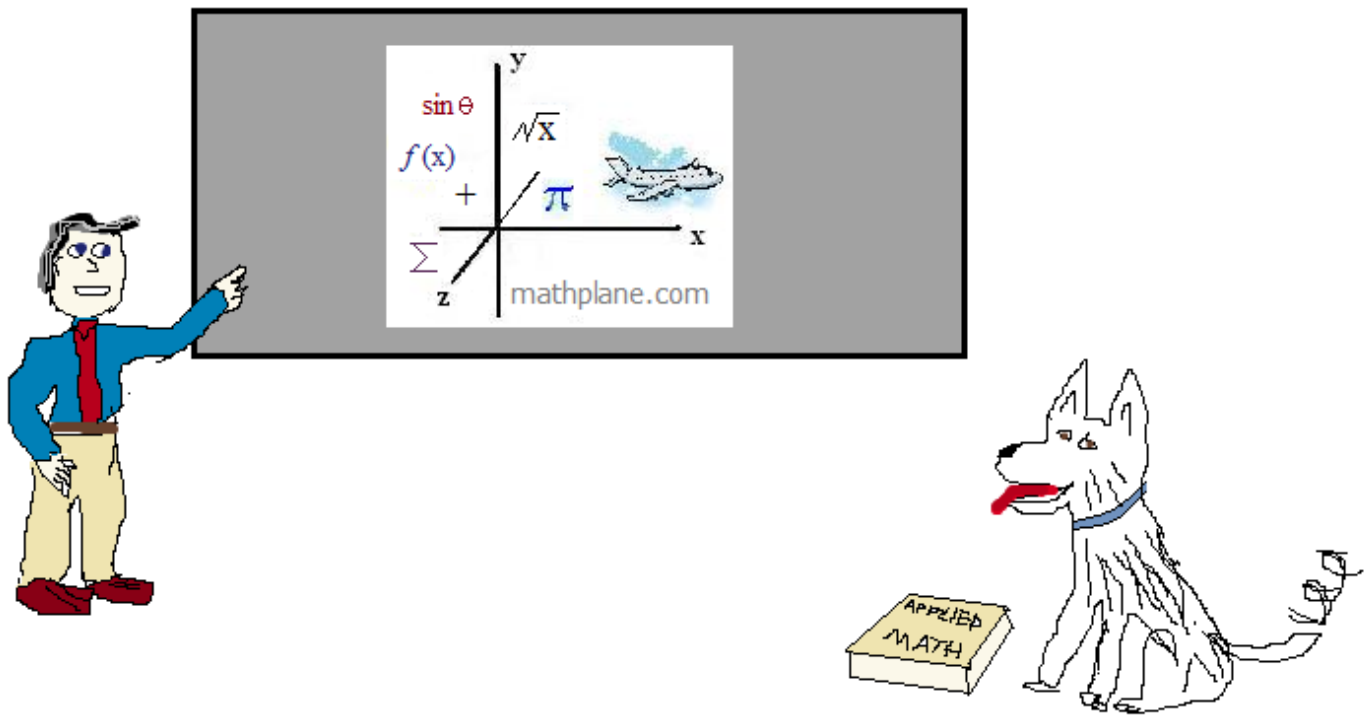slope: $\dfrac{c}{b}$ and $\dfrac{c}{b}$

$\dfrac{e}{d-a}$ and $\dfrac{e}{d-a}$

⟹ opposite sides parallel

(2b, 2c)

(b + d, c + e)

(2d, 2e)

(a + b, c)

(d, e)

(0, 0)   (a, 0)   (2a, 0)

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

Also, at TES and TeachersPayTeachers

And, Mathplane *Express* for mobile at mathplane.ORG